

STRESS LINE GENERATION FOR STRUCTURALLY PERFORMATIVE ARCHITECTURAL DESIGN

Kam-Ming Mark Tam

Massachusetts Institute of Technology
Singapore University of Technology and Design

Caitlin T. Mueller

Massachusetts Institute of Technology

ABSTRACT

Principal stress lines, which are pairs of orthogonal curves that indicate trajectories of internal forces and, therefore, idealized paths of material continuity, naturally encode the optimal topology for any structure for a given set of boundary conditions. Although stress line analysis has the potential to offer a direct, and geometrically provocative approach to optimization that can synthesize both design and structural objectives, its application in design has generally been limited due to the lack of standardization and parameterization of the process for generating and interpreting stress lines. Addressing these barriers that limit the application of the stress line methods, this paper proposes a new implementation framework that will enable designers to take advantage of stress line analysis to inform conceptual structural design. Central to the premise of this research is a new conception of structurally inspired design exploration that does not impose a singular solution, but instead allows for the exploration of a diverse high-performance design space in order to balance the combination of structural and architectural design objectives.

1 INTRODUCTION: DESIGN MOTIVATED BY GEOMETRY AND FORM

In architectural design, structural performance, efficiency, and expressiveness are often goals that can best be achieved when integrated into generative processes in conceptual design. Such efforts, while varied in specific methodologies, attempt to capitalize on the critical relationship between architectural geometry and structural behavior and can lead to innovation in each: efficient structures often entail complex geometric solutions that can be formally compelling. However, the realization of structural-led exploration has generally been unmet due to a lack of advanced design tools that can effectively synthesize architectural and structural considerations (Mueller 2013).

One prominent exception is in the field of surface structures, which have received significant attention from researchers in recent years. The advent of interactive, dynamic, and computational methods for solving three-dimensional equilibrium structures, such as the particle-spring system (Kilian 2005) implemented in parametric tools like Kangaroo (Piker 2013), has combined with advances in digital fabrication to enable designers to generate structurally inspired forms that are highly curvilinear and complex (Block and Rippman 2013). However, the focus has centered primarily on global geometries: little design-oriented research has been conducted to create high-performance topologies for these complex surfaces. Since the final structural performance of any structure is influenced considerably by its topology, which has major impact on visual appearance and constructability, there is a need for knowledge and tools that enable designers to create efficient and elegant topologies for surface structures. This paper proposes such a method, based on the theory of principal stress lines, an increasingly popular concept that nevertheless lacks sufficient investigation in the architectural design realm.

A simple shell geometry discretized with a stress-line-based topology is presented as a motivating example (Figure 1.2). Analysis has found the stress-line-inspired topology to be 20% more efficient than the one based on the mesh grid (Figure 1.1).

2 BACKGROUND ON STRESS LINE

2.1 PRINCIPAL STRESS DIRECTION

Principal stress lines are numerical integration of principal stress directions over a structural body. Designers are interested in principal stress lines because they provide a visualization of the natural force flow of an applied load on the system, which shows the lines of desirable material continuity for a given design domain.

The construction of stress lines begins with the infinite decomposition of a continuum structure into infinitesimal cubical elements to determine the state of stress at each point, which is unique, and is characterized by two normal components and a shear component for 2-D structures (Figure 2.3 and 2.3A). Rotating the element, the state of stress will remain unchanged, but the stress components will correspond to the transformed orientation (Figure 2.4 and 2.4A): principal stress directions are the normal stress

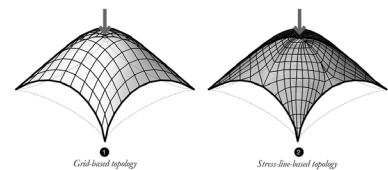


Figure 1
Motivating example of a surface structure with various topologies.

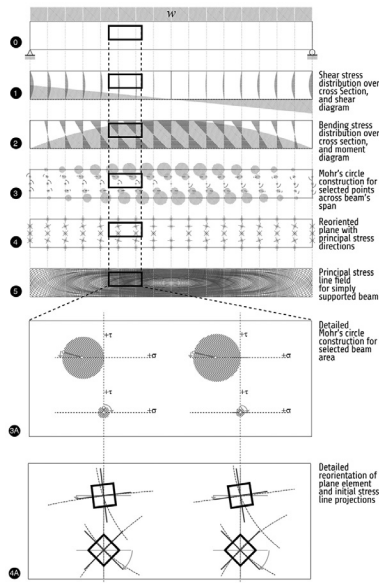


Figure 2
Simply-supported beam under uniformly distributed load.

components corresponding to the element orientation in which the shear stress is zero and normal stresses are maximum. Having determined the principal stress directions for various points across a structural body, these vectors are projected, or integrated, as lines that form the principal stress lines (*Figure 2.5 and 2.4A*)

Mathematically, the required transformation orientation is expressed in the following equation (Hibbeler 2011):

$$\tan 2 = (2 \tau_{xy}) / (\sigma_x - \sigma_y)$$

Therefore, the solution has two roots that are set at 90 degrees apart, which establishes the distinctive quality of stress lines as two orthogonally intersecting families of curves. Additionally, given a design domain for a finite system of isotropic material operating within the elastic range, stress line results are entirely affected by the geometric attributes of the design domain, regardless of the scaling of material properties, applied force, or the object's dimensions. These attributes include the locations and directions of loadings, the positioning and degree of fixities of supports, and the boundary's geometry. Finally, the optimal shape for a design domain is contained in the principal stress field (Chen and Li 2010).

The potent capacity for principal stress fields to suggest optimum topology as a function of geometry regardless of materiality makes stress line techniques a compelling methodology for constructing an all-encompassing framework for topology exploration.

2.1.1 Topology optimization and convergence.

The production of stress-line-based design constitutes a relatively new inquiry that traces its roots to classical studies on structural optimization by Michell (*Figures 3 and 4.1*) (Michell 1904) and to the numerical methods developed to approximate Michell solutions, such as the homogeneous and ground structure methods (*Figures 4.2 and 4.3*).

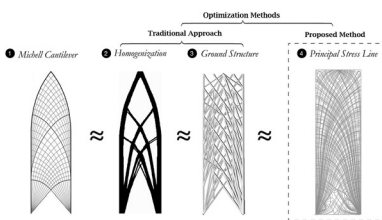


Figure 4
Convergence of optimization results.

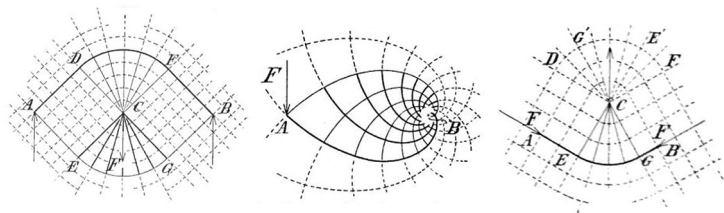


Figure 3
Michell's analytical formulation of optimum trusses for various cases (Michell 1904).

A convergence is typically evident in the results obtained from the various approaches, and from the stress lines generated for the same design boundary (*Figure 4.4*). Since solutions are often infeasible to obtain with traditional analytical and numerical methods (Sokół 2011), the recurring resemblances suggests the potential value of a more direct approach for obtaining optimal structural topologies that uses principal stress analysis.

Evidence that the performance of a structural system can improve as it becomes more similar to the principal stress lines of its design domain is presented here (Figure 5).

2.2 ARCHITECTURAL PRECEDENTS

In spite of a research gap in stress line research, stress-line-inspired designs abound in architectural history. Nervi's designs, such as the Gatti Wool Mill, are heavily influenced by the idea of 'force flow' (Halpern 2013). Similarly, Catalano has capitalized the concept in multiple proposals (Allen and Zalewski 2010). Recent CAD/CAM advances facilitating the production of complexly curved geometries had furthermore renewed interests in stress-line-based design. These examples, however, are exceptional cases led by expert practitioners: the valuable guidance potential of stress lines remains unmet due to the lack of parameterization, standardization, and evaluation on the process for generating, interpreting, and analyzing stress lines.

2.3 PROBLEM AND OBJECTIVE

The research is important because conventional tools integrated with parameterized design interfaces available to designers for generating stress lines, such as Millipede (Michalatos 2014) and Karamba (Preisinger 2015) are typically concerned with the visualization of stress flow, as opposed to the its materialization in practical design applications. With few functionalities available for the incorporation of designers' input, the blackbox approach deprives designers of the capacity to alter the fundamental properties of stress lines that are intrinsic to each constituent stage of generation. Additionally, there is no guarantee that the produced stress lines will lead to usable structural patterns (Figure 6), nor is there documentation evaluating the performance of stress line generation methods.

Addressing these problems, the research is motivated to develop stress line generation into a transparent process that is specifically adapted for design and highly configurable by designers. The design application of stress lines is strongest when designers can actively participate in its generation.

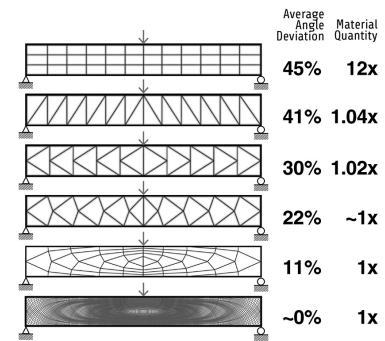


Figure 5
Simply-supported trusses' stress lines conformity and performances.

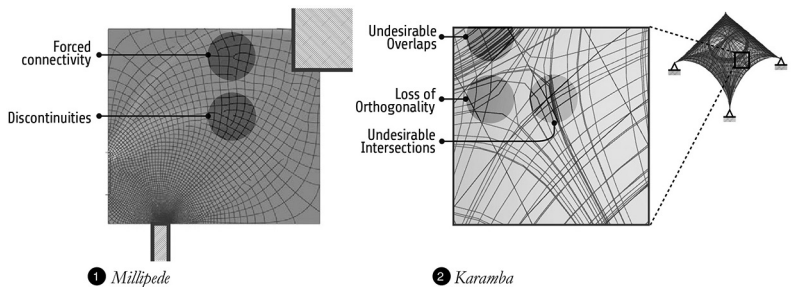


Figure 6
Common problems found in stress lines produced by popular design-oriented tools.

3 STRESS LINE CONSTRUCTION: THEORY

3.1 PROCESS OVERVIEW

3.1.1 Direct or iterative stress line construction.

Structural patterns indicative of the internal stress trajectories of forces can be obtained in two type of approaches: 1) Direct, or 2) Iterative. Direct methods are analytical or graphical: the optimum layout is determined either mathematically by satisfying theoretical constraints, or from geometric characterization of analytical formulations. Consequently, direct methods are prescribed, and do not require finite element analysis (FEA). An example of direct analytical approach is Michell's optimal truss derivation, whereas Mazurek's (Mazurek et al. 2011) graphical description of the same cantilever is an example of a direct geometrical approach (*Figure 7*).

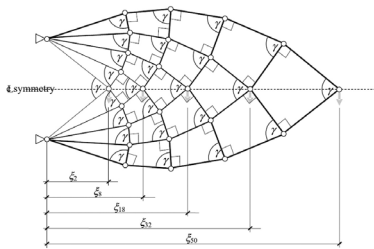


Figure 7
Geometric characterization of the Michell cantilever
(Mazurek 2011).

Existing analytical formulations, however, have only been derived for limited cases. Thus, most stress line construction methods are instead numerical and iterative. Given an initial point, or seed, FEA is used to determine the principal stress directions for that point. A line segment is then drawn along these directions, and its end point becomes the starting point for the subsequent iteration. This process repeats until the stress line reaches the design boundaries; the conclusion of the stress line for one seed leads to stress line construction for the next seed in the sequence (Michalatos 2014).

As expected, the quality of stress lines varies widely according to the parameters in the different stages of production, and on the methods used to calculate the stress directions, which are calculated either analytically or numerically with FEA. To maximize design application, this paper develops an iterative numerical method for constructing stress lines using a common FEA tool, while addressing the known problems inherent in the approach that include (1) low stress line resolution (2) poor stress direction interpolation, and (3) stress line discontinuities (Halpern 2013).

3.1.2 Processing Research.

Following the initial construction of a base stress line field, procedures are then applied to process, or to select stress lines for materialization. Recent relevant research include Michalatos and Kaijima's implementation of the periodic global reparameterization algorithm, which enables the scaling of the principal stress vector field according to an input gradient that may correspond to structural data (*Figure 9*), and Chen and Li's (Chen and Li 2010) subdivision algorithm, which incrementally builds a stress-line-based structure by co-opting a new set of stress line curvatures to reduce the approximation error in each iteration (*Figure 8*). These approaches allude to the participation of the designers, who are empowered to affect the initial stress line, which is not directly materialized, but instead used as a reference on which potential discretized elements are based.

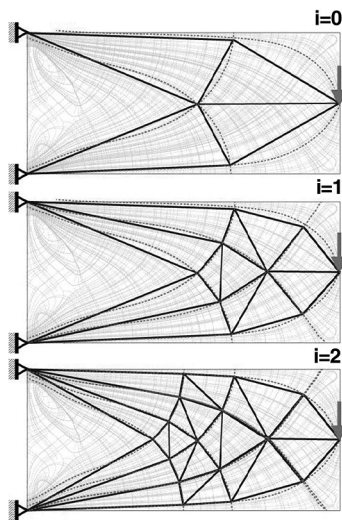


Figure 8
Modified implementation of Chen and Li's method.

3.2 PROPOSED PROCESS OUTLINE

In spite of the recent emergence of interesting stress-line-related research, the procedure dictating the drawing of stress lines has not fundamentally changed. Research on stress line methods is commonly compartmentalized into two approaches: a brief procedure on tracing, followed by an elaborate post-processing procedure.

This paper develops a stress-line-based computation and materialization framework that expands the fundamental procedure to develop the possibilities intrinsic to each stage of the process, dividing the production of stress-line-inspired topology into the three stages of initialization, generation, and processing (Figure 9). By adopting a number of geometric criteria, the proposed framework seeks to minimize the reliance on FEA.

The purpose of initialization is to construct an appropriate mesh topology to characterize the investigated design domain, to conduct the initial analysis from which structural data are obtained to form the basis of the stress line construction, and to create an appropriate seeding plan. In generation, this paper explores different methods for interpolating stress trajectories, and significantly expands on the general tracing algorithm to include rule-based corrections that reduces the numerical noise that is often present in stress line interpolation. Finally, the implementation concludes with strategies for processing and selecting the stress lines based on performance criteria.

3.3 SCOPE AND CASE STUDIES

Although stress-line-based solutions have theoretical application potential in all structural systems, the implementation demonstrated in this paper focuses on both planar and form-found 2.5D membrane structures, applied here on five main structural types: (1-2) planar cantilever and simply supported beams under a point load, and (3-5) regular form-found grid shells with 3-, 4-, and 5- supports. Since members in these systems are subjected only to in-plane stresses, their normal stresses would be constant across their cross section depth contributed primarily by axial forces with negligible bending (Figure 10).

3.4 COMMERCIAL COMPUTATIONAL TOOLS

To maximize the design potential of the proposed framework, the research presented on this paper is developed using popular 3D modeling tool Rhinoceros 3D (Robert McNeel and Associates n.d.), within the visual programming language environment of Grasshopper 3D (Rutten 2013). Structural analyses were conducted using the plugin Karamba, whereas the various surfaces used to implement the proposed framework were initially form-found using Kangaroo Physics (Piker 2013).

3.5 PRINCIPAL STRESS DIRECTION CONFORMITY

A central metric developed in the research—to assess and steer the development of stress-line-based results—is angle conformity, which is used to measure the closeness of a given topology to the base stress line field of the given design domain. Essentially,

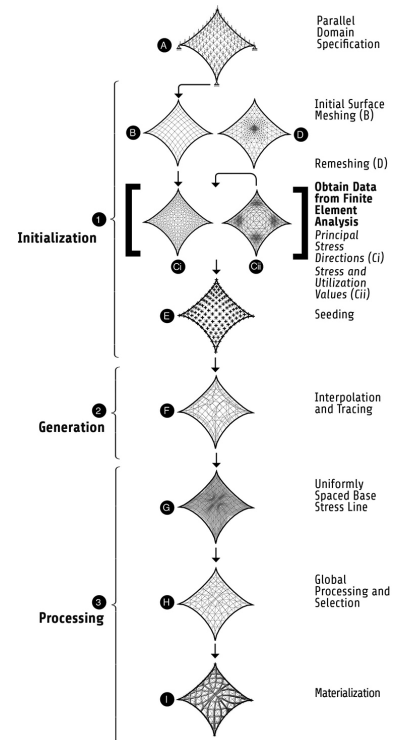


Figure 9
Proposed stress line methodology.

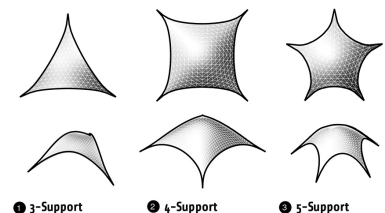


Figure 10
Grid shells explored in this research.

the orientation of each member is compared against its closest set of principal stress directions in a parallel shell analysis using a highly refined mesh to obtain an average value that suggests the angle approximation error (Figure 11).

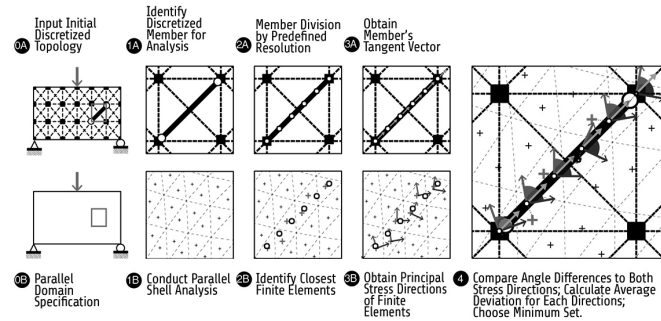


Figure 11
Calculation of angle conformity for sample frame structure.

4 IMPLEMENTATION

4.1 INITIALIZATION

4.1.1 Design domain specification and analysis.

The initial parallel shell analysis is modeled to mirror actual constraints. Typically, the most usable results are obtained when the structure is analyzed with the distributed loading condition used to form-find the shell structure (Figure 12.1). Three types of analytical data are obtained: principal stress directions, principal stress magnitudes, and overall member element utilization.

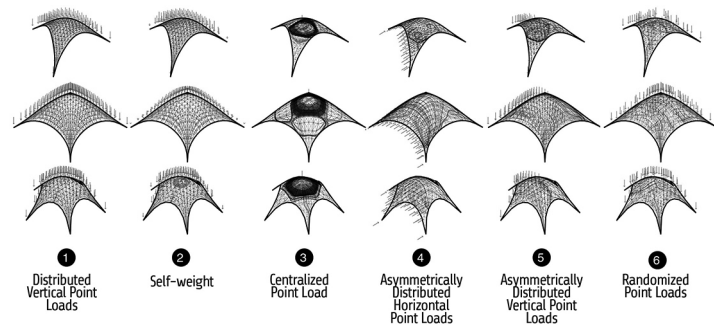


Figure 12
Stress lines for different loading conditions.

4.1.2 Seeding.

Seeding presents an opportunity for designer to incorporate spatial and structural objectives that are determined by nodal configurations, which might correspond pragmatically

to internal programmatic, light, and mechanical constraints. To ensure the inclusion of all stress lines, especially in design cases where circumferential stresses are present, the entire area contained within the design domain should be considered for seeding.

In the absence of particular constraints, the objective of seeding is to determine a collection of starting points from which a uniformly spaced principal stress field can be constructed for later processing and selection. Alternatively, seeding can be strategic or arbitrary, which may respectively correspond to analytical values such as the magnitude of stresses (Figure 13.1), to randomly-generated points (Figure 13.2), or other geometric patterns (Figure 13.3).

4.1.3 Remeshing.

Since the quality of the principal stress direction results is dependent on the resolution of the shell mesh used in the initial analysis, a strategic mesh subdivision algorithm was developed to reveal greater details in mesh elements—or cells—that experience the greatest local variation in principal stress directions. The method minimizes the increases to computational demand that typically follows global mesh subdivision. The principal stress directions of each cell is compared to those of its adjacent cells within a predetermined radius to produce a neighborhood angle deviation value per cell that is then normalized and projected to a targeted subdivision range (Figure 14.2). Each cell is subdivided accordingly to obtain a new set of vertices that are triangulated with a Delaunay algorithm (Delauney 1934) and relaxed using particle spring simulation (Figure 14.3). Analyses have confirmed that the angle conformity for stress lines is higher in the strategically-densified mesh than in the lower-resolution original mesh.

4.2 GENERATION

4.2.1 Interpolation and line segment length.

In a conventional tracing algorithm, the principal stress directions associated to the cell closest to the starting point in the current iteration is used to project the new line segments (Figure 15.1) (Michalatos 2014). This simple method, which is referred to here as a first-order approach, can generate biased line results because the principal stress direction information is generally a value averaged for the center of the respective finite element. Third-order linear interpolation methods are commonly devised to address this bias: the stress direction of each iteration is interpolated from three data points forming a triangular element that contains the targeted point (Figure 15.2) (Chen and Li 2010). While the revised approach achieves improved accuracy, the criteria for selecting reference points are not always consistent, particularly in design cases characterized by complex meshes. A new $(1+n)$ th-order approximation method has been proposed to ensure consistency and accuracy: in each iteration, both the nearest mesh element and n mesh elements that are located within a per-determined radius from the starting point are identified, with their data extracted, weighted according to proximity to produce the interpolated stress directions (Figure 15.3). The improvement to variation in the $(1+n)$ th-order approach has been confirmed in analysis (Figure 16.1).

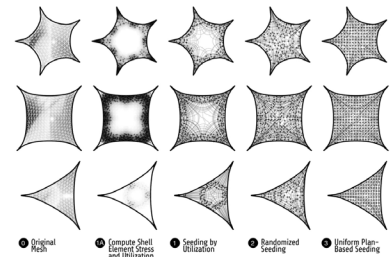


Figure 13
Seeding procedure and results.

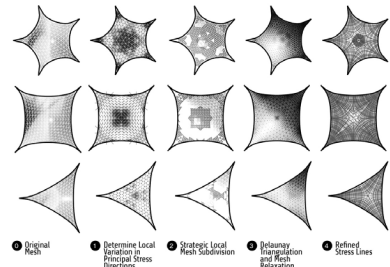


Figure 14
Remeshing procedure and results.

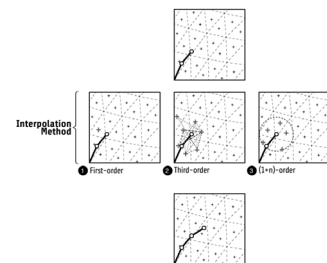


Figure 15
Interpolation procedure explained.

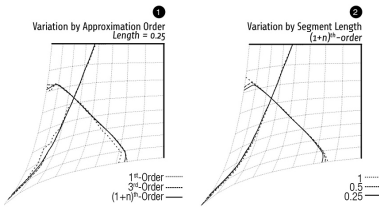


Figure 16
Variation to stress line results by approximation order and segment length.

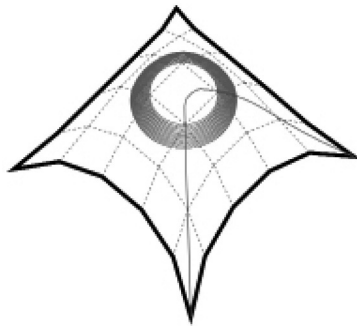


Figure 17
Circumferential stress error in commercial tool.

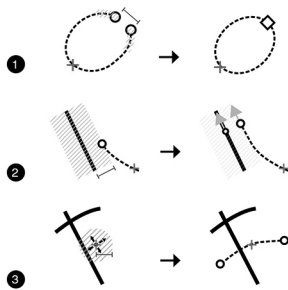


Figure 18
Diagrammatic representation of correction rules' parameters.

When third- or higher-order approaches are used to interpolate principal stress directions, biases in stress line results can further be reduced by decreasing the length of the line segment constructed in each iteration (Halpern 2013), which leads to smoother, and more accurate stress lines (Figure 16.2).

4.2.2 Sequential iterative tracing and rule-based correction.

Conventional tracing algorithms prescribe only the termination condition of the tracing procedure, which is met when a stress line reaches the design boundaries. However, the tracing algorithm may be structured in a number of significantly different ways. For instance, points may be traced sequentially or concurrently, and the two principal stress directions may also be traced independently, or simultaneously in the same iteration. Developed here is a sequential tracing procedure subjected to rule-based corrections where each seeding point is traced independently of the other seeding points in succession.

The proposed approach enables correction of numerical noises as they emerge in the tracing process, thereby ensuring that the resultant stress lines will explicitly meet constructability and spatial requirements. Preliminary implementation of the paper's rule-based corrections are identified here:

Detection of Circumferential Stress: Conventional tracers loop indefinitely when a stress line reaches an area with circumferential stresses (Figure 17). A rule was created here to detect the presence of circumferential stresses, and to close affected stress lines accordingly (Figure 19.1).

Enforce Offset: Approximation errors can produce undesirable intersection between stress lines of the same principal direction. This research proposes a rule that ensures the maintenance of sufficient offsets between adjacent stress lines and is activated when the starting point of currently traced stress line approaches a pre-defined distance from an existing stress line (Figure 18.2).

Bypass Seeding: To eliminate the production of redundant and overlapping stress lines, which increases post-processing burden, the detection of an existing principal stress line nearby at the initial tracing iteration will lead to termination for the respective principal stress direction (Figure 18.3).

The proposed algorithm is structured in a two-layered loop: the outer loop supervises the execution of the seeding plan and tracks the information associated to each stress line, and the inner layer loop traces the stress lines applies rule-based corrections. Specifically, the activation of tracing begins with the input of a seeding plan into the outer loop's engine, which sequentially inputs a seeding point P_j into the inner engine in each j th iteration (Figure 19.A). Upon reception of P_j , the inner engine begins iteration $i=0$. The general tracing procedure begins with the interpolation of the principal stress

vectors for the starting points, followed by the detection of location-based geometric conditions that warrant corrections to the interpolated stress vectors. New line segments are constructed between the starting points, and the points' projections according to the final determined stress directions. These new segments are then examined against the previously constructed stress lines, to determine whether the emergent stress lines qualify for termination rules, which are explained in the following section. The arrival of a projected point to the design boundary will activate a termination tracker that removes the respective point from future iterations. For projected eligible for continuation, they become the starting points of the subsequent the iteration. When a seeding point has been fully traced, the inner engine outputs the produced stress line pair back into the outer engine, which in turn begins the $(j+1)$ th iteration with the input of a new seeding point P_{j+1} (Figure 19.C).

A comparison of stress lines results from the preliminary implementation of the proposed tracing algorithm with the results generated using Karamba's built-in stress line function is provided here (Figure 20).

4.3 POST-PROCESSING

4.3.1 Density and architectural implication.

The theoretically-optimized Michell-like optimum structure has an infinite number of infinitely small bars with infinitely low stress. However, the additional gains in structural efficiency achieved by increasing the number of members in the Michell structure will diminish as density increases (Figure 21) (Mazurek 2011). As such, the ideally optimized stress-line structure, which has an infinite number of members, is neither usable in a practical design context nor significantly improved compared to one of a lower density. Hence the targeted density of the stress-line base structure is ultimately a decision based on constructability and design concerns. This paper proposes a method that seeks to select the stress lines that most contribute to structural performance.

4.3.2 Problem formulation and comparative analysis.

In structural optimization, efficiency is often defined in terms of structural volume or stiffness. The minimization of strain energy, which is calculated by multiplying stiffness by deformation squared, is used in the research for the evaluation of stress-line-based structure (Rozvany 1995; Achtziger 1997) and accounts for structural material efficiency when the members of all compared cases are sized to achieve constant total volume.

Following any modifications to the stress line-based structure, the section areas of structural members are rescaled according to a base case to ensure constant total structural volume for all cases (Figure 22.3). The magnitude of the external load applied to each node is adjusted according to the percentage of the shell's surface area that each node support—estimated by Voronoi decomposition (Tam et al. 2015) (Figure 22.5)—and normalized such that the total applied force remains constant in all compared cases. With the loadings redistributed and the structural members globally resized, an initial

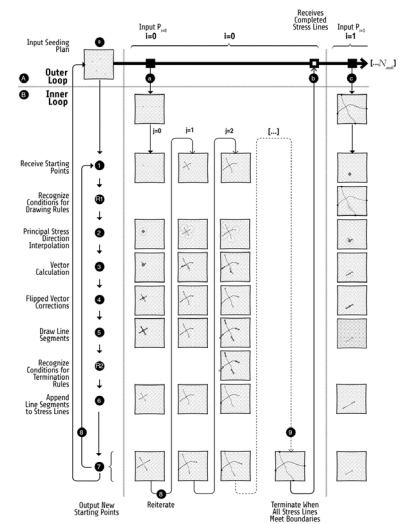


Figure 19
Diagram of proposed tracing algorithm.

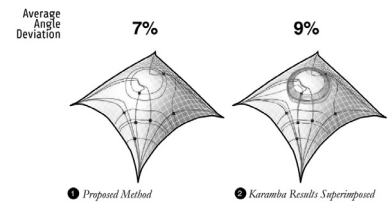


Figure 20
Comparison of visual and angle deviation results of stress lines.

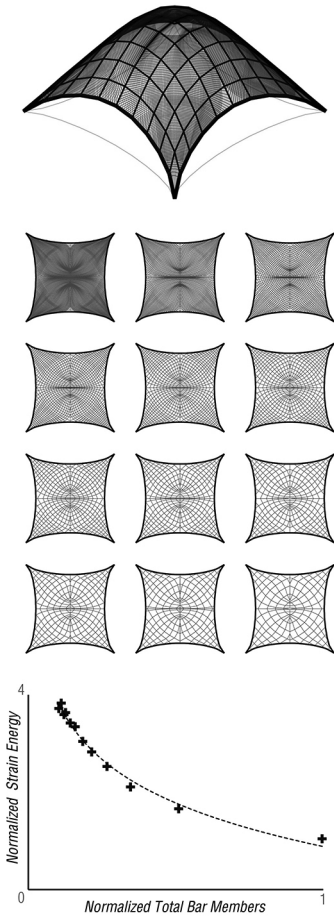


Figure 21
Stress line density and performance.

analysis produces the stress values required to determine the stress ratio for each member, which is calculated by dividing the maximum stress along each member by the maximum stress experienced by any members in the system (*Figure 22.7*). Cross section areas of individual member are then rescaled according to their stress ratio; a final FEA produces the strain energy data used for comparison (*Figure 22.9*). The two-step iteration ensures that each case is compared using reasonably optimized sizing based on stress requirements.

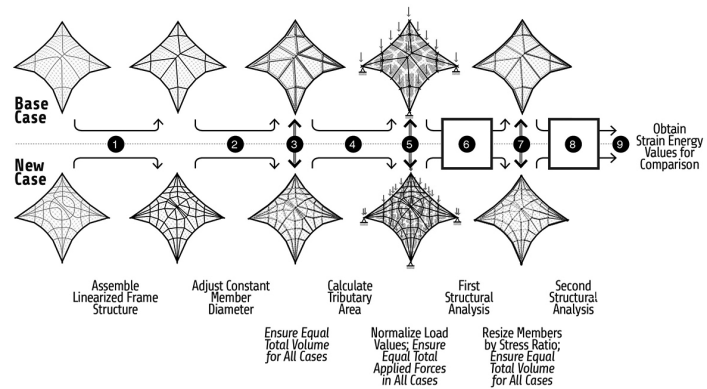


Figure 22
Comparative analysis procedure.

4.3.3 Selection by reduction in angle deviation.

The proposed iterative algorithm incrementally identifies n number of stress lines in each iteration for materialization. Selection is heuristic, and based on fitness objectives that correspond to structural performance. For this paper, approximation error by angle deviation is used as the metric for performance.

Stress lines, which are complexly curved geometries, are not directly able to be materialized. Thus, the procedure extracts only the coordinate position of the intersection points of selected stress lines, while maintaining the same topological connectivity (*Figure 22.1* and *Figure 23.2B*). The geometric simplification maximizes the constructability of the materialized stress lines, and ensures that the resultant structure can be modeled as 3-D frame structures with mostly axial behavior.

The process begins with an initial starting point input by the designer. In each iteration a random subset of n - stress lines intersecting the input points, or lines, are identified and evaluated according to the angle non-conformity of the resultant discretized and linearized structure. The combination minimizing the objective is selected and used as the input lines for the subsequent iteration (*Figure 23.4*). The process repeats as n stress lines intersecting the new input lines are identified. Naturally alternating between the

minimum and maximum principal stress direction in each iteration, the selection method ensures uniform growth to the stress line density. Documentation of the proposed method's capacity to generate considerable improvement to structural performance within only a few iterations is presented here (Figure 24).

An additive approach gives the designer the opportunity to monitor the trade-off between structural performance and the incremental increase in the density of the stress-line-based structure, such that the designer may precisely identify the optimum density particular to the design constraints. Also important is the minimization of computational demand, as the geometry only gradually increases in complexity.

5 CONCLUSION: SUMMARY AND FUTURE WORKS

This paper aims to bring clarity to a developing field within conceptual structural design that is often misunderstood and applied arbitrarily. Specifically, this is achieved by focusing on the fundamental areas in stress line generation that remain open in related research. The basic outline of the stress line method has been reinterpreted, and the many sub-processes involved in stress line generation have been rigorously codified, so that several previously unknown opportunities and issues in the different stages of stress line generation have been discovered.

The presented research has numerous applications that directly expand conceptual structural design for architecture at multiple scales. For example, stress line methods can inform designs of buildings and bridges realized in a number of materials; new digital fabrication techniques could help make geometrically complex stress-line-inspired designs achievable. One specific application currently under investigation by the authors is the use of stress line research to address limitations of current additive manufacturing (or 3D printing) technologies (Figures 26, 27, 28). Because conventionally, material is deposited in layers parallel to the horizontal printing bed, printed specimens are anisotropic, with strength and stiffness that varies significantly depending on the orientation of the applied forces. This problem can potentially be solved by adding materials along three-dimensional paths based on stress lines, both at prototype and end-use scales, resulting in high-performing, geometrically novel structures.

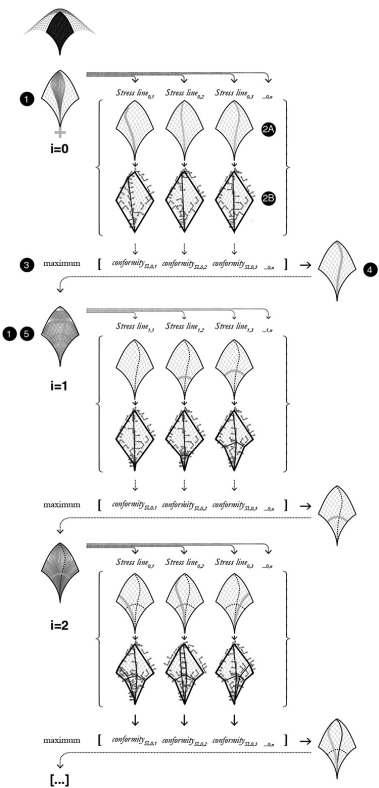


Figure 23
Incremental stress line selection procedure.

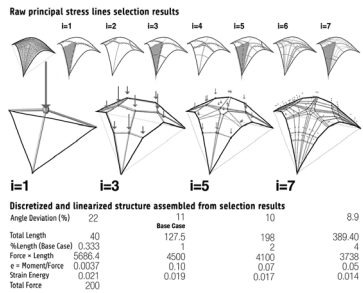


Figure 24
Structural performance of 3-support grid shell topology generated with proposed selection algorithm.

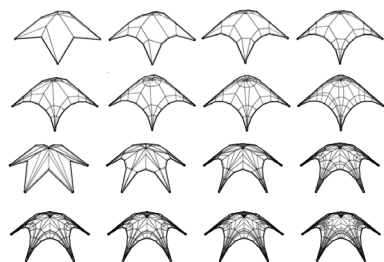


Figure 25
Stress line selection results for grid shell case with 4- and 5- supports.

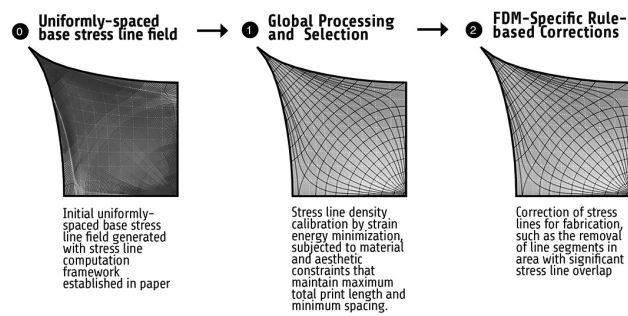


Figure 26
Additive-manufacturing-specific stress line processing.

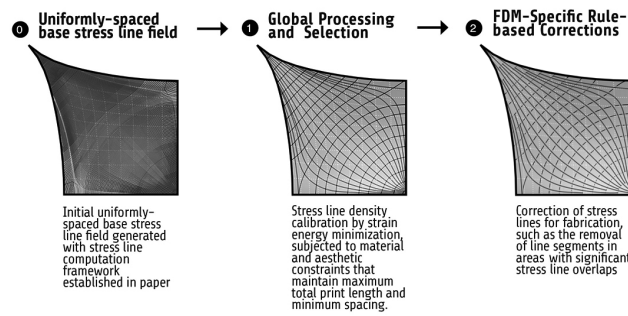


Figure 28
Stress line additive manufacturing fabrication result.

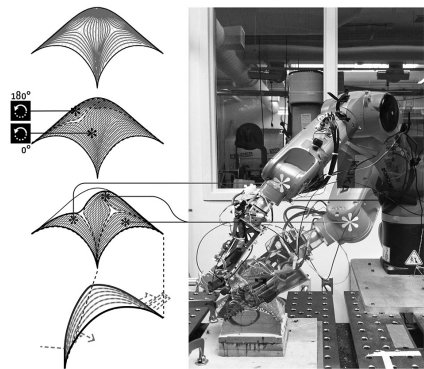


Figure 27
Process to convert stress-line-based paths into robot programs.

REFERENCES

- Achtziger, W. (1997). Topology Optimization of Discrete Structures: An Introduction in View of Computational and Nonsmooth Aspects. In *Topology Optimization in Structural Mechanics*, ed. G.I.N. Rozvany, p. 57-100. Vienna: Springer.
- Adriaenssens, S., J. N. Richardson, R. F. Coelbo, and P. Bouillard. (2014). Discrete Topology Optimization. In *Shell Structures for Architecture: Form Finding and Optimization*, eds. S. Adriaenssens, P. Block, D. Veenendaal, and C. Williams, p. 171-180. New York City: Routledge.
- Allen, E., and W. Zalewski. (2010). *Form and Forces: Designing Efficient, Expressive Structures*. Hoboken, N.J.: John Wiley & Sons.
- Block P., and M. Rippmann. (2013). Funicular Shell Design Exploration. In *Proceedings of the 33rd Annual Conference of the Association for Computer Aided Design in Architecture*, October 24-26, 2013. Cambridge, ON: University of Waterloo School of Architecture.
- Chen, Y., and Y. Li. (2010). Beam Structure Optimization for Additive Manufacturing based on Principal Stress Lines. In *Solid Freeform Fabrication Proceedings*, ed. D.L. Bourell, et al., 666-678. Austin, TX: SFF.
- Delaunay, B. (1934). Sur la Sphère Vide: A la Mémoire de Georges Voronoï. *Bulletin de l'Académie des Sciences de l'URSS, Classe des sciences mathématiques et naturelles*, No. 6: 793-800.
- Halpern, A.B., D. P. Billington, and S. Adriaenssens. (2013). The Ribbed Floor Slab Systems of Pier Luigi Nervi. In *Proceedings of the International Association of Shell and Spatial Structures (IASS) Symposium 2013: Beyond the Limits of Man*. September 23-27, 2013. Wroclaw, PL: Wroclaw University of Technology.
- Hibbeler, R. C. (2011). *Statics and Mechanics of Materials*. 3rd ed. Upper Saddle River, N.J.: Pearson/Prentice Hall.
- Kilian, A., and J. Ochsendorf. (2005). Particle-Spring Systems for Structural Form Finding. *Journal of the International Association For Shell And Spatial Structures* 46(2): 77-85.
- Mazurek, A., W. Baker, and C. Tort. (2011). Geometrical Aspects of Optimum Truss Like Structures. *Structural and Multidisciplinary Optimization* 45(2): 231-42.
- Michalatos, P. (2014). Millipede. Grasshopper. Accessed March 1, 2014. <http://www.grasshopper3d.com/group/millipede>.
- Michalatos, P., and S. Kaijima. (2014). Eigenshells: Structural Patterns on Modal Forms. In *Shell Structures for Architecture: Form Finding and Optimization*, eds. S. Adriaenssens, P. Block, D. Veenendaal, and C. Williams, p. 195-210. New York City: Routledge.
- Michell, A.G.M. (1904). The Limits of Economy of Material in Frame-Structures. *Philosophical Magazine* 8(47): 589-597.
- Mueller, C., and J. Ochsendorf. (2013). An Integrated Computational Approach for Creative Conceptual Structural Design. In *Proceedings of the International Association of Shell and Spatial Structures (IASS) Symposium 2013: Beyond the Limits of Man*. September 23-27, 2013. Wroclaw, PL: Wroclaw University of Technology.
- Piker, D. (2013). Kangaroo: Form Finding with Computational Physics. *Architectural Design* 83(2): 136-37.
- Preisinger, C. (2015). Karamba. Grasshopper. Accessed February 13, 2015. <http://www.grasshopper3d.com/group/karamba>.
- Robert McNeel & Associates. (n.d.) Rhinoceros. Accessed July 17, 2015. <http://www.rhino3d.com/>.
- Rozvany, G.I.N., M.P. Bendsøe, and U. Kirsch. (1995). Layout Optimization of Structures. *Applied Mechanics Reviews* 48(2): 41-119.
- Rutten, D. (2013). *Back Home*. In *I Eat Bugs for Breakfast*. November 10, 2013. Accessed May 11, 2015. <https://ieatbugsforbreakfast.wordpress.com/2013/11/10/back-home/>.
- Sokół, T. (2011). A 99 line code for discretized Michell truss optimization written in Mathematica. *Structural and Multidisciplinary Optimization* 43(2): 181-190.
- Tam, K.M.M., J.R. Coleman, N.W. Fine, and C.T. Mueller. (2015 Forthcoming). Stress Line Additive Manufacturing (SLAM) for 2.5-D Shells. In *Proceedings of the International Association of Shell and Spatial Structures (IASS) Symposium 2015: Future Visions*. September 17-20, 2015. Brasília, BRA: Polytechnic School of the University of São Paulo.

KAM-MING MARK TAM

Kam-Ming Mark Tam is a researcher broadly engaged with computational design in architecture, who is currently an Instructor and a Research Associate at respectively the Singapore University of Technology and Design (SUTD) and the Massachusetts Institute of Technology (MIT). With a research repertoire that encompasses architecture, structural design, urban design and economics, Tam's recent research efforts lie in the development of computational methods for geometric and topological design space exploration, and in the experimentation of novel robotic-enabled and stress-based additive manufacturing techniques for creating performative 3D-printed artifacts. Recently graduated, Tam earned a M. Eng. in Civil Engineering from MIT, and a M.Arch. and a B.A.S (with Economics Minor) at the University of Waterloo, and has practiced in several architectural firms around the world, including Coop Himmelb(l)au in Vienna, and Cannon Design in Vancouver. An active participant at the University of Waterloo's School of Architecture, Tam is the co-founder of the school's community gallery BRIDGE, and the advanced computation research group F_RMLab.

CAITLIN T. MUELLER

Caitlin T. Mueller is an academic who works at the intersection of architecture and structural engineering. She is currently an Assistant Professor at the Massachusetts Institute of Technology's Department of Architecture, in the Building Technology Program, where she co-directs the Structural Design Lab and leads the Digital Structures research group. Broadly focused on the creative synthesis of architecture and structure, her research efforts lie in three main areas: computational techniques for performance-driven conceptual design; structurally-performative digital fabrication, with a focus on additive manufacturing; and architect-engineer collaboration in historical and contemporary projects. Professor Mueller earned a Ph.D. in Building Technology from MIT, a S.M. in Computation for Design and Optimization from MIT, a S.M. in Structural Engineering from Stanford University, and a B.S. in Architecture from MIT, and has practiced at several architecture and engineering firms across the U.S., most recently as a structural designer at Simpson Gumpertz & Heger in Boston.