

Example : Sum of first n natural numbers

We want to prove the following,

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

Initial Case: n=1

$$\sum_{i=1}^1 i = 1 = \frac{1(2)}{2}$$

□

Induction step:

$$\left(\sum_{i=1}^n i = \frac{n(n+1)}{2} \right) \rightarrow \left(\sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2} \right)$$

To prove the induction step,

$$\begin{aligned} \sum_{i=1}^{n+1} i &= (n+1) + \sum_{i=1}^n i \\ &= (n+1) + \frac{n(n+1)}{2} \\ &= \frac{2n+2}{2} + \frac{n(n+1)}{2} \\ &= \frac{2n+2+n^2+n}{2} \\ &= \frac{n^2+3n+2}{2} \\ &= \frac{(n+1)(n+2)}{2} \end{aligned}$$

□

Using the induction hypothesis we get