Contents

Partielle Integration (Produktintegration)

Idee aus der Produktregel (Ableitung)

$$f(x) = u(x)v(x)$$
$$f'(x) = u'(x)v(x) + u(x)v'(x)$$
$$f'(x) - u'(x)v(x) = u(x)v'(x) \mid \text{ integrieren}$$
$$\int_a^b f'(x) - u'(x)v(x) = \int_a^b u(x)v'(x)$$
$$\int_a^b f'(x)\mathrm{d}x - \int_a^b u'(x)v(x)\mathrm{d}x = \int_a^b u(x)v'(x)\mathrm{d}x$$
$$[f(x)]_a^b - \int_a^b u'(x)v(x)\mathrm{d}x = \int_a^b u(x)v'(x)\mathrm{d}x$$
$$[u(x)v(x)]_a^b - \int_a^b u'(x)v(x)\mathrm{d}x = \int_a^b u(x)v'(x)\mathrm{d}x \text{ (Regel für die partielle Integration)}$$

Nebenrechnung

$$[f(x)]_{x=a}^{x=b} = f(x=b) - f(x=a)$$
$$[u(x)v(x)]_a^b = u(b)v(b) - u(a)v(a)$$

Anwendung

$$I = \int_a^b x \cos x \mathrm{d}x$$
$$x = u(x)$$
$$\cos x = v'(x)$$

$$u(x) = x$$
$$u'(x) = 1$$
$$v(x) = \sin x$$
$$v'(x) = \cos x$$

$$I = [x \sin x]_a^b - \int_a^b 1 \cdot \sin x \mathrm{d}x$$
$$I = [x \sin x]_a^b - [-\cos x]_a^b$$
$$I = [x \sin x + \cos x]_a^b$$

Nebenrechnung

$$f(x) = \sin x$$
$$f'(x) = \cos x$$
$$f''(x) = -\sin x$$
$$f'''(x) = -\cos x$$
$$f''''(x) = \sin x \text{ usw.}$$

Aufgabe 252/4

$$I = \int_0^\pi x \sin x \mathrm{d}x$$
$$u(x) = x$$
$$u'(x) = 1$$
$$v(x) = -\cos x$$
$$v'(x) = \sin x$$
$$I = [x \cdot (-\cos x)]_0^\pi - \int_a^\pi -\cos x \mathrm{d}x$$
$$I = [x \cdot (-\cos x)]_0^\pi - [-\sin x]_0^\pi$$
$$I = [x \cdot (-\cos x) + \sin x]_0^\pi$$

Aufgabe 252/5

(partielle Integration mehrfach anwenden)

$$I = \int_a^b x^2 \cos x \mathrm{d}x$$
$$a = -\frac{\pi}{2}$$
$$b = \frac{\pi}{2}$$
$$u(x) = x^2$$
$$u'(x) = 2x$$
$$v(x) = \sin x$$
$$v'(x) = \cos x$$
$$I = [x^2 \sin x]_a^b - \int_a^b 2x \sin x \mathrm{d}x$$
$$I = [x^2 \sin x]_a^b - ([2x(-\cos x)]_a^b - \int_a^b 2(-\cos x) \mathrm{d}x)$$
$$I = [x^2 \sin x]_a^b - ([2x(-\cos x)]_a^b - [2(-\sin x)]_a^b)$$
$$I = [x^2 \sin x]_a^b - [2x(-\cos x) - 2(-\sin x)]_a^b$$
$$I = [x^2 \sin x - 2x(-\cos x) - 2(-\sin x)]_a^b$$
$$I = [x^2 \sin x + 2x \cos x + 2 \sin x]_a^b$$

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Aufgabe 252/6 a)

(partielle Integration mehrfach anwenden)

$$a = -\frac{\pi}{2}$$
$$b = \frac{\pi}{2}$$
$$I = \int_a^b (\sin x)^2 \mathrm{d}x$$

$$I = \int_a^b \sin x \sin x \mathrm{d}x$$
$$u(x) = \sin x$$
$$u'(x) = \cos x$$
$$v(x) = -\cos x$$
$$v'(x) = \sin x$$
$$I = [\sin x \cdot (-\cos x)]_a^b - \int_a^b \cos x \cdot (-\cos x) \mathrm{d}x$$
$$I = [\cos x \cdot (-\sin x)]_a^b - \int_a^b -(\sin x) \cdot (-\sin x) \mathrm{d}x$$
$$I = [\cos x \cdot (-\sin x)]_a^b - [\cos x \cdot (-\sin x)]_a^b - [\cos x \cdot \cos x]_a^b$$
$$I = [\cos x \cdot (-\sin x) - \cos x \cdot (-\sin x) - \cos x \cdot \cos x]_a^b$$
$$I = [-\cos x \cdot \sin x + \cos x \cdot \sin x - \cos x \cdot \cos x]_a^b$$
$$I = [-(\cos x)^2 - \cos x \cdot \sin x + \cos x \cdot \sin x]_a^b$$

Aufgabe 252/7

(Stammfunktion ermitteln)

a)

$$f(x) = x \cdot (x-1)^4$$
$$u(x) = x$$
$$u'(x) = 1$$
$$v(x) = \frac{1}{5}(x-1)^5$$
$$v'(x) = (x-1)^4$$
$$F(x) = x \cdot \frac{1}{5}(x-1)^5 - \frac{1}{30}(x-1)^6$$

$$F(x) = \frac{1}{5}x(x-1)^5 - \frac{1}{30}(x-1)^6 \quad \text{(Musterlösung)}$$

b)

$$\begin{aligned} f(x) &= (2x+5)\cdot(x+2)^8 \\ u(x) &= (2x+5) \\ u'(x) &= 1 \\ v(x) &= \frac{1}{9}(x+2)^9 \\ v'(x) &= (x+2)^8 \\ F(x) &= (2x+5)\frac{1}{9}(x+2)^9 - \frac{1}{90}(x+2)^{10} \end{aligned}$$

$$F(x) = (2x+5)\cdot\frac{1}{9}(x+2)^9 - \frac{2}{90}(x+2)^{10} \text{ (Musterlösung)}$$

c)

$$\begin{aligned} f(x) &= \cos x\cdot(x-1) \\ u(x) &= \cos x \\ u'(x) &= -\sin x \\ v(x) &= \frac{1}{2}(x-1)^2 \\ v'(x) &= (x-1) \\ F(x) &= \cos x\cdot\frac{1}{2}(x-1)^2 - \int(-\sin x)\cdot\frac{1}{2}(x-1)^2\mathrm{d}x \end{aligned}$$

Nebenrechnung

$$\begin{aligned} f(x) &= (-\sin x)\cdot\frac{1}{2}(x-1)^2 \\ u(x) &= -\sin x \\ u'(x) &= -\cos x \\ v(x) &= \frac{1}{6}(x-1)^3 \\ v'(x) &= \frac{1}{2}(x-1)^2 \\ F(x) &= (-\sin x)\frac{1}{6}(x-1)^3 - \int(-\cos x)\frac{1}{6}(x-1)^3\mathrm{d}x \end{aligned}$$

$$F(x) = \cos x + (x-1)\cdot\sin x \text{ (Musterlösung)}$$