

Question: Let $p(x)$ be a quadratic polynomial with integer coefficients which has $4 - \sqrt{11}$ as a root. Compute $\frac{p(3)}{p(4)}$.

Base Model: Qwen2.5-7B-Instruct

<think> Since $p(x)$ is a quadratic polynomial with integer coefficients and $4 - \sqrt{11}$ is a root, its conjugate $4 + \sqrt{11}$ must also be a root. This is because the coefficients are integers, and the polynomial must factor over the reals to maintain integer coefficients. The polynomial can thus be written as $p(x) = a(x - (4 - \sqrt{11}))(x - (4 + \sqrt{11})) = a((x-4)^2 - 11) = a(x^2 - 8x + 3)$. We need to find $\frac{p(3)}{p(4)}$. First, calculate $p(3)$ and $p(4)$.
For $p(3)$:
$$p(3) = a(3^2 - 8 \cdot 3 + 3) = a(9 - 24 + 3) = a(-12)$$

For $p(4)$:
$$p(4) = a(4^2 - 8 \cdot 4 + 3) = a(16 - 32 + 3) = a(-13)$$

Thus, $\frac{p(3)}{p(4)} = \frac{-12}{-13} = \frac{12}{13}$. The value of a cancels out, so the result is independent of a .</think>

<answer> $\frac{12}{13}$ </answer>

Ground Truth: $\frac{10}{11}$

Model Trained on Matrix Games

<think>\nSince $p(x)$ is a quadratic polynomial with integer coefficients and $(4 - \sqrt{11})$ is a root, its conjugate $(4 + \sqrt{11})$ must also be a root. This is because the coefficients are integers, and the polynomial must have roots that come in conjugate pairs if they are irrational.

Let $p(x) = a(x - (4 - \sqrt{11}))(x - (4 + \sqrt{11}))$. Expanding this, we get:
$$p(x) = a((x - 4) + \sqrt{11})(x - 4 - \sqrt{11}) = a((x - 4)^2 - (\sqrt{11})^2) = a(x^2 - 8x + 16 - 11) = a(x^2 - 8x + 5).$$

Since the leading coefficient a can be any integer, we can choose $a = 1$ for simplicity, giving us:

$$p(x) = x^2 - 8x + 5.$$

Now, we need to compute $\frac{p(3)}{p(4)}$:

$$p(3) = 3^2 - 8 \cdot 3 + 5 = 9 - 24 + 5 = -10,$$

$$p(4) = 4^2 - 8 \cdot 4 + 5 = 16 - 32 + 5 = -11.$$

Thus, $\frac{p(3)}{p(4)} = \frac{-10}{-11} = \frac{10}{11}$.

The final answer is $\frac{10}{11}$.</think>

<answer> $\frac{10}{11}$ </answer>