Equilibrium of a Spherical Pendulum by Energy Control

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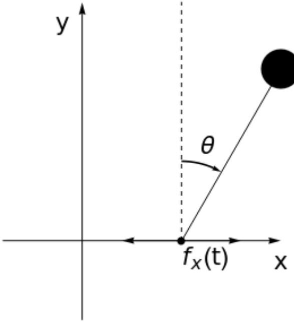


$$x(t) = f_x(t) + l \sin\theta(t), \qquad (1)$$

$$y(t) = l \cos\theta(t). \qquad (2)$$

$$\mathcal{L} = \frac{1}{2} m \left(\dot{x}^2 + \dot{y}^2\right) - mgy, \qquad (3)$$

$$\ddot{\theta} - \frac{g}{l} \sin\theta = -u(t) \cos\theta, \qquad (4)$$

with $u(t) = \ddot{f}_x/l$, the control function

Energy Control

$$\mathcal{H} = \frac{\dot{\theta}^2}{2} + \frac{g}{l}\cos\theta \tag{5}$$

$$\frac{d\mathcal{H}}{dt} = \dot{\theta}\left(\ddot{\theta} - \frac{g}{l}\sin\theta\right) \tag{6}$$

$$\frac{d\mathcal{H}}{dt} = -u(t)\,\dot{\theta}\,\cos\theta \tag{7}$$

$$u(t) = -\mu\,(E_1 - E)\,(\dot{\theta}\cos\theta), \tag{8}$$

2D control

$$x(t) = f_x(t) + l, \sin\theta\cos\phi \tag{9}$$

$$y(t) = f_y(t) + l\sin\theta\sin\phi, \tag{10}$$

$$z(t) = l\cos\theta. \tag{11}$$

$$\ddot{\theta} - \frac{g}{l}\sin\theta - \dot{\phi}^2\,\cos\theta\,\sin\theta = -\cos\theta(\cos\phi\,u_x(t) + \sin\phi\,u_y(t)), \tag{12}$$

$$\ddot{\phi}\sin^2\theta + 2\,\dot{\phi}\,\dot{\theta}\sin\theta\cos\theta = -\sin\theta(\sin\phi\,u_x(t) - \cos\phi\,u_y(t)), \tag{13}$$

with $u_x(t) = \ddot{f_x(t)}/l$ and $u_y(t) = \ddot{f_y(t)}/l$,

Energy Control

$$\frac{d\mathcal{H}}{dt} = -\,\dot{\phi}\,\sin\theta(-\sin\phi\,u_x(t)+\cos\phi\,u_y(t)) \\ -\,\dot{\theta}\,\cos\theta(\cos\phi\,u_x(t)+\sin\phi\,u_y(t)) \tag{14}$$

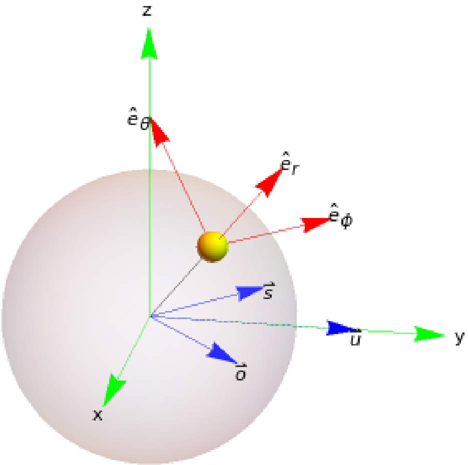
In case u_x,u_y $\parallel$ $(\hat{e}_r)_{cylindrical}$

$$\vec{o}(t) = \begin{cases} u_x(t) = o(t)\cos\phi, \\ u_y(t) = o(t)\sin\phi, \end{cases} \rightarrow \frac{d\mathcal{H}}{dt} = -o(t)\,\dot{\theta}\,\cos\theta$$

In case u_x, u_y $\parallel$ $(\hat{e}_\phi)$

$$\vec{s}(t) = \begin{cases} u_x(t) = -s(t)\sin\phi, \\ u_y(t) = s(t)\cos\phi, \end{cases} \rightarrow \frac{d\mathcal{H}}{dt} = -s(t)\,\dot{\phi}\sin\theta.$$

2D Control Functions



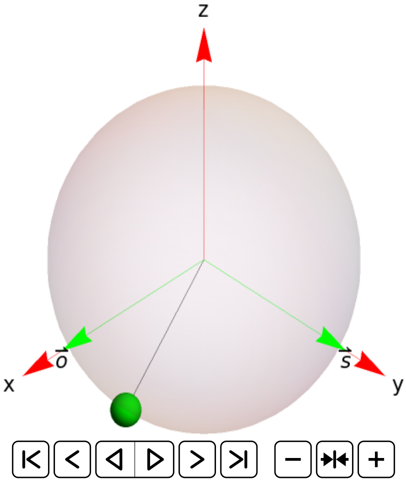
$$o(t) = -\mu\,(E_1 - E)\,(\dot{\theta}\cos\theta), \quad (15)$$

$$s(t) = \nu(\dot{\phi}\sin\theta), \quad (16)$$

The net control, $\vec{u}(t)$,

$$\vec{u}(t) = \vec{o}(t) + \vec{s}(t) \quad (17)$$

Embedded Animation



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