Now we would like to prove that also also the solution of the master equation found in the microscopic approach converges to a solution of the diffusion equation in this limit.
We begin by recalling the distribution we obtained:

$$w_i(t_n) = \binom{n}{n_+} P_+^{\frac{n+i}{2}} P_-^{\frac{n-i}{2}}$$

First we consider the case $P_- = P_+ = \frac{1}{2}$:

$$w_i(t_n) = \binom{n}{n_+} P_+^{\frac{n+i}{2}} P_-^{\frac{n-i}{2}} = \binom{n}{\frac{n+i}{2}} \frac{1}{2^n} = \frac{n!}{\left(\frac{n+i}{2}\right)!\left(\frac{n-i}{2}\right)!} \frac{1}{2^n}$$

By using the Stirling's Formula in the logarithmic form, we obtain:

$$\ln(w_i(t_n)) = \ln(n!) - \ln\left(\left(\frac{n+i}{2}\right)!\right) - \ln\left(\left(\frac{n-i}{2}\right)!\right) - n\ln(2) =$$

$$n\ln n - \cancel{n} + \frac{1}{2}\ln(2\pi n) + O\left(\frac{1}{n}\right) - \frac{n+i}{2}\ln\frac{n+i}{2} + \cancel{\frac{n+i}{2}} - \frac{1}{2}\ln(\pi(n+i)) + O\left(\frac{2}{n+i}\right) -$$

$$-\frac{n-i}{2}\ln\frac{n-i}{2} + \cancel{\frac{n-i}{2}} - \frac{1}{2}\ln(\pi(n-i)) + O\left(\frac{2}{n-i}\right) - n\ln(2) =$$

$$= n\ln n - \frac{n}{2}\ln\frac{n+i}{2} - \frac{n}{2}\ln\frac{n-i}{2} - n\ln 2 - \frac{i}{2}\ln\left(\frac{n+i}{2}\right) + \frac{i}{2}\ln\left(\frac{n-i}{2}\right) +$$

$$+ \frac{1}{2}\ln(2\pi n) - \frac{1}{2}\ln(\pi(n+i)) - \frac{1}{2}\ln(\pi(n-i)) =$$

$$= n\left[\ln\left(\frac{n}{\sqrt{n^2-i^2}}\right)\right] - \frac{i}{2}\ln\left(\frac{n+i}{n-i}\right) + \frac{1}{2}\ln\left(\frac{2\pi n}{\pi(n-i)\,\pi(n+i)}\right)$$

$$= n\ln\left(\frac{1}{\sqrt{1-\frac{i^2}{n^2}}}\right) - \frac{i}{2}\ln\left(\frac{\cancel{n}\left(1+\frac{i}{n}\right)}{\cancel{n}\left(1-\frac{i}{n}\right)}\right) + \frac{1}{2}\ln\left(\frac{2}{\pi}\frac{1}{\frac{n^2}{n}-\frac{i^2}{n}}\right)$$

$$= -\frac{1}{2}n\ln\left(1-\frac{i^2}{n^2}\right) - \frac{i}{2}\ln\left(\frac{1+\frac{i}{n}}{1-\frac{i}{n}}\right) + \frac{1}{2}\ln\left(\frac{2}{\pi n}\frac{1}{1-\frac{i^2}{n^2}}\right)$$

$$= -\frac{1}{2}\ln\left(1-\frac{i^2}{n^2}\right)^n - \frac{i}{2}\ln\left(\frac{1+\frac{i}{n}}{1-\frac{i}{n}}\right) + \frac{1}{2}\ln\frac{2}{\pi n} + \frac{1}{2}\ln\left(\frac{1}{1-\frac{i^2}{n^2}}\right)$$

Now we use the following Taylor expansions's

$$(1+x)^\alpha \simeq 1+\alpha x \qquad \ln(1+x) \simeq x$$

So that we obtain in the limit $n \to +\infty$

$$\simeq -\frac{1}{2}\ln\left(1-\frac{i^2}{n}\right) - \frac{i}{2}\left(\frac{i}{n}\right) + \frac{i}{2}\left(-\frac{i}{n}\right) + \frac{1}{2}\ln\left(\frac{2}{\pi n}\right) + \frac{1}{2}\ln 1$$

$$\simeq \cancel{+\frac{i^2}{2n}} - \cancel{\frac{i^2}{2n}} - \frac{i^2}{2n} + \frac{1}{2}\ln\left(\frac{2}{\pi n}\right)$$

So that one obtains in the end:

$$\ln(w_i(t_n)) = -\frac{i^2}{2n} + \frac{1}{2}\ln\left(\frac{2}{\pi n}\right)$$

$$w_i(t_n) = \sqrt{\frac{2}{\pi n}}\, e^{-\frac{i^2}{2n}}$$