List of exercises

1 Lecture 1

1.1 Sampling random points within D-dimensional domains by hit and miss

Use the hit-and-miss method to estimate numerically the volume of an ellipsoid of parameters a=3,b=2,c=2 and plot the deviation of the estimate from the analytic value as a function of the number of throws used.

Repeat the above procedure to estimate the volume of an ellipsoid of parameters a=3,b=1,c=1. Compare the two results and try to improve the second case if needed.

1.2 Sampling random numbers from a given distribution: inversion method

Use the inversion method to generate random numbers with the following PDF

1.
$$\rho(x) = cx e^{-x^2}$$
, for $x \in \mathbb{R}^+$.

2.
$$\rho(x) = bx^4 \text{ for } x \in [0, 3]$$

Note. First compute analytically the F, F^{-1} and the map $x_i = f(\xi_i)$ and then implement and run the corresponding numerical algorithm. In particular, compute the histogram of the sampled points and compare it with the expected PDF.

2 Lecture 2

2.1 Rejection method

Use the rejection method to generate random numbers that are distributed according to the pdf

$$f(x) = A_f e^{-8(x^2/2 + x^4/4)}.$$

where

$$\int_{-\infty}^{+\infty} f(x)dx = \frac{e}{\sqrt{2}} K_{1/4}(1)$$

and ${\cal K}_{1/4}$ denotes the Bessel function of the second kind.

Hint: One may use a Gaussian function.