

# Diffie-Hellman Algorithm

$$[6^5]^4 = [6^4]^5$$

$$A^B \bmod C =$$

$$( (A \bmod C)^B ) \bmod C$$

## History

- The Diffie-Hellman algorithm was developed by Whitfield Diffie and Martin Hellman in 1976.
- This algorithm was devised not to encrypt the data but to generate same private cryptographic key at both ends so that there is no need to transfer this key from one communication end to another.

# Modular Arithmetic

**Modular exponentiation**

$$A^B \bmod C = ( (A \bmod C)^B ) \bmod C$$

# Modular Arithmetic

**Modular operator**

$$-29 \bmod 3 = 1$$

$$-14 \bmod 2 = 0$$

$$-4 \bmod 9 = 5$$

$$-17 \bmod 7 = 4$$

# Modular Arithmetic Congruence

**modulo**

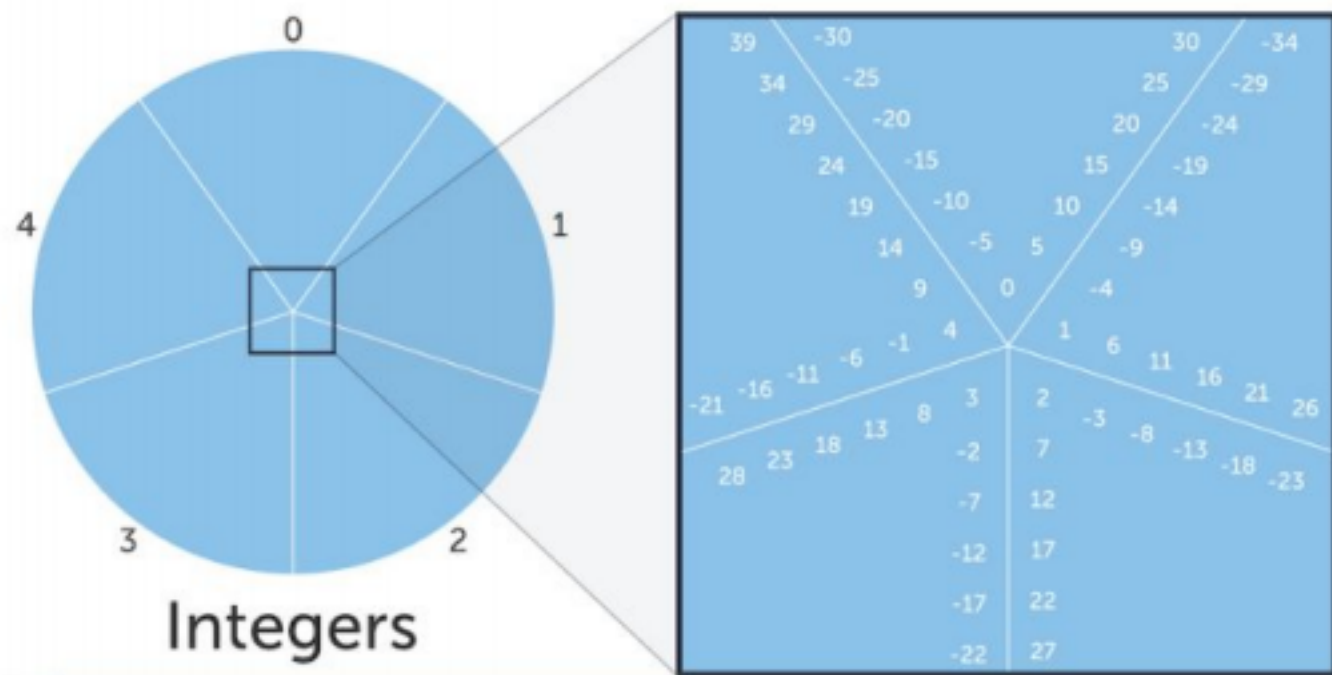
$$A \equiv B \pmod{C}$$

This says that  $A$  is **congruent** to  $B$  modulo  $C$ .

# Modular Arithmetic

Congruence  
modulo

Let's imagine we were calculating mod 5 for all of the integers:



# Modular Arithmetic

**Congruence modulo**

$$A \equiv B \pmod{C}$$

e.g.  $26 \equiv 11 \pmod{5}$

$26 \bmod 5 = 1$  so it is in the equivalence class for 1,

$11 \bmod 5 = 1$  so it is in the equivalence class for 1, as well.

$\equiv$  is the symbol for congruence, which means the values  $A$  and  $B$  are in the same **equivalence class**.



# Modular Arithmetic

## Modular multiplication

$$(A * B) \bmod C = (A \bmod C * B \bmod C) \bmod C$$

Modular Arithmetic Modular

**addition, subtraction**

$$(A + B) \bmod C = (A \bmod C + B \bmod C) \bmod C$$

$$(A - B) \bmod C = (A \bmod C - B \bmod C) \bmod C$$

# Modular Arithmetic

**inverse**

**Example: A=3, C=7**

**Step 1. Calculate  $A * B \bmod C$  for B values 0 through C-1**

$$3 * 0 \equiv 0 \pmod{7}$$

$$3 * 1 \equiv 3 \pmod{7}$$

$$3 * 2 \equiv 6 \pmod{7}$$

$$3 * 3 \equiv 9 \equiv 2 \pmod{7}$$

$$3 * 4 \equiv 12 \equiv 5 \pmod{7}$$

$$3 * 5 \equiv 15 \pmod{7} \equiv 1 \pmod{7} \quad \text{<----- FOUND INVERSE!}$$

$$3 * 6 \equiv 18 \pmod{7} \equiv 4 \pmod{7}$$

5 is the modular inverse of 3 mod 7 since  $5*3 \bmod 7 = 1$