

Control of a Robotic Leg for Walking, Running and Hopping in Irregular Surfaces

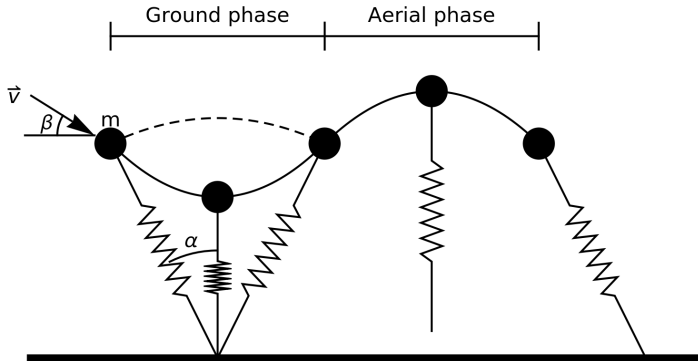
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Running and hopping model



Equations

Ground phase

$$\ddot{x} = x\omega^2 \left(\frac{l}{\sqrt{x^2 + y^2}} - 1 \right), \quad (1)$$

$$\ddot{y} = y\omega^2 \left(\frac{l}{\sqrt{x^2 + y^2}} - 1 \right) - g, \quad (2)$$

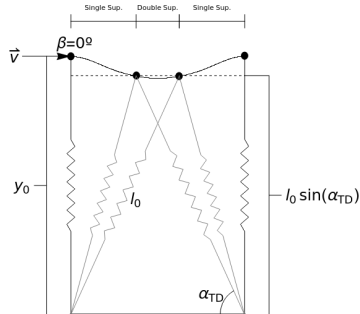
Aerial phase

$$\ddot{x} = 0, \quad (3)$$

$$\ddot{y} = -g. \quad (4)$$

with $\omega = \sqrt{k/m}$, the natural frequency of the spring and l rest length of the spring.

Walking model



Definitions:

- stride - Leg crosses the vertical leg orientation
- step - The moment when the system passes from single support to double support.

Equations

Single support

$$\ddot{x} = \frac{F_1}{m} \frac{x - x_{t1}}{l_1} \quad (5)$$

$$\ddot{y} = \frac{F_1}{m} \frac{y - y_{t1}}{l_1} - g \quad (6)$$

Double support

$$\ddot{x} = \frac{F_1}{m} \frac{x - x_{t1}}{l_1} + \frac{F_2}{m} \frac{x - x_{t2}}{l_2} \quad (7)$$

$$\ddot{y} = \frac{F_1}{m} \frac{y - y_{t1}}{l_1} + \frac{F_2}{m} \frac{y - y_{t2}}{l_2} - g \quad (8)$$

with F_i being the force applied on the mass by the respective leg,

$$F_i = k(l_0 - l_i) \geq 0 \quad i = 1, 2, \quad (9)$$

l_0 is the natural length of the spring, l_i is the respective length,

$$l_i = \sqrt{(x - x_{ti})^2 + (y - y_{ti})^2} \quad i = 1, 2. \quad (10)$$

Since the system is energetically conservative we can change the initial velocity by inverting

$$E = \frac{k(l_0 - y_0)^2}{2} + mgy_0 + m\frac{v_0^2}{2}. \quad (11)$$

Parameters

A scan is made with 3 parameters, $Energy$, y_0 , α in two strides

- $Energy \in [800, 840]$ with 40 subdivisions.
- $\alpha \in [\pi/2 - \pi/5, \pi/2]$ with 30 subdivisions.
- $y_0 \in [l_0 \sin(\alpha), l_0]$ with 25 subdivisions

In all simulations the following parameters remained fixed.

- $\beta = 0$
- $m = 80Kg$
- $l_0 = 1m$
- $k = 14000N/m$

Total number of configurations= $40 \times 30 \times 25 = 30000$

Survival step configurations

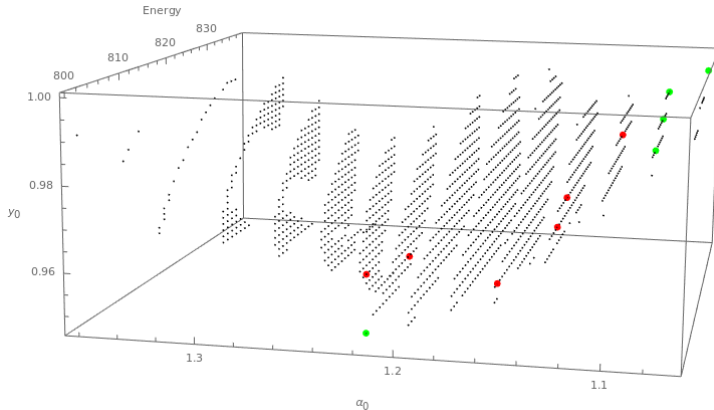
Out of the 30000 configurations in the parameter space, only 8195 were able to complete 2 strides. Of this subset of points, 11 fixed points were found, 5 stable and 6 unstable.

From here a survival test is applied to each of the 8195 configurations that completed 2 strides by incrementing steps instead of strides.

If the simulation fails, the maximum number of steps was assigned to that configuration.

Fixed points and 10 step configurations

Iterating the number of steps where $\Delta t = 0.00075$ so that in maximum it was possible to achieve 10 steps, the configurations that achieve 10 steps can be illustrated in the figure below along with the fixed points.



Touch down control policy

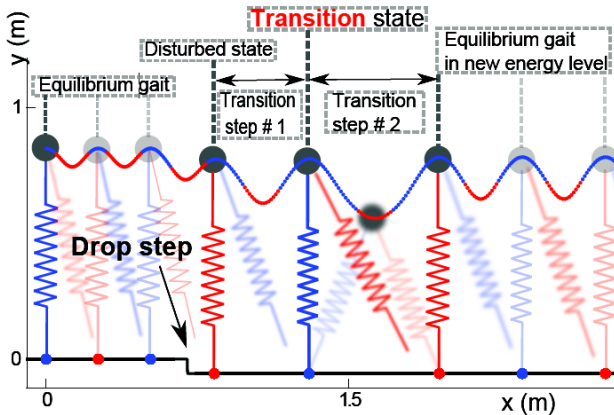
Defining this time $\psi_n = (y_n, \beta_n)$, and an equilibrium state (ψ^*, θ) with $\psi^* = (y^*, \beta^*)$ the desired poincare section, if we find the combination,

$$A(\psi_n, \theta_n) = \psi_{n+1}, \quad (12)$$

$$A(\psi_{n+1}, \theta_{n+1}) = \psi^*. \quad (13)$$

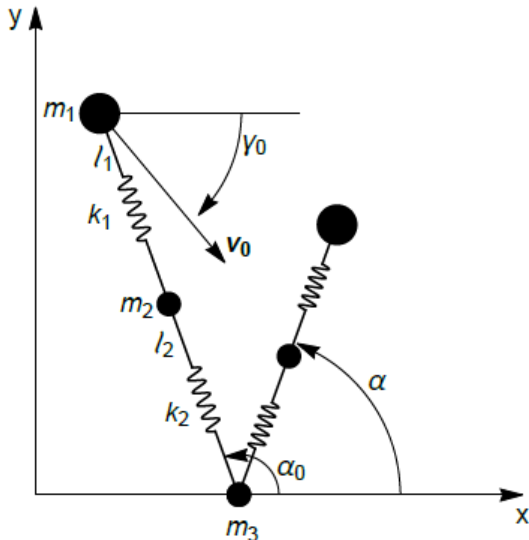
The angle of attack is given by,

$$\theta_n = A^{-1} \Big|_{\psi_n} (\psi_{n+1}) \quad (14)$$



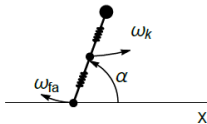
Credit of the image given to: Hamid Reza Vejdani et al. "Touch-down angle control for spring-mass walking", International Conference on Robotics and Automation (ICRA), IEEE (2015), 5101–5106.

Running and hopping model - One knee

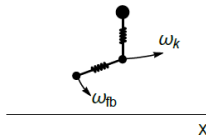


Running and hopping model - One knee

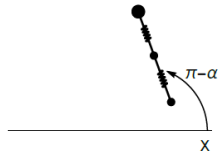
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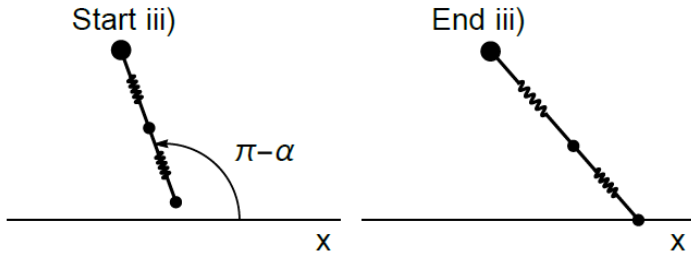
End iia)–Start iib)



End iib)



Running and hopping model - One knee



	Fev	Mar	April	May	June	July	August	Sept
Refining the stability analysis of the two legs walking model								
Development and analysis of the one leg two-springs model (walking and running)								
Development and analysis of the two leg two-springs model (walking and running)								
Thesis writting								