

9 Integration schemes

The harmonic oscillator Implement the Velocity Verlet algorithm and the predictor-corrector algorithm. Consider the harmonic oscillator

$$\begin{aligned}\frac{dq}{dt} &= p, \\ \frac{dp}{dt} &= -\omega^2 q.\end{aligned}\tag{1.22}$$

The exact solution of $p(t)$ vs $q(t)$ we get a circular orbit in phase that rotates by an amount of $\omega\Delta t$ at every time step. Considering this system, compare the two algorithms implemented above in terms of i) energy conservation, plotting $(E(t) - E_0)/E_0$ vs t , E_0 being the initial total energy ii) discrepancy with the analytical solution as a function of time. Verify that the Velocity Verlet is stable (i.e. it follows the analytical solution) for $\omega\Delta t < 2$ checking a few values of ω (e.g. $\omega = 0.01, 1, 100$).