Department of Mathematics and Natural Sciences

MAT 120

ASSIGNMENT 2

SPRING 2021

SET: 15

Please write your Name, ID and Section on the first page of the assignment answer script - you have to do this for both handwritten or LaTeX submission. The last date of submission is "21/03/2021". Solve all problems.

You can only submit a PDF file - image or doc files won't be accepted. Before submitting the PDF, please rename the PDF file in the format - SET_ID_SECTION.

Answer the questions by yourself. Plagiarism will lead to an F grade in the course. ***Total marks is "250".*** *It will be converted to 25 and if you do your work using LaTeX you will get a bonus 50 marks. Which will be converted to 5. So highest marks you can get out of 25 is 30 provided you do everything correct and you submit your assignment in*

Note that you **must evaluate any gamma functions for integer or half-integer arguments**. If the arguments are ***neither*** integers ***nor*** half-integers simply express your answers in terms of gamma functions.

1. Use β and Γ functions to evaluate the following integral

$$\int_0^{\arctan\left(\frac{\pi}{14}\right)} \frac{\sin^3\left(14\tan\left(x\right)\right)\cos^8\left(7\tan\left(x\right)\right)}{\cos^2\left(x\right)}dx$$

Hint: With the help of trigonometric identities and a simple substitution, one can convert the above integral into the trigonometric form of the β function

2. Evaluate the following
$$\int \frac{\cos(4x)}{\sin x}\, dx$$
Show all your working, direct use of formula will not be accepted

3. Evaluate the following
$$\int \frac{e^{5x}}{1+e^{10x}} dx$$
Show all your working, direct use of formula will not be accepted

4. Gaussian distributions have a wide variety of applications, including but not limited to probability theory, machine learning, quantum computing and quantum mechanics.

You are dealing with a quantum particle. The position of a quantum particle is a random Gaussian variable with the **wave function** $\psi(x) = Ae^{-\frac{1}{2}x^2}x^6$, where A is a positive real constant you will find out.

a)Show that $\psi(x)^2$ is even.

To find the value of A, we need to use the concept that the "sum" of all probabilities must add up to 1. Since, our wave function varies continuously, the analogous concept of "sum" is an integral. Physically, this means that the particle must be somewhere. Given that,
$$\int_{-\infty}^{+\infty} \psi(x)^2 dx = 1$$

b)By exploiting the fact that $\psi(x)^2$ is even, rewrite the equation above so that the lower limit is 0 and the upper limit is $+\infty$.
Hint: Think about the property of integral of even functions $\int_{-a}^{a} f(x)\, dx$, **where a is some constant and** $f(x)$ **is even.**

c)Now, find the value of A using part b) and Γ functions. (A is a positive real constant).

The average value of any continuous random variable that you will obtain after numerious trials is known has the "**expectation**" value. Given that the expectation value of the position **squared** (in algebra, it is simply x^2) is,
$$Expected \;\; value = \int_{-\infty}^{\infty} x^2 \psi(x)^2\, dx$$

d) Evaluate the above integral by exploiting the fact that the **integrand** is **even** and with the help of Γ functions to find the expected value of position squared.

(**This question is big but rest assure the answer is not, it uses only simple properties about even integrals and the Γ function.**

5. Do the following tasks using Mathematica.

i) Find the length L of the curve $y = \ln(\sec x),\ \ 0 \le x \le \pi/4$
Use any one of the following formulas

$$L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}\, dx$$

$$L = \int_a^b \sqrt{1 + (\frac{dx}{dy})^2}\, dy$$

ii) Find the area of the surface obtained by rotating the curve $x = 1 + 2y^2,\quad 1 \le y \le 2 about x axis$

iii)Find the area of the surface obtained by rotating the curve $y = \ln x + 1,\quad 0 \le x \le 1$ about y axis

Use the following formulas

$$S = \int_a^b 2\pi y \sqrt{1 + (\frac{dy}{dx})^2}\, dx$$

$$S = \int_a^b 2\pi x \sqrt{1 + (\frac{dx}{dy})^2}\, dy$$