Minimization of the free energy functional in the Bragg-Williams approximation

$$\mathcal{F}_{\mathcal{J}}^{MF} = -\frac{J}{2}\sum_{\langle i,j\rangle} m_i m_j - \sum_i h_i m_i + k_B T \sum_i \left[\frac{1-m_i}{2} \ln\left(\frac{1-m_i}{2}\right) + \frac{1+m_i}{2} \ln\left(\frac{1+m_i}{2}\right) \right]$$

- $\dfrac{\partial \mathcal{F}^{MF}}{\partial m_\alpha} = 0$

Note: $\displaystyle\sum_{\langle i,j\rangle} = \sum_i \sum_{j\in N(i)} = \sum_j \sum_{i\in N(j)}$

$$0 = -\frac{J}{2}\left[\sum_i \sum_{j\in N(i)} \delta_{i\alpha} m_j + \sum_j \sum_{i\in N(j)} m_i \delta_{j\alpha} \right] - \sum_i h_i \delta_{i\alpha} +$$

$$+ k_B T \sum_i \left[-\frac{1}{2}\delta_{i\alpha} \ln\left(\frac{1-m_i}{2}\right) + \frac{1-m_i}{2}\frac{2}{1-m_i}\left(-\frac{1}{2}\delta_{i\alpha}\right) + \right.$$

$$\left. +\frac{1}{2}\delta_{i\alpha} \ln\left(\frac{1+m_i}{2}\right) + \frac{1+m_i}{2}\frac{2}{1+m_i}\left(+\frac{1}{2}\delta_{i\alpha}\right) \right] =$$

$$= -\frac{J}{2}\, 2\sum_{i\in N(\alpha)} m_i - h_\alpha + k_B T\left[-\frac{1}{2}\left[\ln\left(\frac{1-m_\alpha}{2}\right)+1\right] + \frac{1}{2}\left[\ln\left(\frac{1+m_\alpha}{2}\right)+1\right]\right]$$

$$= -J\sum_{i\in N(\alpha)} m_i - h_\alpha + \frac{k_B T}{2} \ln \frac{1+m_\alpha}{1-m_\alpha}$$

Now recall:

$$\tanh^{-1}(x) = \frac{1}{2}\ln\frac{1+x}{1-x} \quad \text{iff } |x|<1$$

Then

$$-J\sum_{i\in N(\alpha)} m_i - h_\alpha + \frac{k_B T}{2} \ln \frac{1+m_\alpha}{1-m_\alpha} = 0$$

$$-J\sum_{i\in N(\alpha)} m_i - h_\alpha + k_B T \tanh^{-1}(m_\alpha) = 0$$

$$\tanh^{-1}(m_\alpha) = \beta h_\alpha + \beta J \sum_{i\in N(\alpha)} m_i$$

$$\boxed{m_\alpha = \tanh\left(\beta\left(h_\alpha + J\sum_{i\in N(\alpha)} m_i\right)\right)}$$