3 Lecture 3

Importance sampling

1. Estimate the integral

$$\int_0^{\pi/2} \sin\, x\, dx \tag{1}$$

by

(i) The crude Monte Carlo method

(ii) The importance sampling technique using a $g(x)$ proportional to $a + bx^2$.

Determine the optimal values of the parameters a and b to generate samples according to $g(x)$ and establish the number of iterations N needed to get an accuracy of $\%1$. Compare N with the number of iterations required to obtain the same accuracy for the crude Monte Carlo method.

(Hint: We suggest to use the hit-and-miss method to sample from $g(x)$.)

2. Let us compute the average

$$\langle f \rangle_\rho = \int_{\mathbb{R}} f(x)\rho(x)dx \tag{2}$$

where $\rho \in \mathcal{N}(0,1)$ and

$$f(x) = e^{-(x-3)^2/2} + e^{-(x-6)^2/2} \tag{3}$$

(a) Show that $\langle f \rangle_\rho$ can be computed in closed form and derive its value.

(b) Construct a regular Monte Carlo approximation based on a normal distribution $\mathcal{N}(0,1)$ by sampling $N = 10^3$ points and estimate the corresponding error.

(c) Compare the above result with the one obtained using the importance sampling approximation based on the uniform distribution $g \in U[-8,-1]$. Use again $N = 10^3$ samples