INSTITUTO SUPERIOR TÉCNICO

PROJECTO MEFT

Thesis Title

Author:
Ludwig BOLTZMANN

Supervisor:
Dr. Jožef ŠTEFAN

Research work performed for the Master in Engineering Physics

at

Research Group Name
Department or School Name

December 29, 2018

1 Introduction

One of the key factors that distinguish humans from other primates is that they are capable of having a bipedal walking/running gait. This aspect served as an evolutionary remark towards our movement because humans could have a much higher mobility aswell as having a bigger vision range than before against their preys and it's predators. This along with the development of technology allowed for humans to have a bigger advantage against natural conditions and therefore contributed for our survival as species. Since the way we walk is unique and very complex [**Mochon & McMahon**] regarding the rest the animal kingdom, a push has been made to reproduce the same type of walking and running in robots and other types of machines.

To accomplish the goal of imitating the same pattern of human walking and running, [**Seyfarth et al. 2006**] describes that in the seventeenth century, Giovanni Afonso Borelli wrote "De motu Animalum", the first treatise on biomechanics, where he compared walking with vaulting over stiff legs although, while for running, he declared that the rebounding effect on compliant legs was important [**Borelli 1685**]. In an attempt to reproduce the same effects of human walking and running two models were created with entirely different dynamics. Walking was developed by [**Alexander 1976; Mochon & McMahon 1980;Cavag** as a inverted pendulum model, while running was devolped by [**Blickhan 1989; McMahon & Cheng 1990**], as a spring-mass model. With these models, several properties could be obtained such as the kinetic and potential energies, the speed and frequency aswell as other aspects for the associated gait. The spring-mass model was able to explain the basics of running, although, regarding walking , it was verified this type of gaiting could not be imitated by using an inverted pendulum model [**Full & Koditchek 1999**]. One of the differences between the model and the experimental data points, for example, was the M shaped ground reaction force that is not present in the inverted pendulum model since delta functions appear at the beggining and ending of one step [**Pandy 2003**].

With this in account, [**Seyfarth et al. 2006**] proposes a two compliant leg model consisting of two springs attached to a mass, being able to reproduce the same effects of walking with a relatively simple description despite the large complexity of the movement. In this paper, the stable parameter map is determined in relation with the variation of physical parameters such as the spring stiffness k. Also, it is shown that walking and running can be encapsulated into the same model with the energy associated to the system being the mediator between one type and the other type of gaiting. After the success, [**Rummel et al. 2010**] uses the same model to determine 5 types of walking gaits, aswell as to test the robustness of the model by determining the regions of attraction along with the saddle region associated to the parameter space.

2 State Of The Art

Starting with [**Blickhan1989**] a running/hopping model is analysed. In this model a spring-mass system is introduced by connecting the Center of Mass (CoM) to a massless spring that can describe the dependency of the different physical variables that charecterize running/hopping. Hopping forward can be described by a non-linear system of equations described by an inverted pendulum spring propagating forward in which there are two phases, the ground phase,

$$\ddot{x} = x\omega^2 \left(\frac{l}{\sqrt{x^2 + y^2}} - 1 \right), \tag{1}$$

$$\ddot{y} = y\omega^2 \left(\frac{l}{\sqrt{x^2 + y^2}} - 1 \right) - g, \tag{2}$$

and the aerial phase, where,

$$\ddot{x} = 0, \tag{3}$$

$$\ddot{y} = -g. \tag{4}$$

In this system, m is the mass, $\omega^2 = \frac{k}{m}$ is the natural frequency of the spring, g is the gravitational acceleration, x is the horizontal displacement, y is the vertical displacement and $l = \sqrt{x_0^2 + y_0^2}$ is the rest length of the spring.
Fig. (1) helps to illustrate the model presented. In this figure, the mass touches the ground, starting the ground phase, with an angle of atack α, with an initial velocity $\vec{v}$ where β is the angle that the velocity makes with the ground. After the spring touches the ground, it is compressed in the normal direction, and there is a propagation to the movement of the mass forward becouse the spring rotates around it's pivot point in the ground. After the spring reaches the same height as when it first touched the ground, it starts the aerial phase where the spring becomes disconnected from the ground until the mass reaches an height of $l \cos \alpha$ restarting the ground phase.

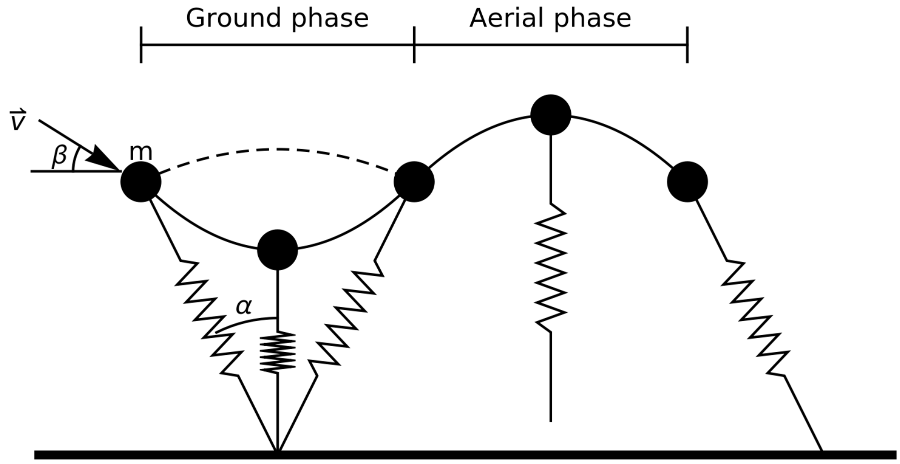

FIGURE 1: caption

The determination of the physical solutions are determined by narrowing the range of the parameters studied within the set of possible solutions. With this parameters, we need a small subset of them, the vector of landing velocity and leg length in this model to determine the state of the system and it's evolution.Since this is a physiological model, some of the parameters cannot be changed. An human subject maximizes the amount of energy that can be stored and delivered elastically [**Blickhan1989**].

This can be achieved by maximizing the contact length and this is why animals use flat angles on the landing velocities.

Analysing [**Vejdani**], a model of two compliant legs is proposed. In this model, when one leg is on the ground, the system is equivalent to a inverted spring pendulum, besides single support, this model also contemplates double support, where the two springs in this phase influence the movement of the CoM in reproducing a bipedal spring-mass walking. The springs accumulate and deliver energy so that the system remains convervative with no energy losses.
In single phase support, the dynamics of the center of mass (CoM) is as follows,

$$\ddot{x} = \frac{F_1}{m}\frac{x - x_{t1}}{l_1}, \tag{5}$$

$$\ddot{y} = \frac{F_1}{m}\frac{y - y_{t1}}{l_1} - g. \tag{6}$$

With double support, the dynamics are different from the previous case since we are in the presence of two springs, this is,

$$\ddot{x} = \frac{F_1}{m}\frac{x - x_{t1}}{l_1} + \frac{F_2}{m}\frac{x - x_{t2}}{l_2}, \tag{7}$$

$$\ddot{y} = \frac{F_1}{m}\frac{y - y_{t1}}{l_1} + \frac{F_2}{m}\frac{y - y_{t2}}{l_2} - g, \tag{8}$$

with F_i being the force applied on the mass by the respective leg,

$$F_i = k(l_0 - l_i) \geq 0 \quad i = 1, 2\,, \tag{9}$$

l_0 is the natural length of the spring, l_i is the respective length,

$$l_i = \sqrt{(x - x_{ti})^2 + (y - y_{ti})^2} \quad i = 1, 2\,. \tag{10}$$

(x, y) are the coordinates of the center of mass (CoM), (x_{ti}, y_{ti}) are the coordenates of the respective toes for the leg 1 and leg 2. Fig. (2) illustrates the model, single support to double support transitions occur when the center of mass drops to a height of $y \sin \alpha$ where α is the angle of attack and the vertical velocity is negative. Double support to single support transitions occur when the spring deflection $l_0 - l_i$ of one of the legs return to 0.

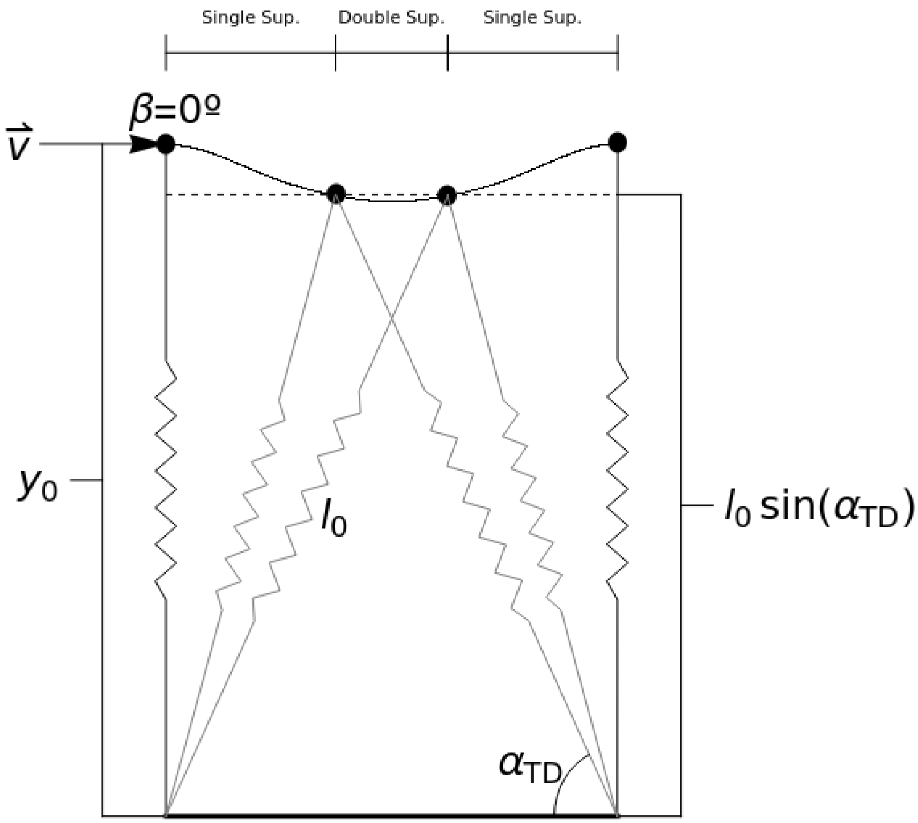

FIGURE 2: teste

Let the Poincare section be defined as the vector $\psi = [y_n, \beta_n]^T$ at the Vertical Leg Orientation (VLO), meaning that everytime the system is in the single support phase and it is at the vertical position we record the vertical position of the CoM y_n and β_n, the angle that the velocity makes with the ground at this phase and this defines the step n of the simulation. This model is energetically conservative, therefore, the energy E is a constant with,

$$E = \frac{k(l_0 - y_n)^2}{2} + mgy_n + m\frac{v_n^2}{2}. \tag{11}$$

Knowing this, we can express the absolute value of the velocity in terms of the energy. This way, in this model, the only parameters in the initial conditions that can alter the stability of the system for a certain angle of attack α, and energy E are β_0, the angle of the initial velocity with the ground and y_0, the initial vertical position of the CoM.

By allowing the simulation to run over one step, we can associate a map, which is called the Poincare map. By defining a function A which iterates the Poincare section at the vertical position, this way,

$$\psi_{n+1} = A\psi_n. \tag{12}$$

By recurrence we can apply the A function $n+1$ times from the initial state ψ_0 to get to the final step $n+1$. Not all solutions are admited, the same criteria were applied