→ A function is said to be defined everywhere if the function has a value for all elements in the domain of that function.

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Similarly
$f(g(a)) = f(1) = a$
$f(g(b)) = f(2) = b$
$f(g(c)) = f(3) = c$
$\Big\} I_B$

• For a function 'f' to be invertible, f should be one to one, onto, and everywhere defined.

• Injection : one to one
• Surjection : onto
• Bijection : Injection + surjection

Let A and B be 2 finite sets with the same no of elements, and let $f: A \to B$ be an everywhere defined function.
(a) If f is one to one, then f is onto
(b) If f is onto, then f is one to one.

26 March:

$f(x) = \frac{x+1}{x}$

(1) Replace $f(x)$ by y
(2) Swap variables x & y
(3) Solve for y
(4) Replace y by $f^{-1}(x)$

eg) $f(x) = x^3+2$
$y = x^3+2$
$x = y^3+2$
$f^{-1}(x) = (x-2)^{1/3}$

eg) $f(x) = 5x-7$
$f^{-1}(x) = \frac{x+7}{5}$

Pigeonhole Principle (Shelton)

n pigeons
m pigeonholes
We have to assign pigeons to pigeonholes where $n > m$.

eg) 10 pigeons
6 pigeonholes
1 hole can contain one or more pigeons

O O O O O O
2 2 2 2 1 1

Now if $n >> m$ (n more than 2m) then atleast 3 will have to be assigned to each hole.

Use this principle → Extended pigeon hole principle

n – pigeons
m – holes

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Every hole will atleast contain $\left\lceil \frac{n-1}{m} \right\rceil + 1$ pigeons
→ ceil fun^c

eg) Show that if 30 dictionaries, in a library contain a total of 61327 pages, then how many pages one of the dictionaries must have

→ $\left\lceil \frac{61326}{30} \right\rceil + 1 = \lceil 2044.2 \rceil + 1 = 2045$ pages

So 1 dictionary must contain atleast 2045 pages

eg) 6 friends have total of 2161 ₹. How much should 1 or more should have

→ $\left\lceil \frac{2161-1}{6} \right\rceil + 1 = 261$

eg) Show that if 7 numbers from 1-12 are choosen then 2 of them will add upto 13

→ $A = \{1, 12\}$
$B = \{2, 11\}$
$C = \{3, 10\}$
$D = \{4, 9\}$
$E = \{5, 8\}$
$F = \{6, 7\}$

Now we have to select 7 numbers
Consider worst case if I select all the first / second element of all sets then I will have 6 elements choosen either $\{1, 2, 3, 4, 5, 6\}$ or $\{12, 11, 10, 9, 8, 7\}$ so I will have a choice of 1 no left which will definitely form a pair with one already selected. Hence 7 nos are required

Q) What is the min no of students required in a DS&T class to be sure that atleast 6 will receive the same grade if there are 5 possible grades A, B, C, D, E

→ 26

Every function is a relation but not every relation is a function

A function may take same value at 2 different elements of A
$f(a)=1$ and $f(a)=2$ X f is not a function here
Identity of a function

A : non empty set then $f: A \to A$
$f(a) = a \ \forall a \in A$
I_A (Identity fun^c on set A) $= \{ (a,a) \mid \forall a \in A \}$

eg) $A = \{1,2,3\}$ $f: A \to A$
$I_A = \{ (1,1), (2,2), (3,3) \}$
$f(1)=1, \ f(2)=2, \ f(3)=3$

Inverse of a function

- f^{-1} is a function from $B \to A$ if and only if f is one to one
- If f^{-1} is a function, then f^{-1} is also one to one
- f^{-1} is everywhere defined if and only if f is onto
- f^{-1} is onto if and only if f is everywhere defined.

$f: A \to B$
$f^{-1}: B \to A$

eg) $A = \{1,2\}$
$B = \{a,b,c\}$
$f(1)=a, \ f(2)=c$
$f: A \to B$
$f^{-1}: B \to A$ Inverse Mapping
$f^{-1}(a) = 1$
$f^{-1}(c) = 2$
But $f^{-1}(b)$ is not defined and does not exist

eg) Next $f: A \to B$, $g: B \to A$ such that $g \circ f = I_A$ and $f \circ g = I_B$ then g is called as the inverse of f and denoted by f^{-1}

$A = \{1,2,3\}$
$B = \{a,b,c\}$
$f(1)=a, \ f(2)=b, \ f(3)=c$
$g(a)=1, \ g(b)=2, \ g(c)=3$

$$\left.\begin{aligned} g\circ f(1) &= g(a) = 1 \\ g\circ f(2) &= g(b) = 2 \\ g\circ f(3) &= g(c) = 3 \end{aligned}\right\} I_A$$

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eg) $a_r = \begin{cases} 1 & 0 \le r \le 2 \\ 3^r & r \geq 3 \end{cases}$

$b_r = \begin{cases} 2^r + 1 & 0 \le r \le 1 \\ r-5 & r \geq 2 \end{cases}$

$a_r + b_r = \begin{cases} 2^r + 1 + 1 & 0 \le r \le 1 \\ 1 & 1 < r \le 2 \\ 1 + r - 5 & r = 2 \\ r - 5 + 3^r & 2 < r \le 3 \end{cases}$ Not possible since natural number only.

Another way to represent numeric functions ≡

* Generating Functions:

$a = \{a_0, a_1, a_2 \ldots a_n\} \rightarrow$ an ∞ seq.

$A(z) = a_0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + \ldots a_r z^r \rightarrow$ an ∞ series

Here every value of the numeric function is 'x' by z^r.

$A(z) = \sum_{i=0}^{i=\infty} a_i z^i = \sum_{r=0}^{\infty} a_r z^r \rightarrow$ term of degree r
[a_0 = constant term].

eg) $a = \{4^0, 4^1, 4^2 \ldots 4^r\}$

$A(z) = 1 + 4z + 4^2 z^2 + 4^3 z^3 + \ldots 4^r z^3 = 1 + 4z + (4z)^2 + (4z)^3$

$= \sum_{r=0}^{\infty} 4^r z^r \qquad = \frac{1}{1-4z}$

eg) $a_r = \alpha^r, \quad r \geq 0$

$A(z) = \alpha^0 + \alpha^1 z^1 + \alpha^2 z^2$

$= \sum_{r=0}^{\infty} \alpha^r z^r = \frac{1}{1-\alpha z}$

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eg) $a_r = 7 \times 3^r$

$A(z) = 7(3^0 + 3^1 z^1 + \cdots 3^r z^r)$

$= 7 \sum_{r=0}^{\infty} 3^r z^r = \frac{7}{1-3z}$

eg) $a_r = 3^r + 2$

$A(z) = \frac{9}{1-3z}$

eg) $a^r = \begin{cases} 0 & r \text{ is odd} \\ 2^{r+1} & r \text{ is even} \end{cases}$ $\quad a^r = 2^r + (-2)^r$

$A(z) = 2 + 0 + 8$

$= 2 + \sum_{r=2}^{\infty} 2^{r+1} z^r$

$(r = r+2)$

$A(z) = \sum_{r=0}^{\infty} 2^r z^r + (-2)^r z^r$

$= \frac{1}{1-2z} + \frac{1}{1+2z}$

$= \frac{1+2z+1-2z}{(1-2z)(1+2z)}$

$= \frac{2}{1-4z^2}$

eg) Calculate a_r from $A(z)$

$\frac{2+3z-6z^2}{1-2z} = A(z)$

$\frac{6}{9} \quad \frac{-3 \pm 2}{2 \times 2}$

$\rightarrow \frac{+(6z^2 - 3z - 2)}{(2z-1)} = A(z).$

$(2z-1)(az-b) = 6z^2 - 3z - 2$

$\{0, 3, 0, 0\}$ $2az^2 - 2bz - az + b$

$\frac{3z(2z-1) - 2}{2z-1} = A(z)$

$2a = 6 \quad a = 3$

$a + 2b = 3$

$b = -2$

$A(z) = 3z - \frac{2}{2z-1} = 3z + \frac{2}{1-2z}$

$3 \cdot$

$= 3z + 2(2^r).$

$\therefore a_r = 2^{r+1} +$

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Q) Let t be an equi △ whose side 1 unit. Show that if 5 points are chosen inside or ~~outside~~ on the △, ~~any 2~~ then 2 of them must be no more than 1/2 unit apart.

→

4 points fixed (3 corners, 1 center)
Then take 1 point anywhere and show that the distance between it and it's nearest point will have a distance less than 1/2

Q) There are 3000 students in a college, which offer 7 distinct courses for a 4 year duration. A student who has taken a course DSGT, learns that the largest classroom can hold only 100 students. What is the problem in this scenario?

6th April

Recurrence Relations, Generating functions, Numeric function

$f : A \to B$

$f(a) = b$

Always start with 0

Discrete Numeric functions : If $a : N \to R$ then $a_0, a_1, a_2 \dots a_r$ denote the values of a at $0, 1, 2, 3 \dots r$

$a(0) = a_0$, $a(1) = a_1$ called sequence.

$\boxed{\begin{array}{l} a(r) = a_r \\ a = \{a_0, a_1 \dots a_r\} \end{array}}$

DNF (sequence)

eg) $a_r = \begin{cases} r^2 & 0 \leq r \leq 3 \\ r+1 & r > 4 \end{cases}$

$a = \{0, 1, 4, 9, 5, 6 \dots \infty\}$

eg) $a_r = r^3 + 2$, $r \geq 0$

$a = \{2, 3, 10 \dots \infty\}$

eg) $a_r = r^3 + 2$, $r \geq 0$

$b_r = 2^r$; $r \geq 0$

$a_r + b_r = r^3 + 2^r + 2 = c_r$

$\therefore c = \{3, 5 \dots \infty\}$

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$A(z) = B(z) + C(z)$

$3z \qquad \frac{2}{1-2z}$

$b_r = \{0, 3, 0 \ldots 0\}$

$c_r = 2(2^r)$

$\therefore a_r = b_r + c_r = \begin{cases} 2\cdot 2^0 + 0 & r=0 \\ 2\cdot 2^1 + 3 & r=1 \\ 2\cdot 2^r + 0 & r \geq 2 \end{cases}$

eg) $A(z) = 3z + 4$

$3z \to B(z)$, $4 \to C(z)$

$b_r = \{0, 3, 0 \ldots\}$

$c_r = \{4, 0, 0 \ldots\}$

$a_r = b_r + c_r = \begin{cases} 4 & r=0 \\ 3 & r=1 \\ 0 & r \geq 2 \end{cases}$

HW eg) Find the DNF if ~~$A(z) = \frac{1}{1+z}$~~

(1) $A(z) = \frac{1}{1+z}$ (2) $A(z) = \frac{3-5z}{1-2z+3z^2}$

$a_r = \{ (-1)^r \quad r \geq 0$

(2) $A(z) = \frac{3-5z}{1-2z+3z^2}$

$= \frac{\cancel{3-5z}\ 1-2z+2-3z}{1-2z+3z^2}$

Types of generating function

(1) Ordinary generating function
(2) Exponential generating function

$A(z) = \sum_{r=0}^{\infty} a_r z^r$

$A(z) = \sum_{r=0}^{\infty} \frac{a_r}{r!} z^r$

eg) $\{0, -1, 0, 1, 0, -1 \ldots\}$

OGF: $A(z) = 0 - 1z + 0z^2 + 1z^3 + 0z^4 \ldots$

EGF: $A(z) = \frac{0}{0!} - \frac{1z}{1!} + \frac{0z^2}{2!} + \frac{1z^3}{3!} + \ldots$

Recurrence Relations

eg) $a_r = a_{r-2} + 4$

eg) Fibonacci series

$a_r = a_{r-1} + a_{r-2}$

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Q) Find the solution of

$a_{r+2} + 2a_{r+1} - 3a_r = 0$ — Linear → Homogeneous and $a_0 = 1$, $a_1 = 2$

Ans: ∴ chara. eqⁿ = $x^2 + 2x - 3 = 0$

$x = \frac{-2 \pm \sqrt{4+12}}{2} = 1, -3$

$x = 1$ and -3

∴ $a_r = A(1)^r + B(-3)^r$

– put $r = 0$.
$1 = A + B$

– put $r = 1$
$2 = A - 3B$

$A - 3B = 2$
$3A + 3B = 3$
$4A = 5$
$A = 5/4$ ∴ $B = -1/4$

∴ Equation becomes $a_n = \frac{5}{4} + \frac{-1}{4}(-3)^r$

Q) Find solution of
$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with $a_0 = 2, a_1 = 5, a_2 = 15$

Ans: chara eqⁿ

$x^3 - 6x^2 + 11x - 6 = 0$

$x = 3, 1, 2$

$a_r = A(3)^r + B(1)^r + C(2)^r$

Put $a_0 = 2$
$2 = A + B + C$

Put $a_1 = 5$
$5 = 3A + B + 2C$

Put $a_2 = 15$
$15 = 9A + B + 4C$

⇒ $3A + B + 2C = 5$
$-A - B - C = -2$
$2A + C = 3$
$C = 3 - 2A$

⇒ $9A + B + 12 - 8A = 15$
$A + B = 3$
$B = 3 - A$

Final $a_r = 2(3)^r + 1 - (2)^r$

$\boxed{a_r = 2(3)^r - 2^r + 1}$

∴ $A + 3 - A + 3 - 2A = 2$
$4/2 = A$ $(A = 2)$ $(B = 1)$ $(C = -1)$

not know

?

$a_n = a_n^{(h)} + a_n^{(p)}$

Homo — Particular solⁿ

Non homogenous linear recurrence relations:

We find total solution = Homogeneous solⁿ + Particular solⁿ

Particular solⁿ depends on function

Cases to find particular solution

(1) If $f(n)$ is a constant

$a_n^{(p)} = P$ (constant) → but diff. value

$a_n^{(p)} = 4$

(2) If $f(n) = C_0 + C_1 n$ then $a_n^{(p)} = P_0 + P_1 n$

Find P_0 and P_1 then substitute

eg) $a_{r+2} - a_{r+1} - 6a_r = 4$

→ To find $a_r^{(h)}$

CE $x^2 - x - 6 = 0$

$x = \frac{1 \pm \sqrt{1+24}}{2} = 3, -2$

$\therefore a_r^{(h)} = A(3)^r + B(-2)^r$

Now $a_r^{(p)} = ?$

Since $f(r) = 4$ = is a constant then solⁿ is also constant.

Let $a_r^{(p)} = p$ → random constant

Now we have to find 'p'.

$a_r^{(p)} = p$

$a_{r+1}^{(p)} = p$

$a_{r+2}^{(p)} = p$

Now just substitute these values in G eqⁿ

$\cancel{p - p} - 6p = 4$

$p = \frac{-2}{3}$

Now total solⁿ $= a_r = A(3)^r + B(-2)^r - \frac{2}{3}$

Calculate value of A and B at this step no prev.

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Q.) $a_{r+2} + 2a_{r+1} - 3a_r = 4$

Here when we solve for $a_r^{(P)}$ we get $0 = 4$ which is absurd. hence we go for second case

$p_0 + p_1 r = a_r^{(P)}$

$a_{r+1}^{(P)} = p_0 + p_1 r + p_1 = (p_0 + p_1) + p_1 r$

$a_{r+2}^{(P)} = p_0 + p_1 r + 2p_1 = (p_0 + 2p_1) + p_1 r$

Now substituting ; $p_0 + 2p_1 + p_1 r + 2((p_0 + p_1) + p_1 r) -$
$3p_0 - 3p_1 r = 4$

$= \cancel{p_0} + 2p_1 + \cancel{p_1 r} + \cancel{2p_0} + 2p_1 + \cancel{2p_1 r} - \cancel{3p_0} - \cancel{3p_1 r} = 4$

$= 4p_1 = 4$

$p_1 = 1$ and $p_0 = 0$ → because pehle cond^n se hi constant wale term we were getting as 0.

or we can also write as,

$(p_0 + 2p_0 + 2p_1 - 3p_0) + r(p_1 + 2p_1 - 3p_1) = 4 + 0(r)$

since we get $p_1 = 1$

and $p_0 + 2p_0 + 2p_1 - 3p_0$

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Q) $a_n = 6a_{n-1} - 9a_{n-2}$ $a_0 = 1, a_1 = 6$

CE ⇒

$x^2 - 6x + 9 = 0$

$x = \frac{6 \pm \sqrt{36-36}}{2}$

$= 3, 3$

Case (2) If roots are repeating then,

$a_n = (A + Bn)\, 3^n$ (where highest degree of 'n' is the no. of roots − 1 of the eqⁿ)

$(A = B = 1)$ — Answer; 3 — value of the repeating root

$\boxed{10^{th}\ April}$

Q) $d_n = 4(d_{n-1} - d_{n-2})$ $d_0 = d_1 = 1$

→ $x^2 - 4x + 4 = 0$

$x = 2, 2$

$d_n = (A + Bn)\, 2^{(n)}$

Put $n = 0$;

$1 = A$

Put $n = 1$

$1 = (1 + B)\, 2$

$\frac{1}{2} - 1 = B$

$B = -1/2$

Case (3) roots are complex

$\alpha \pm i\beta \longrightarrow a_{r+2} + 2c\, a_{r+1} + d a_r = 0$ (where $c^2 < d$ then roots are complex)

$a_r = r_1^{\,r}\,(A \cos r\theta + B \sin r\theta)$

$r_1 = \sqrt{\alpha^2 + \beta^2}$ $\theta = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$

6A. $12P_0 - 17P_1 + 29P_2 = 0$

$12P_0 - 17P_1 = -\frac{29}{4}$

6B $12P_1 - 34P_2 = 0$

$P_1 = \frac{34 \times 1}{4 \times 12} = \frac{17}{24}$

$12P_0 = \frac{-29 \times 6}{4 \times 6} + \frac{17 \times 17}{24}$

$= \frac{289 - 174}{24} = \frac{115}{24 \times 12} = \frac{115}{288} = P_0$

24
12
48
240
488

No initial condition given, so write the final answer.

Cases to find particular solution

(4) An exponential function:

$f(n) = cb^n$ where b is not a characteristic root then

$a_n^{(P)} = Pb^n$

(5) $f(n) = cb^n$ – b is a root with multiplicity m, — no of times root is repeating

$a_n^{(P)} = P.n^m.b^n$

eg) $a_n = 5a_{n-1} - 6a_{n-2} + 7^n$

$a_n^{(h)}$: $CE\ x^2 - 5x + 6 = 0$

$CE = x^2 - 5x + 6 = 0$

$\therefore x = \frac{5 \pm \sqrt{25-24}}{2} = 3, 2$

$\therefore a_n^{(h)} = A(3^n) + B(2^n)$

$a_n^{(P)} = P.7^n$ (since 7 is not a characteristic root)

$a_{n-1}^{(P)} = P.7^{(n-1)}$

$a_{n-2}^{(P)} = P.7^{(n-2)}$

$P.7^n - \frac{5}{7}P.7^n + \frac{6P.7^n}{49} = 7^n$

$49P - 35P + 6P = 49$

$\therefore \boxed{P = \frac{49}{20}}$

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Now substituting. $a_n = A(3)^n + B(2)^n + \frac{49}{20} 7^n$

eg) $a_r - 3a_{r-1} - 4a_{r-2} = 4^r$

$a_r^{(H)}$:

$CE = x^2 - 3x - 4 = 0$

$x = \frac{3 \pm \sqrt{9+16}}{2} = 4, 1$

$\therefore a_r^{(H)} = A(4)^r + B(1)^r$

Since 4 is a characteristic root,

$a_r^{(P)} = P \times r^1 \times 4^r$

$a_{r-1}^{(P)} = \frac{P(r-1)}{4} \times 4^r = \frac{Pr4^r}{4} - \frac{P4^r}{4}$

$a_{r-2}^{(P)} = \frac{P}{16}(r-2) \times 4^r = \frac{Pr4^r}{16} - \frac{P4^r}{8}$

Substituting:

$\cancel{Pr4^r} - \frac{3\cancel{Pr4^r}}{4} + \frac{3}{4}P\cancel{4^r} - \frac{\cancel{Pr4^r}}{4} + 2P\cancel{4^r} = \cancel{4^r}$

$\cancel{P}.$ $\cancel{\frac{3P+2P=1}{4}}$

$\cancel{11}P = 4$

$P = 4/5$

$\therefore Sol = A(4)^r + B(1)^r + \frac{4}{5} r 4^r$

Functions
Recurrence functions
Pigeonhole principal

$\underline{P=2}$

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~~generating functions~~

Q) $a_r - 2a_{r-1} = 3 \cdot 2^r$
$P = 3$

→ Combination of 2 formats

eg) $a_r + a_{r-1} = 3r2^r$
$a_r^{(H)}$: CE - $\cancel{x^2+x=0}$
$\cancel{x}(x+1) = 0$
$x = -1$

$a_r^{(H)} = A(-1)^r$

$a_r^{(P)} = (P_0 + P_1 r) \times 2^r$ since 2 is not a characteristic root.
✓
Case 2 + Case 4

→ Same as above

eg) $a_r - 4a_{r-1} + 4a_{r-2} = (r+1)2^r$
roots = $\cancel{x^2-4x+4}$ $x = 2, 2$ $\therefore m = 2$ = multiplicity
$a_r^{(H)} = (A + Br) \cdot 2^r$

Now $a_r^{(P)} = (P_0 + P_1 r) \times r^2 \times 2^r$ ≡ Particular Solution

Solution of recurrence relation using generating functions

$$A(z) = \sum_{r=0}^{\infty} a_r z^r \longrightarrow GF$$

Suppose we have,

(Q) Solve $a_n - 3a_{r-1} = 2$, $r \geq 1$ with $a_0 = 1$ using GF

→ Multiply by z^r on both sides

$$a_r z^r - 3a_{r-1} z^r = 2z^r$$

Since $r \geq 1$ we take summation from 1 to ∞.

∴ Summing for all $r \geq 1$ on both sides

$$\sum_{r=1}^{\infty} a_r z^r - \sum_{r=1}^{\infty} 3a_{r-1} z^r = 2 \sum_{r=1}^{\infty} z^r$$

$\boxed{1^{st}\text{ term}}$ $A(z) - a_0$

$\boxed{2^{nd}\text{ Term}}$ $3 \sum_{r=1}^{\infty} a_{r-1} z^r = 3 \sum_{r=1}^{\infty} a_{r-1} z^{r-1} (z)$

$$= 3z \sum_{r=0}^{\infty} a_r z^r = 3z\, A(z)$$

$\boxed{3^{rd}\text{ Term}}$: $2 \sum_{r=1}^{\infty} z^r = 2[z + z^2 + z^3 + \ldots]$

$$= 2z[1 + z + z^2 \ldots]$$

$$= \frac{2z}{1-z} \qquad \left(\frac{a}{1-r} = \text{sum of } \infty \text{ GP}\right)$$

Now just adding the 3 terms:

$$A(z) - a_0 - 3zA(z) = \frac{2z}{1-z}$$

$$A(z) - 3zA(z) - \frac{2z}{1-z} = 1$$

$$A(z)(1-3z) = \frac{2z+1-z}{1-z} = \frac{1+z}{1-z}$$

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$n >>> m$ n more than 2m

Extended pigeonhole principle

$$\left\lfloor \frac{n-1}{m} \right\rfloor + 1$$

1) If 30 dictionaries in a lib contain a total of 61327 pages then ~~show that~~ how many pages one of dictionaries will have.

$61327 >>> 30.$ $\left\lfloor \frac{61327-1}{30} \right\rfloor + 1 = 2045$

1 dict must contain 2045 pages atleast.

2) 6 friends $\longrightarrow$ ₹2161 .

$\left\lfloor \frac{2160}{6} \right\rfloor + 1 = 360+1 = 361$ atleast.

3) Show that if 7 no.s from 1-12 are chosen then 2 of them will add up to 13.

$A = \{1, 12\}$ $B = \{2, 11\}$ $C = \{3, 10\}$ $D = \{4, 9\}$
$E = \{5, 8\}$ $F = \{6, 7\}$

1 2 3 4 5 6 11 13

What is min no. of students req in a DSGT class to be sure that atleast 6 will recieve the same grade if there are 5 possible grades a,b,c,d,e

$\cancel{\frac{x-1}{6} + 1 \leq 5}$

Ans

Cases to find particular solution

4. An expanded f^n $f(n) = C \cdot b^n$ b is not a characteristic root
then $a_n^{(p)} = P \cdot b^{(n)}$

$f(n) = 7 \cdot 3^n$

$a_n^{(P)} = P\, 3^n$

5. $f(n) = C \cdot b^n$ - b is a root with multiplicity m

$a_n^{(p)} = p \cdot n^m\, b^n$

$a_n = 5a_{n-1} - 6a_{n-2} + 7^n$

$6a_{n-2} - 5a_{n-1} + a_n = 7^n$

$x^2 - 5x + 6x = 0$

Roots are $x = 3, 2$.

$a_n^{(h)} = A(3)^n + B(2)^n$

$a_n^{(p)} = p \cdot 7^n$

$a_{n-1}^{(p)} = p \cdot 7^{(n-1)}$

$a_{n-1}^{(p)} = p \cdot 7^{(n-2)}$

$p \cdot 7^n - \frac{5}{7} p\, 7^n + \frac{6}{49} p\, 7^n = 7^n$

$p - \frac{5}{7} p + \frac{6}{49} p = 1$

$\therefore \frac{49 - 35 + 6}{49} p = 1$

$p = \frac{49}{20}$

$a_n = A(3)^n + B(2)^n + \frac{49}{20} 7^n$

Q. $a_r - 3a_{r-1} - 4a_{r-2} = 4^r$

$x^2 - 3x - 4 = 0$

$x^2 - 4x + x - 4 = 0$

$x(x-4) + 1(x-4) = 0$

$\therefore x = 4, -1$

$a_n^{(h)} = A(4)^r + B(-1)^r$

$a_r^{(p)} = p \cdot 4^r \cdot r^1$ $\qquad a_n^{(p)} = p \cdot n^m b^n$ [r] characteristic root

$a_{r-1}^{(p)} = p \cdot (r-1) \cdot 4^{r-1}$

$a_{r-2}^{(p)} = p \cdot (r-2) \cdot 4^{r-2}$

$p \cdot r \cdot 4^r - \frac{3}{4}(p_r - p) \cdot 4^r - \frac{4}{16}(p_r - 2p)\, 4^r = 4^r$

$pr - \frac{3}{4}(pr - p) - \frac{1}{4}(p_r - 2p) = 1$

$r\left(1 - \frac{3}{4} - \frac{1}{4}\right) + \frac{3}{4} p + \frac{1}{2} p = 1$

$\therefore \frac{5}{4} p = 1 \qquad \therefore p = \frac{4}{5}$

$a_r - 3a_{r-1} - 4a_{r-2} = p \cdot \frac{4}{5} r \cdot 4^r$

$x = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$

$r_1 = r_1^{n}(A\cos r\theta + B\sin r\theta)$

$r_n = A\left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)^r$

$r_1 = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{5}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{5}{4}} = \frac{\sqrt{6}}{2}$

$A = 1/\sqrt{5}$

$B = \frac{-1}{\sqrt{5}}$

$a_n - 2a_{n-1} + 2a_{n-2} - a_{n-3} = 0$

$x^3 - 2x^2 + 2x - 1 = 0$

$x = 1, \frac{1}{2} \pm \frac{\sqrt{3}i}{2}$

$a_n = A(1)^n + a_1 1^n (B\cos n\theta + C\sin n\theta)$

$r_1 = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$

$\theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

Non homogeneous linear recurring relations

Case to find particular soln.

1. If f(n) is a constan.

Total sol = Homo soln + Particular soln

Depends on f^n

$a_n = a_n^{(h)} + a_n^{(p)}$

Thurs 12-12

Wed LA (20) Fri (10)

1) If f(n) is a constant

$a_n^{(p)} = P$ — constant

$a_{r+2} - a_{r+1} - 6a_r = 4$

To find $a_r^{(h)}$

CE: $x^2 - x - 6 = 0$

$x = 3, -2$

$a_r^{(h)} = A(3)^r + B(-2)^r$

As $f(r) = 4$, which is constant

$a_r^{(p)} = P$ — not sure.

$a_{r+1}^{(p)} = P$

$a_{r+2}^{(p)} = P$

$P - P - 6P = 4$

By substituting val of a_{r+2}, a_{r+1} & a_r in eq.

$-6P = 4$

$P = \frac{-2}{3}$

$a_r = a_r^{(h)} + a_r^{(p)}$

$-6P = 4$

$a_r = a_r^{(h)} + a_r^{(p)}$

$a_r = A(3)^r + B(-2)^r - \frac{2}{3}$

$a_0 = a_1 = 2$

$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ $a_0 = 2, a_1 = 5, a_2 = 15$

$a_2 = 15$

$CE = x^3 - 6x^2 + 11x - 6 = 0$

Roots

$C_0 a_r + C_1 a_{r-1} + C_2 a_{r-2}$

$x = 1, 2, 3$

$a_r = A(1)^n + B(2)^n + C(3)^n$

$2 = A + B + C$ when $n = 0$, $a_0 = 2$

$5 = A + 2B + 3C$ when $n = 1$, $a_1 = 5$

$15 = A + 4B + 9C$ when $n = 2$ & $a_2 = 15$

$A = 1, B = -1, C = 2$

$a_n = 6a_{n-1} - 9a_{n-2}$ $a_0 = 1, a_1 = 6$

$CE = x^2 - 6x + 9 = 0$

$x^2 - 3x - 3x + 9 = 0$

$x = 3, 3$

when roots are repeating the soln will be

$a_n = (A + Bn) \cdot 3^n$

put $n = 0$

$d_n = 4(d_{n-1} - d_{n-2})$ $d_0 = d_1 = 1$

$d_n - 4d_{n-1} + 4d_{n-2} = 0$

CE $x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x = 2, 2$

~~$a_n = A(2)^n + B(2)^n$~~

$d_n = (A + Bn) \cdot 2^n$

$d_0 = 1$

~~$1 = (A + B \cdot 0) 2^0$~~

$1 = A \cdot 2^0$

$A = 1$

$d_1 = 1$ $\therefore 1 = (1 + B) \cdot 2$

$B = -\frac{1}{2}$

$d_n = (1 - \frac{1}{2}$

Case 3 – Roots are complex

$\alpha + i\beta \rightarrow a_{r+2} + 2\gamma a_{r+1} + \delta a_r = 0$ where $\gamma^2 < \delta$

$a_r = r_1^r (A\cos r\theta + B \sin r\theta)$

$r_1 = \sqrt{\alpha^2 + \beta^2}$

$\theta = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$

$a_{r+2} + a_r = 0$

$x^2 + 1 = 0$

$x^2 = -1$

$x = \pm i$

$a_r = r_1^r (A\cos r\theta + B\sin r\theta)$

$r_1 = \sqrt{0^2 + (1)^2} = 1$

$\theta = \tan^{-1}\left(\frac{1}{0}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}$

$a_r = a_{r-1} + a_{r-2}$ $a_0 = 0, a_1 = 1$

$CE = x^2 - x - 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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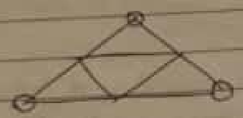
Let T be equilateral Δ side = 1 unit show that if 5 points are chosen lying inside ~~Show~~ or on Δ, then 2 of them must be no more than 1/2 unit apart



There are 3000 students in a college which offers 7 ~~A~~ discrete course of 4 years duration. A student who has taken a course in DSGT learns that the largest classroom can hold only 100 students. What is the problem.

Recurrence Relation

Recurrence Relations, Generating Functions, Numeric Functions

$f : A^{\to a} \to B^{\to b}$

$f(a) = b$

(start with 0 for the natural → real)

Discrete numeric function → a $N \to R$ then $a_0, a_1, a_2 \ldots a_r$ denote the values of fa as

$a(0) = a_0$, $a(1) = a_1$, $a(2) = a_2$ $a(r) = a_r$

$a = \{a_0, a_1, a_2, \ldots a_r\}$ → DNF (sequence)

$a_r = \begin{cases} r^2 & 0 \le r \le 3 \\ r+1 & r \ge 4 \end{cases}$

$a = \{0, 1, 4, 9, 5, 6 \ldots\}$

$a_r = r^3 + 2$, $r \ge 0$

$a_r = r^3 + 2$ $\quad b_r = 2^r$

$r \ge 0$ $\quad r \ge 0$

$a_r + b_r = ?$

$a_r = \begin{cases} 1 & 0 \le r \le 2 \\ 3^r & r \ge 3 \end{cases}$

$b_r = \begin{cases} 2^r + 1 & 0 \le r \le 1 \\ r - 5 & r \ge 2 \end{cases}$

$a_r + b_r = \begin{cases} 1 + 2^r + 1 = 2 + 2^r & 0 \le r \le 1 \\ 1 + r - 5 = r - 4 & r = 2 \\ 3^r + r - 5 & r \ge 3 \end{cases}$

Generating Functions → Alternate way to represent DNF

$a = \{a_0, a_1, a_2 \ldots a_r\}$

$a = \{a_0, a_1, a_2 \ldots a_r\}$

$A(z) = a_0 + a_1 z^1 + a_2 z^2 + a_3 z^3 \ldots a_r z^r +$

$= \sum_{r=0}^{\infty} a_r z^r$ $\quad a_r z^r$ → ~~co~~ term of degree r

Q. $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$ — (i)

$x^2 + 5x + 6 = 0$

$x = -2, -3$.

$a_r^{(h)} = A(-2)^r + B(-3)^r$

$a_r^{(p)} = P_0 + P_1 r + P_2 r^2$

$a_{r-1}^{(p)} = P_0 + P_1 r - P_1 + P_2(r-1)^2$

$= P_0 + P_1 r - P_1 + P_2 r^2 - 2P_2 r + P_2$

$= \cancel{P_0 + P_1(r-2) + P_2(r-2)}$

$a_{r-2}^{(p)} = P_0 + P_1(r-2) + P_2(r-2)^2$

$= P_0 + P_1 r - 2P_1 + P_2 r^2 - 4P_2 r + 4P_2$

substitute in (i). $a_r^{(p)}$; a_{r-1}^{p}, a_{r-2}^{p}

$\therefore P_0 + P_1 r + P_2 r^2 + 5P_0 + 5P_1 r - 5P_1 + 5P_2 r^2 - 10P_2 r + 5P_2$
$+ 6P_0 + 6P_1 r - 12P_1 + 6P_2 r^2 - 24P_2 r + 24P_2 = 3r^2$

$P_0 + 5P_0 - 5P_1 + 5P_2 + 6P_0 - 12P_1 + 24P_2 = 0$ — 1

$r(P_1 + 5P_1 - 10P_2 + 6P_1 - 24P_2) = 0$ — 2

$\cancel{r^2}(P_2 + 5P_2 + 6P_2) = 3\cancel{r^2}$

$12P_2 = 3 \quad P_2 = \frac{1}{4}$

$12P_1 - \frac{34}{4} = 0$

$P_1 = \frac{\cancel{34}\,17}{\cancel{4}\,2 \times 12} \qquad P_1 = \frac{17}{24}$

$\boxed{P_0 = \frac{115}{288}}$

$12P_0 = 17 \times \frac{17}{24} - 29 \times \frac{1}{4}$

Functions; pigeonhole, recurrence relation

$a_r - 2a_{r-1} = 3 \cdot 2^r$

$x - 2 = 0$ $\qquad b = 3$

$x = 2$

$a_n^{(h)} = A \cdot (2)^r$

$a_n^{(p)} =$

Q. $a_r + a_{r-1} = \underbrace{3r}_{\text{poly}} \cdot \underbrace{2^r}_{\text{exp}}$

root $x = -1$

$a_r^{(h)} = A \cdot (-1)^r$ ← Homo soln

$a_r^{(p)} = \underbrace{(p_0 + p_1 r)}_{\text{poly}} \cdot 2^r$ ← not characteristic root

$a_r^{(p-1)} = (p_0 + p_1 (r-1)) \cdot 2^{r-1}$

$\therefore (p_0 + p_1 r) \cdot 2^r - 2(p_0 + p_1(r-1)) \cdot 2^r = 3 \cdot 2^r$

$p_0 + p_1 r - 2p_0 - 2p_1(r-1) = 3$

$p_0 + p_1 r - 2p_0 - 2p_1 r + 2p_1 = 3$

$-p_0 + p_1 r = 3$

$a_r - 4a_{r-1} + 4a_{r-2} = (r+1) \cdot 2^r$

$x^2 - 4x + 4 = 0$

$x = 2, 2$

$a_r^{(h)} = (A + Br) \cdot 2^r$

$a_r^{p} = (p_0 + p_1 r) \cdot r^2 \cdot 2^r$

Solution of RR using Generating Functions

$A(z) = \sum_{r=0}^{\infty} a_r \cdot z^r$

Solve $a_r - 3a_{r-1} = 2$; $r \geq 1$ with $a_0 = 1$ using GF

$a_r - 3a_{r-1} = 2$

Multiply by z^r on both sides

$a_r \cdot z^r - 3a_{r-1} \cdot z^r = 2 z^r$

Summing for all $r \geq 1$ on both sides

$\sum_{r=1}^{\infty} a_r \cdot z^r - \sum_{r=1}^{\infty} 3a_{r-1} \cdot z^r = \sum_{r=1}^{\infty} 2 \cdot z^r$

as $A(z) = a_0 + \sum_{r=1}^{\infty} a_r \cdot z^r$

$\sum_{r=1}^{\infty} a_r \cdot z^r = A(z) - a_0$

2 term $3 \sum_{r=1}^{\infty} a_{r-1} z^{r-1} \cdot z$ $= 3z \sum_{r=1}^{\infty} a_{r-1} z^{r-1}$

$= \sum_{r=0}^{\infty} a_r \cdot z^r$

$= 3z A(z)$

$a = \{4^0, 4^1, 4^2 \cdots 4^r\}$

$A(z) = \sum_{r=0}^{\infty} 4^r z^r = 4^0 + 4^1 z^1 + 4^2 z^2 \ldots 4^r z^r$

$= \frac{1}{1-4z}$

$a_r = \alpha^r \quad ; r \geq 0$

$A(z) = \sum_{r=0}^{\infty} \alpha^r z^r = \frac{1}{1-\alpha z}$

$a_r = 7 \cdot 3^r \qquad A(z) = \sum_{r=0}^{\infty} 7 \cdot 3^r z^r \quad \frac{7}{1-3z}$

$a_r = \begin{cases} 0 & r \text{ is odd} \\ 2^{r+1} & r \text{ is even} \end{cases}$

$a = \{2^1, 0, 2^3, 0, 2^5, 0, 2^7 \ldots\}$

$a_r = x + y = 2^r + (-2)^r$

$(-2)^0 + 2^0 = 2$
$2^1 + (-2)^1 = 0$
$2^2 + (-2)^2 = 4$

$A(z) = \frac{1}{1-2z} + \frac{1}{1+2z}$

1) $\frac{2+3z-6z^2}{1-2z} = A(z) \quad$ Find a_r =

$-6z^2 + 3z + 2$

$= \frac{2 + 3z(1-2z)}{1-2z}$

$= \frac{2}{1-2z} + \frac{3z(\cancel{1-2z})}{\cancel{(1-2z)}}$

$= \frac{2}{1-2z} + 3z$

$\frac{2}{1-2z} \to A(z) \qquad 3z \to B(z)$

$C(z) = \frac{2}{1-2z} \qquad B(z) = 3z$

$a_1 = 3 \quad [a_0 + a_1 z^1 + a_2 z^2 \ldots]$

$a_r = 2 \cdot 2^r$

$b_r = \{0, 3, 0, 0 \ldots\}$

$r \geq 0$

if $r = 1 \quad 2 \cdot 2^r + 3$

rest $\quad 2 \cdot 2^r + 0$

$a_r = 2^{r+1} + 0 \qquad r = 0$

$2^{r+1} + 3 \qquad r = 1$

$2^{r+1} \qquad r \geq 2$

1. $A(z) = \frac{1}{1+z}$

2) $A(z) = \frac{3-5z}{1-2z-3z^2}$

Types of generating functions

1. Ordinary Generating func.
2. Exponential Generating func.

$A(z) = \sum a_r z^r \quad$ — OGF

$A(z) = \sum_{r=0}^{\infty} \frac{a_r}{r!} z^r \quad$ — EGF

$\{0, -1, 0, 1, 0, -1\} \quad$ — OGF

$a_0 \; a_1 \; a_2 \; a_3 \qquad$ EGF

$A(z) = 0 - z + 0z^2 + 1z^3 + 0z^4 \ldots$

$A(z) = \frac{0}{0!} z^0 + \frac{(-1)}{1!} z^1 + \frac{0}{2!} z^2 \ldots \quad$ EGF

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$\therefore A(z) = \dfrac{1+z}{(1-z)(1-3z)}$

If $A(z) = \dfrac{1}{1-az}$ $\quad a_r = a^r$

$A(z) = \dfrac{1}{(1-z)^2}$ $\quad a_r = r+1$

$A(z) = \dfrac{1}{(1-az)^2}$ $\quad a_r = (r+1)a^r$

$A(z) = \dfrac{z}{(1-z)^2}$ $\quad a_r = r$

$A(z) = \dfrac{az}{(1-az)^2}$ $\quad a_r = ra^r$

- Solve by partial fractions

$A(z) = \dfrac{A}{1-z} + \dfrac{B}{1-3z} = \dfrac{A - 3Az + B - Bz}{(1-z)(1-3z)} = \dfrac{(A+B) - z(3A+B)}{(1-z)(1-3z)}$

$A+B=1 \quad -3A-B=1$

$B = -3A-1$

$A - 3A = 2$

$-2A = 2$

$\boxed{A=-1}$ $\quad \boxed{B=2}$

$A(z) = \dfrac{-1}{1-z} + \dfrac{2}{1-3z} = -(1)^r + 2(3^r)$

$$\sum_{r=1}^{\infty} a_{r-1} z^{r-1} = A(z)$$

$$\sum_{r=1}^{\infty} a_r z^r \quad 2z(A(z) - a_0)$$

Q8) $a_r - 2a_{r-1} - 3a_{r-2} = 0$, $r \geq 2$, $a_0 = 3$, $a_1 = 1$ using GF

→ Multiplying z^r on both sides,

$$\underbrace{\boxed{\sum_{r=2}^{\infty} a_r z^r}}_{\text{Term1}} - \underbrace{\boxed{2\sum_{r=2}^{\infty} a_{r-1} z^r}}_{\text{Term2}} - \underbrace{\boxed{3\sum_{r=2}^{\infty} a_{r-2} z^r}}_{\text{Term3}} = 0.$$

$\boxed{\text{Term1}}$: $A(z) - a_0 - a_1 z = A(z) - 3 - z$

$\boxed{\text{Term2}}$: $2z\sum_{r=2}^{\infty} a_{r-1} z^{r-1} \longrightarrow 2z \sum_{r=2}^{\infty}$

$= 2z(A(z) - a_0)$ ~~...~~ $= 2z(A(z) - 3)$

$\boxed{\text{Term3}}$: $3z^2 \sum_{r=2}^{\infty} a_{r-2} z^{r-2} = 3z^2 A(z)$

$\therefore A(z) - z - 3 - 2zA(z) + 6z - 3z^2 A(z) = 0$

$3z^2 A(z) + 5z - 2zA(z) + A(z) - 3 = 0.$

$3z^2 A(z) + A(z)(1 - 2z) - 3 + 5z = 0$

$\cancel{3z^2 A(z) + A(z)(1-2z)} = \dfrac{3 - 5z}{3z^2 - 2z + 1}$

$\therefore A(z) = \dfrac{3 - 5z}{3z^2 - 2z + 1} =$

$3z^2 - 2z + 1 = 0$

$z^2 - \frac{2}{3}z + \frac{1}{3} = 0$

$\cancel{z = \dfrac{\frac{2}{3} \pm \sqrt{\frac{4}{9} - \frac{4 \times 3}{3 \times 3}}}{2}}$

$=$

$z = \dfrac{-2 \pm \sqrt{4 + 12}}{2}$

$= \dfrac{-2 \pm 4}{2}$

$-3, 1$

$A(z)(1 - 2z - 3z^2) - z - 3 + 6z = 0$

$A(z) = \dfrac{5z - 3}{3z^2 + 2z - 1}$

$A, B = 1, 2$

$\cancel{= \dfrac{5z - 3}{(z+3)(z-1)} = \dfrac{A}{z+3} + \dfrac{B}{z-1} = \dfrac{Az - A + Bz + 3B}{(z+3)(z-1)}}$

$\left.\begin{aligned} A + B &= 5 \\ 3B - A &= -3 \end{aligned}\right\}$ $\quad A = 3B + 3 \quad 4B = 2 \quad B = \frac{1}{2} \quad A = \frac{9}{2}$

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Q) Find $a_{r+2} + a_r = 0$

→ CE: $x^2 + 0x + 1 = 0$

$x = \pm i$

$x = \alpha + i\beta$
$= 0 + i(1)$
$x = \alpha - i\beta = 0 - i(1)$
$\therefore \alpha = 0, \beta = 1$

$a_r = r_1^{\,r} (A\cos r\theta + B\sin r\theta)$

$r_1 = \sqrt{0^2 + 1^2} = 1$

$\theta = \tan^{-1}(\quad) = \pi/2$

$\therefore a_r = A\cos r\frac{\pi}{2} + B\sin \frac{r\pi}{2}$

Put

Q) $a_r = a_{r-1} + a_{r-2}$ If $a_0 = 0, a_1 = 1$

→ CE: $x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$

Now this becomes case 1.

$a_r = A\left(\frac{1+\sqrt{5}}{2}\right)^r + B\left(\frac{1-\sqrt{5}}{2}\right)^r$

Put $a_0 = 0$
$0 = A + B$
$A = -B$

Put $a_1 = 1$
$1 = \frac{A}{2} + \frac{\sqrt{5}A}{2} - \frac{A}{2} + \frac{\sqrt{5}A}{2}$

$\frac{1}{\sqrt{5}} = A$

Q) $a_n - 2a_{n-1} + 2a_{n-2} - a_{n-3} = 0$

→ CE: $x^3 - 2x^2 + 2x - 1 = 0$.

0.86603

$x = 1, \frac{1}{2} \pm i\frac{\sqrt{3}}{2} = \frac{1 \pm \sqrt{3}i}{2}$

$a_n = A(1)^n + r_1^{\,n}(B\cos n\theta + C\sin n\theta)$

$r_1 = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{3}$

$a_n = A + B\cos\frac{n\pi}{3} + C\sin\frac{n\pi}{3}$

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Linear recurrence relation with constant coefficients

r and k → non negative int

- $c_0 a_r + c_1 a_{r-1} + c_2 a_{r-2} + \dots c_k a_{r-k} = f(r) \quad r \geq k$

where $c_0, c_1, c_2 \dots c_k$ are constants

This equation is called a linear recurrence relation of order 'k'

('k' tak jaa raha hai)

eg) $a_r = a_{r-2} + 4$

Degree is 1 so linear and order $= k = 2$

Here $c_0 = 1$, $c_1 = 0$, $c_2 = -1$ $f(r) = 4$ → can be constant or any function of 'r'

⟹ Homogenous solutions — 3 cases to find the solution ($f(r) = 0$)

$a_r = f(r)$

⟹ Non homogenous → $f(r) != 0$.

eg 1) $a_{r+2} - a_{r+1} - 6a_r = 0$ → linear recurrent relation → homogenous

<u>Characteristic equation</u> :

order $= 2$

$x^2 - x - 6 = 0 \quad [c_0 x^2 + c_1 x + c_2 = 0] \quad c_0 = 1,\ c_1 = -1,\ c_2 = -6$

$x = \frac{1 \pm \sqrt{1+24}}{2}$

$= \frac{1 \pm 5}{2} \quad (3, -2)$

$x = 3$ and -2 ⟹ Roots of the characteristic eqn.

Case (1) If roots are real and distinct

$a_r = A(-2)^r + B(3)^r$

Now substitute with given values of a_r and solve the simultaneous eqns.

Domain $\rightarrow x$ in $f(x)$
Range $\rightarrow y = f(x)$

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Functions

- Injective : one to one →
- Surjective one to all
- Bijective : all to all

$$A \quad B$$

onto | functions
Range set = co domain set

Composition of functions

$f: A \rightarrow B$, $g: B \rightarrow C$

$g \circ f$ - composition $= g(f(x))$

eg) $f(x) = ax + b$ $g(x) = cx + d$ $a, b, c, d \Rightarrow$ constants

Determine for which value of a, b, c, d $f \circ g = g \circ f$

$\Rightarrow$ $a(cx+d) + b = c(ax+b) + d$

$\cancel{acx} + ad + b = \cancel{acx} + bc + d$

$ad - d = bc - b$

$a(d-1) = b(c-1)$

$\frac{a}{b} = \frac{c-1}{d-1}$

eg) $f(x) = 2x+3$, $g(x) = 3x+4$, $h(x) = 4x$ $x \in R$

$f \circ g$, $g \circ f$, $h \circ g$, $g \circ h$, $f \circ g \circ h$

$f(g(h(x))) = 2(3(4x)+4)+3$
$= 24x + 11$

$f(g(x)) = 6x + 11$
$g(f(x)) = 6x + 13$
$h(g(x)) = 12x + 16$
$g(h(x)) = 12x + 4$

The relation f can be described as set of pairs $\{(a, f(a)) \mid a \in Dom(f)\}$.

a is also called as the argument of function f and $f(a)$ is called value of the function for the argument 'a'.