

Question: Let  $p(x)$  be a quadratic polynomial with integer coefficients which has  $4 - \sqrt{11}$  as a root. Compute  $\frac{p(3)}{p(4)}$ .

Base Model: Qwen2.5-7B-Instruct

<think> Since  $p(x)$  is a quadratic polynomial with integer coefficients and  $4 - \sqrt{11}$  is a root, its conjugate  $4 + \sqrt{11}$  must also be a root. This is because the coefficients are integers, and the polynomial must factor over the reals to maintain integer coefficients. The polynomial can thus be written as  $p(x) = a(x - (4 - \sqrt{11}))(x - (4 + \sqrt{11})) = a((x - 4)^2 - 11) = a(x^2 - 8x + 3)$ . We need to find  $\frac{p(3)}{p(4)}$ . First, calculate  $p(3)$  and  $p(4)$ .  
For  $p(3)$ :  $p(3) = a(3^2 - 8 \cdot 3 + 3) = a(9 - 24 + 3) = a(-12)$   
For  $p(4)$ :  $p(4) = a(4^2 - 8 \cdot 4 + 3) = a(16 - 32 + 3) = a(-13)$   
Thus,  $\frac{p(3)}{p(4)} = \frac{-12a}{-13a} = \frac{12}{13}$ . The value of  $a$  cancels out, so the result is independent of  $a$ .</think>  
<answer>  $\frac{12}{13}$  </answer>

Ground Truth:  $\frac{10}{11}$

Model Trained on Matrix Games

<think>\nSince  $p(x)$  is a quadratic polynomial with integer coefficients and  $(4 - \sqrt{11})$  is a root, its conjugate  $(4 + \sqrt{11})$  must also be a root. This is because the coefficients are integers, and the polynomial must have roots that come in conjugate pairs if they are irrational.  
Let  $p(x) = a(x - (4 - \sqrt{11}))(x - (4 + \sqrt{11}))$ . Expanding this, we get:  
$$p(x) = a((x - 4) + \sqrt{11})((x - 4) - \sqrt{11}) = a((x - 4)^2 - (\sqrt{11})^2) = a(x^2 - 8x + 16 - 11) = a(x^2 - 8x + 5).$$
  
Since the leading coefficient  $a$  can be any integer, we can choose  $a = 1$  for simplicity, giving us:  
$$p(x) = x^2 - 8x + 5.$$
  
Now, we need to compute  $\frac{p(3)}{p(4)}$ :  
$$p(3) = 3^2 - 8 \cdot 3 + 5 = 9 - 24 + 5 = -10,$$
  
$$p(4) = 4^2 - 8 \cdot 4 + 5 = 16 - 32 + 5 = -11.$$
  
Thus,  $\frac{p(3)}{p(4)} = \frac{-10}{-11} = \frac{10}{11}$ .  
The final answer is  $\frac{10}{11}$ .</think>  
<answer>\n $\frac{10}{11}$ \n</answer>