Find Intersected Point of Involute curve and a Specific Circle

nuomizong

19/06/2015

1 Explanation

Figure 1 shows the principle of **Involute Curve**. In figure 2, **A** is the starting point. When |OA| turns an anti-clockwise angle θ, the arc length of |AC| should equal to the length of |CP| according to the definition of **Involute Curve**. If Point P is also located on circle of radius r, then the turned angle θ can be solved analytically.

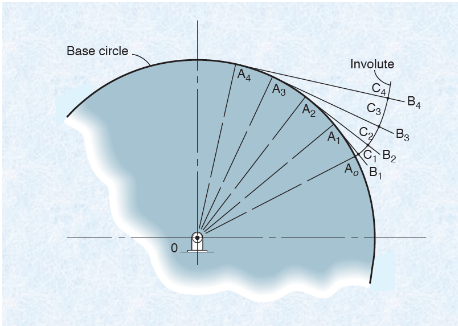

Figure 1: Involute curve definition

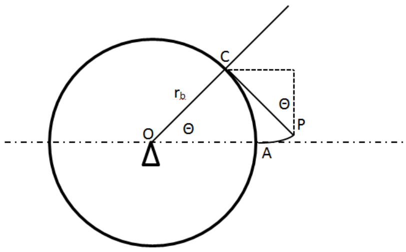

Figure 2: Involute

2 Algorithms

After $|OA|$ turns an anti-clockwise angle θ, so we have:

$$|\mathrm{CA}| = s = r_b\theta \tag{1}$$

where r_b means Base Circle.

Because Point C lies on Base Circle, there we can compute its coordinates:

$$\begin{cases} x_c = r_b\cos(\theta) \\ y_c = r_b\sin(\theta) \end{cases} \tag{2}$$

Because $|CA| = |CP|$, so the Point P can be calculated using:

$$\begin{cases} x_p = x_c + s\sin(\theta) \\ y_p = y_c - s\cos(\theta) \end{cases} \tag{3}$$

As a result of Point P lies on another circle, so we have:

$$x_p^2 + y_p^2 = r^2 \tag{4}$$

Solve all aforementioned equations, we conclude:

$$\theta = \sqrt{(\frac{r}{r_b})^2 - 1} \tag{5}$$

Finally, Point P is determined via:

$$\begin{cases} x_p = r_b\cos(\theta) + r_b\theta\sin(\theta) \\ y_p = r_b\sin(\theta) - r_b\theta\cos(\theta) \end{cases} \tag{6}$$