SPRING2021
MAT120
ASSIGNMENT-05

April 18, 2021

Name:Nur-E-Jannat
Student ID:20210002
Section:07
Set:03

1. $\int_0^\pi \int_0^1 \int_0^{\pi/6} xy\sin(yz)dzdydx$

$\int_0^{\pi/6} xy\sin(yz)dz$

$= xy\int_0^{\pi/6}\sin(yz)dz$

Let,

$u = yz$

$\Rightarrow du = ydz$

$\Rightarrow dz = \frac{1}{y}du$

z	0	$\pi/6$
u	0	$\pi y/6$

$$\begin{aligned}
& By substituting, \\
& xy\int_0^{\pi/6}\sin(u)\cdot\frac{1}{y}du \\
= \ & xy\cdot\frac{1}{y}\int_0^{\pi y/6}\sin(u)du \\
= \ & x[-\cos(u)]_0^{\pi y/6} \\
= \ & x\left[-\left(\cos\left(\frac{\pi y}{6}\right)-1\right)\right] \\
= \ & x\left(-\cos\left(\frac{\pi y}{6}\right)+1\right) \\
\int_0^1 \ & x\left(-\cos\left(\frac{\pi y}{6}\right)+1\right)dy \\
x \ & \left(\int_0^1 1dy - \int_0^1\cos\left(\frac{\pi y}{6}\right)dy\right)
\end{aligned}$$

$\int_0^1 1dy$
$= [y]_0^1$
$= 1$
$\int_0^1 \cos\left(\frac{\pi y}{6}\right) dy$
$Let,$
$u = \frac{\pi y}{6}$
$\Rightarrow du = \frac{\pi}{6}dy$
$\Rightarrow dy = \frac{6}{\pi}du$

y	0	1
u	0	$\pi/6$

$\int_0^1 \cos\left(\frac{\pi y}{6}\right) dy$
By substituting,
$\int_0^{\pi/6} \cos(u)\frac{6}{\pi}du$
$= \frac{6}{\pi}[\sin(u)]_0^{\pi/6}$

$= \frac{6}{\pi}\left(\sin\left(\frac{\pi}{6}\right) - \sin(0)\right)$

$= \frac{6}{\pi} \times \frac{1}{2}$

$$= \frac{3}{\pi}$$

$$\int_0^1 x\left(-\cos\left(\frac{\pi y}{6}\right)+1\right)dy$$
$$= x\left(1-\frac{3}{\pi}\right)$$

$$\int_0^\pi x\left(1-\frac{3}{\pi}\right)dx$$
$$= \left(1-\frac{3}{\pi}\right)\cdot\int_0^\pi x dx$$
$$= \left(1-\frac{3}{\pi}\right)\cdot\left[\frac{x^2}{2}\right]_0^\pi$$
$$= \left(1-\frac{3}{\pi}\right)\cdot\frac{\pi^2}{2}$$
$$= \frac{\pi^2}{2}-\frac{3\pi}{2}$$
$$= \frac{\pi^2-3\pi}{2}$$

$$\int_0^\pi \int_0^1 \int_0^{\pi/6} xy\sin(yz)dzdydx$$

$$= \frac{\pi^2-3\pi}{2}$$

2. $\int_{-1}^{1}\int_{0}^{1-x^2}\int_{0}^{y} ydzdydx$

$= \int_{-1}^{1}\int_{0}^{1-x^2} y[z]_0^y dydx$

$= \int_{-1}^{1}\int_{0}^{1-x^2} y^2 dydx$

$= \int_{-1}^{1}\left[\frac{y^3}{3}\right]_0^{1-x^2} dx$

$= \int_{-1}^{1}\left(\frac{\left(1-x^2\right)^3}{3} - 0\right) dx$

$= \frac{1}{3}\int_{-1}^{1}\left(1 - 3x^2 + 3x^4 - x^6\right) dx$

$= \frac{1}{3}\left[\int_{-1}^{1} 1dx - \int_{-1}^{1} 3x^2dx + \int_{-1}^{1} 3x^4dx - \int_{-1}^{1} x^6dx\right]$

$= \frac{1}{3}\int_{-1}^{1} 1dx - \int_{-1}^{1} x^2dx + \int_{-1}^{1} x^4dx - \frac{1}{3}\int_{-1}^{1} x^6dx$

$= \frac{1}{3}[x]_{-1}^{1} - \left[\frac{x^3}{3}\right]_{-1}^{1} + \left[\frac{x^5}{5}\right]_{-1}^{1} - \frac{1}{3}\left[\frac{x^7}{7}\right]_{-1}^{1}$

$= \left(\frac{1}{3} \times 2\right) - \left(\frac{1}{3} + \frac{1}{3}\right) + \left(\frac{1}{5} + \frac{1}{5}\right) - \left(\frac{1}{3} \times \left(\frac{1}{7} + \frac{1}{7}\right)\right)$
$= \frac{2}{3} - \frac{2}{3} + \frac{2}{5} - \frac{2}{21}$
$= \frac{32}{105}$

3. $\int_0^{\sqrt{2}} \int_0^{x} \int_0^{2-x^2} xyz dz dy dx$

$= \int_0^{\sqrt{2}} \int_0^{x} xy \left[\frac{z^2}{2}\right]_0^{2-x^2} dy dx$

$= \int_0^{\sqrt{2}} \int_0^{x} xy \cdot \frac{\left(2-x^2\right)^2}{2} dy dx$

$= \int_0^{\sqrt{2}} \frac{x\left(2-x^2\right)^2}{2} \cdot \left[\frac{y^2}{2}\right]_0^{x} dx$

$= \int_0^{\sqrt{2}} \frac{x^3\left(2-x^2\right)^2}{4} dx$

$= \frac{1}{4} \int_0^{\sqrt{2}} x^3 \left(2 - x^2\right)^2 dx$

Let, $u = x^3$

$\Rightarrow x = u^{1/3}$

$\Rightarrow dx = \frac{1}{3} u^{-\frac{2}{3}} du$

$x^2 = u^{2/3}$

$\Rightarrow \left(2 - x^2\right)^2 = \left(2 - u^{2/3}\right)^2$

x	0	$\sqrt{2}$
u	0	$2\sqrt{2}$

By substituting,

$\frac{1}{4}\int_0^{2\sqrt{2}} u\left(2-u^{2/3}\right)^2 \cdot \frac{1}{3}u^{-2/3}du$

$= \frac{1}{12}\int_0^{2\sqrt{2}} u^{1/3}\left(4-4u^{2/3}+u^{4/3}\right)du$

$= \frac{1}{12}\int_0^{2\sqrt{2}} \left(4u^{1/3}-4u+u^{5/3}\right)du$

$= \frac{1}{12}\left[\int_0^{2\sqrt{2}} 4u^{1/3}du - \int_0^{2\sqrt{2}} 4udu + \int_0^{2\sqrt{2}} u^{5/3}du\right]$

$\int_0^{2\sqrt{2}} 4u^{1/3}du$

$= 4\left[\frac{3}{4}u^{4/3}\right]_0^{2\sqrt{2}}$

$= 4 \times \frac{3}{4}(2\sqrt{2})^{4/3}$

$= 12$

$\int_0^{2\sqrt{2}} 4udu$

$= 4\left[\frac{u^2}{2}\right]_0^{2\sqrt{2}}$

$= 4 \times \frac{(2\sqrt{2})^2}{2}$

$= 16$

$$\int_0^{2\sqrt{2}} u^{5/3} du$$
$$= \left[\frac{3}{8} u^{8/3}\right]_0^{2\sqrt{2}}$$
$$= \frac{3}{8}(2\sqrt{2})^{8/3}$$
$$= 6$$

$$\frac{1}{12}\left[\int_0^{2\sqrt{2}} 4u^{1/3} du - \int_0^{2\sqrt{2}} 4u du + \int_0^{2\sqrt{2}} u^{5/3} du\right]$$
$$= \frac{1}{12}(12 - 16 + 6)$$
$$= \frac{1}{6}$$
$$\int_0^{\sqrt{2}} \int_0^{x} \int_0^{2-x^2} xyz dz dy dx = \frac{1}{6}$$

4. $v = \int_0^1 \int_0^{1-x} \int_0^{\sqrt{y}} dzdydx$

$= \int_0^1 \int_0^{1-x} [z]_0^{\sqrt{y}} dydx$

$= \int_0^1 \int_0^{1-x} \sqrt{y} dydx$

$= \int_0^1 \left[\frac{2}{3} y^{3/2}\right]_0^{1-x} dx$

$= \int_0^1 \frac{2}{3}(1-x)^{3/2} dx$

$\int_0^1 \frac{2}{3}(1-x)^{3/2} dx$
Let, $u = 1 - x$
$\Rightarrow du = -1dx$
$\Rightarrow dx = -du$

x	0	1
u	1	0

By substituting, $\int_1^0 -\frac{2}{3}(u)^{3/2} du$
$= -\frac{2}{3} \left[\frac{2}{5} u^{5/2}\right]_1^0$
$= -\frac{4}{15} \times \left(0 - \frac{2}{5}\right)$
$= \frac{4}{15}$

Untitled-5 * - Wolfram Mathematica 12.1

File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

5. a)

$dx + e^{3x} dy = 0$

$y' = \frac{1}{-e^{3x}}$

```
ClearAll["Global`*"]
```

In[109]:= $\text{DSolve}\left[y'[x] == \frac{1}{-e^{3x}}, y[x], x\right]$

Out[109]= $\left\{\left\{y[x] \to \frac{e^{-3x}}{3} + c_1\right\}\right\}$

In[110]:= 5. b)

```
ClearAll["Global`*"]
```

In[120]:= $y[x] = \left(\frac{e^{-3x}}{3}\right) c;$

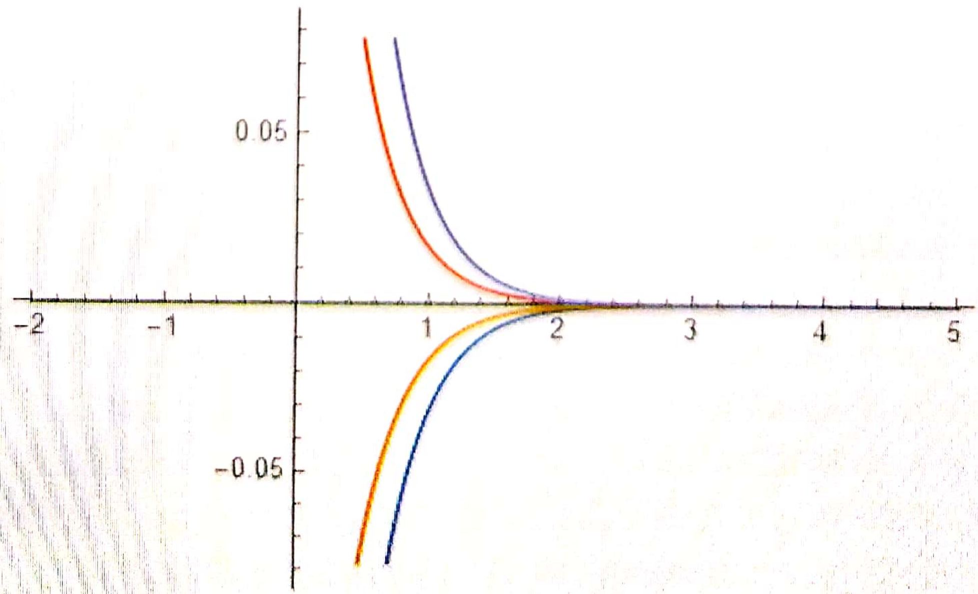
```
Table[y[x], {c, -2, 2}]
```

Out[121]= $\left\{-\frac{2}{3} e^{-3x}, -\frac{1}{3} e^{-3x}, 0, \frac{e^{-3x}}{3}, \frac{2 e^{-3x}}{3}\right\}$

In[112]:= $y[x] = \left(\frac{e^{-3x}}{3}\right) c;$

```
Plot[Evaluate[Table[y[x], {c, -2, 2}]], {x, -2, 5}]
```

Out[113]=



In[163]:=
```
ClearAll["Global`*"]
```