Course Scheduling in the Computer Science and Optimisation Department of Monford College

Filipe Almeida, up201708999
Ines Silva, up201806385
Nuno Marques,up201708997
Optimisation, M.EIC

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1 The course scheduling problem

The problem as described has two components: assigning courses to faculties and scheduling course-faculty pairs into time slots. Our solutions aims to maximize happiness for faculties with considerations to seniority and student preferences.

For **stage 1** there are **8 different courses** named CSO1 through CSO8. Each course has a fixed number of classes that must be assigned to one of the **8 full-time instructors** or to the contingent faculty.

For **stage 2** we imported the solution from stage 1, as we felt this was the simpler approach. With our courses assigned to faculties, this stage's goal is to schedule every class. There are two different scheduling groups **MWF** (Monday, Wednesday, Friday) and **TuTh** (Tuesday, Thursday), each group has **7 time slots** at different times. The contingent faculty courses can be scheduled in either group but full-time faculties have a specific scheduling group.

Both stages take a number of faculty and student preferences as expressed in detail below in the form of hard and soft **constraints**.

2 Stage 1 - Assign courses to the faculty

2.1 Indices and Sets

1. I, corresponding to the **instructors** that will be teaching the classes, including both full-time and contingent faculty;
2. C, corresponding to the **courses** to be taught in the considered semester.

2.2 Parameters

1. $p_{i,c}$, corresponding to the **preference** of instructor i to course
2. l_i, corresponding to required **course load** for instructor i;
3. s_i, corresponding to a **seniority** coefficient given to instructor i according to their rank and years of service;
4. $q_{i,c}$, having a value of 1 if instructor i is **qualified** to teach course c, and a value of 0 otherwise;
5. n_c, corresponding to the required **number of classes** for the Spring Semester for course c.

2.3 Decision Variables

1. $N_{i,c}$, corresponding to the number of classes that instructor i teaches of course c.
2. A, corresponding to a penalty attributed based on the number of members of the full-time faculty that are assigned to more than two classes of course CSO3;
3. B, corresponding to a penalty attributed based on the number of members of the full-time faculty that are assigned to classes of more than two different courses.

2.4 Objective Function

$$\max Z = (\sum_{i \in I, c \in C} N_{i,c} * (0.5 * s_i + p_{i,c})) - (A + B) * 5 \tag{1}$$

The objective function is made of two parts. First we want to maximize the sum of happiness and seniority for each instructor.Here we decided to halve the values of seniority to keep both happiness and seniority on the same scale. This value is then subtracted by the sum of penalties, that we want to minimize, multiplied by five so that each penalty has the half the value of a max seniority and happiness assignment.

2.5 Constraints

1. Each full-time faculty member's actual course load must be equal to his/her required course load per semester:
$$\sum_{c \in C} N_{i,c} = l_i, \forall i \in I \tag{2}$$
2. The number of classes assigned to the faculty needs to be equal to the number of classes required for each course:
$$\sum_{i \in I} N_{i,c} = n_c, \forall c \in C \tag{3}$$
3. Classes can only be taught by instructors who are qualified to do so:
$$N_{i,c} \leq q_{i,c} * N_{i,c}, \forall i \in I, c \in C \tag{4}$$
4. Ideally, the number of preparations for each full-time faculty would be no more than two:
$$A = \sum_{i \in I \setminus \{\text{contingent}\}} \begin{cases} 1, & if \sum_{c \in C} \begin{cases} 1, & \text{if } N_{i,c} \geq 1 \\ 0, & \text{otherwise} \end{cases} \geq 3 \\ 0, & \text{otherwise} \end{cases} \tag{5}$$
This is a soft constraint so we made it a penalty by summing the amount of courses each instructor has with one or more classes and then we count the amount of times this sum is equal or greater than three for each instructor.
5. Ideally, no more than two classes of the course CSO3 should be assigned to any full-time faculty:
$$B = \sum_{i \in I \setminus \{\text{contingent}\}} \begin{cases} 1, & \text{if } N_{i,cso3} \geq 3 \\ 0, & \text{otherwise} \end{cases} \tag{6}$$
This is another soft constraint made into a penalty by counting the amount of instructors with equal or more than three CSO3 clases.

2.6 Solution

With this solution penalties A and B were both null, as no soft restriction was broken, and thus our objective function for **stage 1** resulted in a value of:

$$Z = 149$$

Table 1: Found solution for the course assignment problem. The value of each cell represents the number of classes from the corresponding course that the respective instructor was assigned to.

instructors	courses							
	CSO1	CSO2	CSO3	CSO4	CSO5	CSO6	CSO7	CSO8
Barbosa	1	0	2	0	0	0	0	0
Castro	0	0	0	1	0	1	0	0
Gerardo	0	0	0	0	0	2	0	1
Lameiras	1	2	0	0	0	0	0	0
Machado	0	0	1	0	0	0	0	0
Pedro	0	2	1	0	0	0	0	0
Queiros	0	2	0	0	1	0	0	0
Soeiro	0	0	0	0	0	3	0	0
contingent	0	0	1	0	0	0	1	0

Table 2: Happiness values for each faculty.

instructors	happiness		
	Happiness	Max Possible Happiness	Happiness + 0.5 * Seniority
Barbosa	14	15	17
Castro	10	10	14
Gerardo	15	15	19.5
Lameiras	15	15	18
Machado	5	5	10
Pedro	15	15	24
Queiros	15	15	24
Soeiro	15	15	22.5
Total	104	105	**149**

3 Stage 2 - Assign each course-faculty pair to a class time

3.1 Indices and Sets

1. D, corresponding to the **days** of the week for which classes are going to be scheduled in;
2. D_{MWF}, corresponding to **days** of Monday, Wednesday, Friday;
3. D_{TuTh}, corresponding to the **days** of Tuesday and Thursday;
4. S, corresponding to the **time slots** to schedule classes to, on each day;
5. C, corresponding to the **courses** to be taught in the considered semester.
6. I, corresponding to the **instructors** that will be teaching the classes, including both full-time and contingent faculty;
7. I_{MWF}, corresponding to the full-time **instructors** that will be teaching the classes on **Mondays**, **Wednesdays** and **Fridays**;
8. I_{TuTh}, corresponding to the full-time **instructors** that will be teaching the classes on **Tuesdays** and **Thursdays**.

3.2 Parameters

1. $n_{i,c}$, corresponding to the **number of classes** from course c that instructor i is assigned to;
2. $p_{i,s}$, corresponding to the **preference** of instructor i for time slot s, for **Mondays**, **Wednesdays** and **Fridays**;
3. $q_{i,s}$, corresponding to the **preference** of instructor i for time slot s, for **Tuesdays** and **Thursdays**;
4. s_i, corresponding to a **seniority** coefficient given to instructor i according to their rank and years of service.

3.3 Decision Variables

1. $X_{d,s,i,c}$, corresponding to a boolean value indicating whether instructor i will be teaching course c on time slot s of day d;
2. A, corresponding to a penalty applied when instructors are assigned to classes on more than two consecutive time slots;
3. B, corresponding to a penalty applied when two classes of the same course are being taught on the same time slot;
4. C, corresponding to a penalty applied when a class is scheduled to begin before 9h00 or end after 16h00.
5. D, corresponding to a penalty applied when a more than one class is scheduled for the same time slot.

3.4 Objective Function

$$\begin{aligned} \max Z = &\Big(\sum_{d1 \in D_{MWF}, s \in S, i \in I, c \in C} X_{d1,s,i,c} * (0.5 * s_i + p_{i,s})\Big) \\ &+ \Big(\sum_{d2 \in D_{TuTh}, s \in S, i \in I, c \in C} X_{d2,s,i,c} * (0.5 * s_i + q_{i,s})\Big) \\ &- (A + B + C) * 5 - D \end{aligned} \tag{7}$$

Similar to the first stage we sum the combined happiness and seniority of faculties and subtract the penalties. This time we separated the problem in MWF and TuTh groups so we sum each separately and add them together. For this stage we also account for contingent happiness. Penalty **D** has reduced importance compared to the rest as we felt that was more appropriate.

3.5 Constraints

1. The total number of scheduled classes of course c, across all days d and slots s, for each instructor i, must be equal to the number of classes of course c that instructor i was assigned to:

$$\sum_{d \in D, s \in S} X_{d,s,i,c} = n_{i,c}, \forall i \in I, c \in C$$

2. Each instructor i can only teach, at most, one class in the same time slot:

$$\sum_{c \in C} X_{d,s,i,c} \leq 1, \forall d \in D, s \in S, i \in I$$

3. Instructors that only teach on Tuesdays and Thursdays can't be assigned to classes on Mondays, Wednesdays or Fridays

$$X_{d,s,i,c} = 0, \forall d \in D_{MWF}, s \in S, i \in I_{TuTh}, c \in C$$

4. Instructors that only teach on Mondays, Wednesdays and Fridays can't be assigned to classes on Tuesdays or Thursdays

$$X_{d,s,i,c} = 0, \forall d \in D_{TuTh}, s \in S, i \in I_{MWF}, c \in C$$

5. No more than 3 classes can be scheduled for the same time slot s on the same day d

$$\sum_{i \in I, c \in C} X_{d,s,i,c} \leq 3, \forall d \in D, s \in S$$

6. Ideally, no faculties teach more more than two consecutive classes

$$A = \sum_{d \in D, \in S \setminus \{7\}, i \in I \setminus \{\text{contingent}\}} \sum_{c \in C} \begin{cases} 1, & \text{if } X_{d,s,i,c} + X_{d,s+1,i,c} + X_{d,s+2,i,c} \geq 3 \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

We made this soft restriction into a penalty that sums that amount of times an instructor teaches more than 3 classes consecutively.

7. Ideally, no classes of the same course are scheduled for the exact same time slot in order to give students more choice.

$$B = \sum_{d \in D, s \in S, c \in C} \begin{cases} 1, & \text{if } \sum_{i \in I} X_{d,s,i,c} \geq 2 \\ 0, & \text{otherwise} \end{cases}$$

Another soft restriction made into a penalty by summing the amount of instructors giving the same class in the same day and time slot.

8. Ideally, no classes are scheduled to begin before 9h00 and end after 16h00 due to student preference.

$$C = \sum_{d \in D_{MWF}, s \in \{6,7\}, i \in I, c \in C} X_{d,s,i,c} + \sum_{d \in D_{TuTh}, s \in \{1,6,7\}, i \in I, c \in C} X_{d,s,i,c} \tag{9}$$

9. Ideally, no classes are scheduled for the same time slot in order to give students more choice

$$D = \sum_{d \in D, s \in S} \begin{cases} 1, & \text{if } \sum_{i \in I, c \in C} X_{d,s,i,c} \geq 2 \\ 0, & \text{otherwise} \end{cases}$$

We felt this penalty shouldn't have the same weight as the rest so it's the only one not multiplied by a factor of five in the objective function, this is because we felt this was the least important constraint, but if all things are equal more choice for students is always better.

3.6 Solution

Solution for the course scheduling problem. The value of each cell represents the day and time each course-instructor pair was assigned to.

This solution has two penalties, one for constraint C and another for constraint D. For constraint C we see *Machado* gives a class before 9h00, this is because his combined seniority and happiness minus penalty C overcome other options. Not surprising as *Machado* has zero preference for other time slots. This penalty has a value of five. Penalty D has one penalty point for having two classes on Thursday at 11h25. Penalty C and D combined for a total of **6 penalty points**.

With the sum of combined happiness and seniority at **145 points** we get the result of our objective function for **stage 2** by subtracting the penalties for a result of:

$$Z = 139$$

Table 3: Monday, Wednesday, Friday

	Monday	Wednesday	Friday
9h10	CSO4-Castro		
10h20	CSO2-Pedro	CSO2-Queiros	CSO2-Pedro
11h30	CSO5-Queiros	CSO3-Pedro	CSO3-contingent
13h30	CSO2-Queiros	CSO3-Barbosa	CSO1-Barbosa
14h40	CSO6-Castro	CSO3-Barbosa	CSO7-contingent
15h50			
17h25			

Table 4: Tueday, Thursday

	Tuesday	Thursday
8h15	CSO3-Machado	
9h50	CSO6-Gerardo	CSO2-Lameiras
11h25	CSO6-Gerardo	CSO8-Gerardo CSO1-Lameiras
13h00	CSO2-Lameiras	CSO6-Soeiro
14h35	CSO6-Soeiro	CSO6-Soeiro
16h10		
18h00		

Table 5: Happiness values for each faculty.

instructors	happiness		
	Happiness	Max Possible Happiness	Happiness + 0.5 * Seniority
Barbosa	15	15	18
Castro	2	10	6
Gerardo	15	15	19.5
Lameiras	15	15	18
Machado	5	5	10
Pedro	15	15	24
Queiros	15	15	24
Soeiro	15	15	22.5
Contingent	2	2	3
Total	99	107	**145**

4 Conclusions

The results were obtained using IBM's CPLEX Optimizer. During development of the model we tested various weights to penalties, seniority and happiness but in the end we were happy we these results.

The first stage was a single point off **maximum possible happiness** while avoid any penalty. The second stage both faculties and student had to make concessions. This resulted in two student preference penalties and reduced happiness for Castro. The stage 2 solution achieved **99 happiness points of a possible 107** with only **6 penalty points** for a problem with a large number of faculty and student considerations. The final solution provided a great amount of choice for students while achieving high happiness for faculties.