

A simple algorithm capable of stable gait in a noisy environment?

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Abstract:

In this paper, we present an algorithm based on a fixed aperture angle between the stance leg and the flying leg. In noiseless environments, the algorithm is able to deliver unbounded walking, with stable gaits typically found in grounded walking or grounded running. Stable solutions may have irrational period and deliver rather different phase velocity diagrams with apex return maps returning to the same point after several steps. The algorithm is also capable of delivering long walks under the constant influence of substantial noise levels up to 10 cm, suggesting the possibility of being used alongside with active control in the future as gait instabilities are shown to build up slowly.

1. Introduction

A walking robot that can actually perform tasks, meaning that it was not simply designed for producing a specific gait, is a very important step towards bringing robotics out of the factory floor and into our daily lives. Pioneering work was kicked-off at Carnegie Mellon university in the 80's, with the Raibert hopper [1] being one of the most famous examples having been extensively analyzed by different authors [2].

In recent years, a nascent field in Robotics has focused on humanoid robotics, whereby bipedal systems with human comparable sizes and weights have been analyzed in detail [3]. However, most of the algorithms proposed control each leg in isolation [4] rather than one leg relative to the other [5][6].

The objective of this paper is to identify a simple theoretical algorithm that can set the bases so that, in the future, it can be extended to control a full-scale robot capable of:

- Walking at typical human walking/running speeds (up to 6-7 m/s) [7]
- Producing low peak forces with low or slowly changing instabilities
- Operating on a noisy terrain, typical of an outdoors environment

The model used in our simulations is a spring mass model with two massless legs and a point mass. While this model is not extensible to a real robot, it is a good starting point to understand the dynamics of a legged system without the additional complications introduced by simulating a rigid body

2. Model used in simulations

In figure [A], a simplified diagram presents a visual representation of the fixed aperture algorithm applied to the system under the influence of gravity. Restricting the motion to a 2D plane, the Center of Mass (CoM) dynamics during stance can be described by are:

$$m\ddot{x} = -k(l_0 - l_1) \cos \alpha_1 - k(l_0 - l_2) \cos \alpha_2 \quad (1)$$

$$m\ddot{y} = -mg + k(l_0 - l_1) \sin \alpha_1 + k(l_0 - l_2) \sin \alpha_2, \quad (2)$$

Where α_1 is the angle of attack the leading leg and $\alpha_2 = \alpha_1 - \Phi_0$.

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During the flight phases, the spring mass terms are zero (equivalent to setting $k=0$) and the CoM performs a parabolic trajectory under the effect of gravity. Both legs will have the same stiffness and the only control variable used in the simulations will be the aperture leg between both legs.

Given that this algorithm is designed mostly for walking and grounded running, it is based on a trailing leg that is in stance and a leading leg that is in flight. Thus, the control rules applied will be the following:

- a) Both legs are flying: the trailing leg will have a 90-degree angle with the ground, and the leading leg will have an angular offset equal to the aperture angle
- b) One leg is in stance: the foot on the ground will have no slippage and will not be actuated by the algorithm, while the foot in flight will have an angular offset equal to the aperture angle
- c) Both legs on the ground: the algorithm takes no action whatsoever

The rest length of each leg is represented by l_0 . The lengths l_1 and l_2 are the current length of each leg, measured by $l_i = ((x_{Fi} - x_{CoM})^2 + (y_{Fi} - y_{CoM})^2)^{1/2}$, where (x_{Fi}, y_{Fi}) describes the current position of a given foot and (x_{CoM}, y_{CoM}) the current position of the CoM.

3. Simulations

The following parameters were set constant across all simulations: $m = 80$ kg, $K_r = K_l = 12,000$ N/m, $g = 9.81$ m/s², $l_0 = 1$ m while multiple initial conditions of horizontal speed and vertical position were tested with different aperture angles (the control parameter being tested). All simulations were performed in Matlab with absolute and relative precision set to 10^{-11} using ode45 as the integration algorithm.

4. Results obtained

4.1 Noiseless Environment

The first analysis performed was a sweep across multiple conditions and aperture angles. All simulations started at the apex position, with a total of 40,131 different simulations ran:

- a) Initial horizontal speed: 21 speeds ranging from 0 to 7 m/s
- b) Initial vertical position: 21 speeds ranging from 0.8 to 1.2 m
- c) Aperture angle between legs: one tested for each aperture angle from 0 to 90 degrees

To determine a stable gait, the following tests were used: the model returns to apex height (either after 1 step or several steps), the basin of attraction is stable around that point and the elastic potential energy is zero at that point. The results presented in the table of figure [B] show the difficulty of obtaining stable gaits from any initial condition, with only 0.8% of all combinations tested achieving a stable gait.

By segmenting the initial conditions, it is possible to infer that some energy bands are more likely to have stable gaits than others. On a first inspection, the best initial conditions seem to be located in the $[0, 1[$ m/s for initial horizontal speed and $[1, 1.1[$ m for initial vertical position. However, a closer inspection reveals that the majority of the stable solutions found are either stationary (no movement in x) or actually correspond to a movement walking backwards after the flying foot collides with the ground.

Furthermore, Figure [C] shows that although the initial energy plays a role in delivering stable gaits, it is also clear that the end result depends substantially on the way that energy is injected into the system (either through more initial gravitational potential or more kinetic energy). In addition, it is possible to see that the shape of the bands where stable gaits were found changes from one energy band to another.

Finally, figure [D] shows the velocities phase diagram for two different initial conditions. In both cases presented, the phase diagram is somewhat unexpected as the limit cycles are not completely stable (there is a precession across time). This kind of behavior is not present in one legged systems such as the Raibert Hopper [1] [2]. Moreover, their shapes can change substantially across different initial conditions. Also of notice in figure [E], is the fact that all vertical forces are symmetric in time.

4.2 Noisy Environment

To test the resilience of the algorithm, we selected one of the stable configurations and made it transverse an environment whereby a uniform random noise was overlaid on top of the ground level. The simulation was then initiated and ran for up to 60 seconds (or less in the case the walker fell to the ground). This was repeated 1,000 times with noise amplitudes ranging from 0 to 10 cm. The noise was always present during the simulation, not being limited to a single step-up step-down.

In figure [F] we present the results obtained with the detail of the horizontal displacement achieved, the time walked, the number of steps and the final horizontal speed. It is important to refer that the stability of the model (measured by either travelled speed, number of steps or horizontal displacement) substantially decreases as the noise level is increased. Moreover, final backwards gait was also detected.

5. Final Discussion

The results obtained are broadly in line with [5], in the sense that symmetrical forces were obtained in both papers, stable solutions were typically associated with grounded gaits, and leg compression rates were within comparable ranges. This is somewhat remarkable as the previous paper was done with quails and we the presented simulations use weight and length parameters at human scale. Additionally, in this paper we presented the new fact that phase diagrams change substantially across different initial conditions and aperture angle controls, with a behavior typical of an irrational period.

In noisy environments, under constant noise, the algorithm stability decreased substantially faster than in the case of a single step-up step-down disturbance [5], to the extent that backwards travelling was initiated under some conditions. Nonetheless, walking times were always above several seconds, opening the possibility of acting on these instabilities through active control. In these simulations we have kept the walker under the constant influence of the noise (sampling rate 0.1 s) whereas previous papers [5] have only considered a step-up step down localized noise.

Future work to extend these findings into a walking robot could focus on:

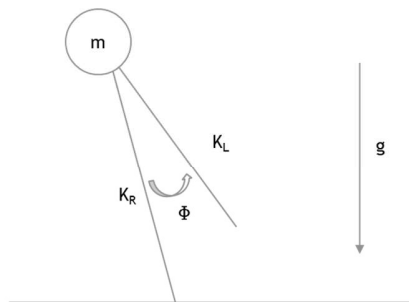
- a) Introducing variable stiffness through feed-forward leg placement or muscle pre-activation (as most legged animals do when they adjust their gait to remove instabilities)
- b) Introducing the ability to add/subtract energy from the system through actuation (enabling for example our robot to go up and down a ramp)
- c) Finding analytically when a gait will be stable/periodic (even not capable of locating the next apex)
- d) Predicting analytically apex points for any initial condition (e.g. planets XYZ orbits are well defined although until today it is impossible to derive a time dependent equation)

6. Acknowledgements

We would like to thank professor Hartmut Geyer for the valuable insights provided this semester.

Figure A: Model used in simulations

Each leg is a spring of equal stiffness



$m = 80 \text{ kg}$, $K_R=K_L=12,000 \text{ N/m}$, $g=9.81 \text{ m/s}^2$, $l_0 = 1 \text{ m}$
 Φ = fixed control

The algorithm

Both legs flying	<ul style="list-style-type: none"> Falling body under gravity field (leading leg with 90 angle of attack)
One leg in the air	<ul style="list-style-type: none"> One foot on the ground with no slippage, not controlled by the algorithm Another foot at angle Φ measured from the leg in stance
Two legs in stance	<ul style="list-style-type: none"> Algorithm takes no action whatsoever

Figure B: Success Rates achieving a stable gait for multiple aperture angles tested across different initial conditions (across initial vertical position and initial horizontal speed) without noise

Initial vertical position (m) - >

Vx0 (m/s)	0.8-0.9	0.9-1	1-1.1	1.1-1.2	Total Geral
0-1		0.0%	21.7%	0.5%	7.4%
1-2	0.0%	0.4%	1.7%	0.7%	0.6%
2-3	0.0%	2.6%	0.1%	2.2%	1.2%
3-4	0.0%	1.8%	0.0%	0.1%	0.5%
4-5	0.0%	1.1%	0.0%	0.5%	0.4%
5-6	0.0%	2.1%	0.0%	0.0%	0.5%
6-7	0.0%	2.2%	0.0%	0.4%	0.7%
Total Geral	0.0%	1.6%	1.0%	0.5%	0.8%

Figure C: Initial conditions where a stable gait was found for at least one aperture angle. The red dotted lines highlight the stable solutions found in four different energy bands. No noise applied.

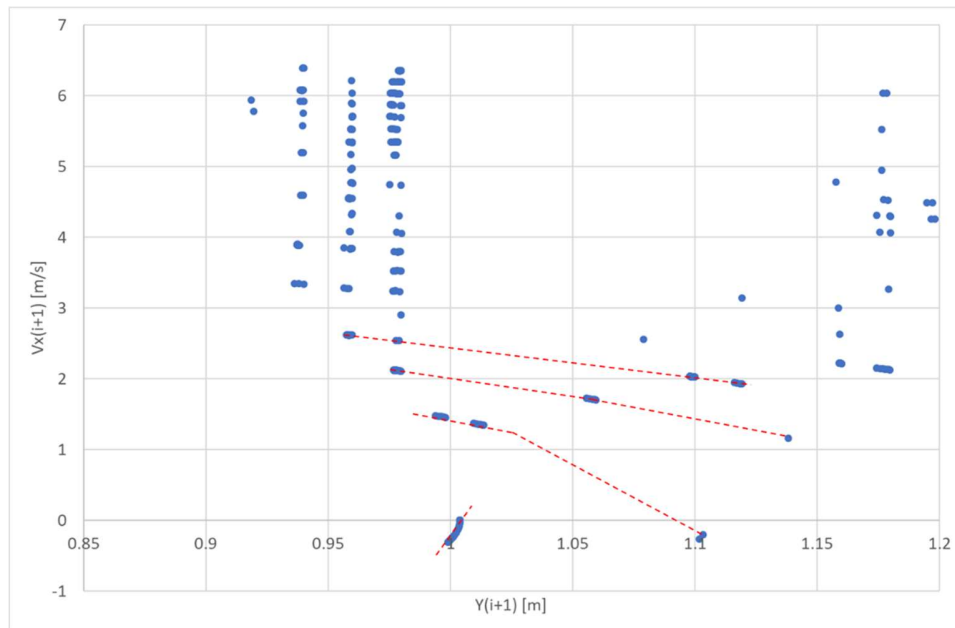
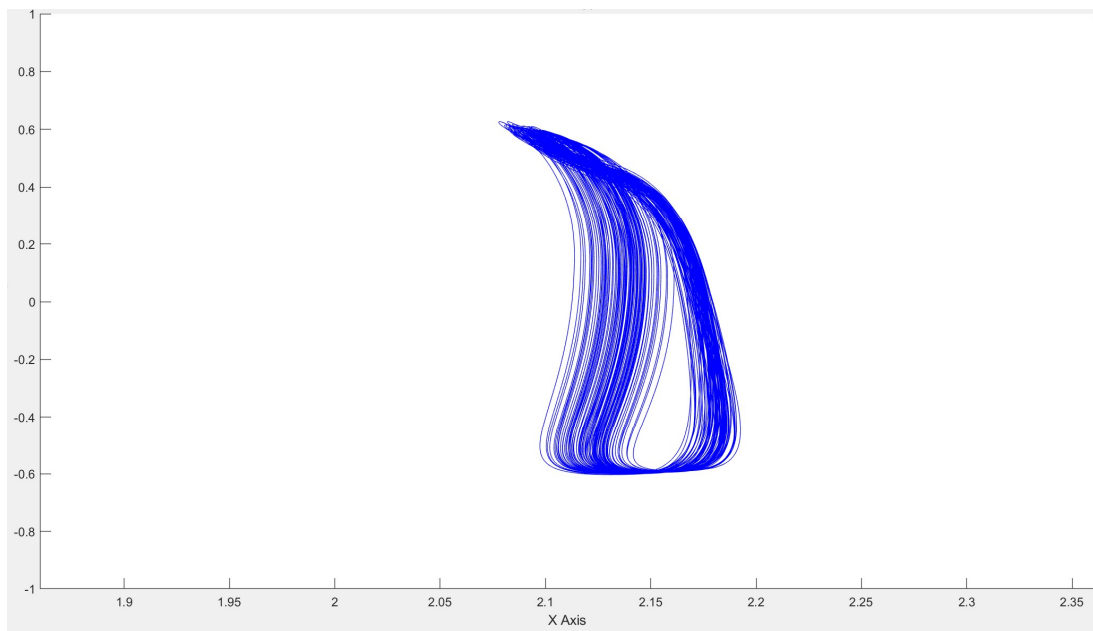
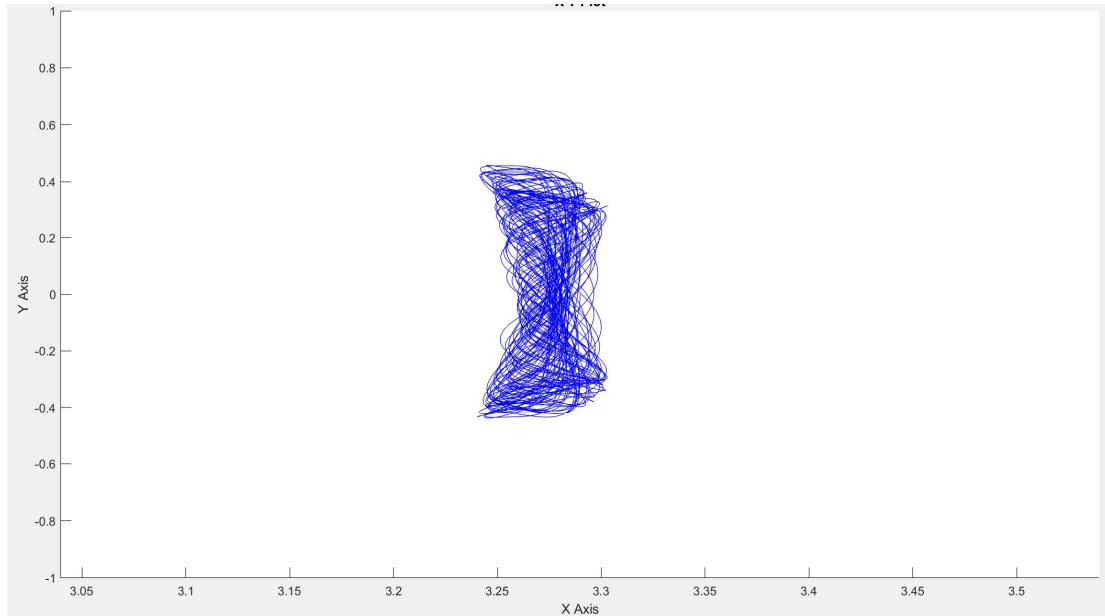


Figure D1: Examples of Velocity Phase Diagrams (V_x vs V_y) obtained for different initial conditions



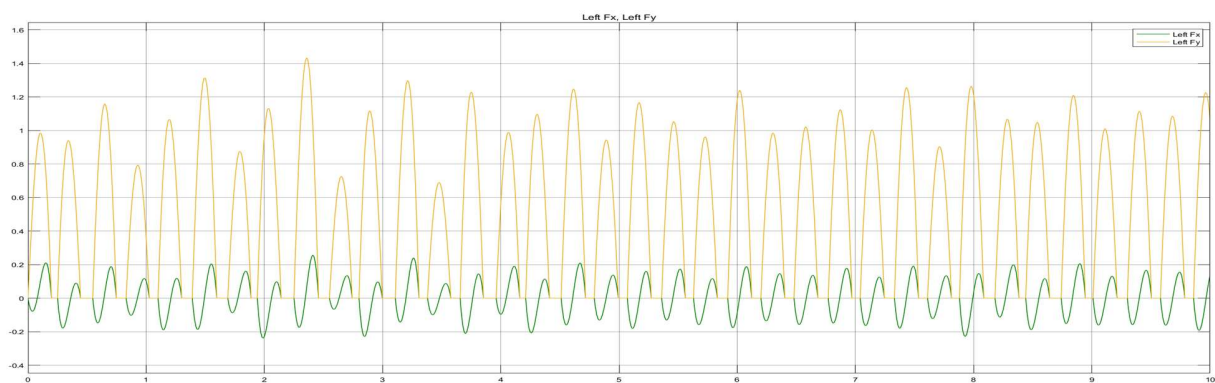
Initial conditions and control parameter: $\Phi = 31$ degrees, $V_{x0} = 2.11$ m/s, $Y_0 = 0.98$ m ($E_0 = 947$ J) no noise applied.

Figure D2: Examples of Velocity Phase Diagrams (V_x vs V_y) obtained for different initial conditions



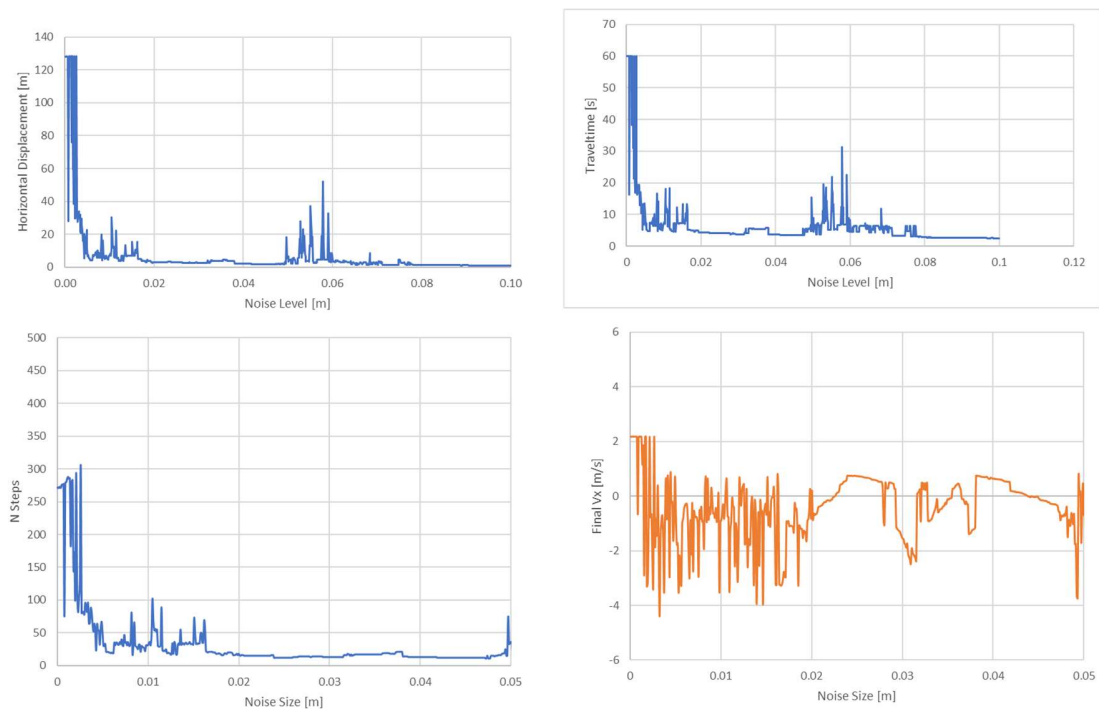
Initial conditions and control parameter: $\Phi = 28.25$ degrees, $V_{x0} = 3.29$ m/s, $Y_0 = 0.96$ m ($E_0 = 1,186$ J). No noise applied.

Figure E: Vertical and Horizontal forces obtained for the left leg as a weight ratio



Initial conditions and control parameter: $\Phi = 28.25$ degrees, $V_{x0} = 3.29$ m/s, $Y_0 = 0.96$ m ($E_0 = 1,186$ J). No noise applied.

Figure F: Results obtained for different levels of noise overlaid on top of the ground level for a configuration that was stable in a noiseless environment.



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