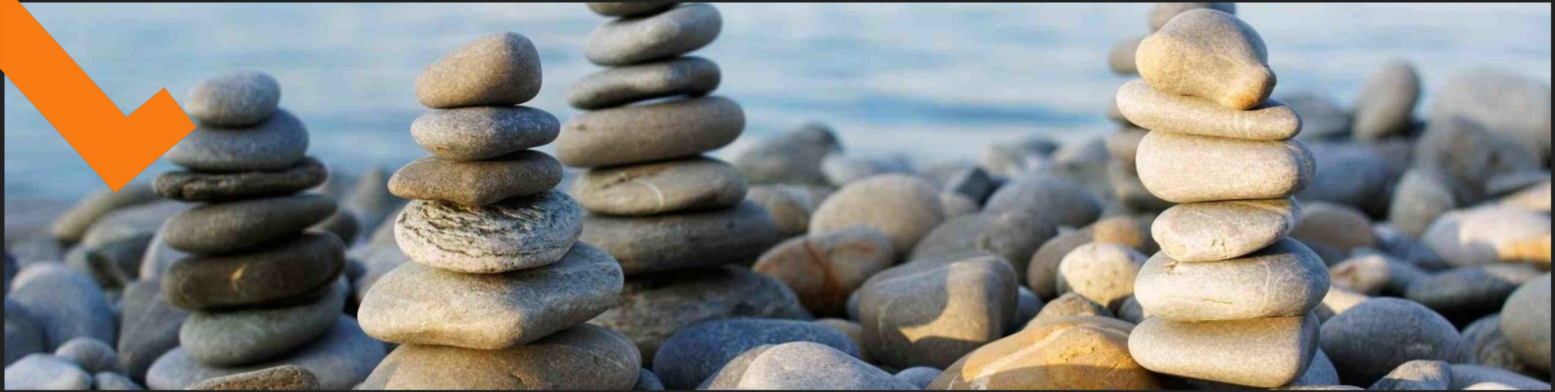
Lambdas & Pebbles



What is Lambda Calculus?

- Allows one to describe computation from a functional point of view.
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Historical Side Note

- Allows one to describe computation from state machine point of view.
- Invented by Alan Turing in the mid 1930s.

Grammar of Lambda Calculus

```
<Expression> ::= <identifier>

<Expression> ::= λ<identifier>.<Expression>

<Expression> ::= <Expression> <Expression>

<Expression> ::= (<Expression>)
```

What’s the point?

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- Lambda calculus can encode any computable function.
    - Equivalent to a turing machine.
- It’s simplicity and compositional power is **<u>awesomeness in its purest form</u>**.

A Lambda Expression that takes 2 Inputs

$$(\lambda x.\lambda y.\ x + y)$$

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$$(\lambda x.\lambda y.\ x + y)\ \ 13\ \ 9$$

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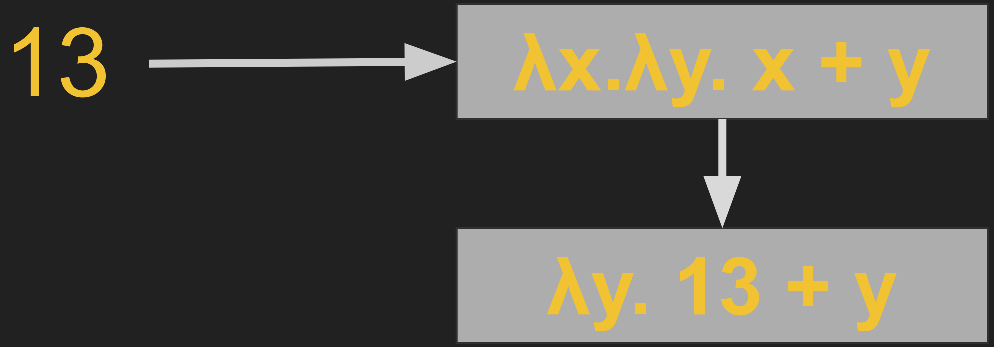

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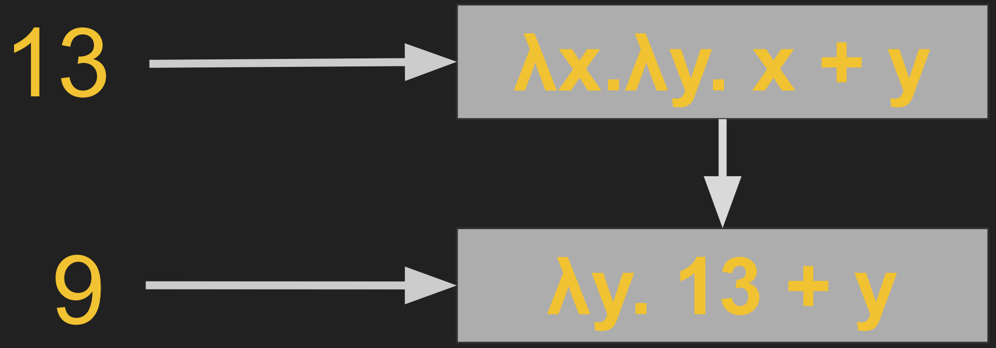

A Lambda Expression that takes 2 Inputs

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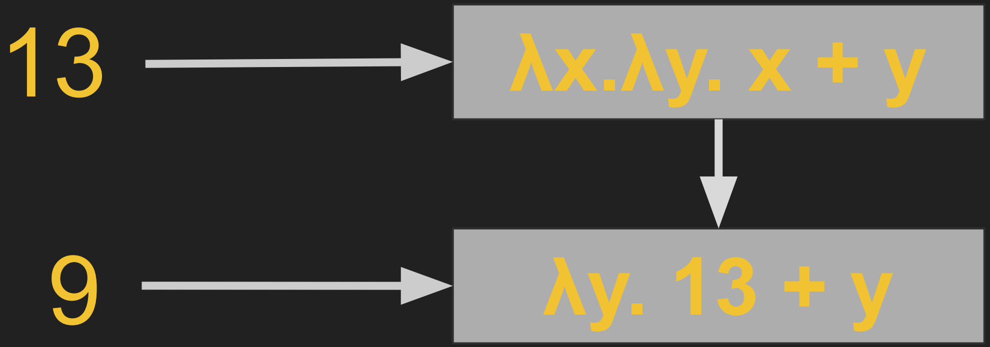

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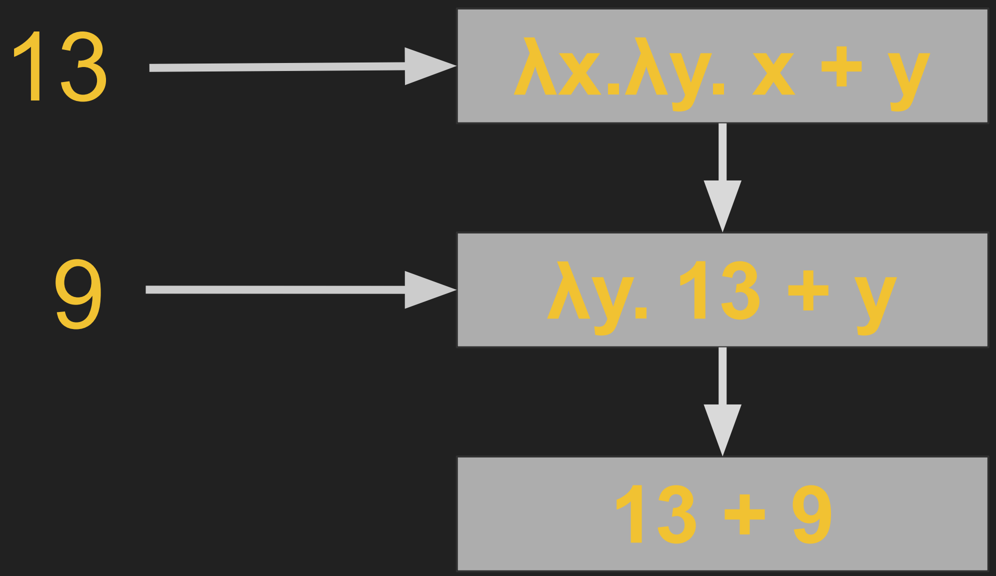

A Lambda Expression that takes 2 Inputs

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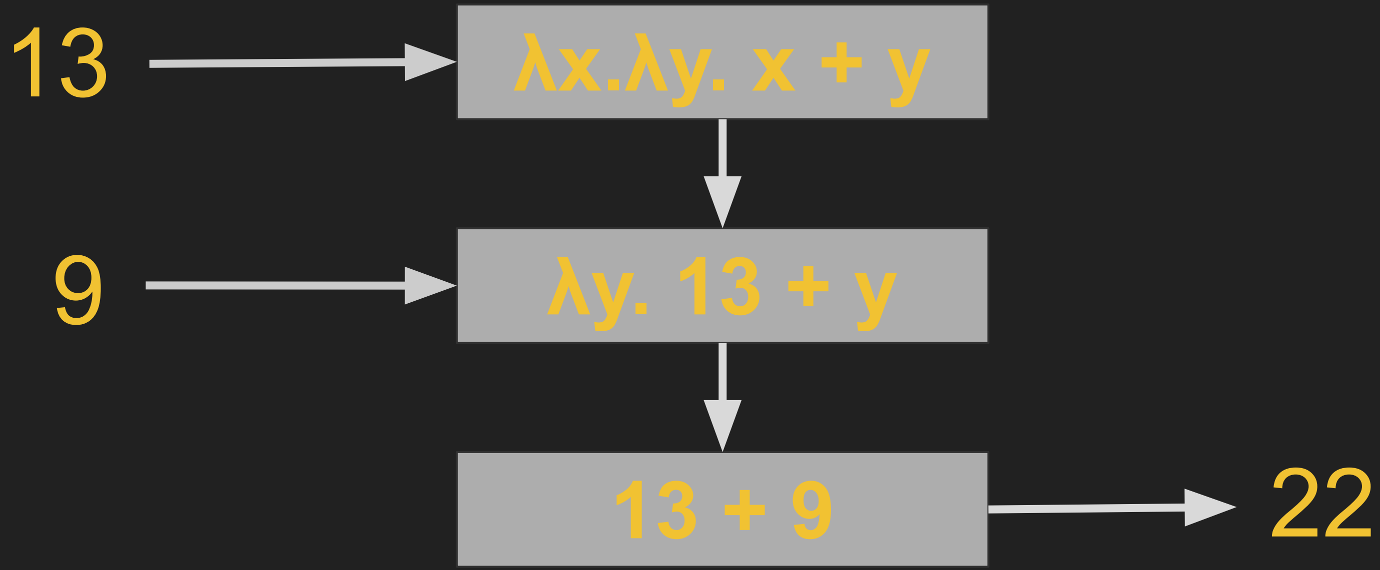

Church Numerals

- Given all we have are functions. How do we encode the notion of number?

Church Numerals

- Given all we have are functions. How do we encode the notion of number?
  - Really no choice… we have to use functions and function applications!

First 5 Church Numerals

0: $\lambda f.\lambda x.\ x$

1: $\lambda f.\lambda x.\ f\ x$

2: $\lambda f.\lambda x.\ f\ (f\ x)$

3: $\lambda f.\lambda x.\ f\ (f\ (f\ x))$

4: $\lambda f.\lambda x.\ f\ (f\ (f\ (f\ x)))$

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Let’s encode these in Scheme!

Arithmetic with Church Numerals

- How do we operate with these numerals?

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$f\ \underbrace{fff.....fff}_{n}$

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SUCC: $\lambda n.\lambda f.\lambda x.\ f\ (n\ f\ x)$ $\qquad f\underbrace{fff.....fff}_{n}$

ADD: $\lambda m.\lambda n.\lambda f.\lambda x.\ m\ f\ (n\ f\ x)$

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MULT: $\lambda m.\lambda n.\lambda f.\lambda x.\ m\ (n\ f)\ x$

Arithmetic with Church Numerals

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    - Lambda Functions!

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ADD: λm.λn.λf.λx. m f (n f x) $\underbrace{ff..ff}_{m}\ \underbrace{ff..ff}_{n}$

MULT: λm.λn.λf.λx. m (n f) x $\underbrace{\underbrace{ff..ff}_{n}..\underbrace{ff..ff}_{n}}_{m}$

Arithmetic with Church Num

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Let’s encode these in Scheme!

And now….

Y Combinator

$$\lambda f.(\lambda x.f(x\ x))(\lambda x.f(x\ x))$$

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- Also called a fixed point combinator
    - $x = f(x)$; x is a fixed point of f

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$$Y\ g = (\lambda f.(\lambda x.f(x\ x))(\lambda x.f(x\ x)))\ g$$

$$\longrightarrow \quad (\lambda x.g(x\ x))(\lambda x.g(x\ x))$$

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⟶ (λx.g(x x))(λx.g(x x))

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⟶ g(g(g((λx.g(x x))(λx.g(x x))))) ⟶ ……

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  - *x* = *f*(*x*); *x* is a fixed point of *f*

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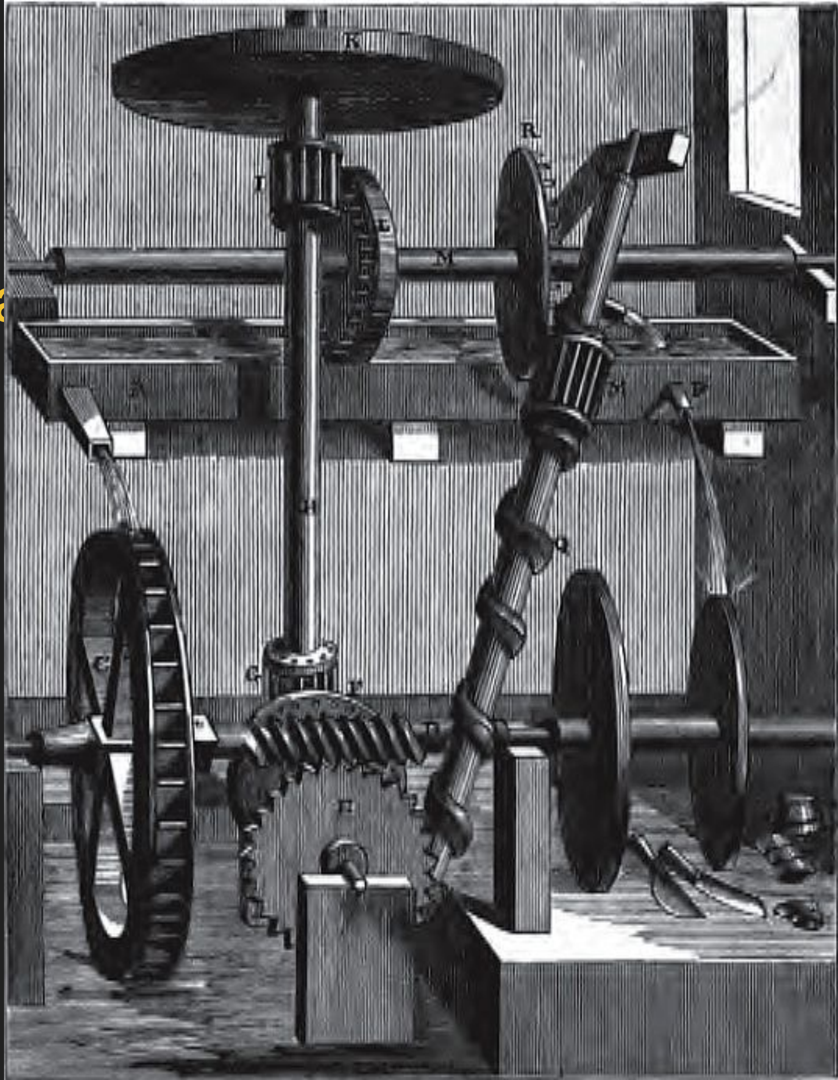

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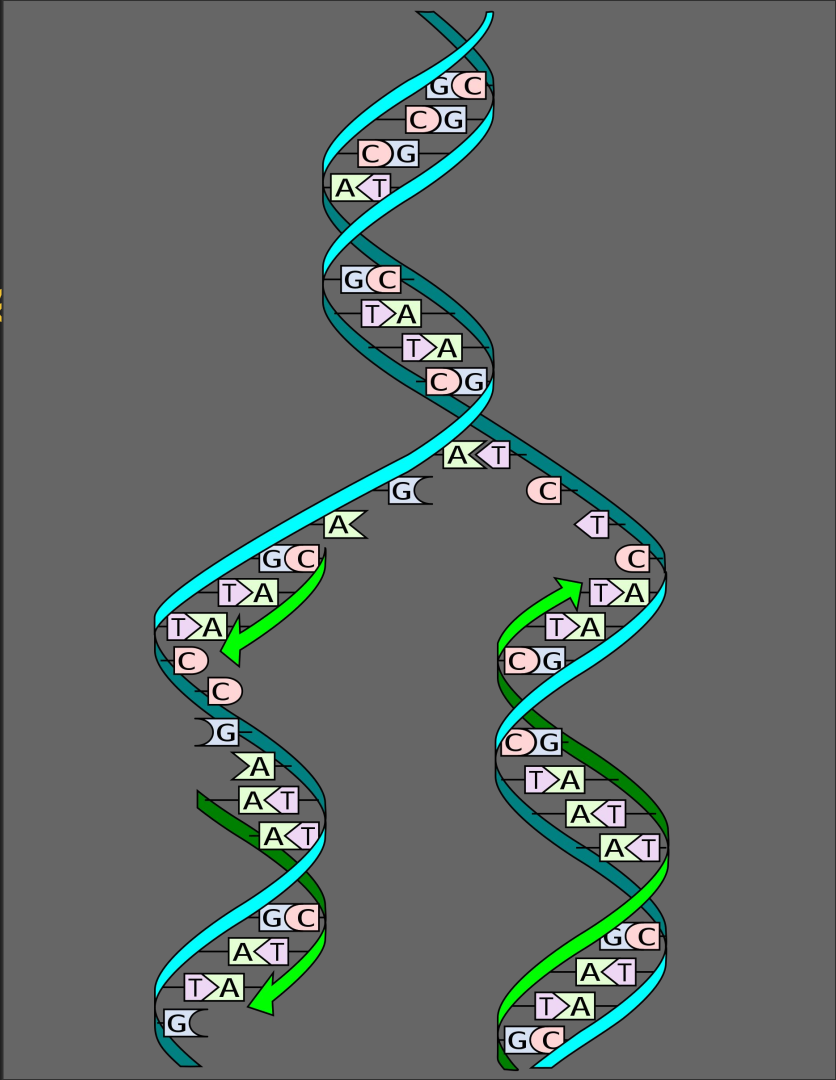

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$Y(g) = g(Y(g)) = \ldots$

Let's do factorial in Scheme!

(caveat: using the Z combinator)

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$$Y(g) = g(Y(g)) = \ldots$$

Historical Side Note

Y Combinator is an American seed accelerator, started in March 2005. Y Combinator is consistently ranked at the top of U.S. accelerators

https://www.ycombinator.com

Why did you choose the name "Y Combinator?"

The Y combinator is one of the coolest ideas in computer science. It's also a metaphor for what we do. It's a program that runs programs; we're a company that helps start companies.

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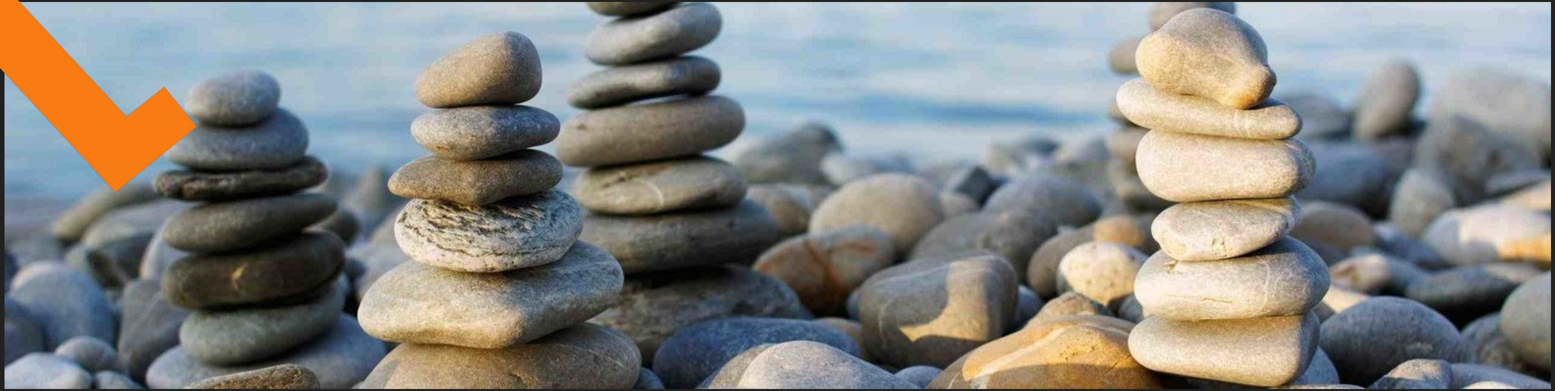

$$\lambda f.(\lambda x.f(x\ x))(\lambda x.f(x\ x))$$