Resolution of the translated Gaussian integral in 1 dimension

We want to evaluate

$$\int_{\mathbb{R}} e^{-\frac{a}{2}x^2+bx}\,dx \qquad a>0,\ b\in\mathbb{R}.$$

The strategy is to propose a change of variable that is a translation towards the stationary point of the exponential exponent.

$$\frac{d}{dx}\left(-\frac{a}{2}x^2+bx\right)=-ax+b=0 \longrightarrow x=\frac{b}{a}$$

We then propose the change of variables

$$y=x-\frac{b}{a}$$

So that:

$$\int_{\mathbb{R}} e^{-\frac{a}{2}x^2+bx}\,dx=\int_{\mathbb{R}} e^{-\frac{a}{2}\left(y+\frac{b}{a}\right)^2+b\left(y+\frac{b}{a}\right)}\,dy=\int_{\mathbb{R}} e^{-\frac{a}{2}y^2-\frac{b^2}{2a}-\cancel{by}+\cancel{by}+\frac{b^2}{a}}\,dy=$$

$$=e^{\frac{b^2}{2a}}\int_{\mathbb{R}} e^{-\frac{a}{2}y^2}\,dy=e^{\frac{b^2}{2a}}\sqrt{\frac{2\pi}{a}}$$

$\square$