SPRING2021
MAT120
ASSIGNMENT-03

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Section:07
Set:4

1. Given that,

The portion of the curve,

$$y = \sqrt{4 - x^2}(-1 \leq x \leq 1)$$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{4 - x^2}\right)$

$= \frac{1}{2}\left(4 - x^2\right)^{-\frac{1}{2}} \cdot (-2x)$

$= -\frac{x}{\sqrt{4-x^2}}$

$\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{4-x^2}$

$$
\begin{aligned}
S &= \textstyle\int_{-1}^{1} 2\pi \left(\sqrt{4-x^2}\right) \cdot \sqrt{1+\frac{x^2}{4-x^2}} dx \\
&= \textstyle\int_{-1}^{1} 2\pi \left(\sqrt{4-x^2}\right) \cdot \sqrt{\frac{4}{4-x^2}} dx \\
&= \textstyle\int_{-1}^{1} 2\pi \left(\sqrt{4-x^2}\right) \cdot \frac{2}{\sqrt{4-x^2}} dx \\
&= \textstyle\int_{-1}^{1} 2\pi \left(\sqrt{4-x^2}\right) \cdot \frac{2}{\sqrt{4-x^2}} dx \\
&= \textstyle\int_{-1}^{1} (4\pi) dx \\
&= 4\pi [x]_{-1}^{1} \\
&= 8\pi
\end{aligned}
$$

$2.x = e^t(\sin t + \cos t)$
$y = e^t(\cos t - \sin t)$
$(1 \le t \le 4)$
$x' = \frac{d}{dt}\left[e^t(\sin t + \cos t)\right]$
$= e^t \sin t + e^t \cos t + e^t \cos t - e^t \sin t$
$= 2e^t \cos t$
$y' = \frac{d}{dt}\left[e^t(\cos t - \sin t)\right]$
$= -e^t \sin t + e^t \cos t - e^t \sin t - e^t \cos t$
$= -2e^t \sin t$
$L = \int_a^b \sqrt{(x')^2 + (y')^2} dt$
$= \int_1^4 \sqrt{(2e^t \cos t)^2 + (-2e^t \sin t)^2} dt$
$= \int_1^4 \sqrt{4e^{2t} \left(\cos^2 t + \sin^2 t\right)} dt$
$= \int_1^4 \sqrt{4e^{2t}} dt \left[\cos^2 t + \sin^2 t = 1\right]$
$= \left[2e^t\right]_1^4$
$= \left(2e^4 - 2e^1\right)$
$= 2\left(e^4 - e\right)$

3.Given that,

$$x = \frac{1}{8}y^1 + \frac{1}{4}y^{-2}$$
$$\frac{dx}{dy} = \frac{d}{dy}\left(\frac{1}{8}y^4 + \frac{1}{4}y^{-2}\right)$$
$$= \frac{1}{8} \times 4y^3 + \frac{1}{4}(-2) \times y^{-3}$$
$$= \frac{y^3}{2} - \frac{y^{-3}}{2}$$
$$\left(\frac{dx}{dy}\right)^2 = \left(\frac{y^3}{2} - \frac{y^{-3}}{2}\right)^2$$
$$= \left(\frac{y^3}{2}\right)^2 - 2 \times \frac{y^3}{2} \times \frac{y^{-3}}{2} + \left(\frac{y^{-3}}{2}\right)^2$$
$$= \frac{y^6}{4} - \frac{1}{2} + \frac{y^{-6}}{4}$$
$$\left(\frac{dx}{dy}\right)^2 + 1 = \frac{y^6}{4} - \frac{1}{2} + \frac{y^{-6}}{4} + 1$$
$$= \left(\frac{y^3}{2} + \frac{y^{-3}}{2}\right)^2$$
$$L = \int_1^4 \sqrt{\left(\frac{y^3}{2} + \frac{y^{-3}}{2}\right)^2} dy$$
$$= \int_1^4 \left(\frac{y^3}{2} + \frac{y^{-3}}{2}\right) dy$$
$$= \left[\frac{y^4}{8} - \frac{y^{-2}}{4}\right]_1^4$$
$$= 0.14$$

4.Given that, $x = 2\sqrt{1-y}$

$\frac{dx}{dy} = \frac{d}{dy}(2\sqrt{1-y})$

$= 2 \cdot \frac{1}{2}(1-y)^{-\frac{1}{2}} \cdot (-1)$

$= -\frac{1}{\sqrt{1-y}}$

S $= \int_a^b 2\pi(2\sqrt{1-y})\sqrt{1+\left(\frac{dx}{dy}\right)^2} dy$

$= \int_{-1}^0 2\pi(2\sqrt{1-y}) \cdot \sqrt{\frac{2-y}{1-y}} dy$

$= \int_{-1}^0 2\pi(2\sqrt{1-y}) \cdot \frac{\sqrt{2-y}}{\sqrt{1-y}} dy$

$= \int_{-1}^0 4\pi \cdot \sqrt{2-y} dy$

$=4\ \pi \times \left[-\frac{2}{3}(2-y)^{3/2}\right]_{-1}^0$

$= -\frac{8\pi}{3}\left[(2-y)^{3/2}\right]_{-1}^0$

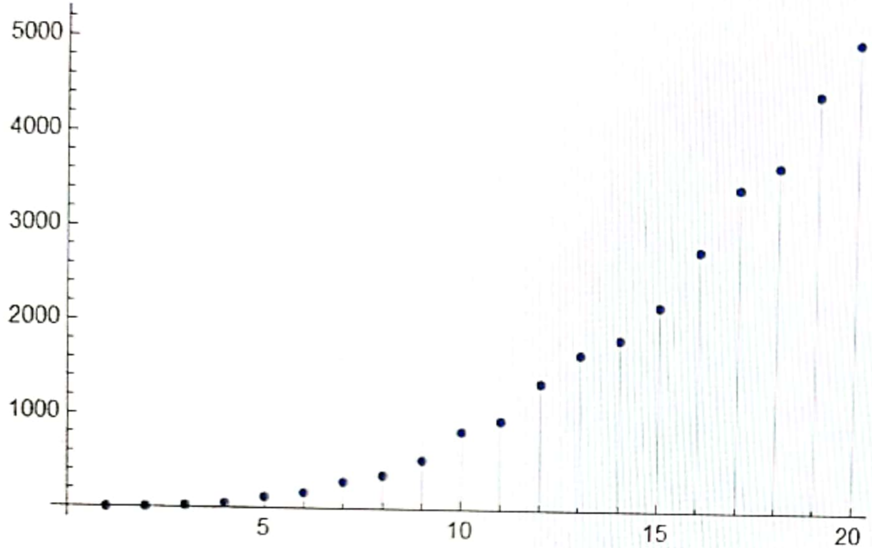
$= -19.83$

In[ ]:= Table[Prime[numbers]^2, {numbers, 20}]

Out[ ]= {4, 9, 25, 49, 121, 169, 289, 361, 529, 841, 961, 1369, 1681, 1849, 2209, 2809, 3481, 3721, 4489, 5041}

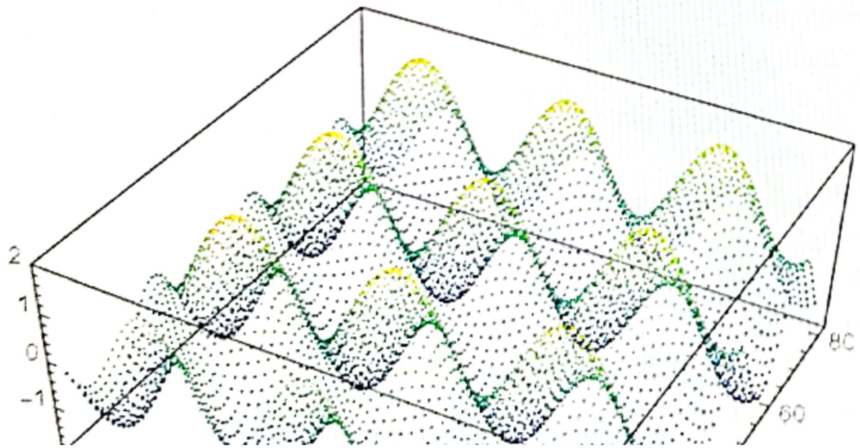
In[ ]:= ListPlot[Table[Prime[numbers]^2, {numbers, 20}], Filling → Axis]

Out[ ]=



In[ ]:= ListPointPlot3D[Table[Sin[a] + Cos[b], {a, -10, 10, 0.25}, {b, -10, 10, 0.25}], ColorFunction → "BlueGreenYellow"]

Out[ ]=



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