Numerical Linear Algebra Homework Project 3: Eigenvalues and Eigenvectors

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Problem 1

(1)

$$A = \begin{bmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{bmatrix} = \left[\begin{array}{c|c} a_{11} & \mathbf{x}^T \\ \hline \mathbf{x} & \hat{A}_1 \end{array}\right] \tag{1}$$

$$\mathbf{u}_1 = \mathbf{x} + \operatorname{sgn}(x_1)\|\mathbf{x}\|_2 \mathbf{e}_1 = \begin{bmatrix} -1-\sqrt{2} \\ -1 \\ 0 \end{bmatrix} \tag{2}$$

$$\hat{R}_1 = \mathbb{1}_3 - \frac{2}{\|\mathbf{u}_1\|_2^2}\mathbf{u}_1\mathbf{u}_1^T \tag{3}$$

$$R_1 = \left[\begin{array}{c|c} 1 & \mathbf{0}^T \\ \hline \mathbf{0} & \hat{R}_1 \end{array}\right] \tag{4}$$

$$R_1 A = \left[\begin{array}{c|c} a_{11} & \mathbf{x}^T \\ \hline -\operatorname{sgn}(x_1)\|\mathbf{x}\|_2 & \\ 0 & \hat{R}_1\hat{A}_1 \\ 0 & \end{array}\right] \tag{5}$$

$$\hat{A}_1 = \left[(\hat{A}_1)_1, (\hat{A}_1)_2, (\hat{A}_1)_3\right] \tag{6}$$

$$\hat{R}_1\hat{A}_1 = \left[(\hat{R}_1\hat{A}_1)_1, (\hat{R}_1\hat{A}_1)_2, (\hat{R}_1\hat{A}_1)_3\right] \tag{7}$$

$$(\hat{R}_1\hat{A}_1)_i = (\hat{A}_1)_i - \frac{2}{\|\mathbf{u}_1\|_2^2}\mathbf{u}_1\mathbf{u}_1^T(\hat{A}_1)_i \tag{8}$$

$$R_1 A = \begin{bmatrix} 4 & -1 & -1 & 0 \\ \sqrt{2} & -2\sqrt{2} & -2\sqrt{2} & \sqrt{2} \\ 0 & -2\sqrt{2} & 2\sqrt{2} & 0 \\ 0 & -1 & -1 & 4 \end{bmatrix} \tag{9}$$

$$R_1 A R_1 = \left[\begin{array}{c|ccc} a_{11} & \|\mathbf{x}\|_2 & 0 & 0 \\ \hline \|\mathbf{x}\|_2 & & & \\ 0 & & \hat{R}_1\hat{A}_1\hat{R}_1 & \\ 0 & & & \end{array}\right] \tag{10}$$

$$\left[(\hat{R}_1\hat{A}_1\hat{R}_1)\right]_i = \left[(\hat{R}_1\hat{A}_1)\right]_i - \frac{2}{\|\mathbf{u}_1\|_2^2}\mathbf{u}_1\mathbf{u}_1^T\left[(\hat{R}_1\hat{A}_1)\right]_i \tag{11}$$

$$R_1AR_1 = \left[\begin{array}{cccc} 4 & \sqrt{2} & 0 & 0 \\ \sqrt{2} & 4 & 0 & \sqrt{2} \\ 0 & 0 & 4 & 0 \\ 0 & \sqrt{2} & 0 & 4 \end{array}\right] \tag{12}$$

$$\hat{R}_1\hat{A}_1\hat{R}_1 = \left[\begin{array}{ccc} 4 & 0 & \sqrt{2} \\ 0 & 4 & 0 \\ \sqrt{2} & 0 & 4 \end{array}\right] = \left[\begin{array}{cc} (\hat{R}_1\hat{A}_1\hat{R}_1)_{11} & \mathbf{y}^T \\ \mathbf{y} & \hat{A}_2 \end{array}\right] \tag{13}$$

$$\mathbf{u}_2 = \mathbf{y} + \operatorname{sgn}(y_1)\|\mathbf{y}\|_2\mathbf{e}_1^{(2)} = \sqrt{2}\begin{bmatrix}1\\1\end{bmatrix} \tag{14}$$

$$\hat{R}_2 = \mathbb{1}_2 - \frac{2}{\|\mathbf{u}_2\|_2^2}\mathbf{u}_2\mathbf{u}_2^T \tag{15}$$

$$(\hat{R}_2\hat{A}_2)_i = (\hat{A}_2)_i - \frac{2}{\|\mathbf{u}_2\|_2^2}\mathbf{u}_2\mathbf{u}_2^T(\hat{A}_2)_i \tag{16}$$

$$\hat{R}_2\hat{A}_2 = \left[\begin{array}{cc} 0 & -4 \\ -4 & 0 \end{array}\right] \tag{17}$$

$$\hat{R}_2\hat{A}_2\hat{R}_2 = \left[\begin{array}{cc} 4 & 0 \\ 0 & 4 \end{array}\right] \tag{18}$$

$$R_2R_1AR_1R_2 = \begin{bmatrix} 4 & \sqrt{2} & 0 & 0 \\ \sqrt{2} & 4 & -\sqrt{2} & 0 \\ 0 & -\sqrt{2} & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \tag{19}$$