Articulated leg

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Abstract

A model for a walking one-legged robot.

1 Introduction

Our goal is to develop a control mechanism for a one-legged robot, robust to noise, and able to run under the influence of a gravitational field. To save energy, the controls act during the flight phase so that the robot motors only need to produce enough torque to move the "small" masses corresponding to the knee and foot. During the stance phase the control is purely passive, and the robot gears are either locked (the knee) or move freely (the hip). The algorithm targets a specific horizontal speed that is introduced at the beginning of the simulation and manages the attack angle in such a way that the available energy is used to provide that horizontal speed during the flight phase.

... irregular floor ...

2 The control model

A one-legged robot model consists of three masses connected by two springs (Fig. 1). The mass m_1 is the robot body mass or hip mass, and m_2 and m_3 are the masses of the knee and of the foot. The three masses are connected by two springs with stiffness constantes k_1 and k_2. This is a model for an articulated robot leg.

As the robot is a gravity field, the lagragian of the robot is

$$\begin{aligned} L = &\tfrac{1}{2}m_1(\dot{x}_1^2+\dot{y}_1^2)+\tfrac{1}{2}m_2(\dot{x}_2^2+\dot{y}_2^2)+\tfrac{1}{2}m_3(\dot{x}_3^2+\dot{y}_3^2)-m_1gy_1-m_2gy_2-m_3gy_3 \\ &-\tfrac{1}{2}k_1(\ell_1-\ell_{10})^2-\tfrac{1}{2}k_2(\ell_2-\ell_{20})^2, \end{aligned} \tag{1}$$

where $\ell_1^2=(x_1-x_2)^2+(y_1-y_2)^2$, $\ell_2^2=(x_2-x_3)^2+(y_2-y_3)^2$, ℓ_{10} and ℓ_{20} are the rest lengths of the springs, g is the acceleration of gravity and (x_i,y_i) are the position coordinates of the masse m_i. The mechanical energy of the robot is

$$\begin{array}{l} H=\frac{1}{2}m_1(\dot{x}_1^2+\dot{y}_1^2)+\frac{1}{2}m_2(\dot{x}_2^2+\dot{y}_2^2)+\frac{1}{2}m_3(\dot{x}_3^2+\dot{y}_3^2)+m_1gy_1+m_2gy_2+m_3gy_3 \\ +\frac{1}{2}k_1(\ell_1-\ell_{10})^2+\frac{1}{2}k_2(\ell_2-\ell_{20})^2. \end{array} \tag{2}$$

We further consider that the robot leg is at the floor or above it, in the sense that $y_3\geq f(x_3)$, where $y=f(x)$ is the function describing the floor. For a straight floor, we take $f(x)=0$.

The lagrangian (1) of the articulated robot leg depends of 9 variables. However, for control proposes, some of the variables will not be independent, as the three masses move subject to different constraints. For each case, we will derive different equations of motion and control parameters. In any case, (1) is always the starting Lagrangian and the energy (2) will be used to evaluate the mechanical energy of the system.

We derive now the sequence of controls and parameter that we will use to simulate gaiting and running. There are essentially three major sequential phases in the control of the robotic leg. These three phases will control one footstep and then the algorithms applies recursively.

Control phase i). We initiate the robot leg in the position described in Fig. 1, making an angle α_0 with the horizontal direction. In this position, the mass m_3 is at the fixed position $(x_3,y_3=f(x_3))$. During this phase, the leg is not articulated, there is no bending of the knee. The initial state of the leg is characterised by an initial speed $v_0=|\mathbf{v}_0|$ at m_1 and the initial attack angle of the robot is α_0, with $\pi/2<\alpha_0<\pi$. The initial velocity makes an angle γ_0 with the horizontal direction, and the initial velocities of masses m_2 and m_3 are assumed to be zero. Depending on the the angle γ_0, at the initial time, the mass m_1 has a speed perpendicular to the robotic leg and a radial speed along the leg. Therefore, the magnitude of the initial speed and γ_0 determine the initial energy of the robotic leg. To insure that the robotic leg move to the right, we consider that $\gamma_0\in[-(\pi-\alpha_0),\alpha_0]$. During this phase, the total energy (2) is conserved.

During this phase, the leg rotates around the fixed position (x_3,y_3) of the foot, reaching a prescribed angle of attack α ($<\pi/2$). From the initial condition until the final angular position α, the leg takes some time t_i (Fig. 1).

To simulate the motion during phase i) and due to the fact that the leg is not articulated, we have the following dependence of the coordinates of the three masses:

$$\left\{\begin{array}{lcl} x_1-x_3 &=& (\ell_1+\ell_2)\cos\theta \\ y_1-y_3 &=& (\ell_1+\ell_2)\sin\theta \\ x_2-x_3 &=& \ell_2\cos\theta \\ y_2-y_3 &=& \ell_2\sin\theta \end{array}\right. \quad \left\{\begin{array}{lcl} \dot{x}_1 &=& (\dot{\ell}_1+\dot{\ell}_2)\cos\theta-(\ell_1+\ell_2)\dot{\theta}\sin\theta \\ \dot{y}_1 &=& (\dot{\ell}_1+\dot{\ell}_2)\sin\theta+(\ell_1+\ell_2)\dot{\theta}\cos\theta \\ \dot{x}_2 &=& \dot{\ell}_2\cos\theta-\ell_2\dot{\theta}\sin\theta \\ \dot{y}_2 &=& \dot{\ell}_2\sin\theta+\ell_2\dot{\theta}\cos\theta, \end{array}\right. \tag{3}$$

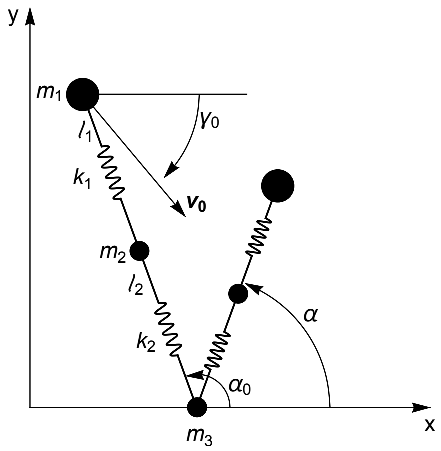

Figure 1: Initial phase (α_0) of the robotic leg, and final position (α) of leg at phase i) of control. The parameters of the simulation are: $k_1 = k_2 = 12\,000$ N/m , $m_1 = 60$ kg, $m_2 = m_3 = 1$ kg, $\ell_1 = \ell_2 = 0.5$ m, $|\mathbf{v}_0| = 4.0$ m/s, $\gamma_0 = -50°$, $\alpha = 70°$, $\alpha_0 = 110°$ and $g = 9.8$ m/s^2.

where we have introduced the condition that the foot of the robot is fixed. Introducing (3) into the lagrangian (1), and solving the Euler-Lagrange equations in order to the higher order derivatives, the equations of motion of the robotic leg during the phase i) are

$$\left\{\begin{array}{rcl} \ddot{\ell}_1 & = & \dfrac{1}{m_1 m_2}\left(-k_1(m_1+m_2)(\ell_1-\ell_{10})+k_2 m_1(\ell_2-\ell_{20})+m_1 m_2 \ell_1 \dot{\theta}^2\right) \\ \ddot{\ell}_2 & = & \dfrac{1}{m_2}\left(-g m_2 \sin\theta + k_1(\ell_1-\ell_{10}) - k_2(\ell_2-\ell_{20}) + m_2 \ell_2 \dot{\theta}^2\right) \\ \ddot{\theta} & = & -\dfrac{g\cos\theta(m_1\ell_1+(m_1+m_2)\ell_2)}{m_1\ell_1^2+2m_1\ell_1\ell_2+(m_1+m_2)\ell_2^2} \\ & & -\dfrac{2\theta'\left(m_1(\ell_1+\ell_2)\ell_1'+(m_1\ \ell_1+(m_1+m_2)\ell_2)\ell_2'\right)}{m_1\ell_1^2+2m_1\ell_1\ell_2+(m_1+m_2)\ell_2^2} \end{array}\right. \tag{4}$$

According to te previous assumptions, the initial conditions for equation (4) are

$$\begin{array}{l} \ell_1(0) = \ell_{10},\ \dot{\ell}_1(0) = v_0\cos(\alpha_0-\gamma_0) \\ \ell_2(0) = \ell_{20},\ \dot{\ell}_2(0) = 0 \\ \theta(0) = \alpha_0,\ \dot{\theta}(0) = -v_0\dfrac{\sin(\alpha_0-\gamma_0)}{\ell_{10}+\ell_{20}}. \end{array}$$

To synthesise, during this phase, the robot leg is not articulated, the initial conditions are $\mathbf{v}_0$ and α_0, and the control parameter is the attack angle α. Moreover, the time span of phase i) is t_i, determined by the attack angle α. During this phase the energy (2) is conserved.

Control phase ii). In the second control phase ii), the foot of the robotic leg leaves the ground through the control of two rotors in the robot. Therefore, we have two simultaneous control movements: the rotation of the foot around the knee and the rotation of knee around the hip. We consider that the angular speeds of foot and knee are new control parameters ω_f and ω_k, corresponding to cross radial speeds $v_f = \ell_2 \omega_f$ and $v_k = \ell_1 \omega_k$ of the foot and knee. As phase i) precedes phase ii), we must have initially, $\omega_f < 0$ and $\omega_k > 0$. As the rotation around the knee is opposite to the direction of motion, at some stage of the control, we must reverse the speed of the foot. So, we define a subphase iia), where $\omega_f = \omega_{fa} < 0$, and a second subphase iib), where $\omega_f = \omega_{fb} > 0$ (Fig. 2).

In the subphase iia), we define a new control recoil angle $\theta_r < \pi + \alpha$. In this case, the foot arm angle will change in time according to the law $\theta_f(t) = \pi + \alpha + \omega_{fa}(t - t_i)$, where $\theta_f(t)$ is the angle that the foot arm makes with the x-axis. As at the initial stage of phase iia), the mass m_3 has zero speed, solving the equation $\theta_r = \pi + \alpha + \omega_{fa}(t - t_i)$, the subphase iia) has the duration $t_{iia} = (t - t_i) = (\theta_r - \pi - \alpha)/\omega_{fa}$. After this time, the angular speed of the foot becomes $\omega_{fb} > 0$ and subphase iib) is initiated. Imposing the condition that at $t = t_{iia} + t_i$ the angle of the knee arm is $\theta_k = 3\pi/2$, we have $\theta_k(t_{iia}) = 3\pi/2 = \pi + \alpha + \omega_k(t_{iia} - t_i)$, and the angular speed of the knee is chosen to be $\omega_k = \omega_{fa}(\pi/2 - \alpha)/(\theta_r - \pi - \alpha)$.

We impose now that the end of phase ii) occurs when the attack angle of the leg is $\pi - \alpha$. So, during control phase iib), the knee arm angle varies in the interval $\theta_k \in [3\pi/2, 2\pi - \alpha]$. With the control condition $\theta_k(t) = 3\pi/2 + \omega_k(t - t_{iia})$, the time span of phase iib) is $t_{iib} - t_{iia} = (\pi/2 - \alpha)/\omega_k$. As at the end of phase ii), the two arm of the leg must be aligned, we must have $2\pi - \alpha = \theta_r + \omega_{fb}(t_{iib} - t_{iia})$, which implies that $\omega_{fb} = \omega_k(2\pi - \alpha - \theta r)/(\pi/2 - \alpha)$.

To calculate the equations of motion of the robot during the control phase ii), we define the rotation matrix

$$R(\omega, t) = \begin{pmatrix} \cos \omega t & -\sin \omega t \\ \sin \omega t & \cos \omega t \end{pmatrix}. \tag{5}$$

The controls of the rotors on the hip and knee are such that

$$\begin{pmatrix} \dot{x}_3(t) - \dot{x}_2(t) \\ \dot{y}_3(t) - \dot{y}_2(t) \end{pmatrix} = R(\omega_f, t - t^*) \begin{pmatrix} \dot{x}_3(t^*) - \dot{x}_2(t^*) \\ \dot{y}_3(t^*) - \dot{y}_2(t^*) \end{pmatrix} \tag{6}$$

and

$$\begin{pmatrix} \dot{x}_2(t) - \dot{x}_1(t) \\ \dot{y}_2(t) - \dot{y}_1(t) \end{pmatrix} = R(\omega_k, t - t^*) \begin{pmatrix} \dot{x}_2(t^*) - \dot{x}_1(t^*) \\ \dot{y}_2(t^*) - \dot{y}_1(t^*) \end{pmatrix} \tag{7}$$

where, $t^* = t_i$ for the subphase iia) and $t^* = t_{iia}$ for the subphase iib). Analogously, $\omega_f = \omega_{fa}$ for subphase iia), and $\omega_f = \omega_{fb}$ for subphase iib).

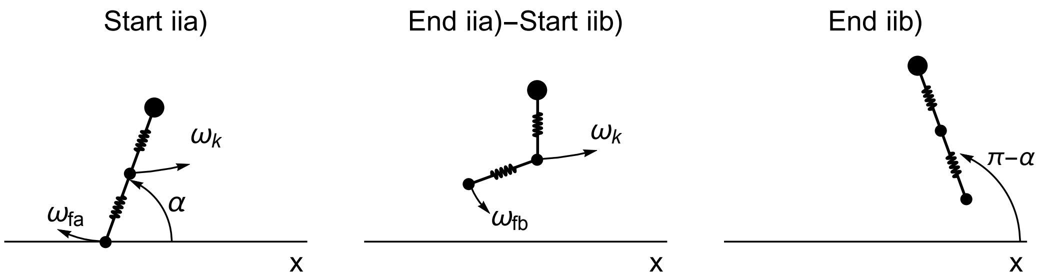

Figure 2: Control phase ii), for the same parameters as in Fig. 1. The angular speeds of the foot are $\omega_{fa} < 0$ during subphase iia) and $\omega_{fb} > 0$ during subphase iib). The angular speed of the knee is $\omega_k > 0$. The bending angle θ_r of the foot is an additional control parameter. In the figure, $\theta_r = 200^\circ$ at the end of subphase iia).

By integration, from (5)-(7), it follows that

$$\begin{pmatrix} x_3(t) - x_2(t) \\ y_3(t) - y_2(t) \end{pmatrix} = \begin{pmatrix} x_3(t^*) - x_2(t^*) \\ y_3(t^*) - y_2(t^*) \end{pmatrix} + S(\omega_f, t - t^*) \begin{pmatrix} \dot{x}_3(t^*) - \dot{x}_2(t^*) \\ \dot{y}_3(t^*) - \dot{y}_2(t^*) \end{pmatrix} \tag{8}$$

and

$$\begin{pmatrix} x_2(t) - x_1(t) \\ y_2(t) - y_1(t) \end{pmatrix} = \begin{pmatrix} x_2(t^*) - x_1(t^*) \\ y_2(t^*) - y_1(t^*) \end{pmatrix} + S(\omega_k, t - t^*) \begin{pmatrix} \dot{x}_2(t^*) - \dot{x}_1(t^*) \\ \dot{y}_2(t^*) - \dot{y}_1(t^*) \end{pmatrix} \tag{9}$$

where

$$S(\omega, t) = \frac{1}{\omega} \begin{pmatrix} \sin \omega t & -1 + \cos \omega t \\ 1 - \cos \omega t & \sin \omega t \end{pmatrix}. \tag{10}$$

The initial conditions for phase ii) are calculated by (3) and the solutions of equations (4), with $t = t_i$.

From relations (5)-(10) and the initial conditions $\dot{x}_3(t_i) = \dot{y}_3(t_i) = 0$ and $y_3(t_i) = 0$, the lagrangian (1) becomes dependent of the coordinates of the mass m_1. Therefore, calculating the Euler-Lagrange equations, the equations of motion of the robotic leg during phase ii) are

$$\left\{ \begin{array}{lcl} \ddot{x}_1 & = & f_x(t) \\ \ddot{y}_1 & = & -\dfrac{m_1 + m_2 + m_3}{m_1 + m_2} g + f_y(t), \end{array} \right. \tag{11}$$

where

$$\left\{ \begin{array}{lcl} f_x(t) & = & \dfrac{\omega_k (m_2 + m_3)(\dot{x}_1(t^*) - \dot{x}_2(t^*))(\sin \omega_k (t - t^*) + \cos \omega_k (t - t^*))}{m_1 + m_2 + m_3} \\ & & + \dfrac{\omega_f m_3 \dot{x}_2(t^*)(\sin \omega_f (t - t^*) + \cos \omega_f (t - t^*))}{m_1 + m_2 + m_3} \\ f_y(t) & = & \dfrac{\omega_k m_2 ((\dot{y}_2(t^*) - \dot{y}_1(t^*)) \sin \omega_k (t - t^*) + (\dot{x}_1(t^*) - \dot{x}_2(t^*)) \cos \omega_k (t - t^*))}{m_1 + m_2} \\ & & + \dfrac{\omega_f m_2 (-\dot{y}_2(t^*) \sin \omega_f (t - t^*) + \dot{x}_2(t^*) \cos \omega_f (t - t^*))}{m_1 + m_2}. \end{array} \right. \tag{12}$$

The functions $f_x(t)$ and $f_y(t)$ describe the action of the rotors or controls. During subphase iia), $\omega_f = \omega_{fa}$ and $t^* = t_i$, and during subphase iib), $\omega_f = \omega_{fb}$ and $t^* = t_{iia}$. The positions of the masses m_2 and m_3 are determined by (8) and (9).

The robotic leg is controllable if, during this phase, the foot of the leg is always above the ground. This will be analysed in detail in the next section.

Control phase iii). We have assumed that at the end of the control phase ii), the angle of attack of the robot leg is $\pi - \alpha$ and that the robotic leg has no floor contact. Therefore, the final control cycle is the free fall of the non articulated robotic leg (Fig. 3). The end of the control phase iii) is calculated according the condition $y_3(t) = 0$, which occurs for some time $t = t_{iii}$. In this case, by the lagrangian (1), the equations of motion of the non articulated robotic leg are

$$\left\{\begin{array}{lcl}
\ddot{\ell}_1 & = & \dfrac{1}{m_1 m_2}\left(-k_1(m_1+m_2)(\ell_1-\ell_{10})+k_2 m_1(\ell_2-\ell_{20})+m_1 m_2 \ell_1 \dot{\theta}^2\right) \\
\ddot{\ell}_2 & = & \dfrac{1}{m_2 m_3}\left(k_1 m_3(\ell_1-\ell_{10})-k_2(m_2+m_3)(\ell_2-\ell_{20})+m_2 m_3 \ell_2 \dot{\theta}^2\right) \\
\ddot{\theta} & = & -\dfrac{2\theta'\left(m_1((m_2+m_3)\ell_1+m_3\ell_2)\ell_1'+m_3(m_1\ell_1+(m_1+m_2)\ell_2)\ell_2'\right)}{m_1(m_2+m_3)\ell_1^2+2m_1m_3\ell_1\ell_2+(m_1+m_2)m_3\ell_2^2} \\
\ddot{x}_3 & = & \dfrac{k_2(\ell_2-\ell_{20})\cos(\theta)}{m_3}+\dfrac{2m_1m_2\ell_1\theta'\sin(\theta)\left(\ell_1\ell_2'-\ell_2\ell_1'\right)}{m_1(m_2+m_3)\ell_1^2+2m_1m_3\ell_1\ell_2+(m_1+m_2)m_3\ell_2^2} \\
\ddot{y}_3 & = & -g+\dfrac{k_2(\ell_2-\ell_{20})\sin(\theta)}{m_3}+\dfrac{2m_1m_2\ell_1\theta'\cos(\theta)\left(\ell_2\ell_1'-\ell_1\ell_2'\right)}{m_1(m_2+m_3)\ell_1^2+2m_1m_3\ell_1\ell_2+(m_1+m_2)m_3\ell_2^2}.
\end{array}\right. \tag{13}$$

The initial conditions for the equations of motion (13) are calculated from the solutions of equations (11) and the relations (3). At the initial stage of phase iii), the angle θ and its derivative is calculated with the coordinates of the masses m_1 and m_3. To be more specific, denoting by $\bar{x}_i$ and $\bar{vx}_i$, $\bar{y}_i$ and $\bar{vy}_i$, with $i = 1,2,3$, the final positions and velocities of the three masses at phase ii), the nontrivial initial conditions at phase iii) of $\dot{\ell}_i$ and θ for equations (13) are

$$\begin{array}{l}
\dot{\ell}_1^* = \frac{(\bar{x}_1-\bar{x}_2)(\bar{vx}_1-\bar{vx}_2+(\bar{y}_1-\bar{y}_2)(\bar{vy}_1-\bar{vy}_2)}{\ell_1^*} \\
\dot{\ell}_2^* = \frac{(\bar{x}_2-\bar{x}_3)(\bar{vx}_2-\bar{vx}_3+(\bar{y}_2-\bar{y}_3)(\bar{vy}_2-\bar{vy}_3)}{\ell_2^*} \\
\dot{\theta}^* = \frac{(\bar{x}_1-\bar{x}_3)(\bar{vy}_1-\bar{vy}_3)-(\bar{y}_1-\bar{y}_3)(\bar{vx}_1-\bar{vx}_3)}{(\ell_1^*+\ell_2^*)^2} + \frac{(\bar{x}_2-\bar{x}_3)(\bar{vy}_2-\bar{vy}_3)-(\bar{y}_2-\bar{y}_3)(\bar{vx}_2-\bar{vx}_3)}{(\ell_2^*)^2}.
\end{array}$$

At the end of the control phase iii), we restart the control cycle at phase i). At this stage, the control cycle is reinitialised with new initial conditions.

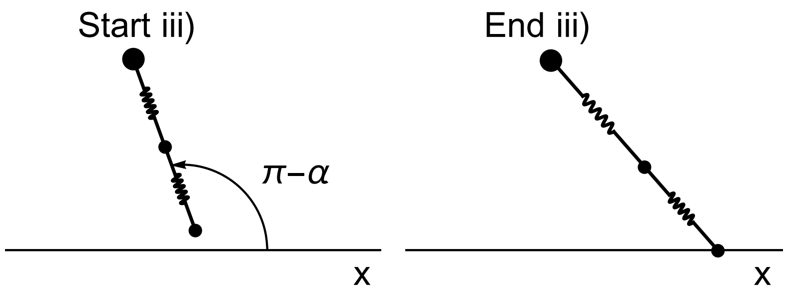

Figure 3: Control phase iii), for the same parameter values as in Figs. 1 and 2. The control parameters are listed in Table 1

3 Stability

To analyse the controllability of the robotic leg, we have tested numerically the regions of the control parameters $|\mathbf{v}_0|$, γ_0 and α such that the foot leg does not hit the ground during the control phase ii), for a fixed choice of the other parameters of Fig. 1. The controllability parameter regions are depicted in Fig. 4. As a controllable reference case, we take the robotic leg with the parameters in table 1.

Parameter	Robot leg	Control
m_1	60 kg	
m_2	0.5 kg	
m_3	0.5 kg	
ℓ_1	0.5 m	
ℓ_2	0.5 m	
k_1	12000 N/m	
k_2	12000 N/m	
$\|\mathbf{v}_0\|$		4.0 m/s
γ_0		$-50°$
α		70°
α_0		110°
θ_r		200°
ω_{fa}		-20 rad/s

Table 1: Reference mechanic parameters of the robotic leg, and corresponding control parameters.

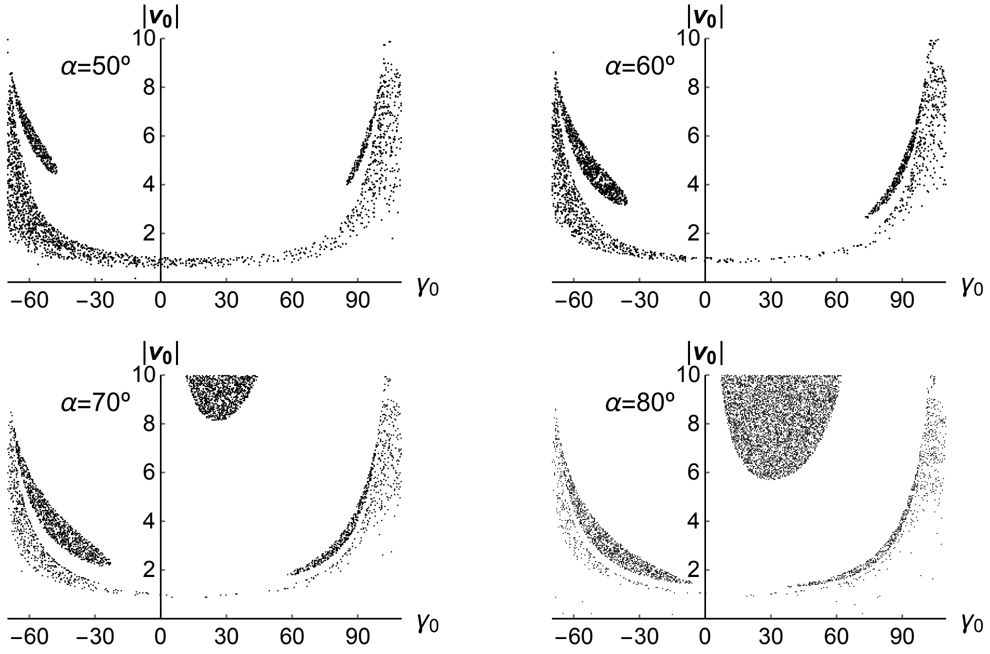

Figure 4: Regions of the parameter space $(\gamma_0, |\mathbf{v}_0|)$ and several values of the attack angle α for which the robotic leg is above the ground, during control phase ii). The parameter values of the simulation are: $k_1 = k_2 = 12000$ N/m, $m_1 = 60$ kg, $m_2 = m_3 = 1$ kg, $\ell_1 = \ell_2 = 0.5$ m and $\alpha_0 = 110°$.

4 Simulations

5 Conclusions

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References

[Vejdani *et al*, 2015] H. R. Vejdani, A. Wu, H. Geyer, and J. W. Hurst, Touch-down angle control for spring-mass walking, 2015 IEEE International Conference on Robotics and Automation (ICRA) Washington State Convention Center Seattle, Washington, May 26-30, 2015.