Introduction to Paraxial Fluids of Light - 2

Nuno A. Silva, Tiago D. Ferreira
INESC TEC and University of Porto, Portugal

Last week

A mean-field model for a quantum fluid: BECs
The NLSE in nonlinear optics
The hydrodynamic interpretation of light

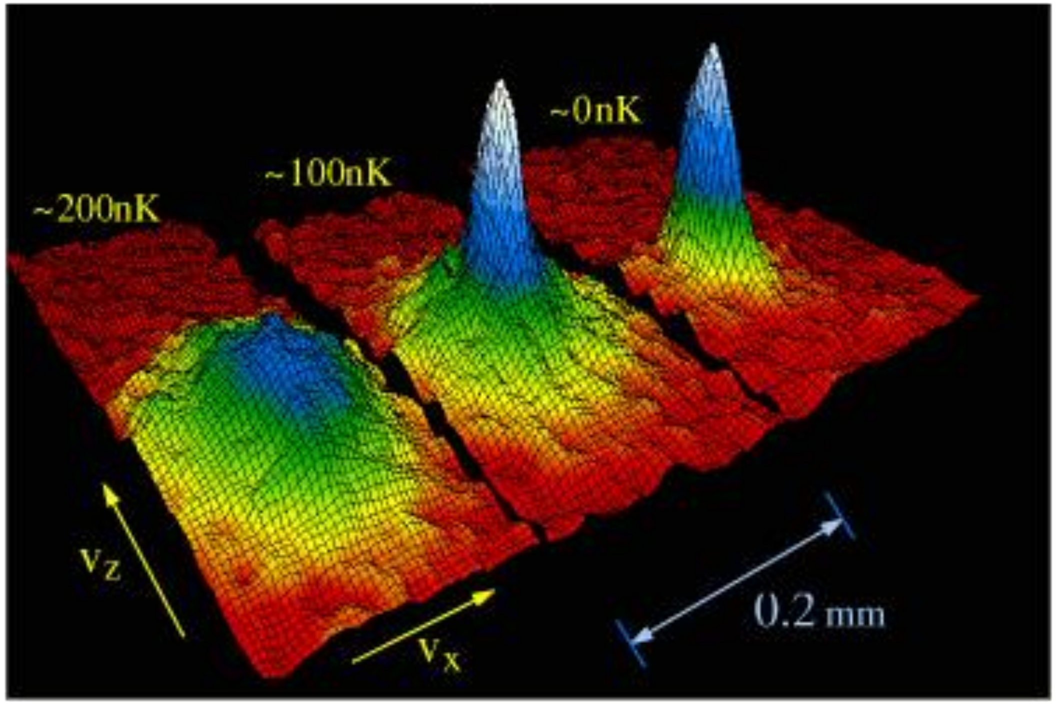
Last week

A mean-field model for a quantum fluid: BECs

NLSE model for a quantum fluid

Macroscopic quantum wave function $\hat{\psi}(\mathbf{r}, t)$

2 D velocity distributions



NLSE model for a quantum fluid

System hamiltonian

$$\hat{H} = \int \hat{\psi}^{\dagger}(\mathbf{r},t)\left[\frac{\mathbf{p}^2}{2M} + V_{\mathrm{ext}}(\mathbf{r},t)\right]\hat{\psi}(\mathbf{r},t)d\,\mathbf{r} + \hat{H}_{\mathrm{int}}.$$

$$\hat{H}_{\mathrm{int}} = \frac{1}{2}\int d\,\mathbf{r}\int d\,\mathbf{r}'\,\hat{\psi}^{\dagger}(\mathbf{r},t)\hat{\psi}^{\dagger}(\mathbf{r}',t)V(\mathbf{r}-\mathbf{r}')\hat{\psi}(\mathbf{r},t)\hat{\psi}(\mathbf{r}',t),$$

NLSE model for a quantum fluid

System hamiltonian

$$\hat{H} = \int \hat{\psi}^{\dagger}(\mathbf{r},t)\left[\frac{\mathbf{p}^2}{2M} + V_{\text{ext}}(\mathbf{r},t)\right]\hat{\psi}(\mathbf{r},t)d\mathbf{r} + \hat{H}_{\text{int}}.$$

$$\hat{H}_{\text{int}} = \frac{1}{2}\int d\mathbf{r}\int d\mathbf{r}'\,\hat{\psi}^{\dagger}(\mathbf{r},t)\hat{\psi}^{\dagger}(\mathbf{r}',t)V(\mathbf{r}-\mathbf{r}')\hat{\psi}(\mathbf{r},t)\hat{\psi}(\mathbf{r}',t).$$

Mean-field Theory

$$\hat{\psi}(\mathbf{r},t) = \Phi(\mathbf{r},t) + \delta\hat{\psi}(\mathbf{r},t), \quad \Phi(\mathbf{r},t) \equiv \langle\hat{\psi}(\mathbf{r},t)\rangle,$$

$$\frac{\partial}{\partial t}\Phi(\mathbf{r},t) = -\frac{i}{\hbar}\left[\frac{\mathbf{p}^2}{2M} + V_{\text{ext}}(\mathbf{r},t) + \int \Phi^{\dagger}(\mathbf{r}',t)V(\mathbf{r}-\mathbf{r}')\Phi(\mathbf{r},t)d\mathbf{r}'\right]\Phi(\mathbf{r},t).$$

NLSE model for a quantum fluid

Binary collisions only

$$V(\mathbf{r}-\mathbf{r}') = g\,\delta(\mathbf{r}-\mathbf{r}')\,, \quad g = 4\pi\hbar^2\frac{a}{M}.$$

NLSE model for a quantum fluid

Binary collisions only

$$V(\mathbf{r}-\mathbf{r}') = g\,\delta(\mathbf{r}-\mathbf{r}')\;, \quad g = 4\pi\hbar^2\frac{a}{M}.$$

Nonlinear Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Phi(\mathbf{r},t) = H\Phi(\mathbf{r},t)\;, \quad H \equiv \left[-\frac{\hbar^2\nabla^2}{2M} + V_{\mathrm{ext}}(\mathbf{r},t) + g\,|\Phi(\mathbf{r},t)|^2\right].$$

Last week

A mean-field model for a quantum fluid: BECs
The NLSE in nonlinear optics

NLSE model for a (paraxial) light fluid

Maxwell's wave equation

$$\boldsymbol{\nabla}^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 \boldsymbol{P}}{\partial t^2}$$

NLSE model for a (paraxial) light fluid

Maxwell's wave equation

$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 \boldsymbol{P}}{\partial t^2}$$

Nonlinear polarization

$$\begin{aligned}\boldsymbol{P}(\boldsymbol{r},t) &= \epsilon_0\, \boldsymbol{\chi}\left[\boldsymbol{E}(\boldsymbol{r},t)\right] \boldsymbol{E}(\boldsymbol{r},t) \\ &= \underbrace{\epsilon_0\, \boldsymbol{\chi}^{(1)} \cdot \boldsymbol{E}(\boldsymbol{r},t)}_{\boldsymbol{P}^{(1)}(\boldsymbol{r},t)} + \underbrace{\epsilon_0\, \boldsymbol{\chi}^{(2)} : \boldsymbol{E}(\boldsymbol{r},t)^2}_{\boldsymbol{P}^{(2)}(\boldsymbol{r},t)} + \underbrace{\epsilon_0\, \boldsymbol{\chi}^{(3)} : \boldsymbol{E}(\boldsymbol{r},t)^3}_{\boldsymbol{P}^{(3)}(\boldsymbol{r},t)} + \ldots\end{aligned}$$

NLSE model for a (paraxial) light fluid

Maxwell's wave equation

$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 \boldsymbol{P}}{\partial t^2}$$

Nonlinear polarization

$$\begin{aligned}\boldsymbol{P}(\boldsymbol{r},t) &= \epsilon_0\,\chi\,[\boldsymbol{E}(\boldsymbol{r},t)]\,\boldsymbol{E}(\boldsymbol{r},t)\\ &= \underbrace{\epsilon_0\,\chi^{(1)}\cdot\boldsymbol{E}(\boldsymbol{r},t)}_{\boldsymbol{P}^{(1)}(\boldsymbol{r},t)} + \underbrace{\epsilon_0\,\chi^{(2)}:\boldsymbol{E}(\boldsymbol{r},t)^2}_{\boldsymbol{P}^{(2)}(\boldsymbol{r},t)} + \underbrace{\epsilon_0\,\chi^{(3)}:\boldsymbol{E}(\boldsymbol{r},t)^3}_{\boldsymbol{P}^{(3)}(\boldsymbol{r},t)} + \dots\end{aligned}$$

Centro-symmetric media (no even orders) + isotropy (susceptibility becomes a number): at lowest nonlinear order (third order, Kerr/cubic terms)

$$\nabla^2\mathcal{E}(\mathbf{r}) + \frac{\omega^2}{c^2}\left[1+\chi^{(1)}(\omega)\right]\mathcal{E}(\mathbf{r}) = -\frac{3}{4}\frac{\omega^2}{c^2}\chi^{(3)}(\omega)\left|\mathcal{E}(\mathbf{r})\right|^2\mathcal{E}(\mathbf{r}).$$

NLSE model for a (paraxial) light fluid

Paraxial approximation (small angle from the propagation axis z)

$$\mathcal{E}(\mathbf{r}_\perp, z) = \mathcal{E}_0(\mathbf{r}_\perp, z)\, e^{ik(\omega)z}$$

NLSE model for a (paraxial) light fluid

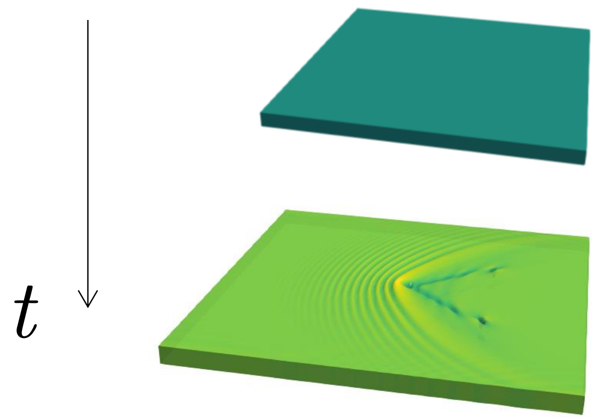
Paraxial approximation (small angle from the propagation axis z)

$$\mathcal{E}(\mathbf{r}_\perp, z) = \mathcal{E}_0(\mathbf{r}_\perp, z)\, e^{ik(\omega)z}$$

Nonlinear Schrödinger equation

$$i\,\partial_z\,\mathcal{E}_0(\mathbf{r}_\perp, z) = \left[-\frac{1}{2k}\boldsymbol{\nabla}_\perp^2 - \frac{i\alpha}{2} - k\,\frac{\delta n(\mathbf{r}_\perp, z)}{n_0} - \frac{3}{8}\frac{k}{n_0^2}\,\chi^{(3)}(\omega)\,\left|\mathcal{E}_0\left(\mathbf{r}_\perp, z\right)\right|^2\right]\mathcal{E}_0(\mathbf{r}_\perp, z). \quad (2.10$$

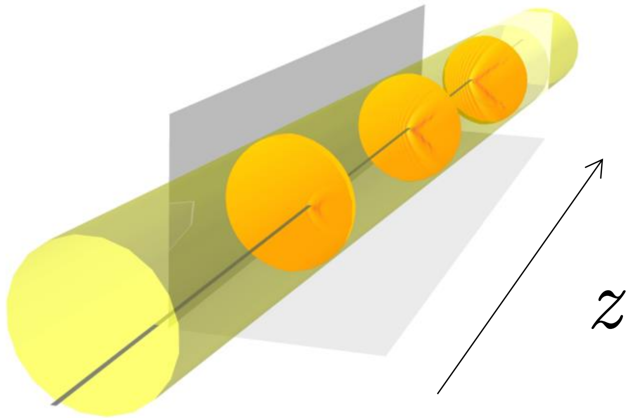
Quantum Fluids



$$i\hbar\partial_t\Psi + \frac{\hbar^2}{2m}\nabla^2\Psi + g\left|\Psi\right|^2\Psi + V_{ext}\Psi = 0$$

Time t
Density $\left|\Psi\right|^2$
Interaction via collisions g
Atomic mass m
Potential (Confinement, Barrier) V_{ext}

Light in propagating geometries



$$i\partial_z\Omega_p + \tfrac{1}{2}\nabla_\perp^2\Omega_p + V\Omega_p + G\left|\Omega_p\right|^2\Omega_p = 0$$

Propagation Distance z
Intensity $\left|\Omega_p\right|^2$
Interaction via nonlinearity G
Effective mass through diffraction k_p
Linear refractive index V

Last week

A mean-field model for a quantum fluid: BECs
The NLSE in nonlinear optics
The hydrodynamic interpretation of light

Hydrodynamic interpretation of light

$$i\partial_z E + \frac{1}{2}\nabla_\perp^2 E + V\left(\boldsymbol{r}_\perp\right) E + g\left|E\right|^2 E = 0$$

Madelung transformation

$$E\left(\boldsymbol{r}_\perp, z\right) = \sqrt{\rho\left(\boldsymbol{r}_\perp, z\right)}e^{i\phi\left(\boldsymbol{r}_\perp, z\right)}$$

Hydrodynamic interpretation of light

Set of Hydrodynamic like equations

$$\boldsymbol{v} = \nabla \phi, \qquad V_B \equiv \frac{\nabla^2 \sqrt{\rho}}{2\sqrt{\rho}}$$

Hydrodynamic interpretation of light

Set of Hydrodynamic like equations

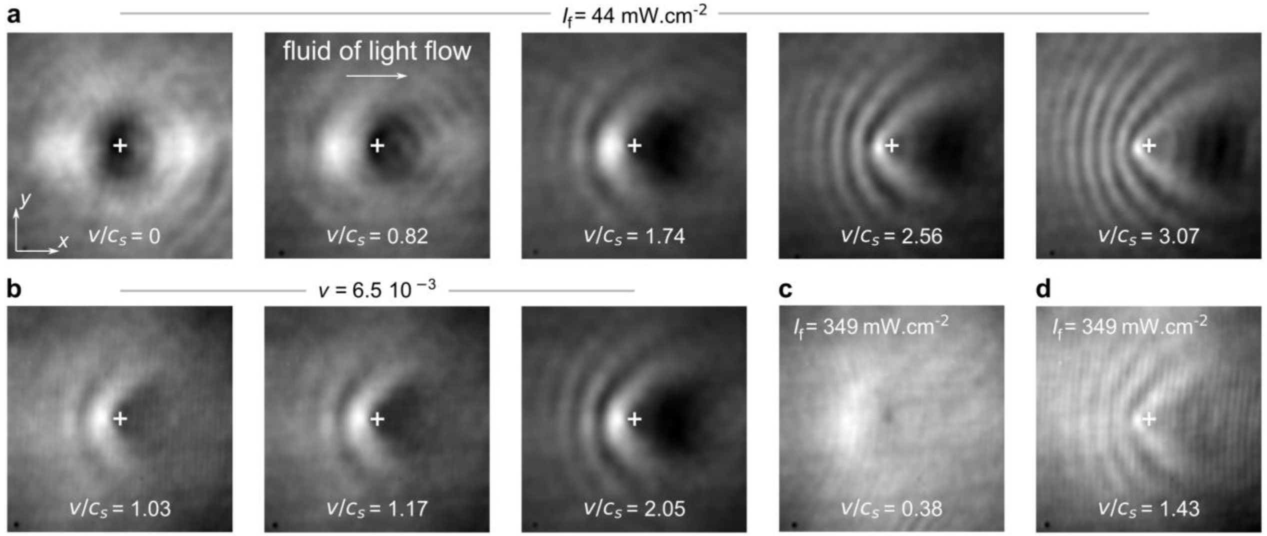
$$\boldsymbol{v} = \nabla\phi, \qquad V_B \equiv \frac{\nabla^2\sqrt{\rho}}{2\sqrt{\rho}}$$

$$\partial_z \rho + \nabla\left(\rho v\right) = 0$$

$$\partial_z v + \left(v \cdot \nabla\right) v = \nabla\left(V + V_B + g\rho\right)$$

Hydrodynamic interpretation of light

Light shall behave like a fluid



But we said quantum fluid, where is the quantum part?

$$i\partial_z E + \frac{1}{2}\nabla_\perp^2 E + V\left(r_\perp\right)E + g\left|E\right|^2 E = 0$$

In this self-interaction term, that emulates BECs

Superfluid properties

Small excitations spectra

Bogoliubov dispersion relation

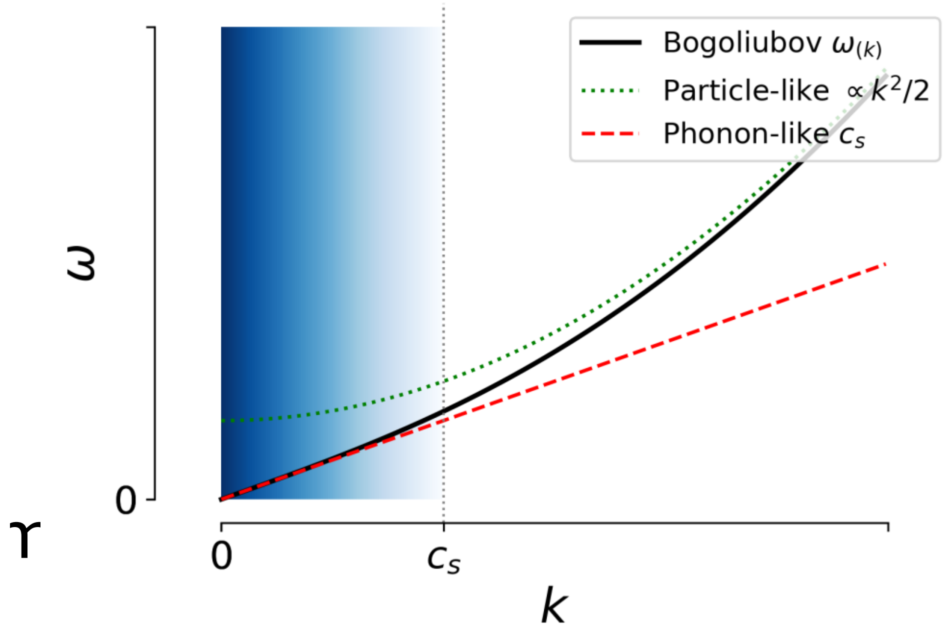
$$\omega_{\mathrm{Bog}}(\boldsymbol{k}_\perp) = \sqrt{k_\perp^2\left(\frac{k_\perp^2}{4} + G|\Omega_p^0|^2\right)}.$$

Superfluid properties

Small excitations spectra

Bogoliubov dispersion relation

$$\omega_{\mathrm{Bog}}(\boldsymbol{k}_{\perp}) = \sqrt{k_{\perp}^{2}\left(\frac{k_{\perp}^{2}}{4} + G|\Omega_{p}^{0}|^{2}\right)}.$$



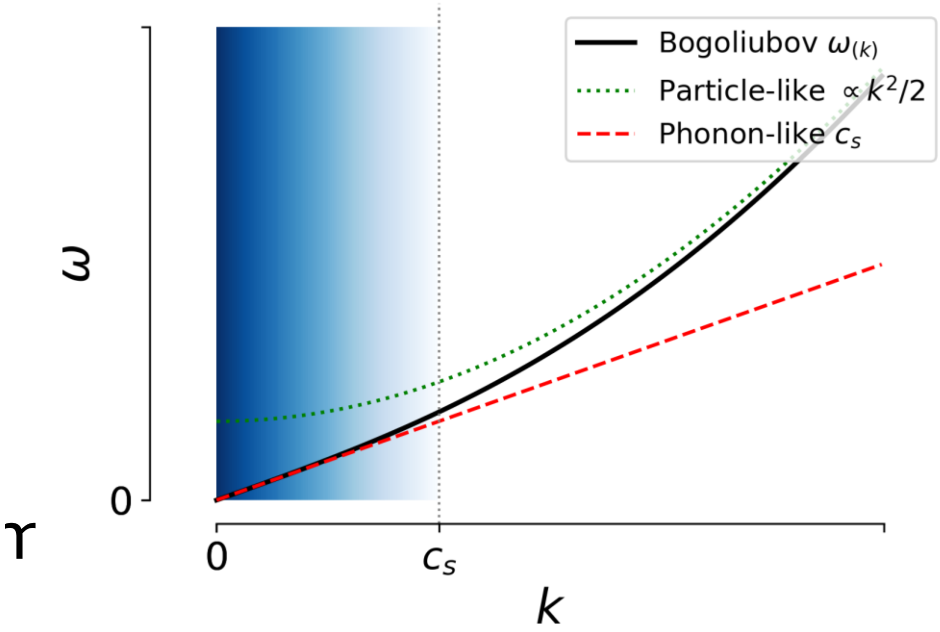
Superfluid properties

Small excitations spectra

Bogoliubov dispersion relation

$$\omega_{\text{Bog}}(\boldsymbol{k}_\perp) = \sqrt{k_\perp^2\left(\frac{k_\perp^2}{4} + G|\Omega_p^0|^2\right)}.$$

Landau Criteria $v < c_s$



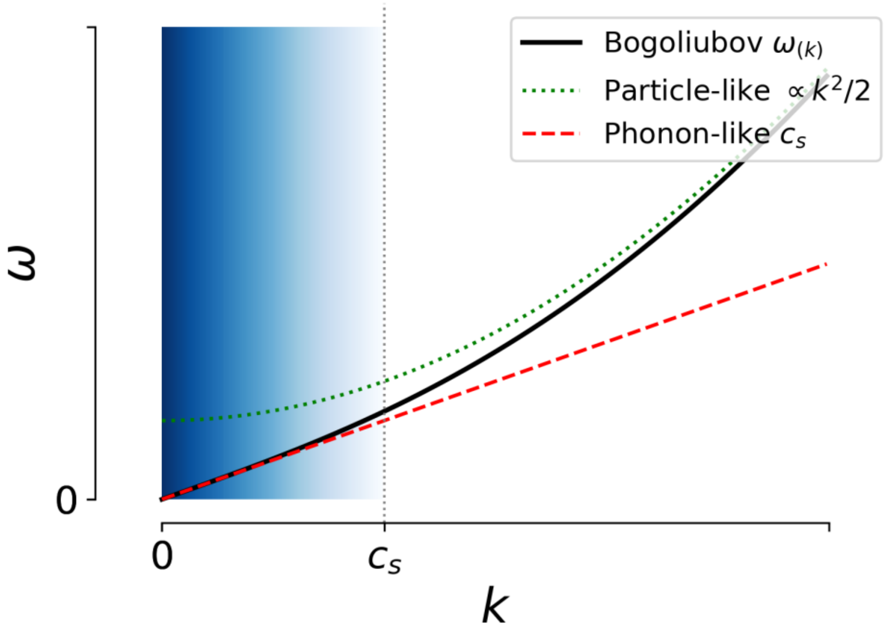
Superfluid properties

Small excitations spectra

Bogoliubov dispersion relation

$$\omega_{\text{Bog}}(\boldsymbol{k}_\perp) = \sqrt{k_\perp^2 \left(\frac{k_\perp^2}{4} + G|\Omega_p^0|^2 \right)}.$$

Landau Criteria $v < c_s$



Superfluid properties

absence of drag, loss of momentum

Why?

Exercise 1. Compute the Bogoliubov Dispersion relation

The Bogoliubov dispersion relation is the dispersion relation for small excitations on top of a uniform background.

Exercise 2. Explore the Landau Criteria and try to understand why small perturbations on top of a constant background fluid are forbidden below the sound velocity.

This week

Conserved Quantities in the NLSE
Solutions of the NLSE
Overview of some results of the literature

Conserved quantities in the NLSE

To understand the dynamics of a given physical model, conserved quantities are a must in any researcher's toolbox.

Conserved quantities in the NLSE

Leibniz proposed the conservation of kinetic energy;

Newton the conservation of momentum,

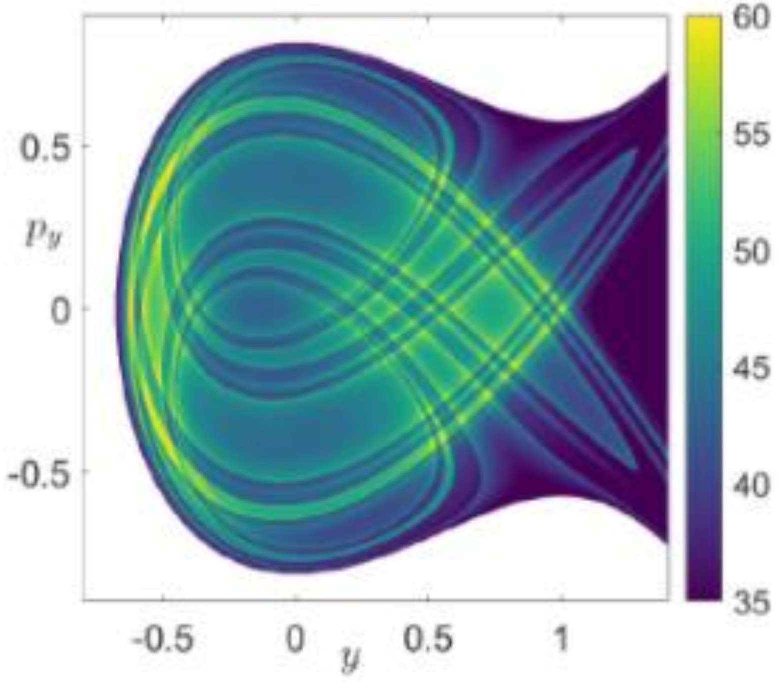
Bernoulli’s principle lead to the discovery of Lagrangian and Hamiltonian formalisms;

Émile du Châtelet proposed the conservation of total energy;

Emmy Noether introduced the Noether’s theorem.

Lagrangian mechanics

Elegant way to derive the equations of motion of a physical system from a points configuration in the phase space;



Lagrangian mechanics

Elegant way to derive the equations of motion of a physical system from a points configuration in the phase space;

$$S = \int_{t_1}^{t_2} L \, \mathrm{d}t \,, \qquad \delta S = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_j}\right) = \frac{\partial L}{\partial q_j}$$

Lagrangian mechanics

Elegant way to derive the equations of motion of a physical system from a points configuration in the phase space;

$$S = \int_{t_1}^{t_2} L \,\mathrm{d}t \,, \qquad \delta S = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_j}\right) = \frac{\partial L}{\partial q_j}$$

Equivalent to Hamiltonian via Legendre's transformation

$$\mathcal{H} = \sum_i \dot{q}^i \frac{\partial \mathcal{L}}{\partial \dot{q}^i} - \mathcal{L} = \sum_i \dot{q}^i p_i - \mathcal{L}$$

Lagrangian mechanics

Lagrangian density of the NLSE $\quad i\frac{\partial\psi}{\partial z} + \frac{1}{2}\nabla_\perp^2\psi + F(|\psi|^2)\psi + V(r,z)\psi = 0$

$$\mathcal{L} \;=\; \frac{i}{2}\left(\psi^*\psi_z - \psi\psi_z^*\right) - \frac{1}{2}\nabla_\perp\psi\nabla_\perp\psi^* + G\left(|\psi|^2\right),$$

$$G(\lambda) = \int_0^\lambda F(\lambda)d\lambda,$$

Action $\quad S\left\{\psi,\psi^*\right\} = \int\limits_{z_0}^{z_1}\int\limits_{all\ space} \mathcal{L}\,d\mathbf{x}\,dz.$

Noether's Theorem

If for a given infinitesimal transformation (call it a symmetry)

$$\begin{aligned} z &\mapsto z^{'} = z + \delta z(\mathbf{x}, z, \psi) \\ \mathbf{x} &\mapsto \mathbf{x}^{'} = \mathbf{x} + \delta \mathbf{x}(\mathbf{x}, z, \psi) \\ \psi(\mathbf{x}, z) &\mapsto \psi(\mathbf{x}^{'}, z^{'}) = \psi(\mathbf{x}, z) + \delta\psi(\mathbf{x}, z) \end{aligned}$$

The action is conserved, then there is an associated current that is also conserved:

$$\int \left[\frac{\partial \mathcal{L}}{\partial \psi_z} \left(\psi_z \delta z + \nabla_\perp \psi \cdot \delta \mathbf{x} - \delta\psi \right) + \frac{\partial \mathcal{L}}{\partial \psi_z^*} \left(\psi_z^* \delta z + \nabla_\perp \psi^* \cdot \delta \mathbf{x} - \delta\psi^* \right) - \mathcal{L} \delta t \right] d\mathbf{x}.$$

Consequences in mechanics

Time invariance – conservation of the Hamiltonian/Total energy

Space invariance – conservation of the total momentum

Consequences in the NLSE

Conservation under a phase transformation (gauge transformation)

$$\psi(\vec{x}, \tilde{t}) \mapsto \psi(\vec{x}, \tilde{t})' = \psi(\vec{x}, \tilde{t})\, e^{i\gamma(\vec{x})\tilde{t}},$$

Leads to the conservation of the norm of the wave function

$$N = \int |\psi|^2 d\vec{x}.$$

Consequences in the NLSE

Conservation under a space transformation

$$\vec{x} \mapsto \vec{x}' = \vec{x} + \delta\vec{x}\,(\vec{x}, \tilde{t}, \psi)\,,$$

Leads to the conservation of the momentum

$$M \;=\; \frac{i}{2}\int \left[\psi\nabla_\perp\psi^* - \psi^*\nabla_\perp\psi)\right] d\mathbf{x}$$

Consequences in the NLSE

Conservation under a time translation

$$\tilde{t} \mapsto \tilde{t}' = \tilde{t} + \delta\tilde{t}\,(\vec{x}, \tilde{t}, \psi)$$

Leads to the conservation of the energy/hamiltonian

$$H = \int \left[\frac{1}{2}\nabla_\perp\psi\nabla_\perp\psi^* - G(|\psi|^2)\right] d\mathbf{x}$$

This week

Conserved Quantities in the NLSE
Solutions of the NLSE

Solutions of the NLSE

The most common solution is the typical plane wave solution:

$$E = Ae^{i(\boldsymbol{k}_0 \cdot \boldsymbol{r} - \omega_0 z)}$$

Let us substitute into the NLSE

$$i\partial_z E + \frac{1}{2}\nabla_\perp^2 E + g\left|E\right|^2 E = 0$$

Solutions of the NLSE

Seems like a solution...

$$\omega_0 = \frac{k_0^2}{2} - gA^2$$

But is it stable?

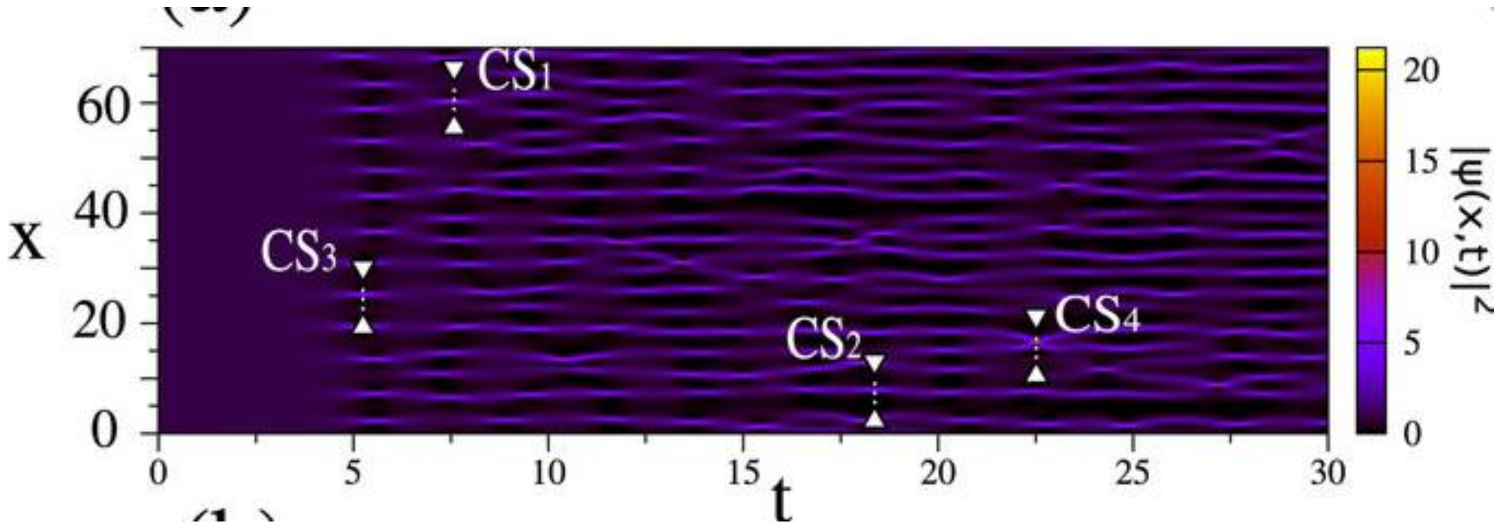
Apply small amplitude excitations, derive the dispersion relation (Bogoliubov theory)

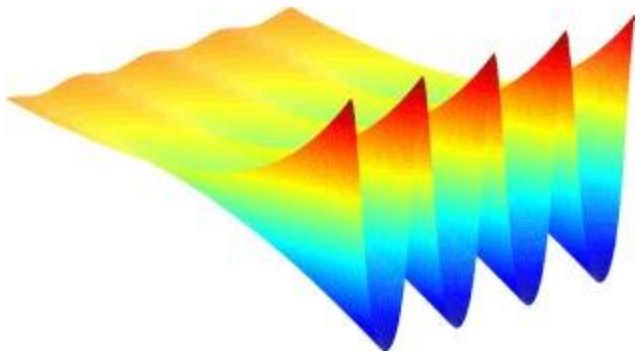
$$\psi(\vec{r},\tilde{t}) = \left(A + B(\vec{r})\, e^{-i\omega\tilde{t}} + C^*(\vec{r})\, e^{i\omega\tilde{t}}\right) e^{i\alpha(\tilde{t})}, \qquad \omega(k) = \sqrt{k^2\left(\frac{k^2}{2} - gA^2\right)}$$

Modulation instability

$$\omega(k) = \sqrt{k^2\left(\frac{k^2}{2} - gA^2\right)}$$

For g>0 small amplitude oscillations increase of amplitude exponentially over the time, leading to the instability and breaking the plane wave solution.





Plane wave solutions

$$\omega\left(k\right)=\sqrt{k^2\left(\frac{k^2}{2}-gA^2\right)}$$

For g<0 the plane wave solution is stable;

This is the reason why we study quantum fluids of light with defocusing nonlinearities – the system is still interesting and doesn’t burn any material in our lab!

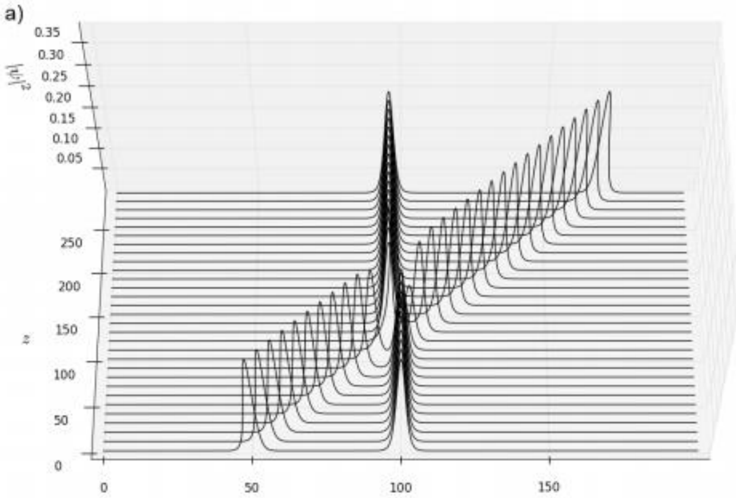
Soliton Solutions

It is possible however to demonstrate (Inverse scattering technique) that the NLSE also admits self-localized solutions of the type

$$\begin{aligned}\psi(x,z) &= 2\nu\mathrm{sech}\left[2\nu\left(x-\bar{x}_0-\mu z\right)\right]\exp\left\{i\mu\left(x-\bar{x}_0\right)+i\delta(z)\right\}\\ \delta(z) &= (2\nu^2-\mu^2/2)z+\delta_0,\end{aligned}$$

Self-localized solutions: Doesn't diffract

Particle-like behaviors: collisions, interactions

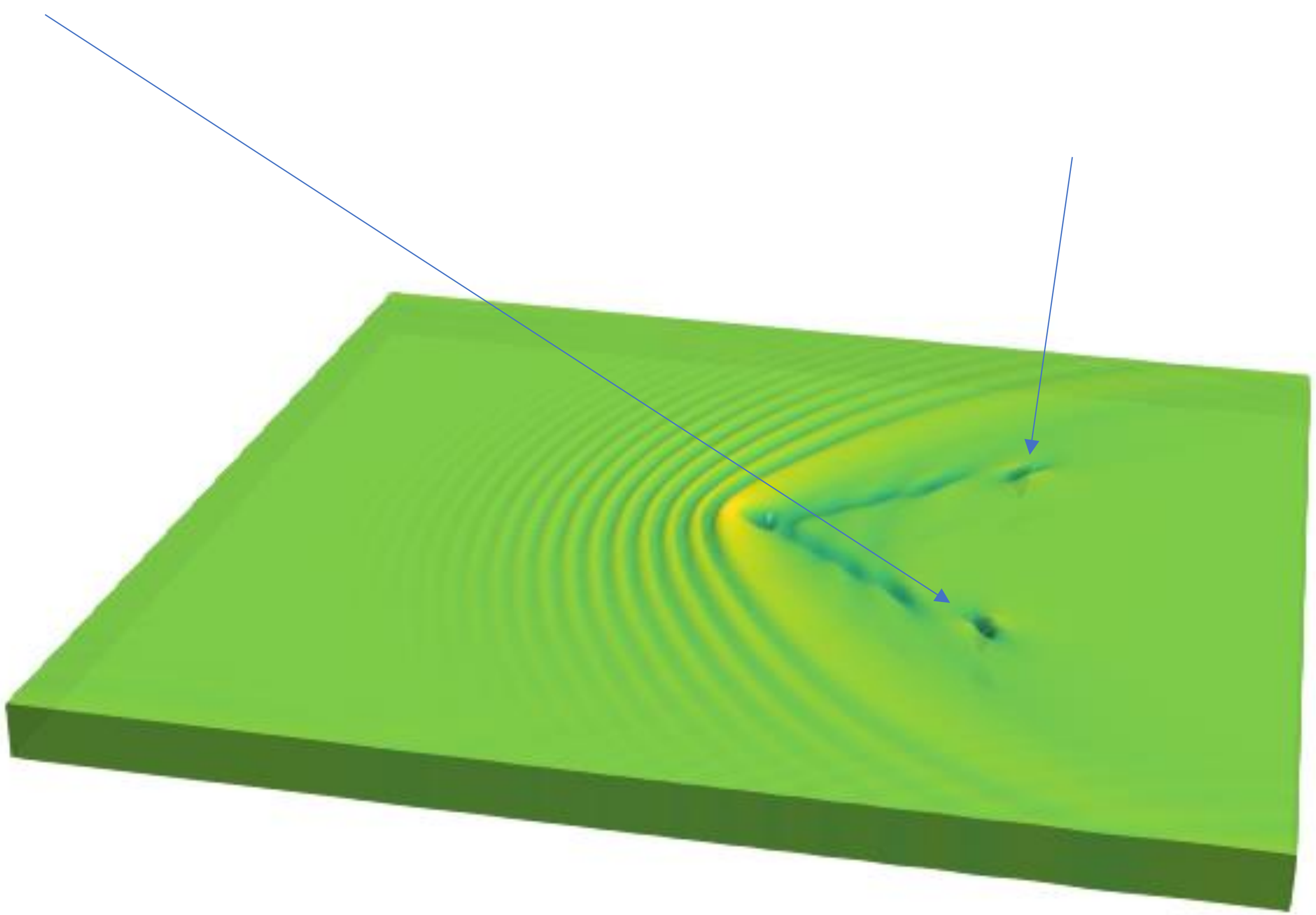


Vortex solutions

Self-defocusing g<0

Self-localized

Particle-like



But most of the time…

Analytical solutions of the NLSE are not easy to obtain.

First order differential equation that contains nonlinear terms: it happens that it can be integrated, but most of the times this is not the case;

Therefore any research involving these theories often require the use computational methods and hopefully (for a timely research) high-performance solutions.

This week

Conserved Quantities in the NLSE
Solutions of the NLSE
Overview of some results of the literature

Historical Note

PHYSICAL REVIEW A VOLUME 60, NUMBER 5 NOVEMBER 1999

Bogoliubov dispersion relation and the possibility of superfluidity for weakly interacting photons in a two-dimensional photon fluid

Raymond Y. Chiao* and Jack Boyce†
Department of Physics, University of California, Berkeley, California 94720-7300
(Received 3 May 1999; revised manuscript received 22 July 1999)

The Bogoliubov dispersion relation for the elementary excitations of the weakly interacting Bose gas is shown to hold for the case of the weakly interacting photon gas (the ''photon fluid'') in a nonlinear Fabry-Perot cavity. The chemical potential of a photon in the two-dimensional photon fluid does not vanish. The Bogoliubov relation, which is also derived by means of a linearized fluctuation analysis in classical nonlinear optics, implies the possibility of a new, superfluid state of light. The theory underlying an experiment in progress to observe sound waves in the photon fluid is described, and another experiment to measure the critical velocity of this superfluid is proposed. [S1050-2947(99)08411-5]

PACS number(s): 42.50.Ct, 42.65.Sf, 67.90.+z, 05.30.Jp

$$i\frac{\partial E_f}{\partial z} + \frac{1}{2n_e k_f}\nabla_\perp^2 E_f + k_f \Delta n E_f + i\frac{\alpha}{2}E_f = 0,$$

$$\omega_z = \sqrt{\frac{k_\perp^2}{2k_f n_e}\left(\frac{k_\perp^2}{2k_f n_e} + 2k_f \Delta n\right)}.$$

ELSEVIER
Optics Communications
Volume 179, Issues 1–6, 25 May 2000, Pages 157-166

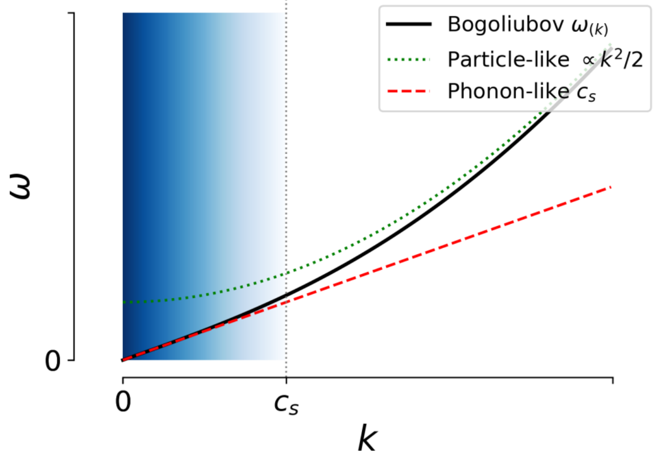
Bogoliubov dispersion relation for a `photon fluid': Is this a superfluid? [1]

Raymond Y. Chiao

Landau criteria

Dissipation-less regime for

$$v < c_s$$



Historical Note

PHYSICAL REVIEW A VOLUME 60, NUMBER 5 NOVEMBER 1999

Bogoliubov dispersion relation and the possibility of superfluidity for weakly interacting photons in a two-dimensional photon fluid

Raymond Y. Chiao* and Jack Boyce†
Department of Physics, University of Calif
(Received 3 May 1999; revised ma

The Bogoliubov dispersion relation for the elementar shown to hold for the case of the weakly interacting ph Perot cavity. The chemical potential of a photon in the Bogoliubov relation, which is also derived by means of a optics, implies the possibility of a new, superfluid stat progress to observe sound waves in the photon fluid is critical velocity of this superfluid is proposed. [S1050-29

PACS number(s): 42.50.Ct, 42.65.Sf, 67.90.+z, 05.30.Jp

ELSEVIER

Optics Communications
Volume 179, Issues 1–6, 25 May 2000, Pages 157-166

Bogoliubov dispersion relation for a \`photon fluid': Is this a superfluid? [1]

Raymond Y. Chiao

Paraxial Fluids of Light (2014 onwards)

PROCEEDINGS OF THE ROYAL SOCIETY A

rspa.royalsocietypublishing.org

Research

Cite this article: Carusotto I. 2014 Superfluid light in bulk nonlinear media. *Proc. R. Soc. A* **470**: 20140320.
http://dx.doi.org/10.1098/rspa.2014.0320

Received: 17 April 2014
Accepted: 11 June 2014

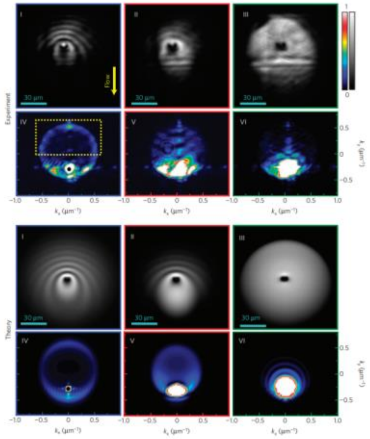
Superfluid light in bulk nonlinear media

Iacopo Carusotto

INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, via Sommarive 14, Povo 38123, Italy

We review how the paraxial approximation naturally leads to a hydrodynamic description of light propagation in a bulk Kerr nonlinear medium in terms of a wave equation analogous to the Gross–Pitaevskii equation for the order parameter of a superfluid. The main features of the many-body collective dynamics of the fluid of light in this propagating geometry are discussed: generation and observation of Bogoliubov sound waves in the fluid of light is first described. Experimentally accessible manifestations of superfluidity are then highlighted. Perspectives in view of realizing analogue models of gravity are finally given.

Polariton Fluids (2006 onwards)



nature physics LETTERS

PUBLISHED ONLINE: 20 SEPTEMBER 2009 | DOI: 10.1038/NPHYS1364

Superfluidity of polaritons in semiconductor microcavities

Alberto Amo[1]*, Jérôme Lefrère[1], Simon Pigeon[2], Claire Adrados[1], Cristiano Ciuti[2], Iacopo Carusotto[3], Romuald Houdré[4], Elisabeth Giacobino[1] and Alberto Bramati[1]*

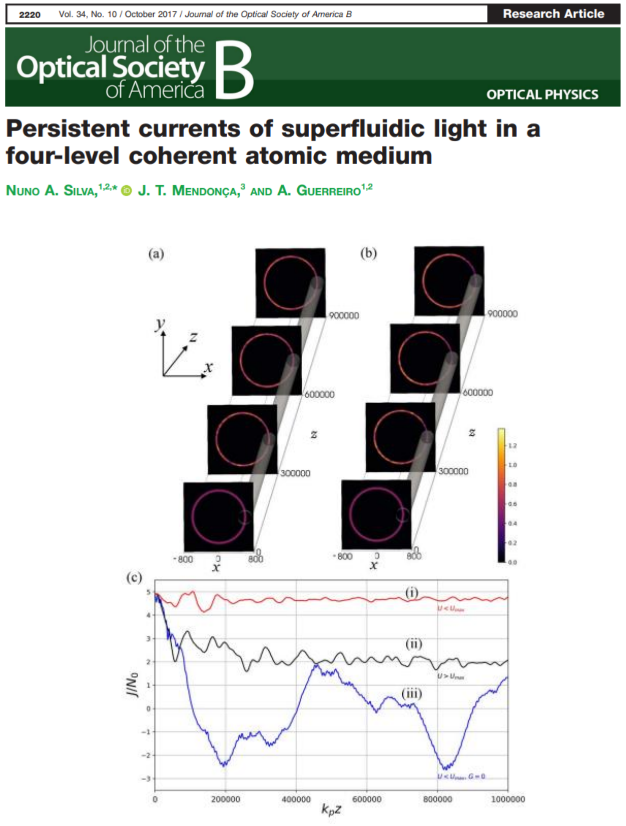
Pulsed version (2010 onwards)

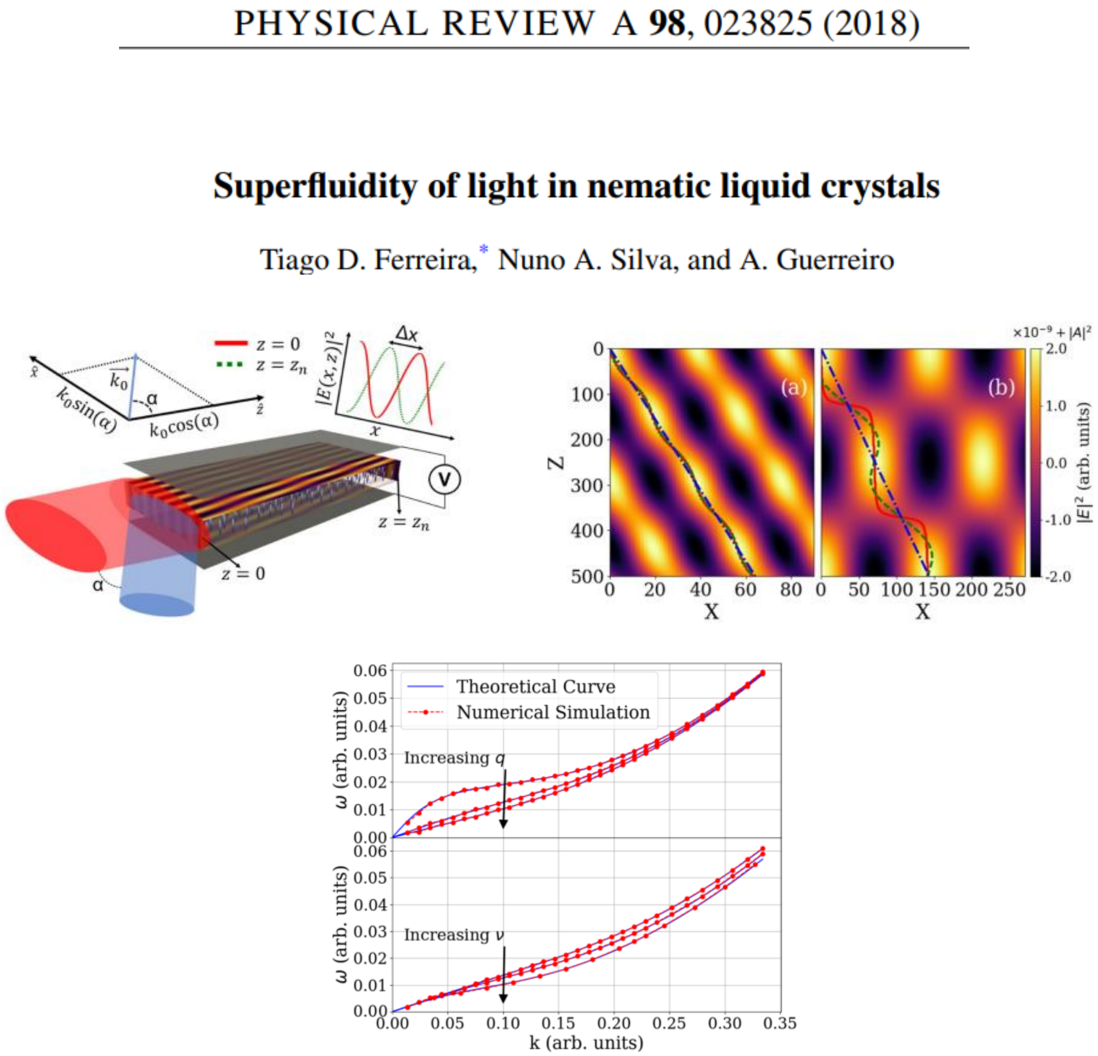
Superfluid Motion of Light

Patricio Leboeuf and Simon Moulieras
Phys. Rev. Lett. **105**, 163904 – Published 14 October 2010

Paraxial Fluids at INESC TEC

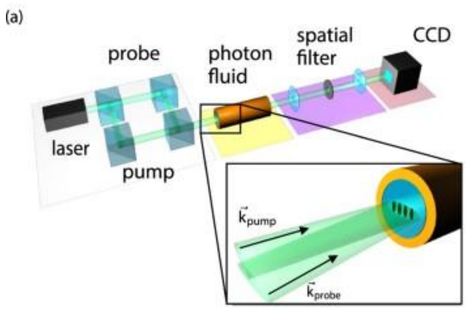
Theoretical work since 2017





Overview – Experimental Groups

Thermo-optic Nonlocal media



Extreme Light

University of Glasgow

Team leader: Danielle Faccio

Analogue Gravity, Superradiance

Photorefractive Crystals

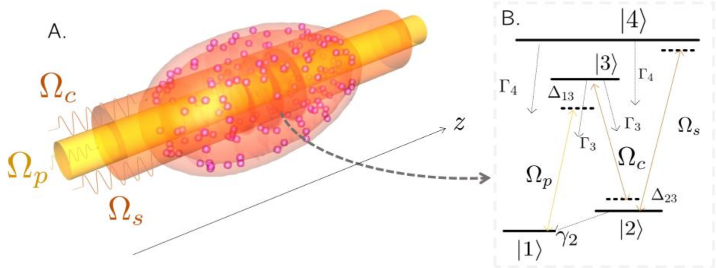
UNIVERSITÉ CÔTE D'AZUR

cnrs

Claire Michel, Matthieu Bellec

INESCTEC

Hot Atomic Vapours



Laboratoire Kastler Brossel
Physique quantique et applications

Team leader: Quentin Glorieux

Paraxial and polariton fluids, turbulence

Constructing an Analogue Quantum Simulator

Constructing an Analogue Quantum Simulator

Methanol
+
Graphene

optica

Experimental characterization of nonlocal photon fluids

DAVID VOCKE,[1,5] THOMAS ROGER,[1] FRANCESCO MARINO,[2] EWAN M. WRIGHT,[1,3] IACOPO CARUSOTTO,[4] MATTEO CLERICI,[1] AND DANIELE FACCIO[1,*]

Rb
Vapor

PHYSICAL REVIEW LETTERS **121,** 183604 (2018)

Editors' Suggestion

Observation of the Bogoliubov Dispersion in a Fluid of Light

Q. Fontaine, T. Bienaimé, S. Pigeon, E. Giacobino, A. Bramati, and Q. Glorieux[*]
Laboratoire Kastler Brossel, Sorbonne Université, CNRS, ENS-PSL Research University, Collège de France, Paris 75005, France

Photorefractive
media

nature COMMUNICATIONS

ARTICLE

DOI: 10.1038/s41467-018-04534-9 OPEN

Superfluid motion and drag-force cancellation in a fluid of light

Claire Michel[1], Omar Boughdad[1], Mathias Albert[1], Pierre-Élie Larré[2,3] & Matthieu Bellec[1]

Constructing an Analogue Quantum Simulator

Methanol + Graphene

optica

Experimental characterization of nonlocal photon fluids

David Vocke,[1,5] Thomas Roger,[1] Francesco Marino,[2] Ewan M. Wright,[1,3] Iacopo Carusotto,[4] Matteo Clerici,[1] and Daniele Faccio[1,*]

Rb Vapor

PHYSICAL REVIEW LETTERS 121, 183604 (2018)

Editors' Suggestion

Observation of the Bogoliubov Dispersion in a Fluid of Light

Q. Fontaine, T. Bienaimé, S. Pigeon, E. Giacobino, A. Bramati, and Q. Glorieux*

Laboratoire Kastler Brossel, Sorbonne Université, CNRS, ENS-PSL Research University, Collège de France, Paris 75005, France

Photorefractive media

nature COMMUNICATIONS

ARTICLE

DOI: 10.1038/s41467-018-04534-9 OPEN

Superfluid motion and drag-force cancellation in a fluid of light

Claire Michel[1], Omar Boughdad[1], Mathias Albert[1], Pierre-Élie Larré[2,3] & Matthieu Bellec[1]

$$i\partial_z E_f + \frac{1}{2n_e k_f}\nabla_\perp^2 E_f - k_f \Delta n_{max} \frac{|E_f|^2}{|E_f|^2 + I_{sat}} E_f = 0$$

SBN Crystal

Saturable Non-linearity

- Anisotropic Response

+ Low laser power (μW)

Constructing an Analogue Quantum Simulator

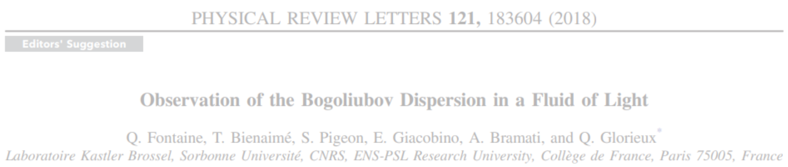
Methanol
+
Graphene


optica

Experimental characterization of nonlocal photon fluids

David Vocke,[1,5] Thomas Roger,[1] Francesco Marino,[2] Ewan M. Wright,[1,3] Iacopo Carusotto,[4] Matteo Clerici,[1] and Daniele Faccio[1,*]

Rb
Vapor


PHYSICAL REVIEW LETTERS 121, 183604 (2018)

Editors' Suggestion

Observation of the Bogoliubov Dispersion in a Fluid of Light

Q. Fontaine, T. Bienaimé, S. Pigeon, E. Giacobino, A. Bramati, and Q. Glorieux*

Laboratoire Kastler Brossel, Sorbonne Université, CNRS, ENS-PSL Research University, Collège de France, Paris 75005, France

Photorefractive
media (SBN)


nature COMMUNICATIONS

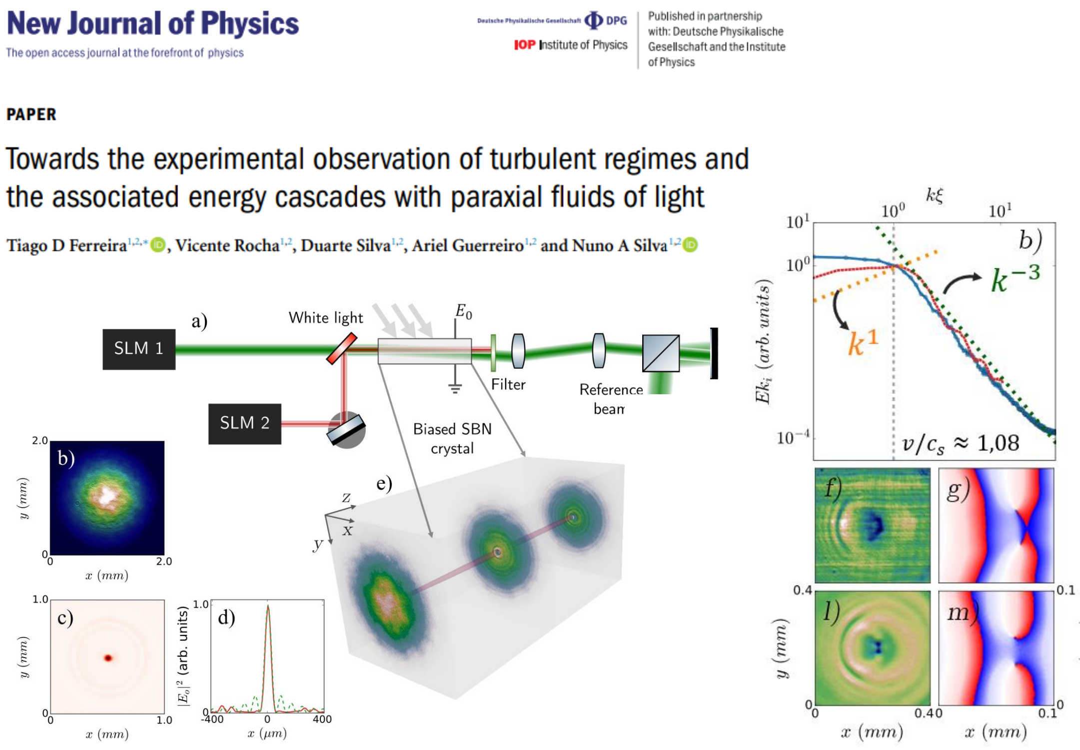
ARTICLE

DOI: 10.1038/s41467-018-04534-9 OPEN

Superfluid motion and drag-force cancellation in a fluid of light

Claire Michel[1], Omar Boughdad[1], Mathias Albert[1], Pierre-Élie Larré[2,3] & Matthieu Bellec[1]

2022


New Journal of Physics

The open access journal at the forefront of physics

Published in partnership with: Deutsche Physikalische Gesellschaft and the Institute of Physics

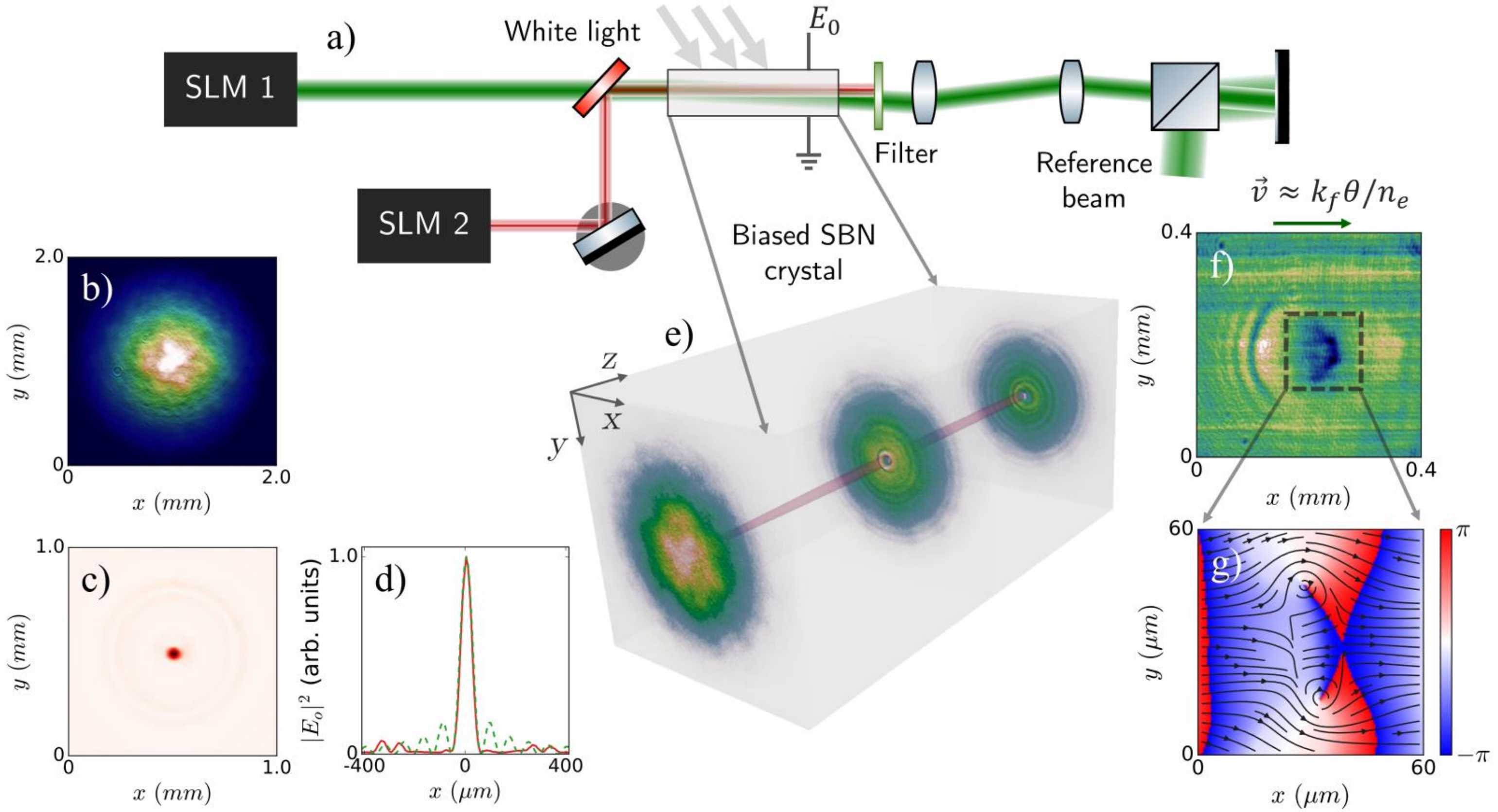
PAPER

Towards the experimental observation of turbulent regimes and the associated energy cascades with paraxial fluids of light

Tiago D Ferreira[1,2,*], Vicente Rocha[1,2], Duarte Silva[1,2], Ariel Guerreiro[1,2] and Nuno A Silva[1,2]

Experimental Work with Photorefractive Crystals

Probing quantum turbulence

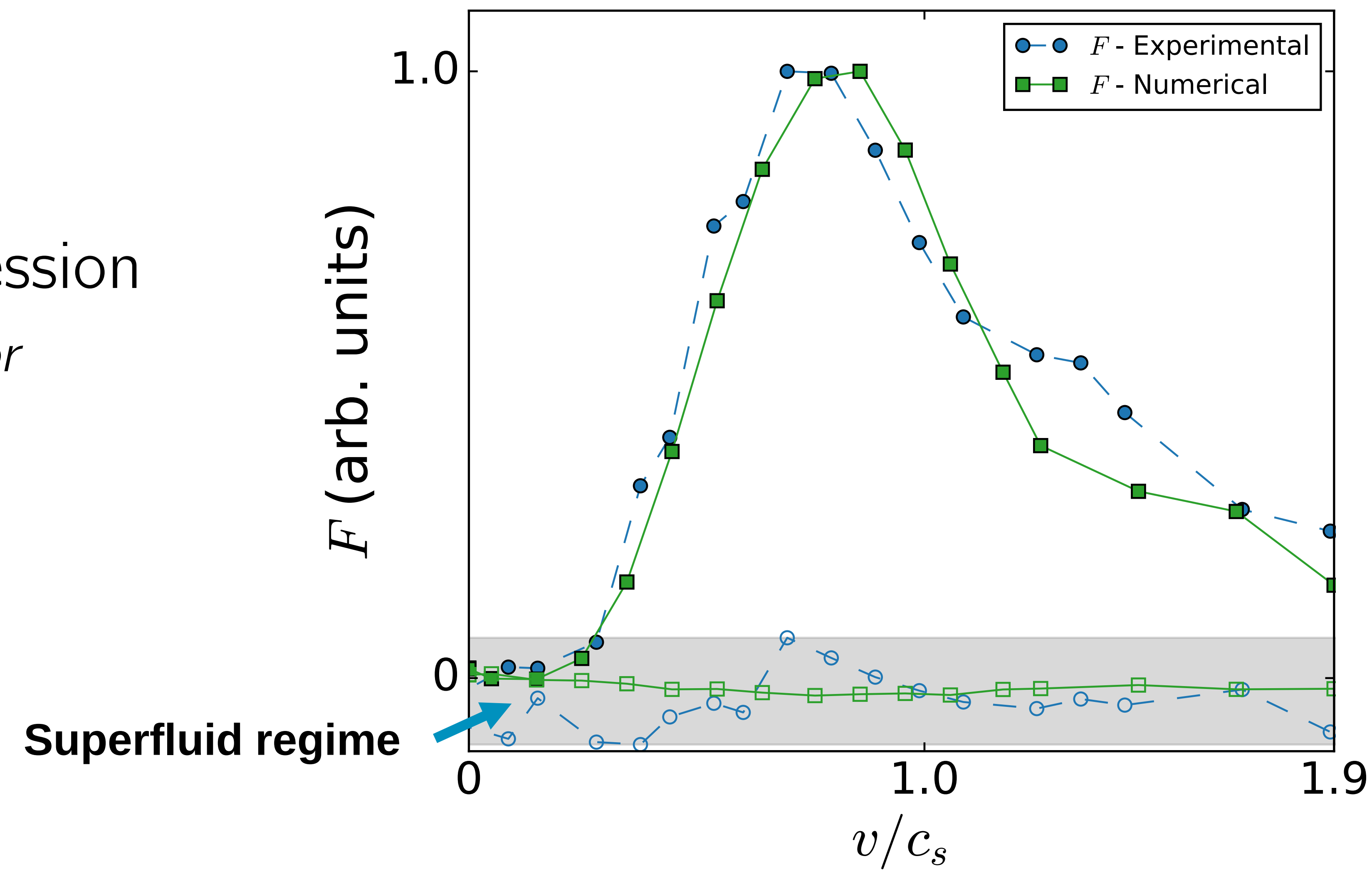


"Towards the experimental observation of turbulent regimes and the associated energy cascades with paraxial fluids of light."
New Journal of Physics 24.11 (2022): 113050.
Ferreira, Tiago D., Rocha, V., Silva, D., Guerreiro, A., & Silva, N. A.

Probing 2D quantum turbulence

Drag-force suppression

Superfluid-like behavior



Probing 2D quantum turbulence

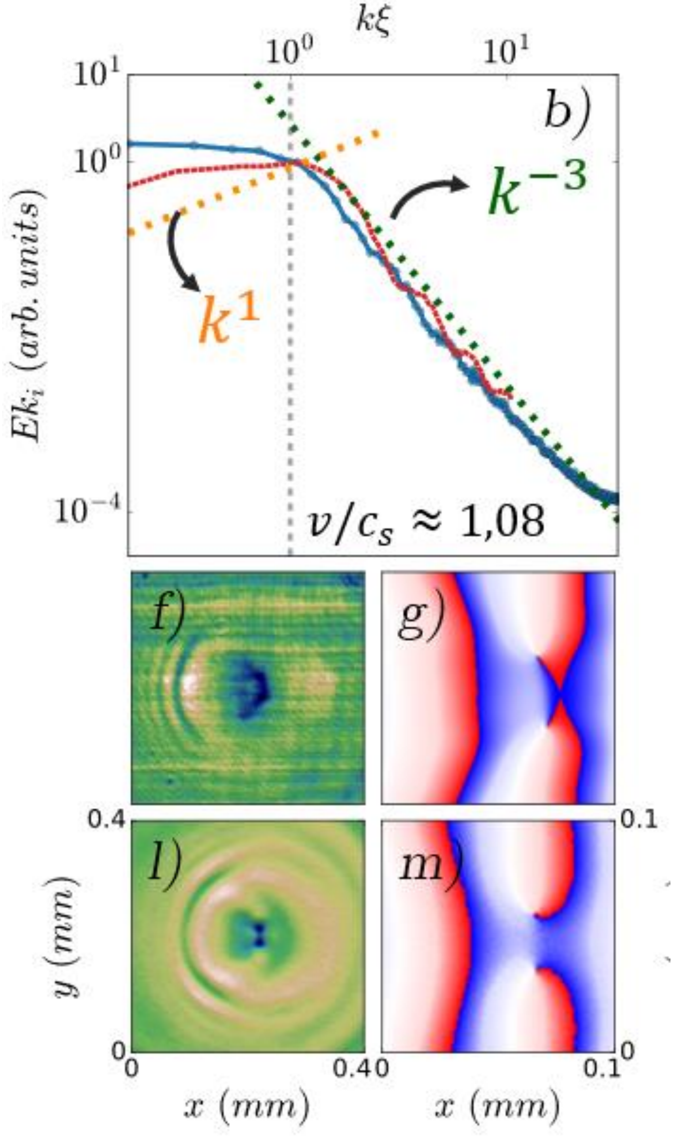
Observation of vortex structures, signature of that quantum turbulence features a vortex turbulence regime

Characteristic k^-3 power-law of the direct energy cascade (Worlds first)

$$\vec{u}(r_\perp, t) = |\psi(r_\perp, z)| \cdot \vec{v}(r_\perp, z) = \vec{u^i}(r_\perp, z) + \vec{u^c}(r_\perp, z)$$

$$\nabla \cdot \vec{u^i}(r_\perp, t) = 0$$
$$\nabla \times \vec{u^c}(r_\perp, t) = 0$$

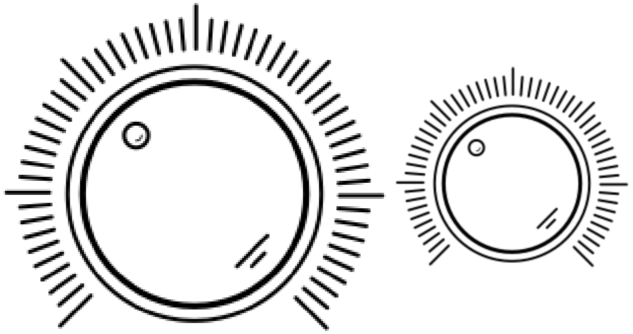
$$E^i(k) = \frac{k}{2} \int_0^{2\pi} d\phi_k \left| \vec{u^i}(\vec{k}, t) \right|^2$$



Constructing an Analogue Quantum Simulator

Constructing the Next-Gen Analogue Quantum Simulator

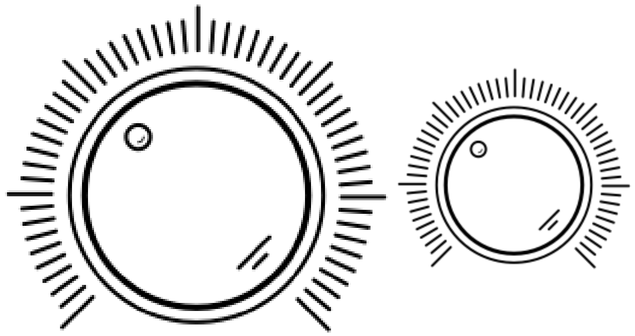
Constructing the Next-Gen Analogue Quantum Simulator



+Tunability

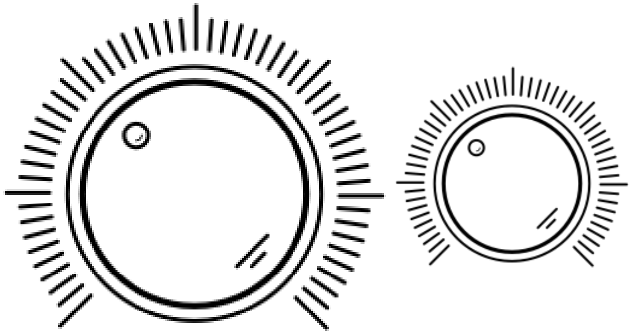
Constructing the Next-Gen Analogue Quantum Simulator

Arbitrary State Generation



+Tunability

Constructing the Next-Gen Analogue Quantum Simulator

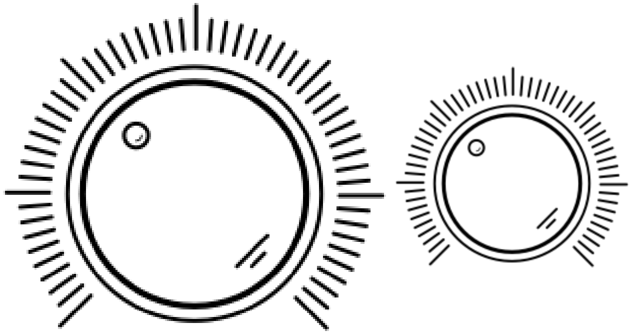


+Tunability

Arbitrary State Generation

Easy to access and validate

Constructing the Next-Gen Analogue Quantum Simulator



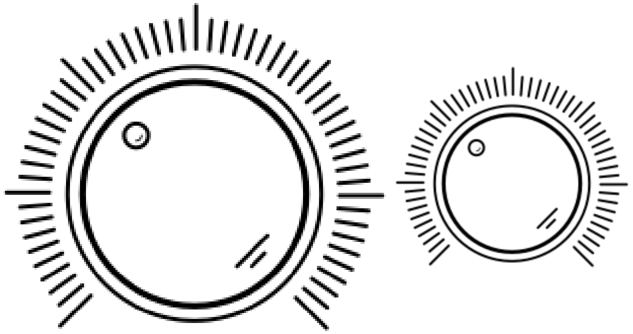
+Tunability

Arbitrary State Generation

Easy to access and validate

Unlimited Propagation Distance

Constructing the Next-Gen Analogue Quantum Simulator



+Tunability

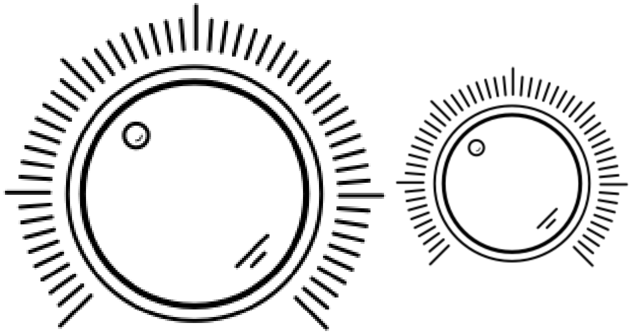
Arbitrary State Generation

Easy to access and validate

Unlimited Propagation Distance

Access to Intermediate time States

Constructing the Next-Gen Analogue Quantum Simulator



+Tunability

Arbitrary State Generation

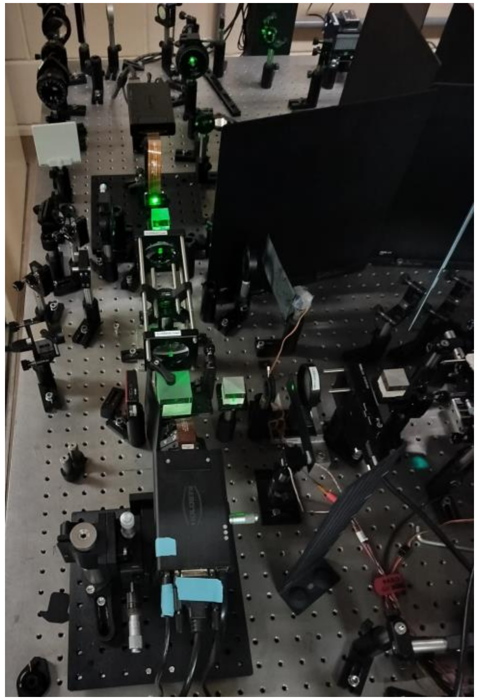
Easy to access and validate

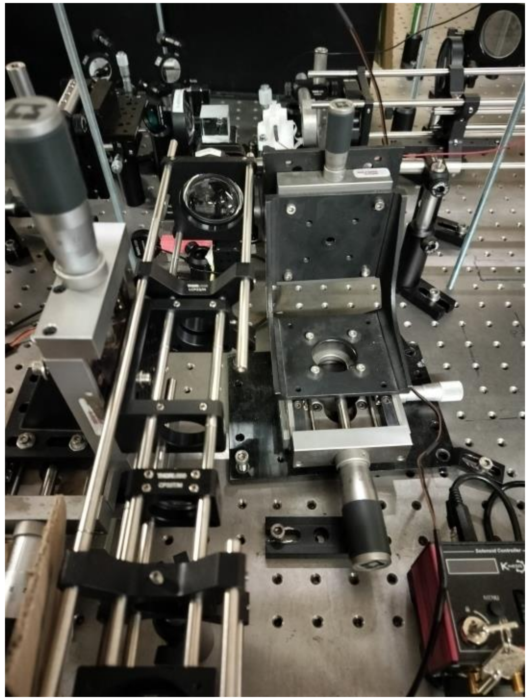
Unlimited Propagation Distance

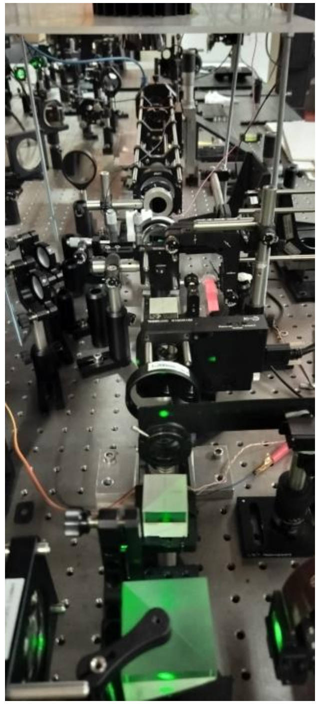
Access to Intermediate time States

Arbitrary Control of Multiple Beams

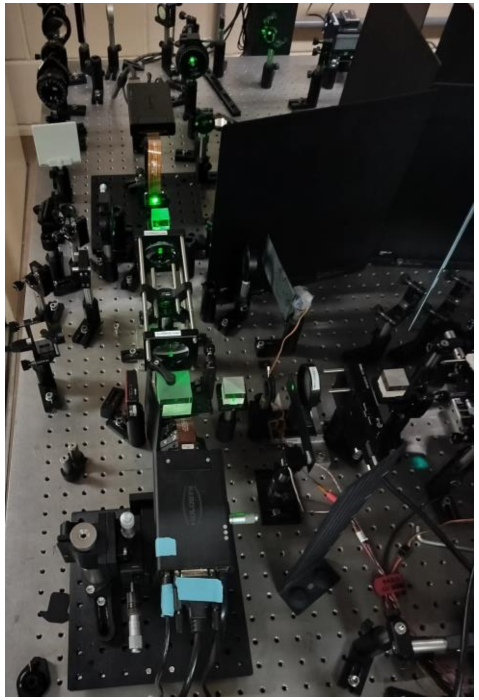
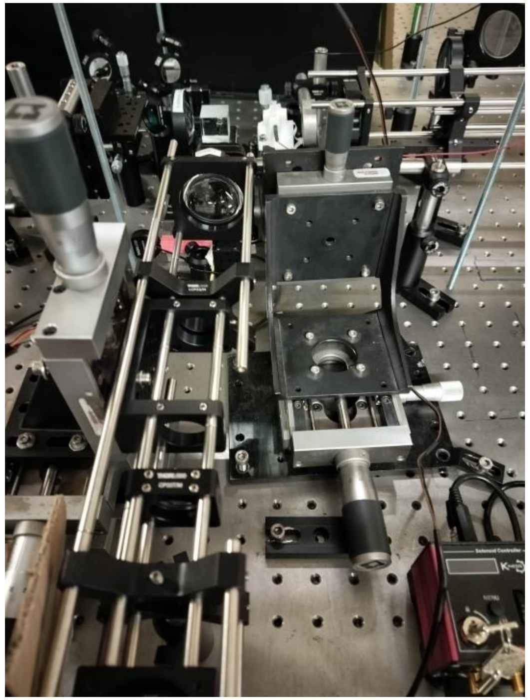
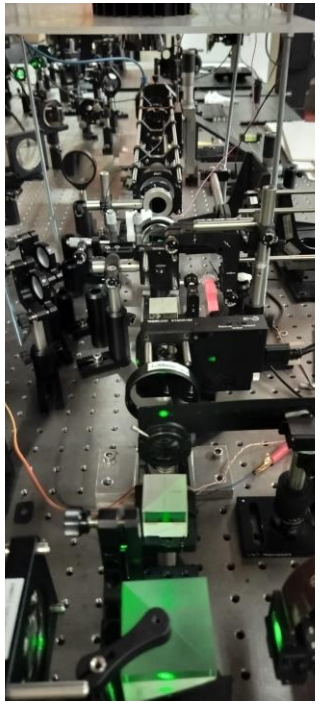
Constructing the Next-Gen Analogue Quantum Simulator





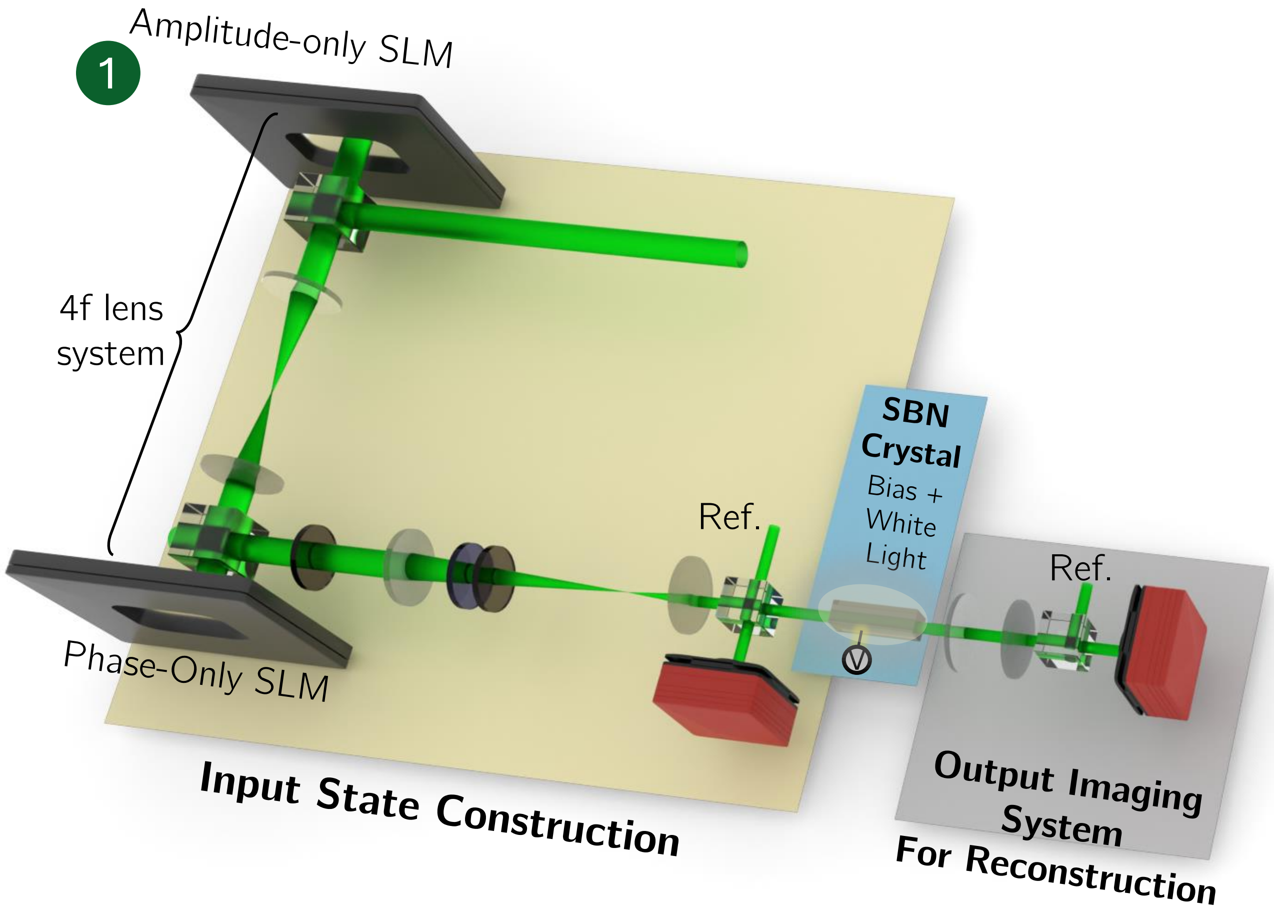


Constructing the Next-Gen Analogue Quantum Simulator

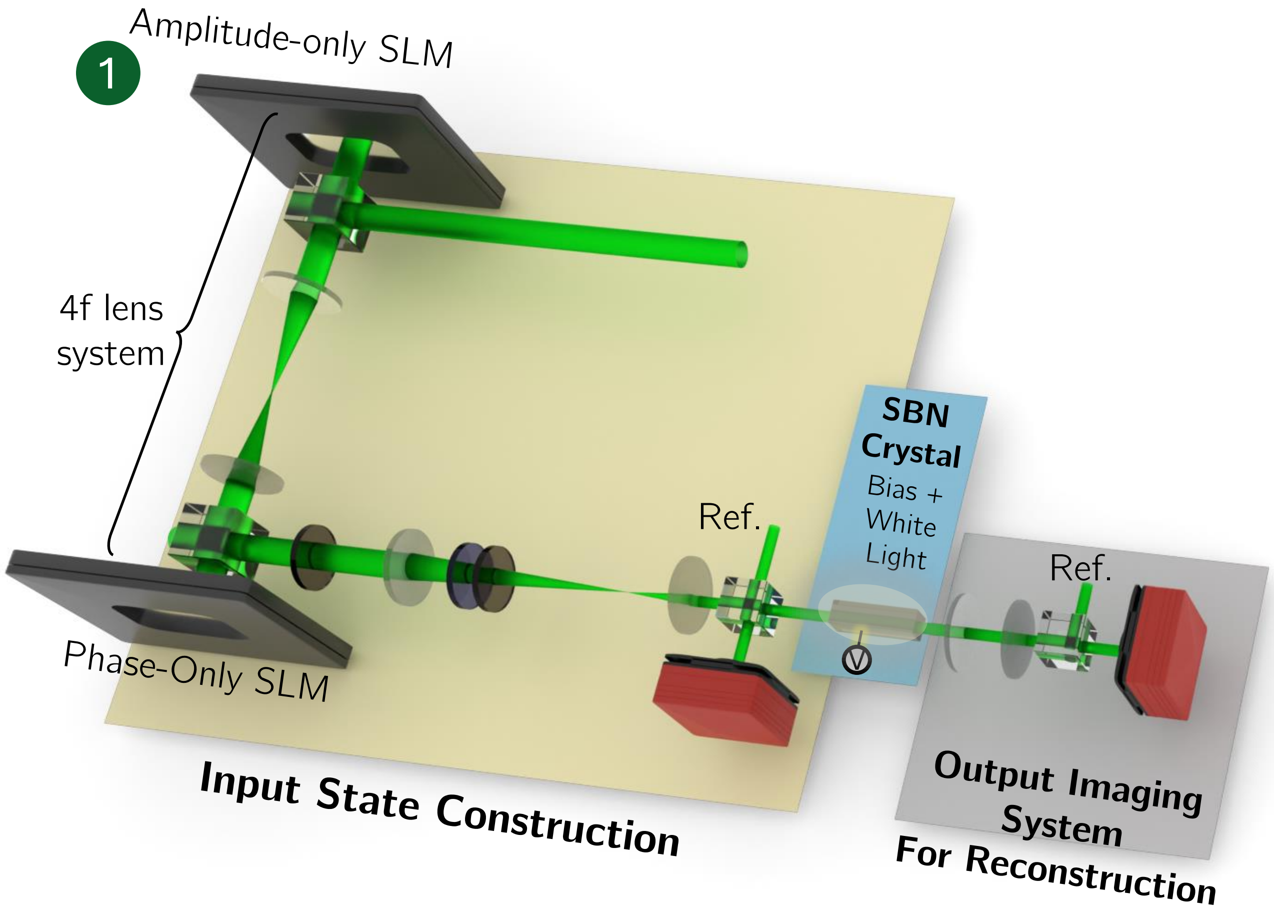

Tiago Ferreira

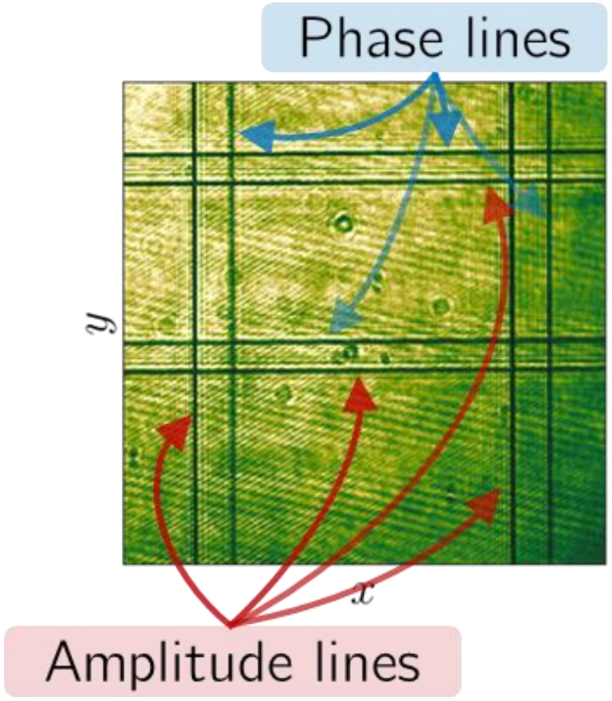
Constructing the Next-Gen Analogue Quantum Simulator



Tiago D Ferreira et al. Physical Review Letters 133.24 (2024): 243802.

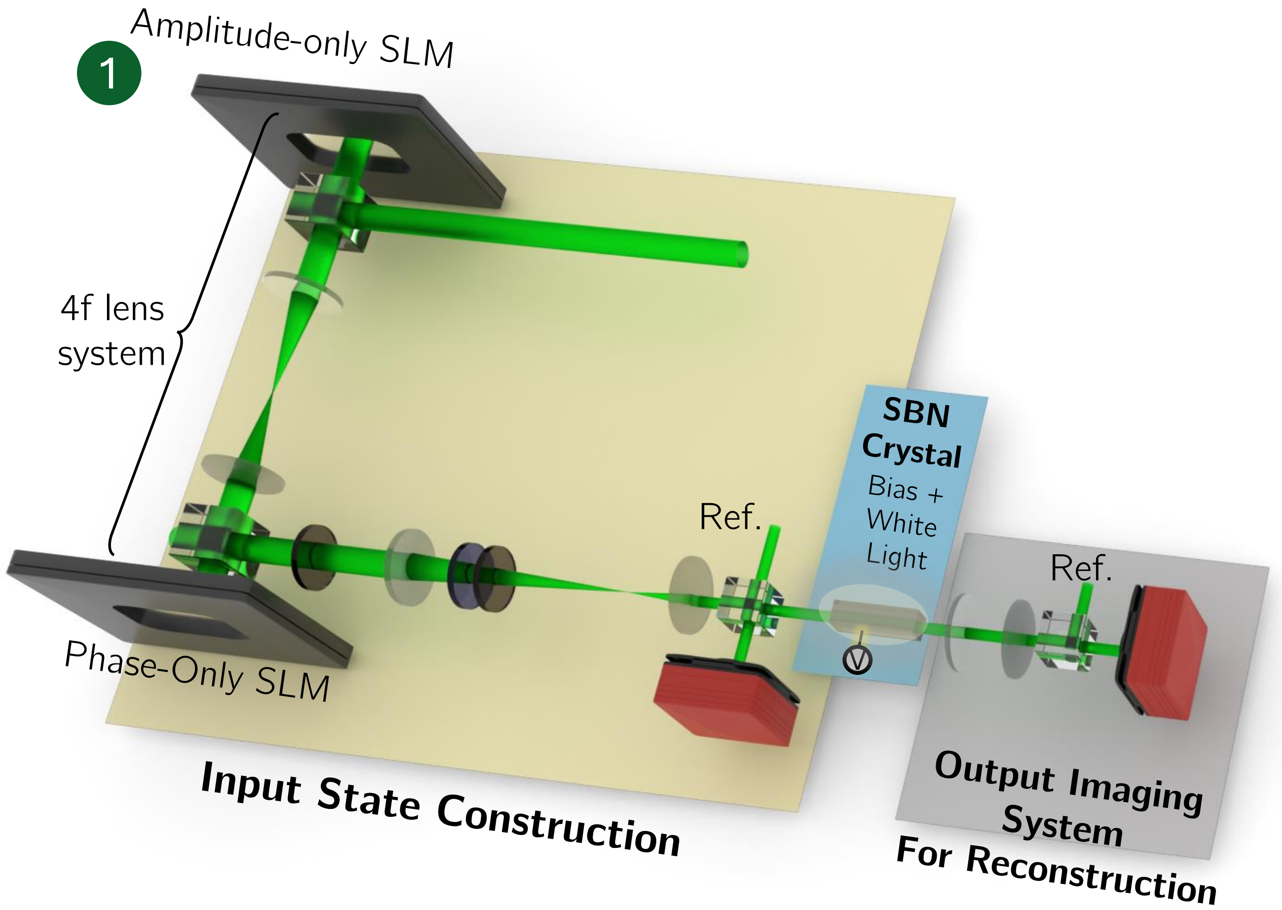
Constructing the Next-Gen Analogue Quantum Simulator

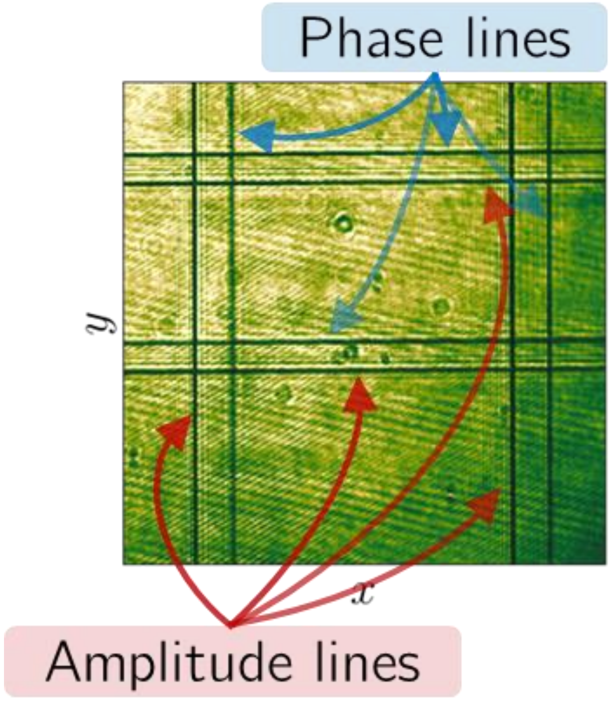




Tiago D Ferreira et al. Physical Review Letters 133.24 (2024): 243802.

Constructing the Next-Gen Analogue Quantum Simulator

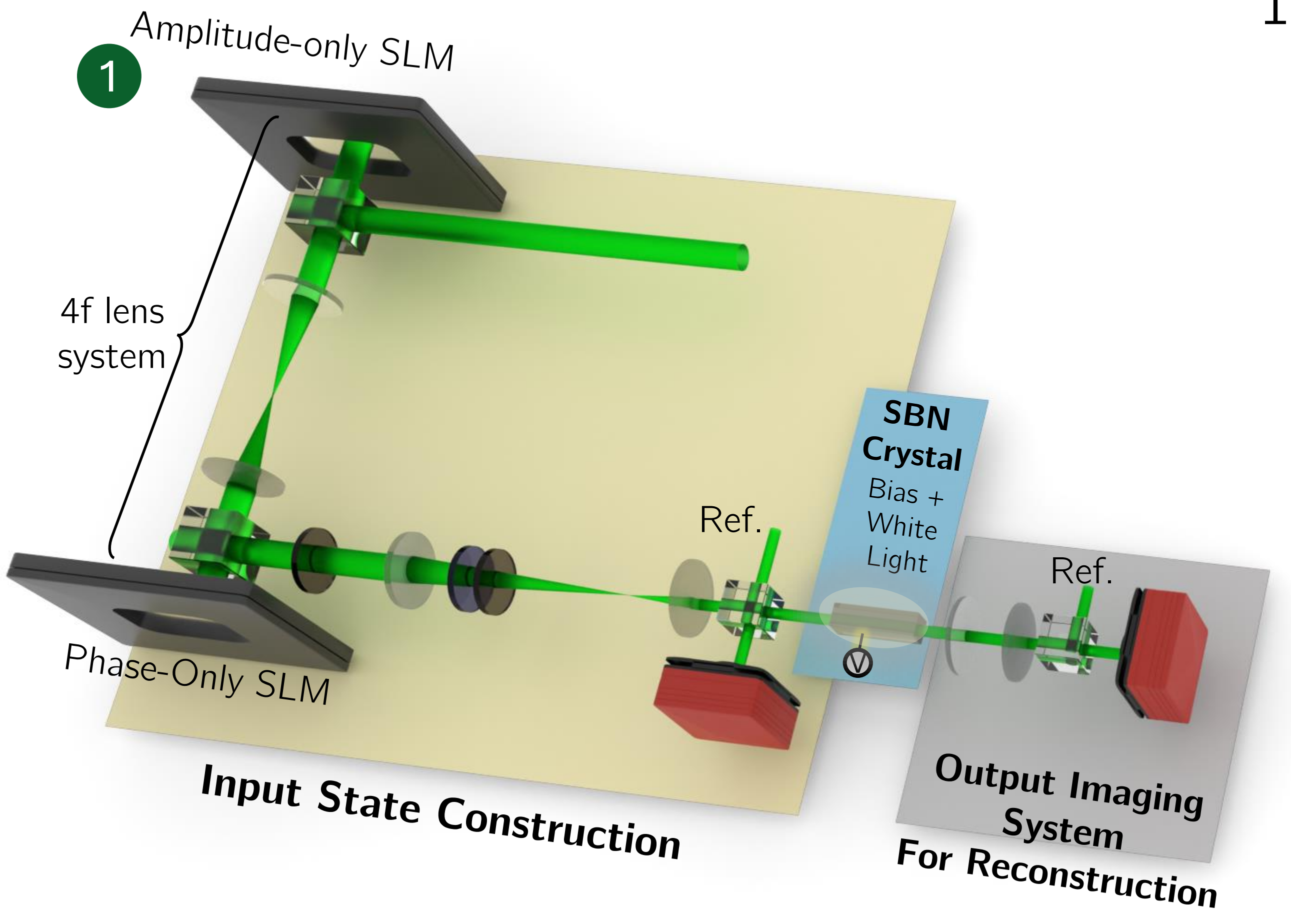




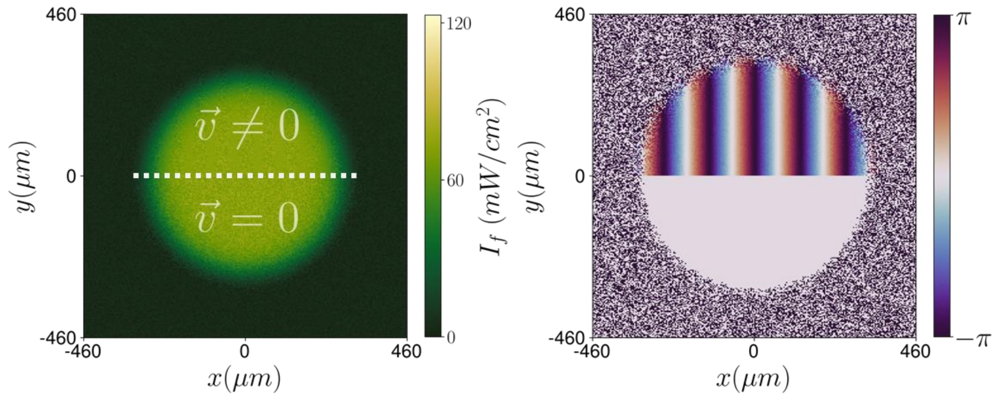
Matched at single-pixel level

Tiago D Ferreira et al. Physical Review Letters 133.24 (2024): 243802.

Constructing the Next-Gen Analogue Quantum Simulator

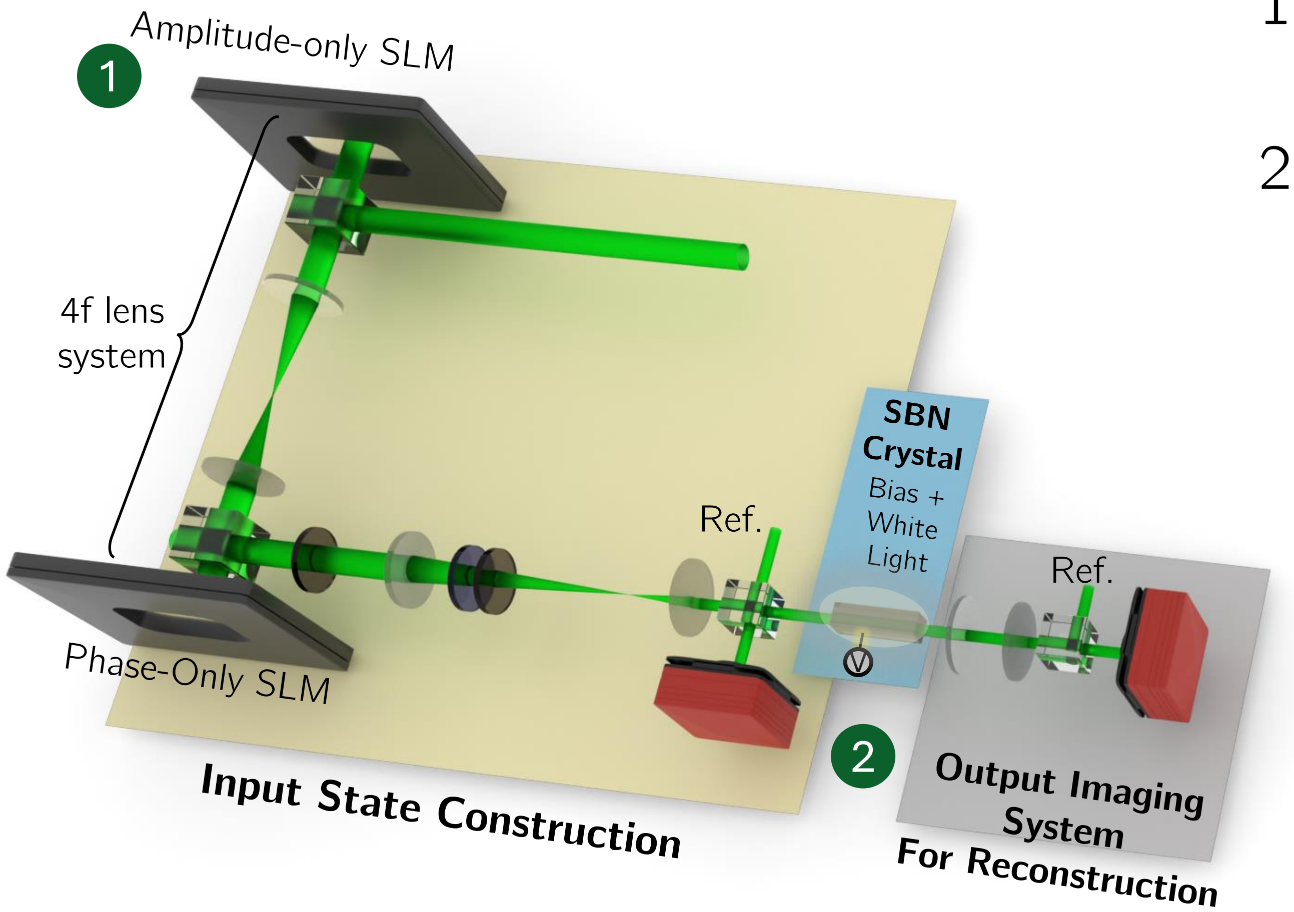


1. Create an arbitrary complex field profile $E(z=0)$

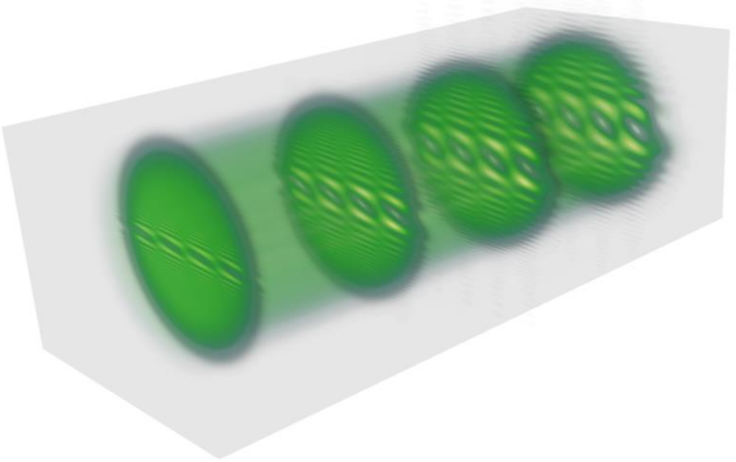


Tiago D Ferreira et al. Physical Review Letters 133.24 (2024): 243802.

Constructing the Next-Gen Analogue Quantum Simulator

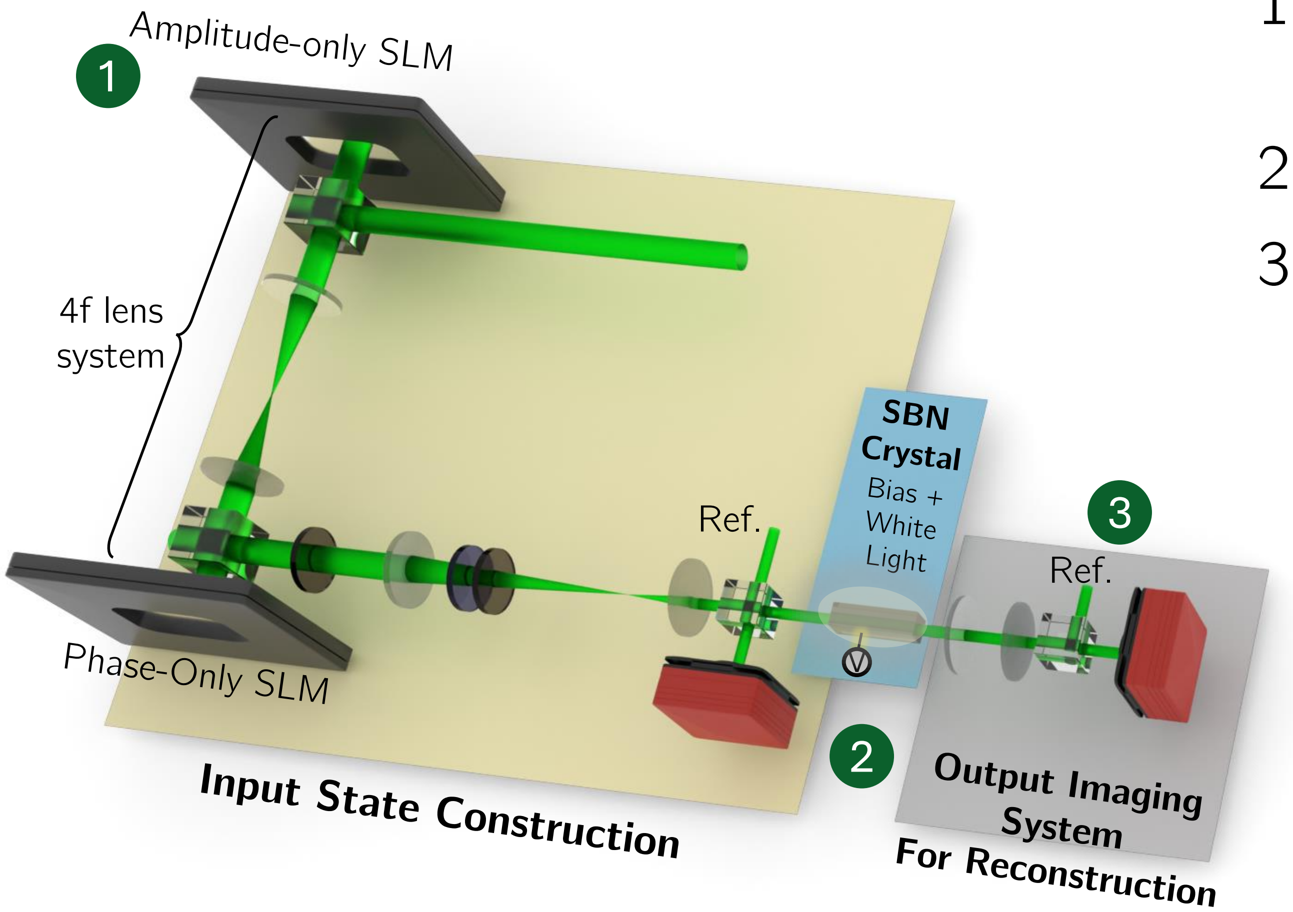


1. Create an arbitrary complex field profile $E(z=0)$
2. Let it evolve

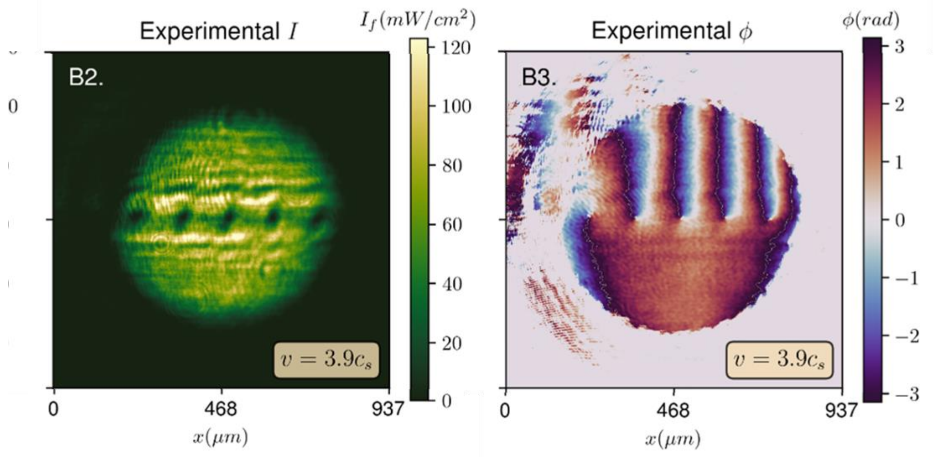


Tiago D Ferreira et al. Physical Review Letters 133.24 (2024): 243802.

Constructing the Next-Gen Analogue Quantum Simulator

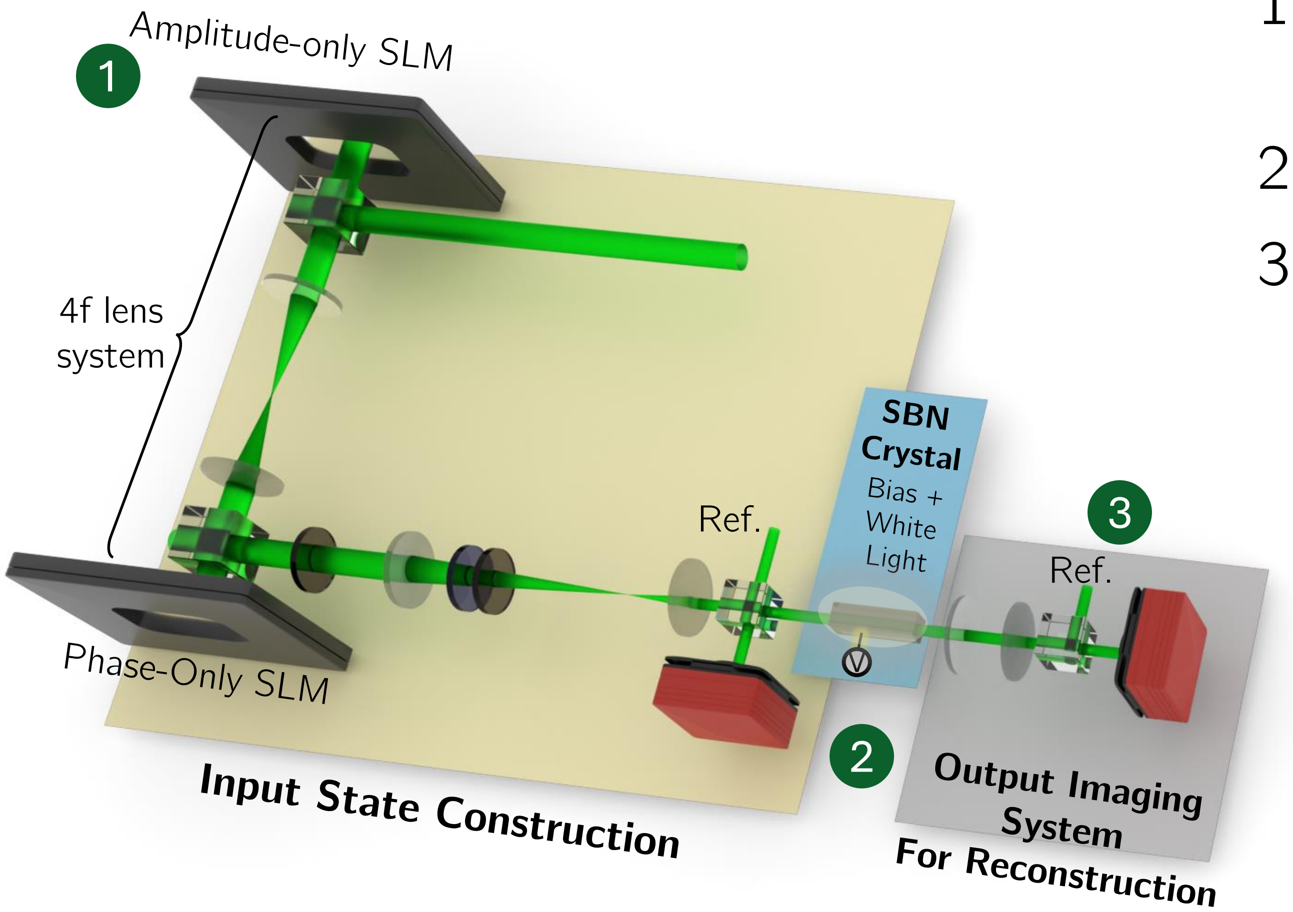


1. Create an arbitrary complex field profile $E(z = 0)$
2. Let it evolve
3. Recover the complex field (Digital Off-Axis Holography) at the output of the crystal $E(z = z_f)$



Tiago D Ferreira et al. Physical Review Letters 133.24 (2024): 243802.

Constructing the Next-Gen Analogue Quantum Simulator



1. Create an arbitrary complex field profile $E(z = 0)$
2. Let it evolve
3. Recover the complex field (Digital Off-Axis Holography) at the output of the crystal $E(z = z_f)$

Evolution of the Complex Field under GPE

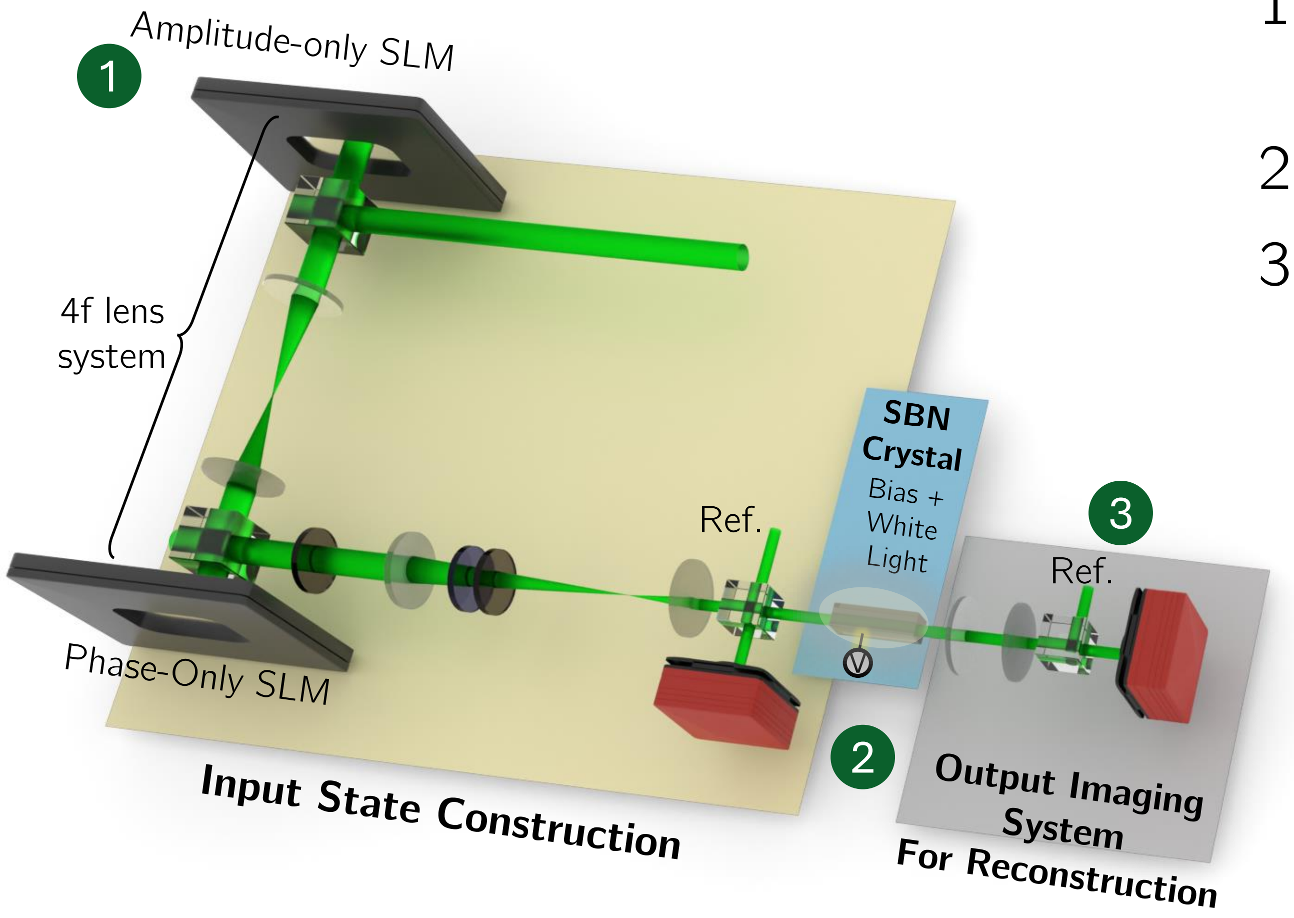
$$\psi(t = 0) \rightarrow \psi(t = t_f)$$

With an optical analogue

$$E(z = 0) \rightarrow E(z = z_f)$$

Tiago D Ferreira et al. Physical Review Letters 133.24 (2024): 243802.

Constructing the Next-Gen Analogue Quantum Simulator



1. Create an arbitrary complex field profile $E(z = 0)$
2. Let it evolve
3. Recover the complex field (Digital Off-Axis Holography) at the output of the crystal $E(z = z_f)$

Evolution of the Complex Field under GPE

$$\psi(t = 0) \rightarrow \psi(t = t_f)$$

With an optical analogue

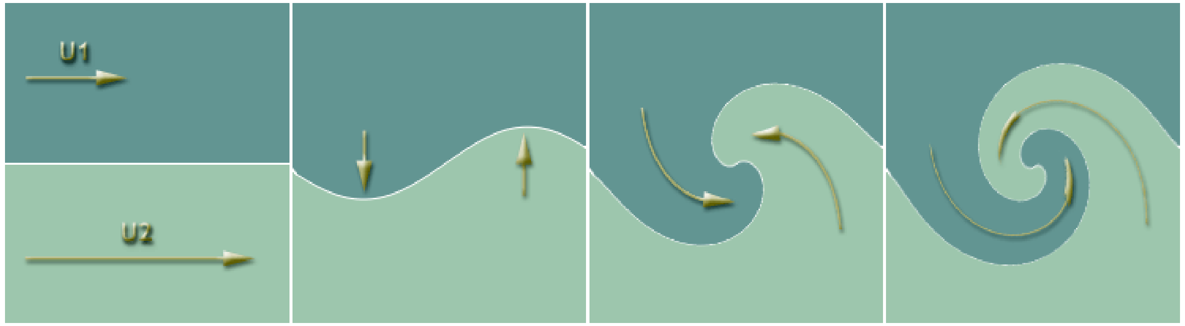
$$E(z = 0) \rightarrow E(z = z_f)$$

Numerical simulation of the input state for streamlined validation

Tiago D Ferreira et al. Physical Review Letters 133.24 (2024): 243802.

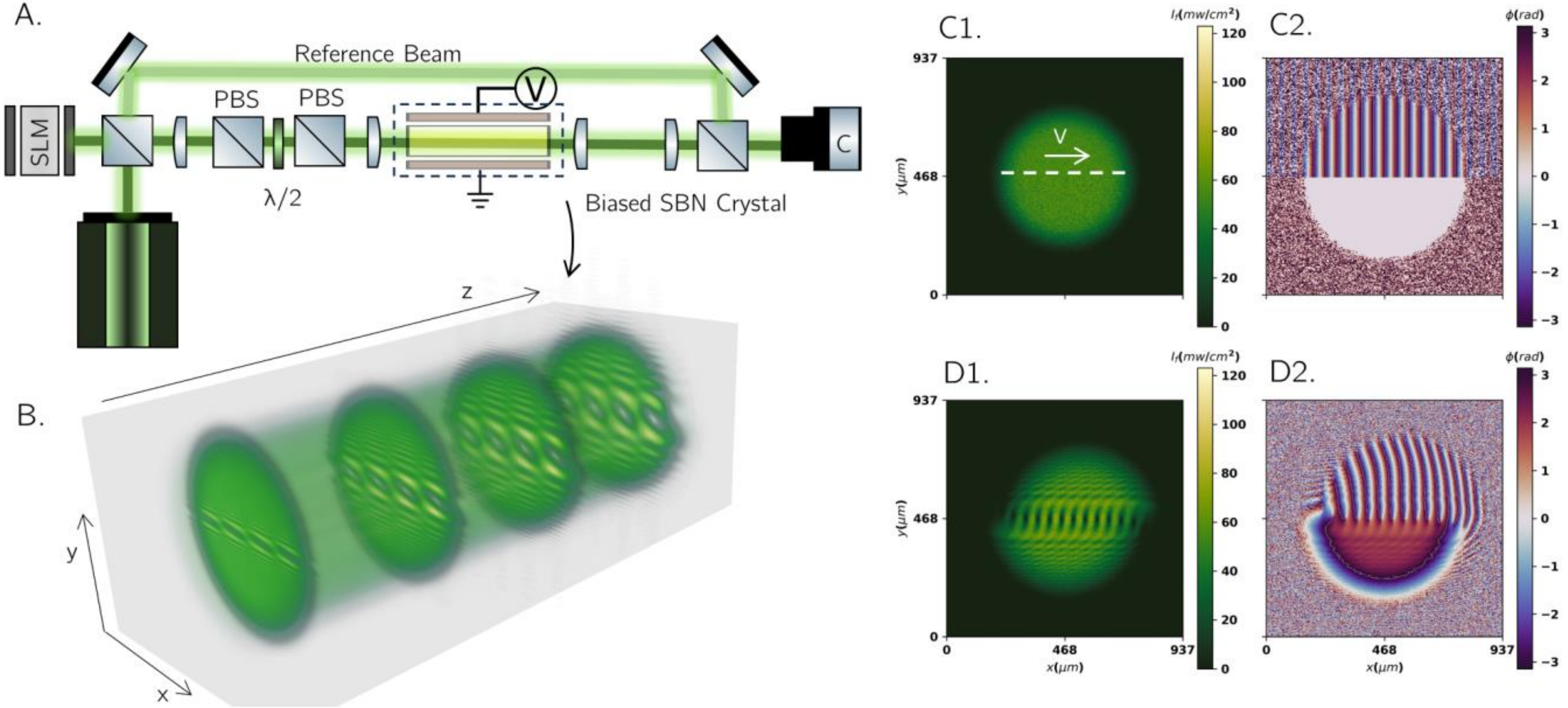
Arbitrary State Generation and Validation

Kelvin-Helmholtz Instability - generating quantum-like turbulence with no defect



Tiago D Ferreira et al. Physical Review A 109, 043704 (2024)

Kelvin-Helmholtz Instability

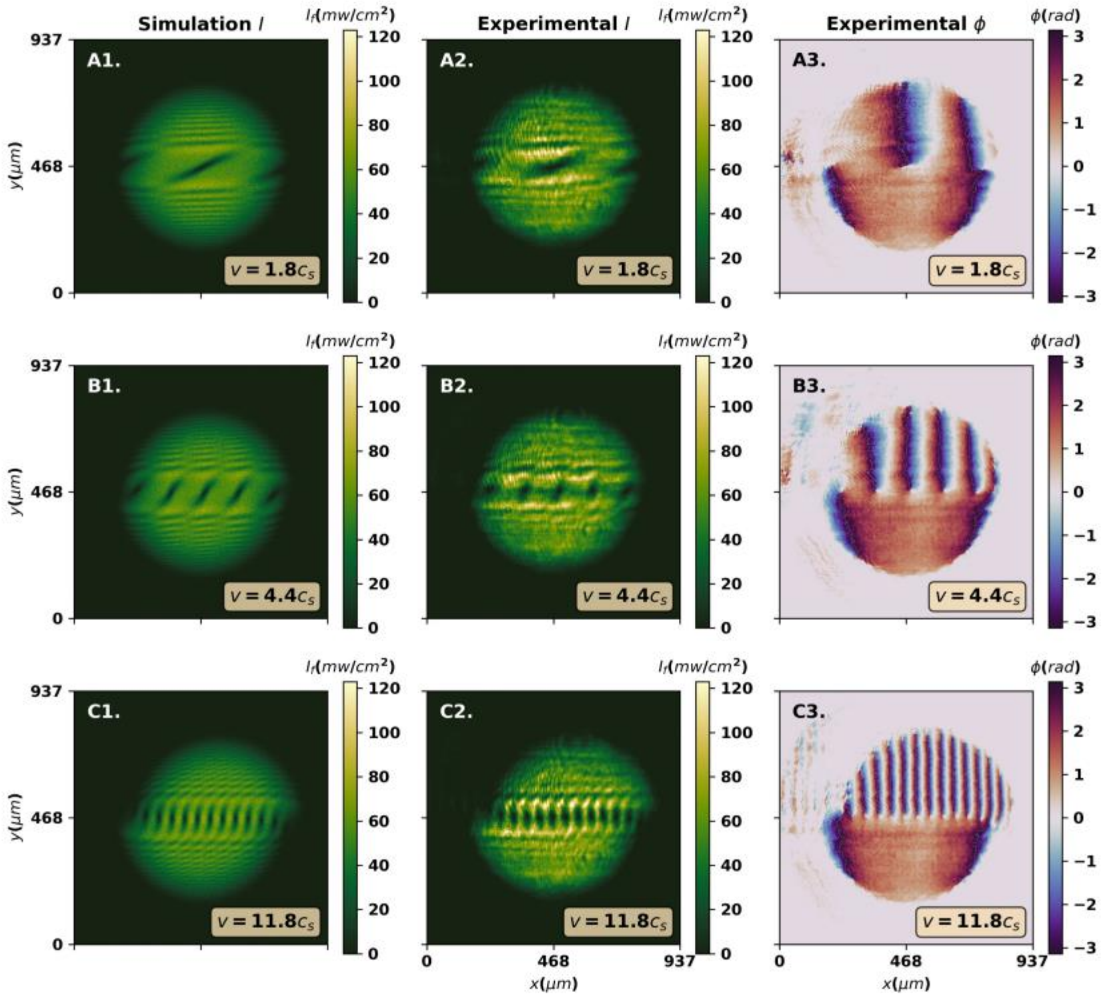


Exploring the dynamics of the Kelvin-Helmoltz instability in paraxial fluids of light
Accepted in PRA,2023
Tiago D. Ferreira, Jakub Garwoła, and Nuno A. Silva

Kelvin-Helmholtz Instability

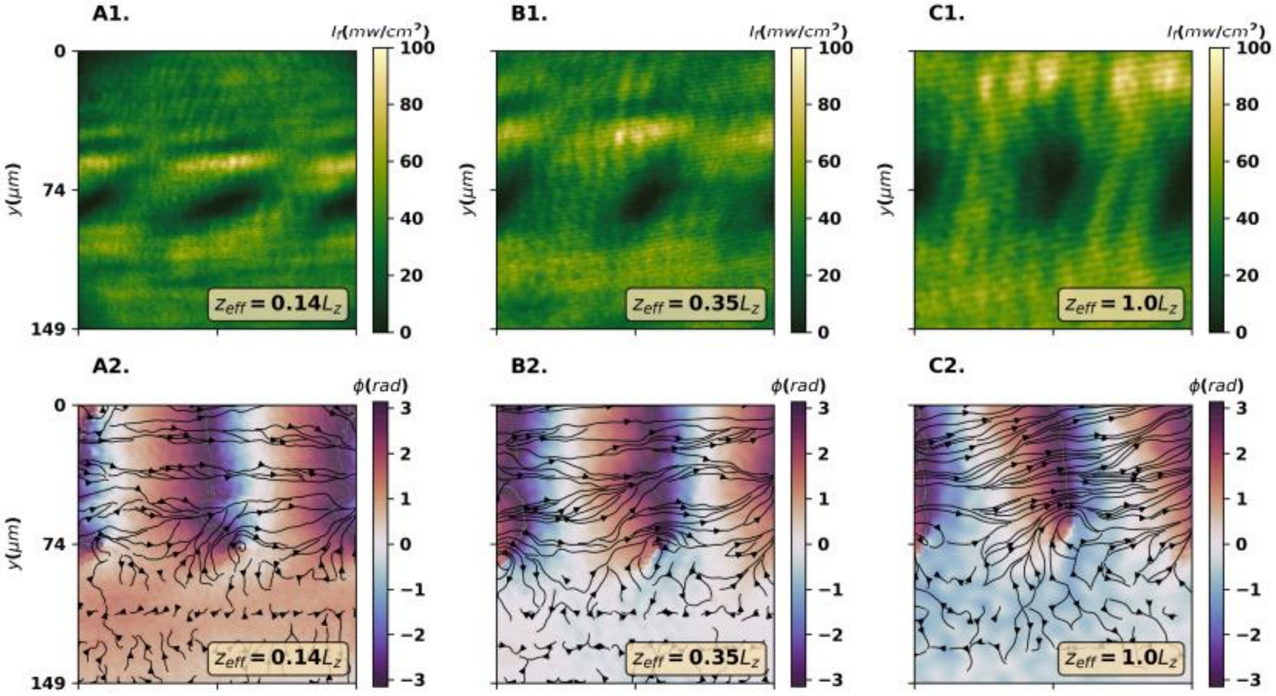
Formation of a vortex-sheet

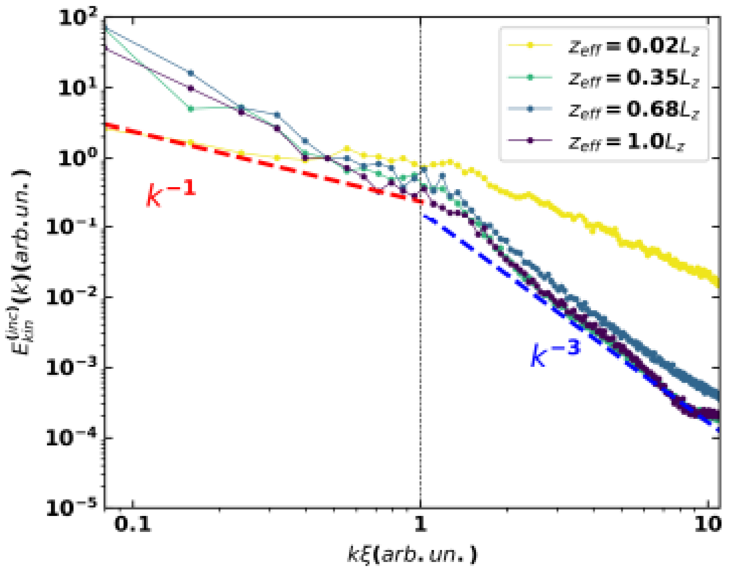
Observation of vortex formation as a seed for quantum turbulence



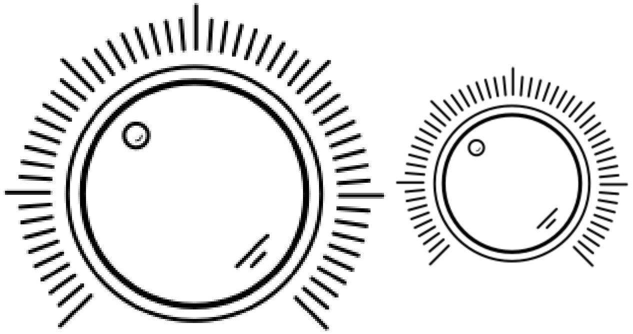
Kelvin-Helmholtz Instability

Observation of vortex formation as a seed for quantum turbulence





Constructing the Next-Gen Analogue Quantum Simulator



+Tunability

Arbitrary State Generation

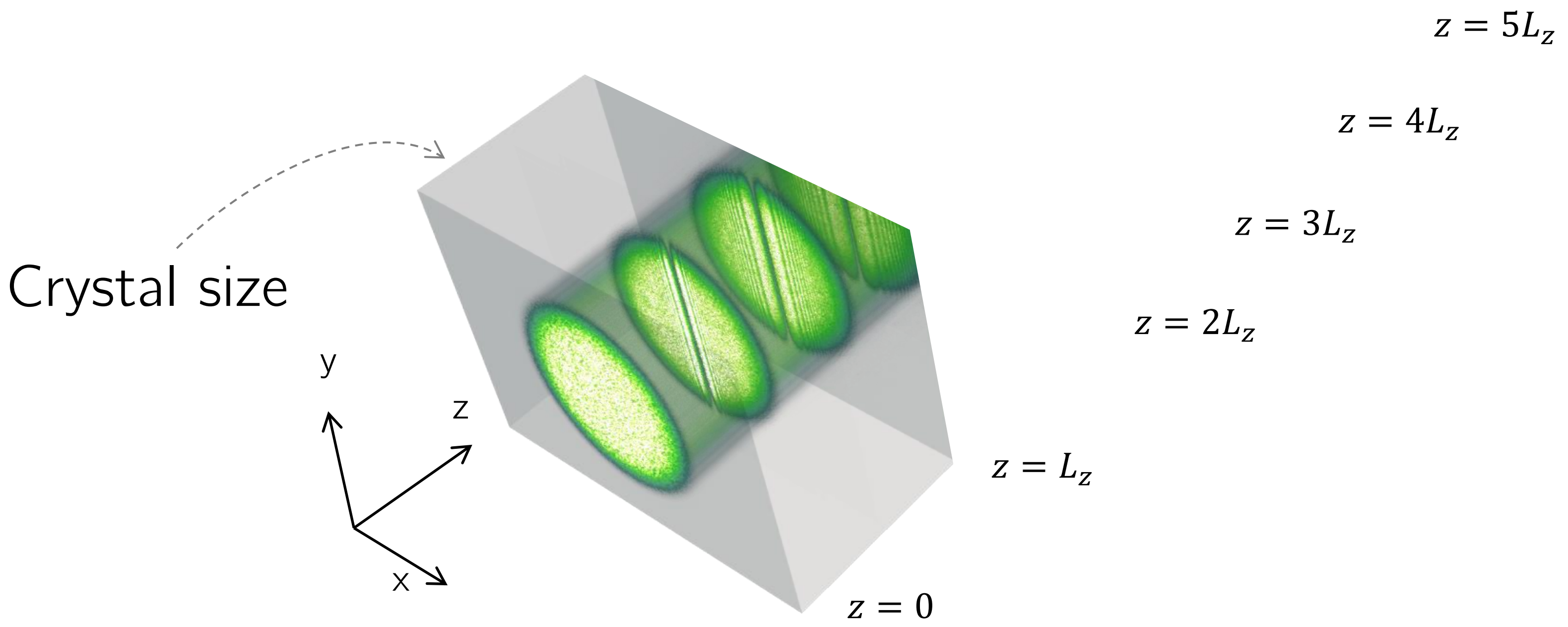
Easy to access and validate

Unlimited Propagation Distance

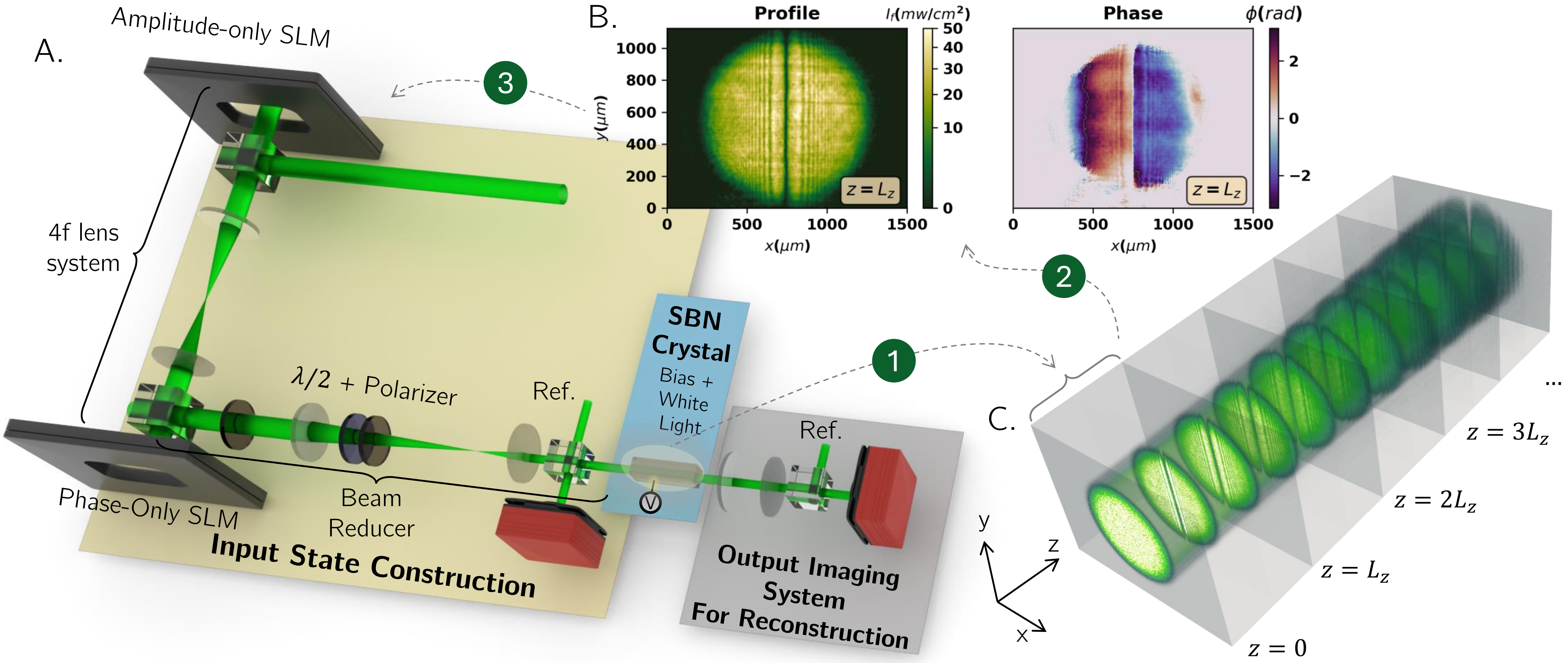
Access to Intermediate time States

Arbitrary Control of Multiple Beams

Limited Propagation distance



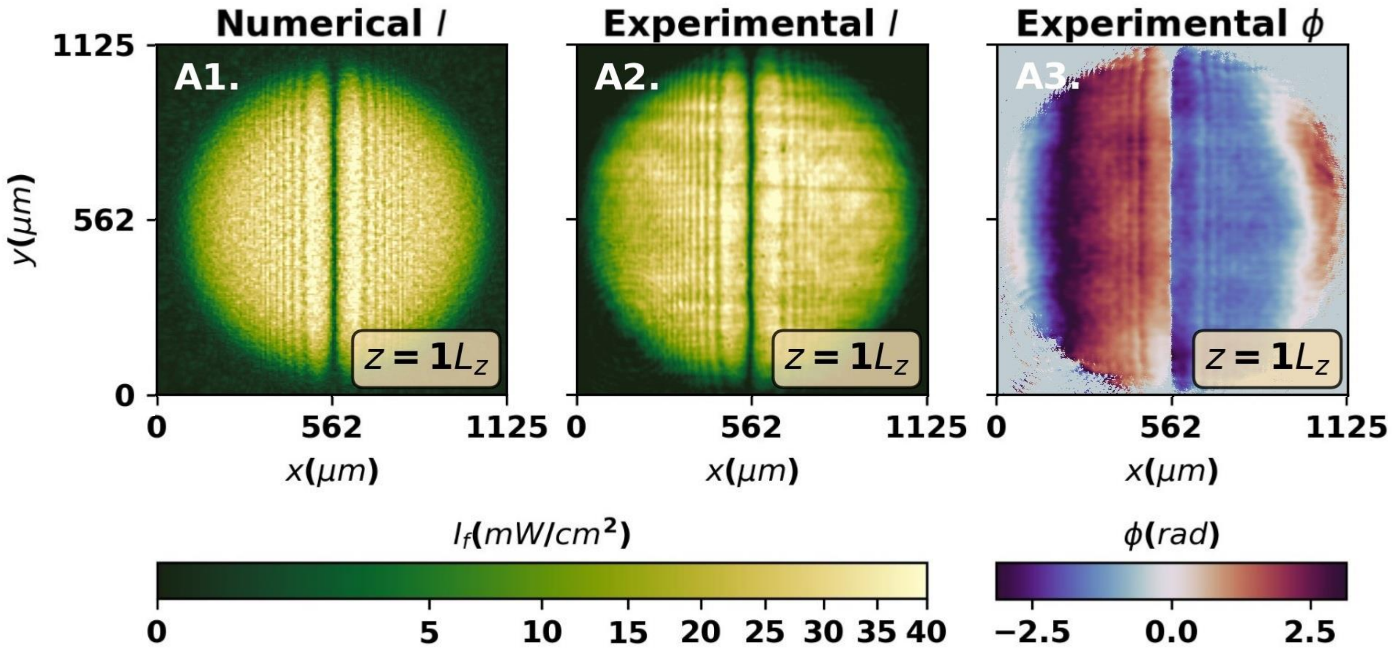
Reinjection with Digital Feedback Loop



Tiago D Ferreira et al. arXiv:2312.12336 (2023), to appear in PRL

Reinjection to assess more dynamics

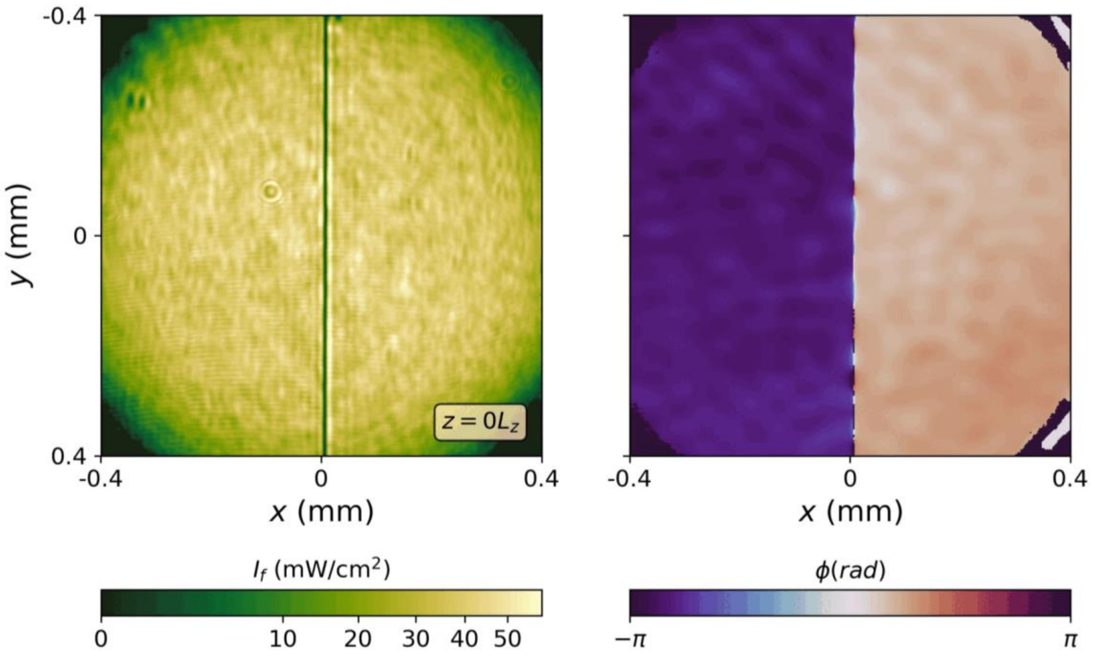
Ex.1: Phase Slip - Dark Soliton and shock waves expansion



Old setup

Reinjection to assess more dynamics

Ex.1: Phase Slip - Dark Soliton and shock waves expansion + **snake instability and vortex pair decay**

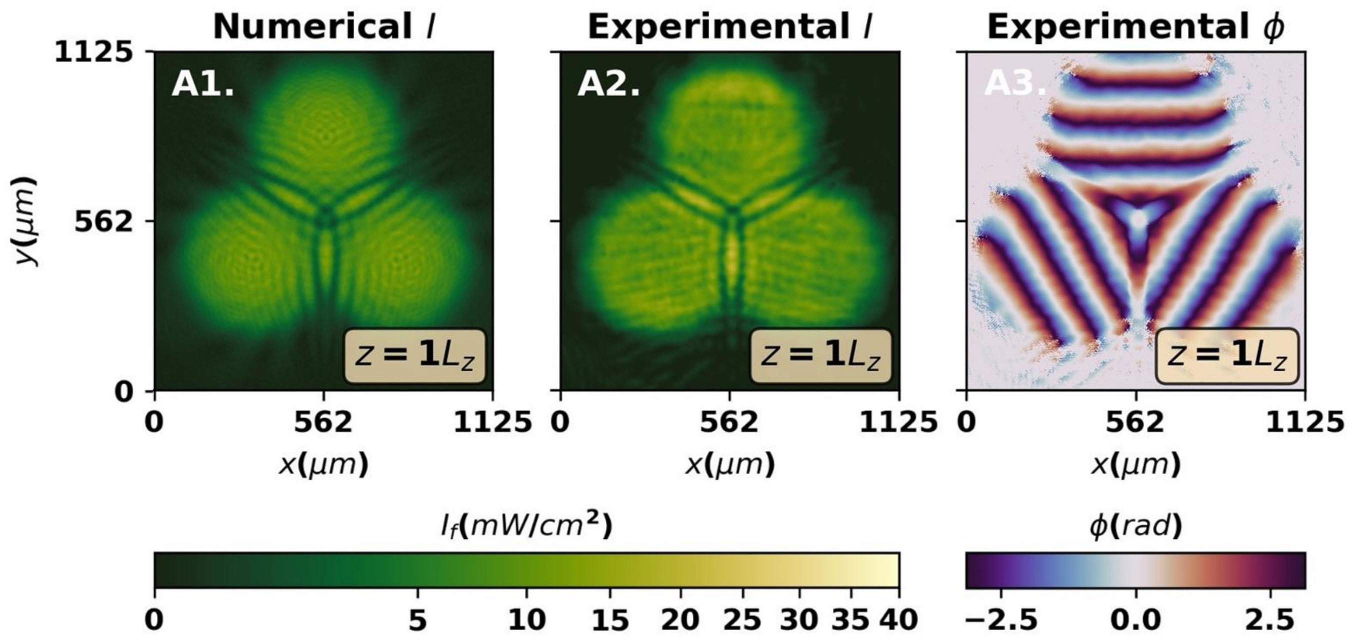


New method, Up to 7 passages

Access to Intermediate time States

Reinjection to assess more dynamics

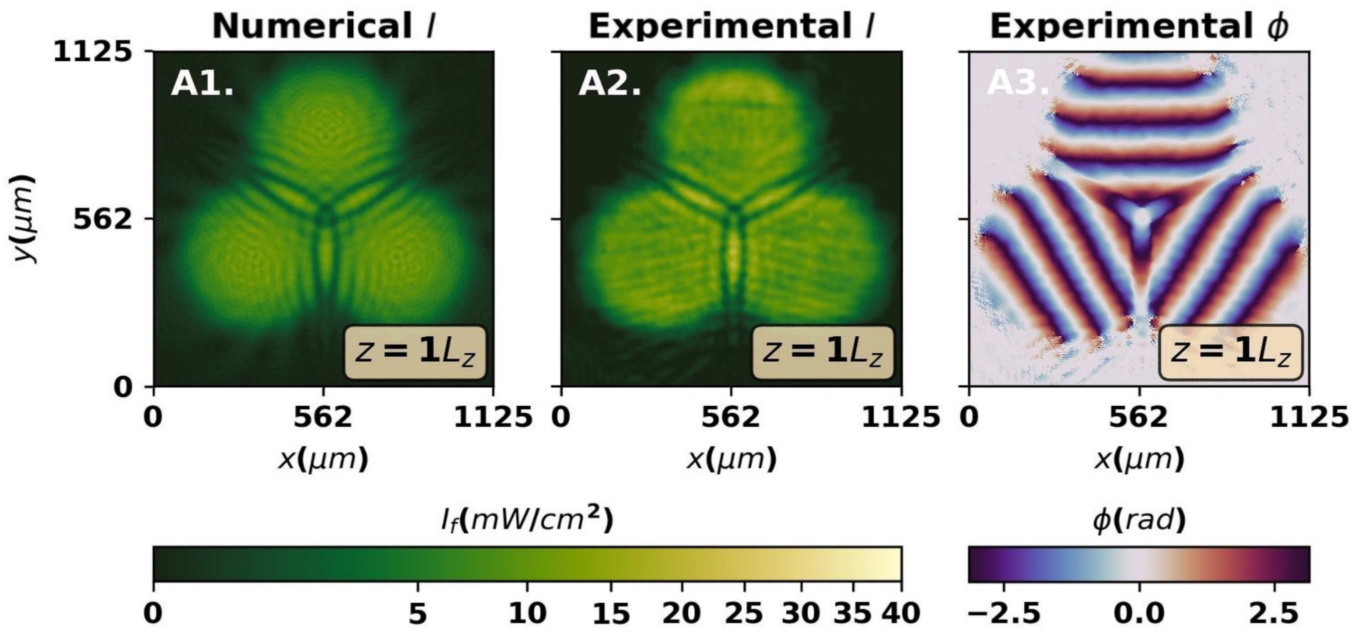
Ex.2: Vortex lattice generation and dynamics



Old setup

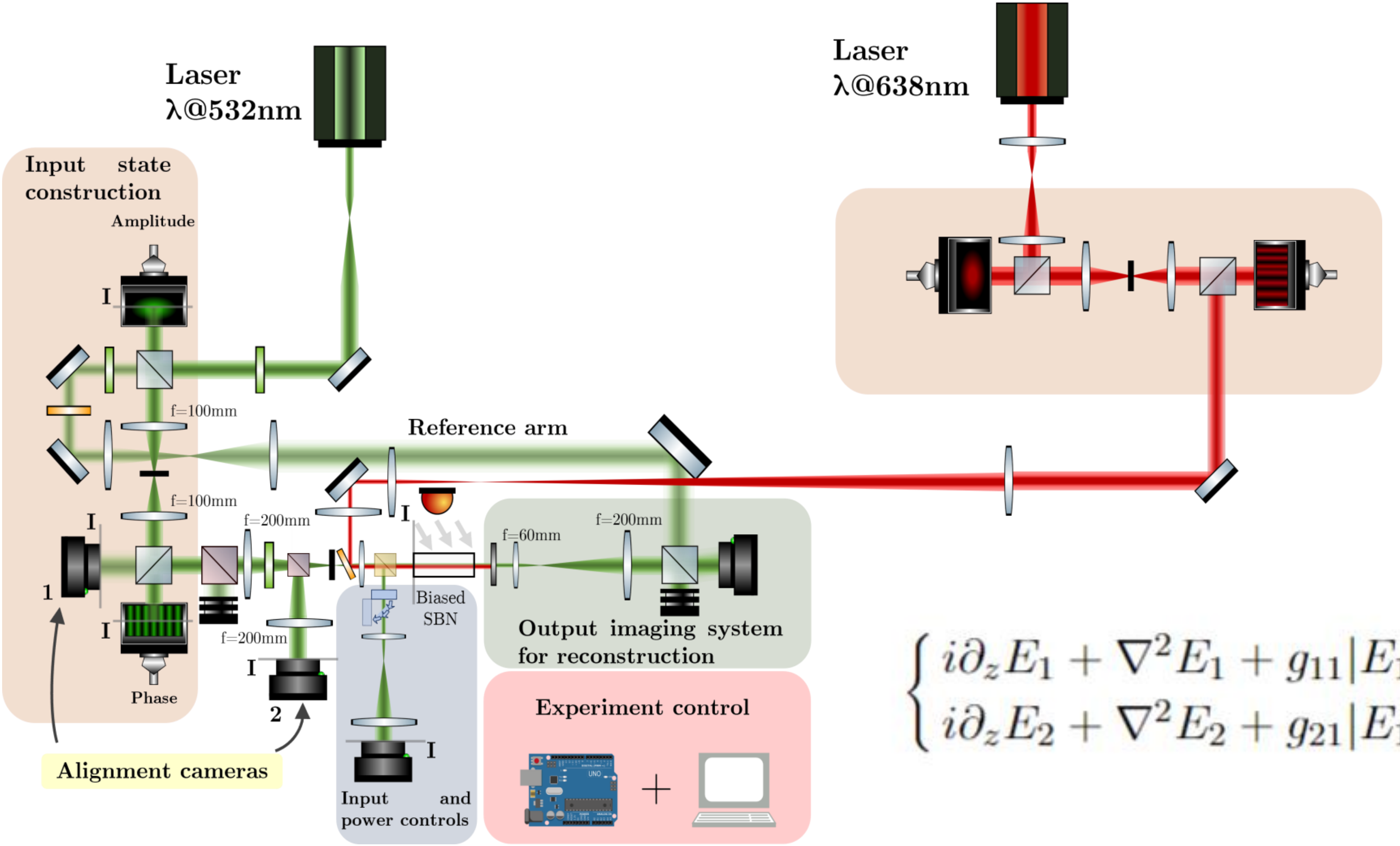
Reinjection to assess more dynamics

Ex.2: Vortex lattice generation and dynamics



New method,
Up to 7 passages

Multiple Beams with arbitrary control

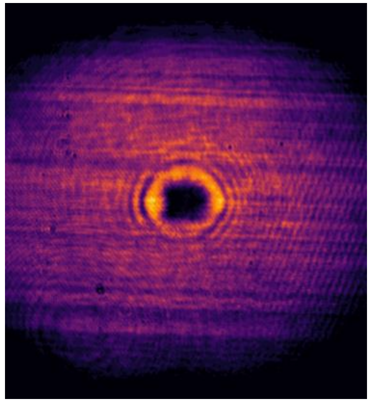


$$\begin{cases} i\partial_z E_1 + \nabla^2 E_1 + g_{11}|E_1|^2 E_1 + \boxed{g_{12}|E_2|^2 E_1} = 0 \\ i\partial_z E_2 + \nabla^2 E_2 + g_{21}|E_1|^2 E_2 + g_{22}|E_2|^2 E_2 = 0 \end{cases}$$

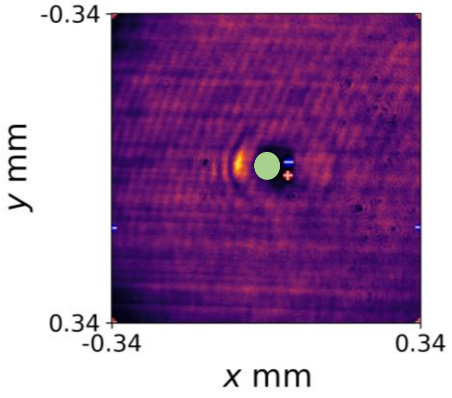
Multiple Beams with arbitrary control

Optical Defects

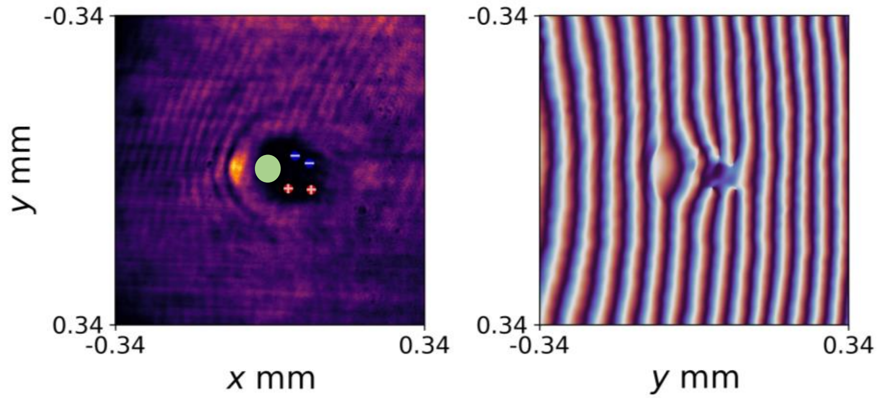
$$\begin{cases} i\partial_z E_1 + \nabla^2 E_1 + g_{11}|E_1|^2 E_1 + \boxed{g_{12}|E_2|^2 E_1} = 0 \\ i\partial_z E_2 + \nabla^2 E_2 + g_{21}|E_1|^2 E_2 + g_{22}|E_2|^2 E_2 = 0 \end{cases}$$



First passage



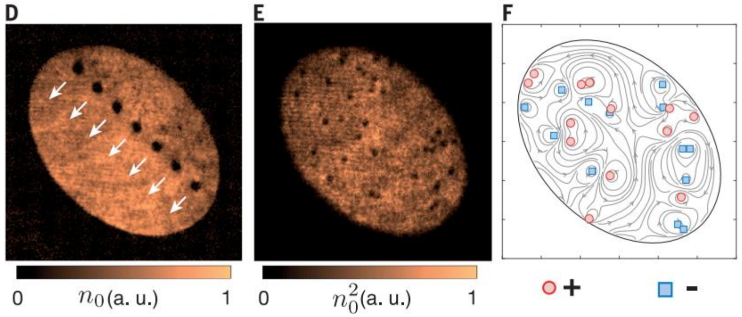
Second passage



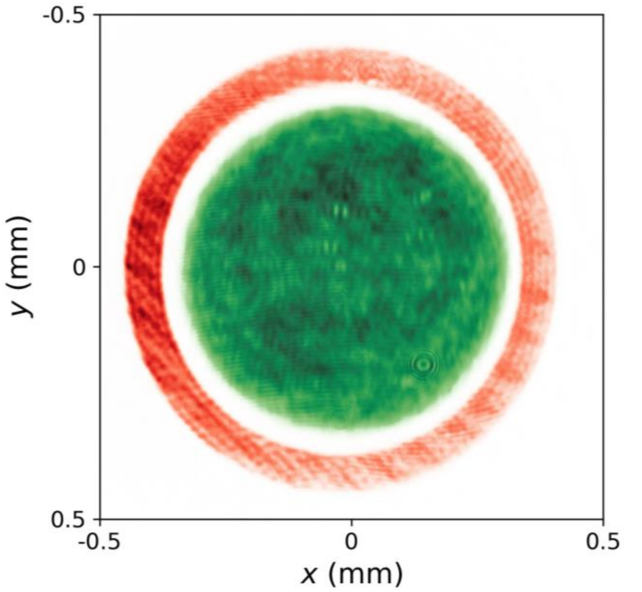
Multiple Beams with arbitrary control

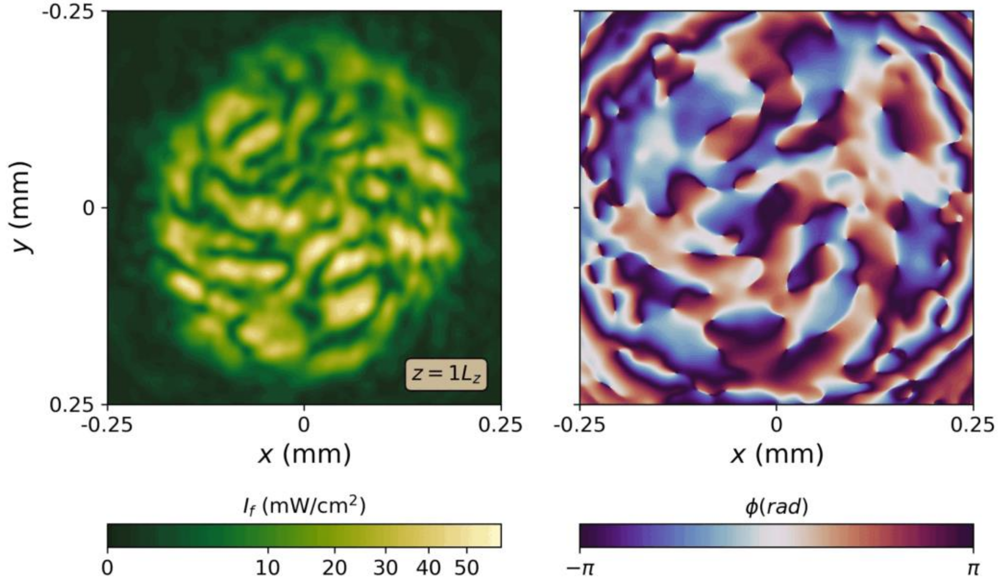
Optical Defects

Trapping Potentials



Gauthier, Guillaume, et al. "Giant vortex clusters in a two-dimensional quantum fluid." *Science* 364.6447 (2019): 1264-1267.



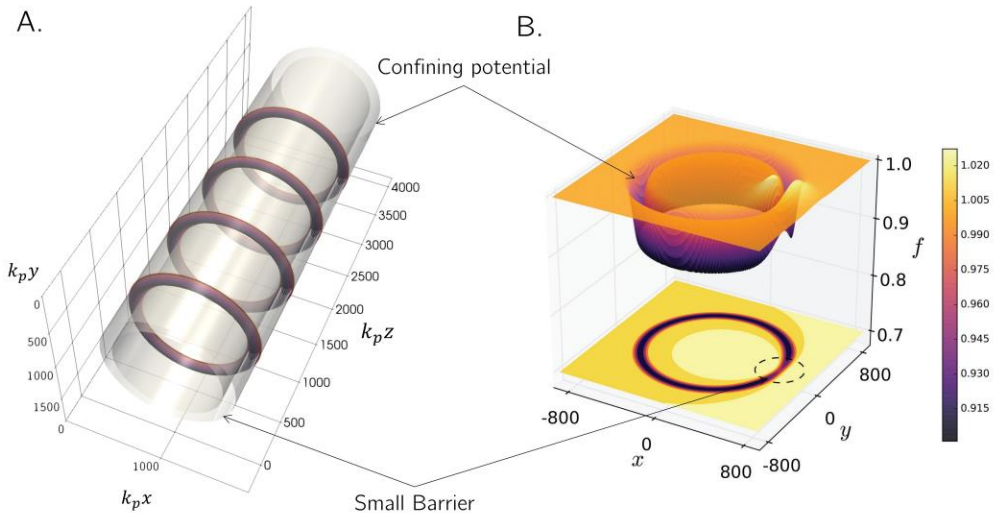


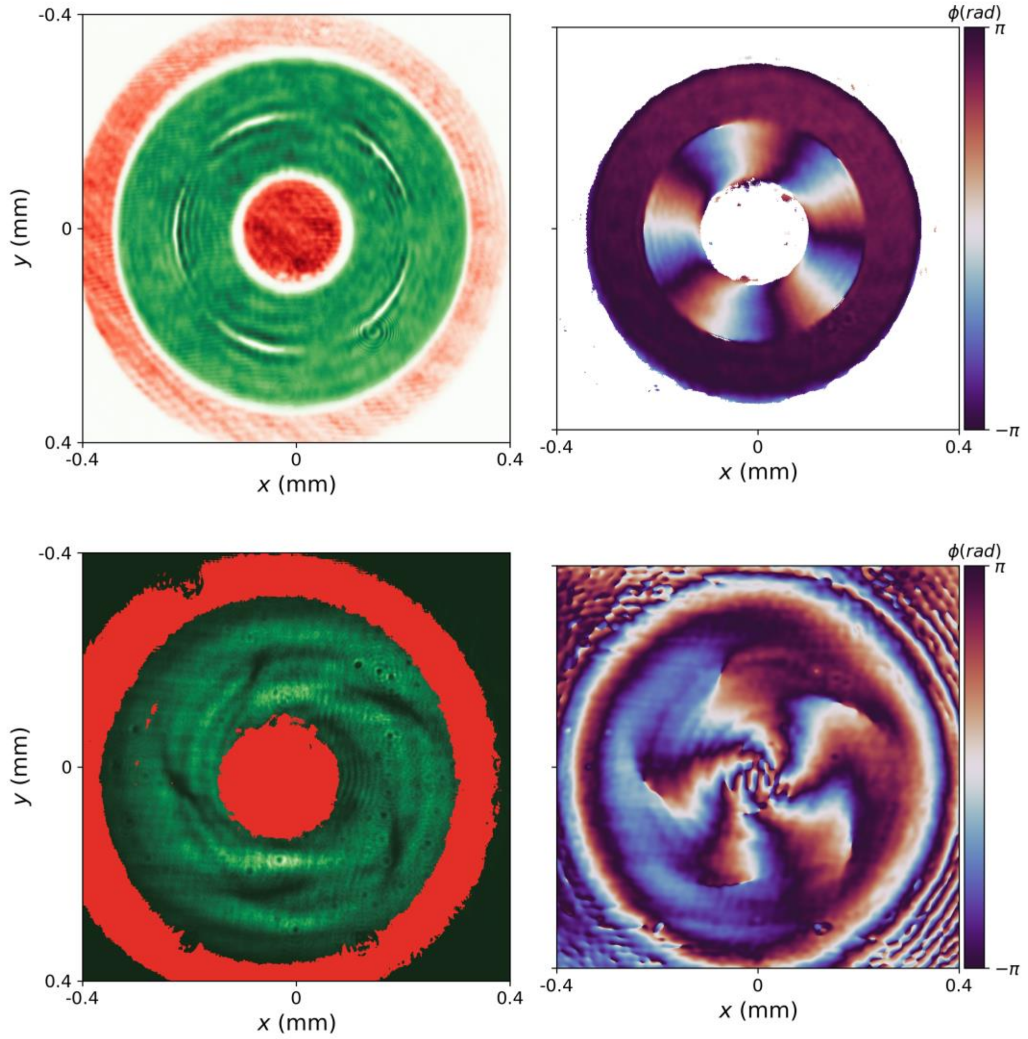
Multiple Beams with arbitrary control

Optical Defects

Trapping Potentials

Non-trivial Geometries





Multiple Beams with arbitrary control

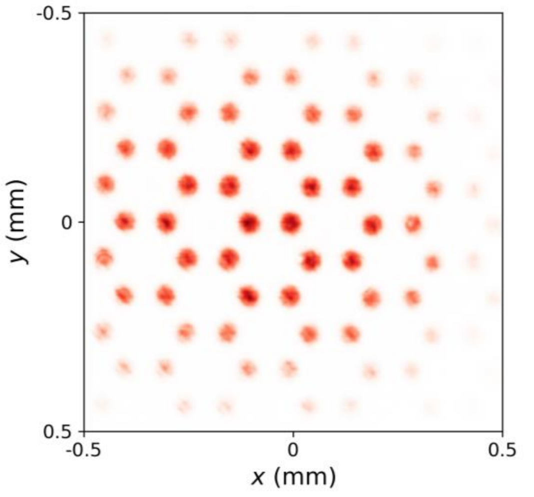
Optical Defects

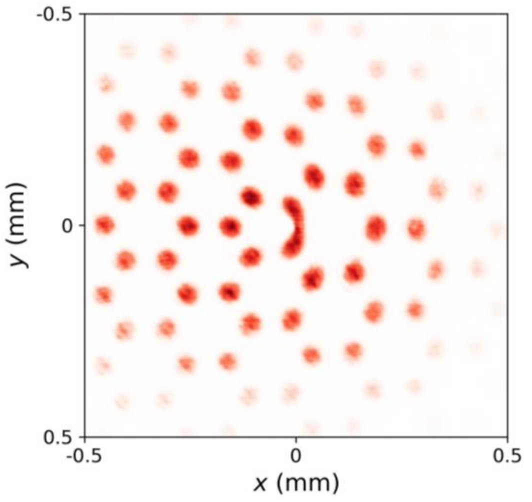
Trapping Potentials

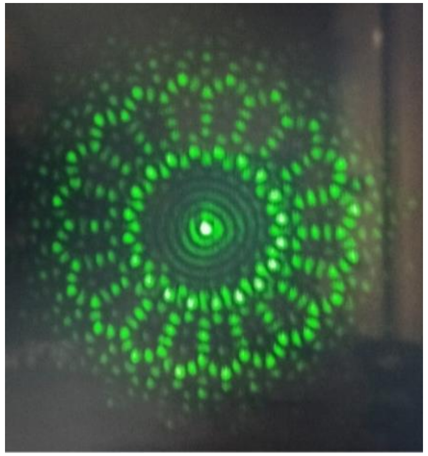
Non-trivial Geometries

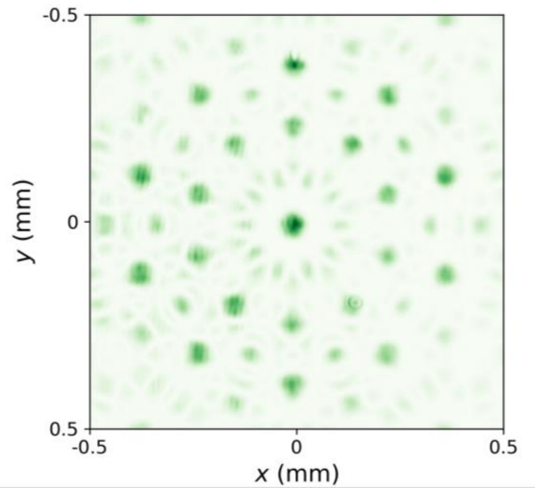
Crystalline Structures

(periodic, defects, quasi-crystal)









Multiple Beams with arbitrary control

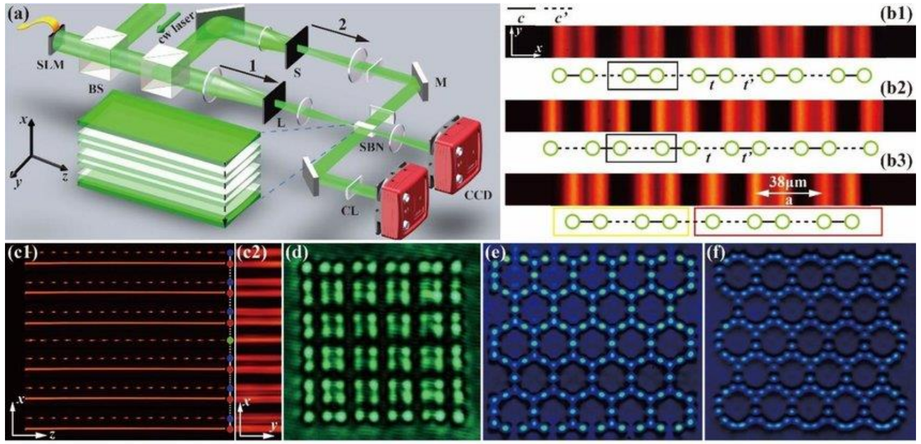
Optical Defects

Trapping Potentials

Non-trivial Geometries

Crystalline Structures

(periodic, defects, quasi-crystal)



e.g. Study topological phenomena

Xia, Shiqi, et al. "Topological phenomena demonstrated in photorefractive photonic lattices." Optical Materials Express 11.4 (2021): 1292-1312.

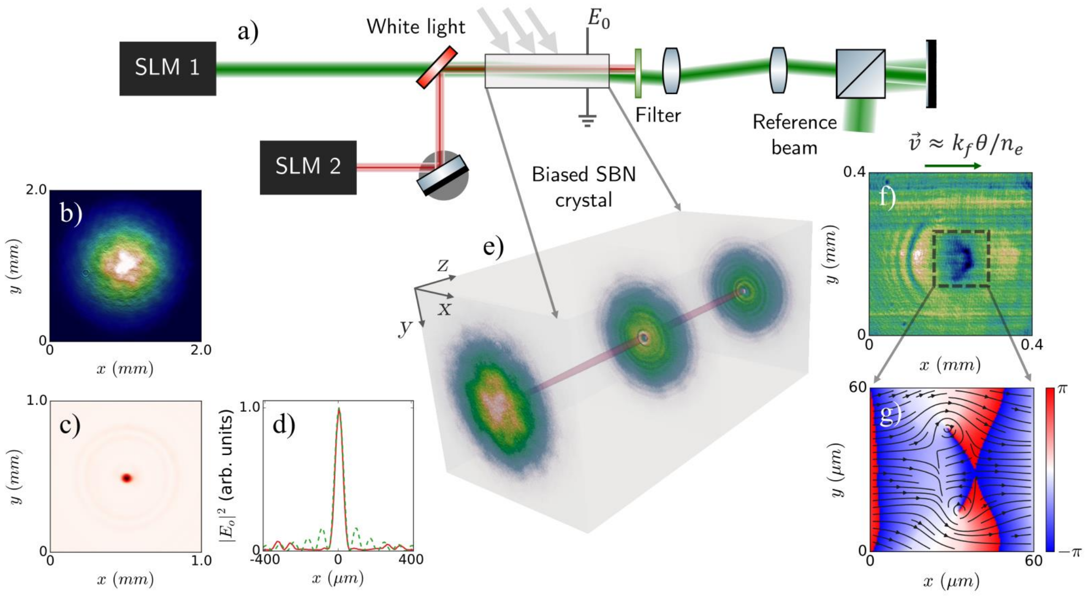
An Analogue Simulator for Quantum Matter

Some convincing metrics

Some convincing metrics

Typical Numerical Simulation

- 0,2GFlop Per time step for 1024x1024 grid
- Around 10000 time steps
- **Total of 2TFlop**



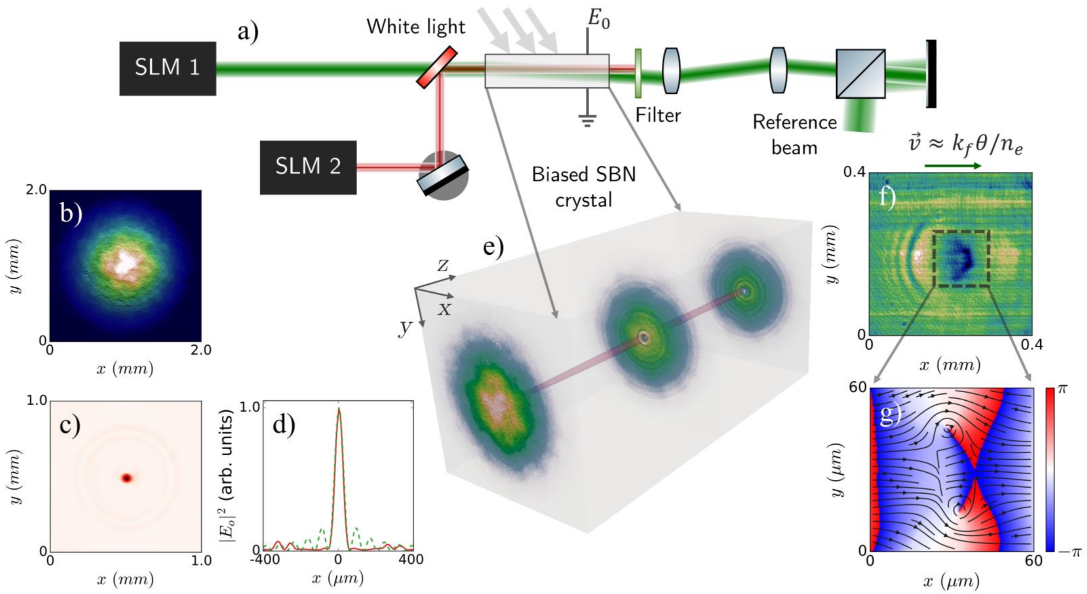
Some convincing metrics

Typical Numerical Simulation

- 0,2GFlop Per time step for 1024x1024 grid
- Around 10000 time steps
- **Total of 2TFlop**

If 1ms time to perform a simulation (possible with fast SLMs)

- **Equivalent performance: 2PetaFlop/s**



Some convincing metrics

Typical Numerical Simulation

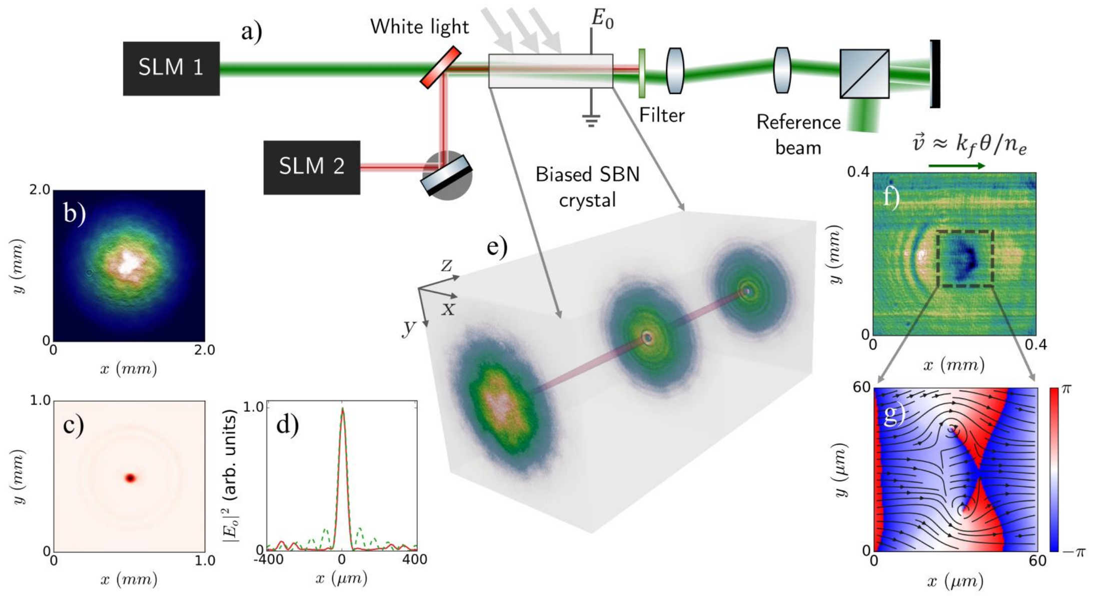
- 0,2GFlop Per time step for 1024x1024 grid
- Around 10000 time steps
- **Total of 2TFlop**

If 1ms time to perform a simulation (possible with fast SLMs)

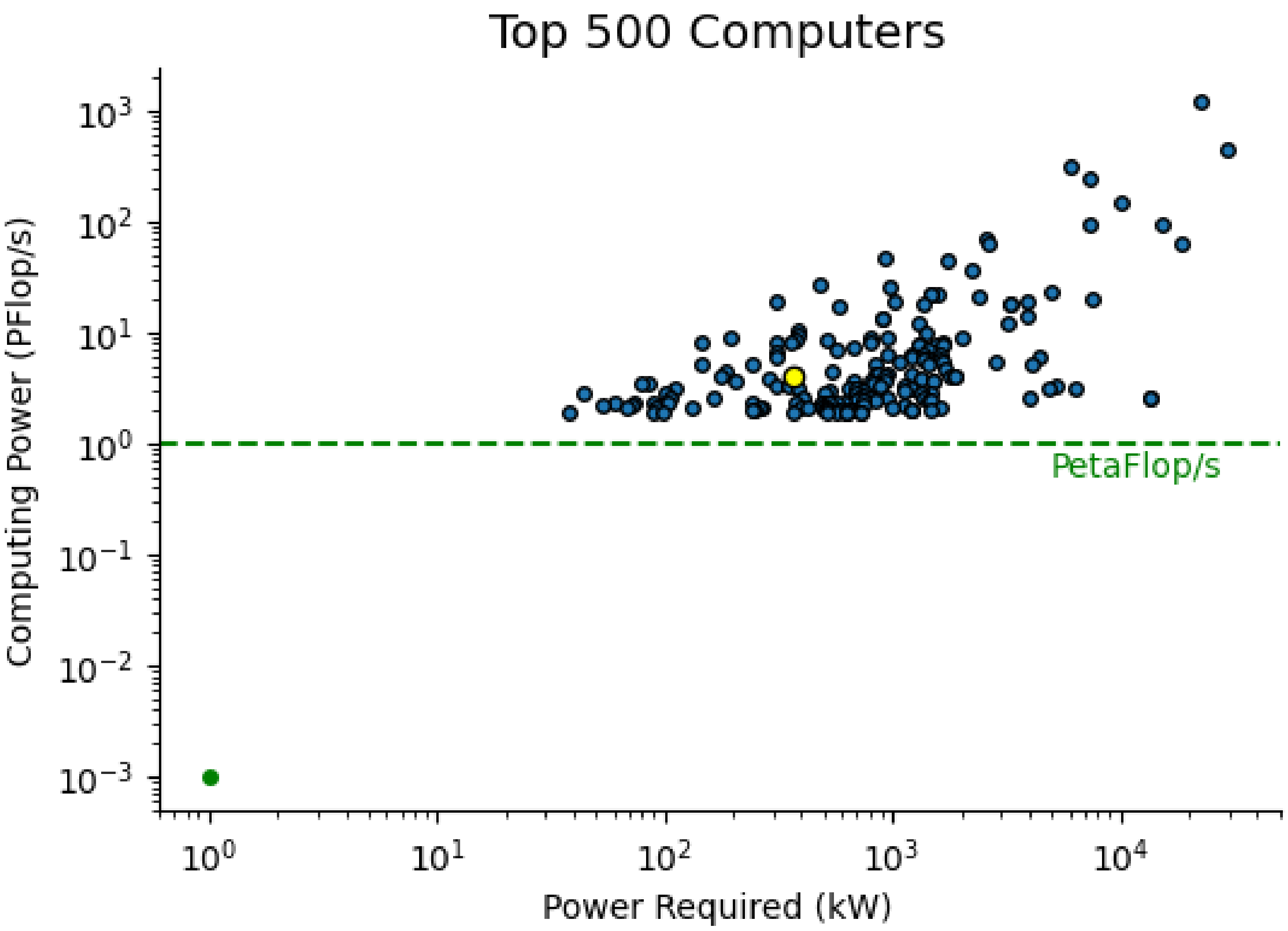
- **Equivalent performance: 2PetaFlop/s**

Low power consumption

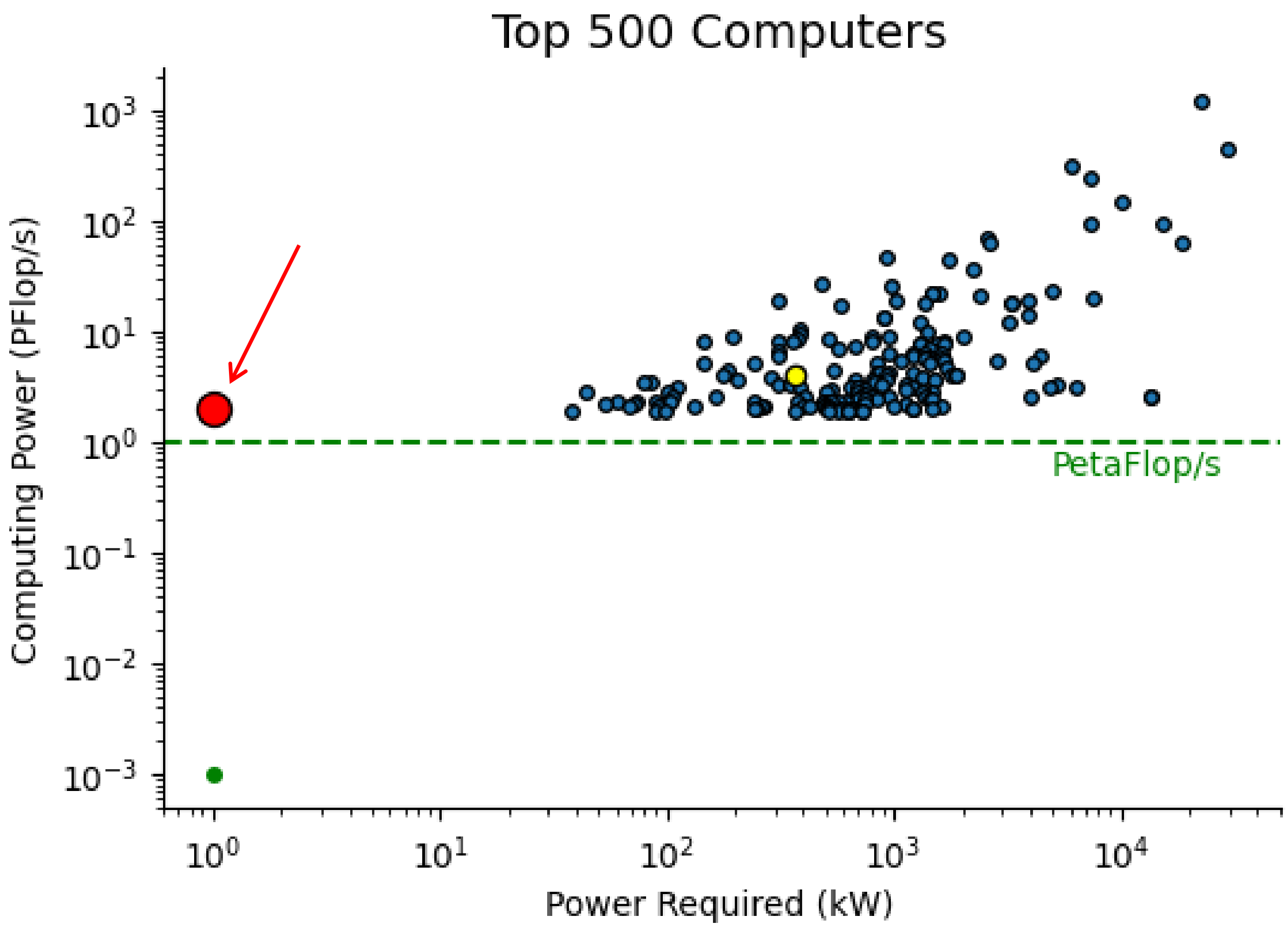
- **Less than 1kW**



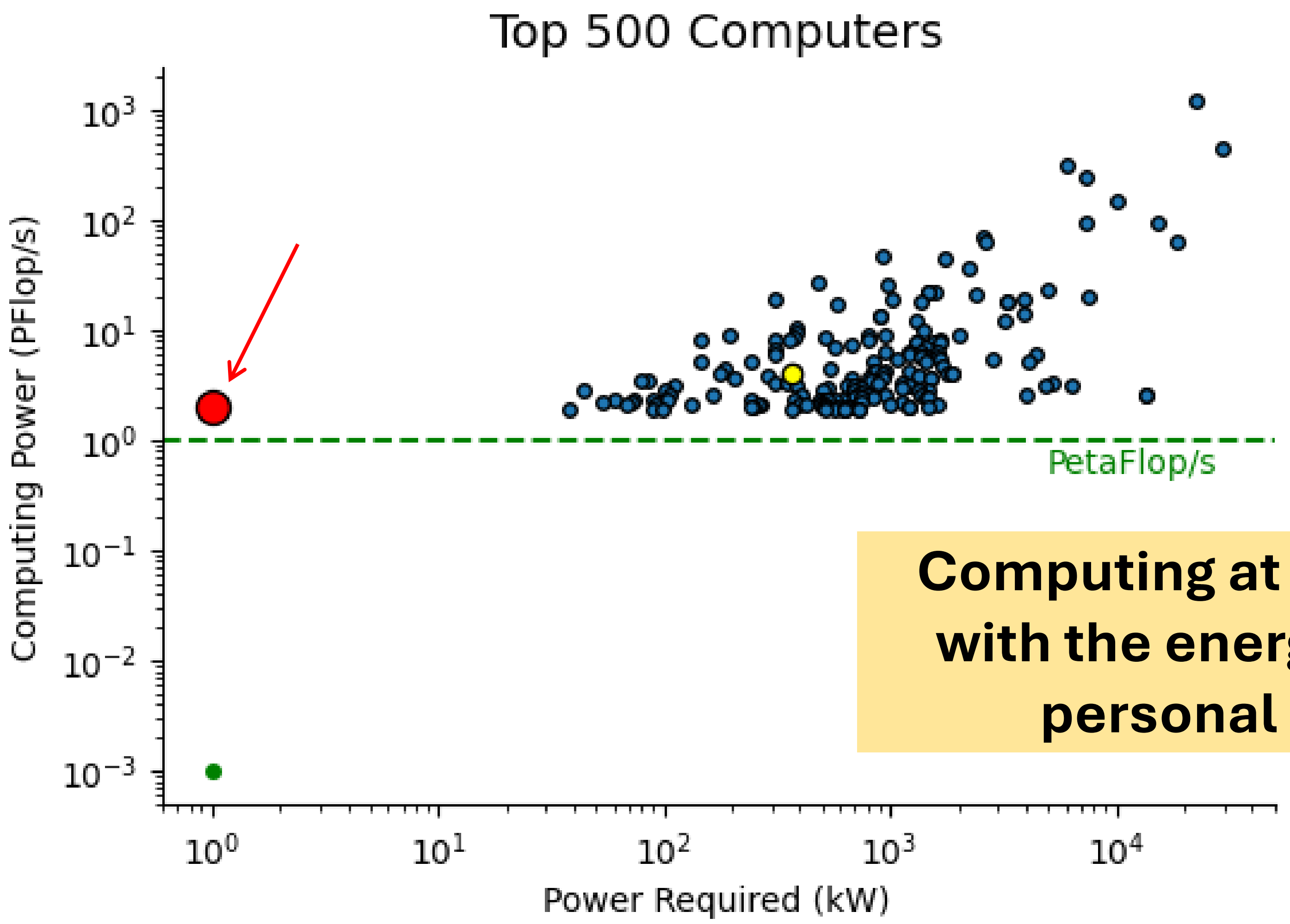
Some convincing metrics



Some convincing metrics



Some convincing metrics



Computing at Top 500 Speed with the energy impact of a personal computer

Next week

Do the exercises

Try to read the suggested documents

Theorists – numerical tools

Experimentalists – experimental tools