SPRING2021
MAT120
ASSIGNMENT-01

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Name:Nur-E-Jannat
Student ID:20210002
Section:07
Set:7

1) $\int_0^{+\infty}(1-x)e^{-x}dx$

Let, $u=(1-x)$

or, $du=-1dx$

$dv=e^{-x}dx$

or, $v=-e^{-x}$

We know, $\int udv=uv-\int vdu$

$\int(1-x)e^{-x}dx$

$=(1-x)\left(-e^{-x}\right)-\int\left(-e^{-x}\right)(-1)dx$

$=-e^{-x}+xe^{-x}+e^{-x}+c$

$=xe^{-x}+c$

$\int_0^{\infty}(1-x)e^{-x}dx$

$=\lim_{t\to\infty}\left(xe^{-x}\right)_0^t$

$=\lim_{t\to\infty}\left(te^{-t}\right)$

$=0$

It in convergent.

$2)Let,$

$B = \int \cos^n x dx$

$B = \int \cos^{n-1}(x) \cdot \cos(x) dx$

$\int u dv = uv - \int v du$

$u = \cos^{n-1}(x)$

$or, du = (n-1)\cos^{n-2}(x) \cdot \sin x dx$

$dv = \cos(x) dx$

$or, v = -\sin(x)$

$\int \cos^{n-1}(x) \cdot \cos x dx$

$= \cos^{n-1}(x) \cdot \sin(x)$

$- \int \sin x \cdot (n-1)\cos^{n-2}(x) \sin x dx$

$= \cos^{n-1}(x)\sin(x) + \int (n-1)\cos^{n-2}(x)\sin^2 x dx$

$= \cos^{n-1}(x)\sin(x) + \int (n-1)\cos^{n-2}(x)\left(1 - \cos^2 x\right) dx$

$= \cos^{n-1}(x)\sin(x) + \int (n-1)\cos^{n-2}(x) dx$

$- \int (n-1)\cos^n(x) dx$

$B = \cos^{n-1}(x)\sin(x) + (n-1)\int \cos^{n-2} x dx$
$-(n-1)B$
$or, B + (n-1)B = \cos^{n-1}(x)\sin(x)$
$+(n-1)\int \cos^{n-2} x dx$
$or, n \cdot B = \cos^{n-1}(x)\sin(x) + (n-1)\int \cos^{n-2} x dx$
$or, n\int \cos^{n}(x)dx = \cos^{n-1}(x)\sin(x)$
$+(n-1)\int \cos^{n-2}(x)dx$
$\int \cos^{n}(x)dx = \frac{1}{n}\cos^{n-1}(x)\sin x + \left(\frac{n-1}{n}\right)\int \cos^{n-2}(x)dx$

$$\int_0^{\pi/2} \cos^6 x dx$$

Apply integral reduction,

$\left[\frac{\cos^5 x \sin x}{6}\right]_0^{\pi/2} + \frac{5}{6} \cdot \int_0^{\pi/2} \cos^4 x dx$

$$\int_0^{\pi/2} \cos^4 x dx = \frac{3\pi}{16}$$

$\left[\frac{\cos^5 x \sin x}{6}\right]_0^{\pi/2} + \left[\frac{5}{6} \times \frac{3\pi}{16}\right]$

$$= \left(\frac{1}{6}\cos^5 x \sin x\right)_0^{\pi/2} + \frac{5\pi}{32}$$

Computing the boundaries,

$$\int_0^{\pi/2} \cos^6 x dx = 0 + \frac{5\pi}{32} = \frac{5\pi}{32}$$

3)$\int \frac{2x^2-1}{(4x-1)\left(x^2+1\right)}dx$

$\frac{2x^2-1}{(4x-1)\left(x^2+1\right)} = \frac{A}{4x-1} + \frac{Bx+C}{x^2+1}$

Multiply both sides by the least common denominator $(4x-1)\left(x^2+1\right)$,

$(4x-1)\left(\left(x^2+1\right)\left[\frac{2x^2-1}{(4x-1)\left(x^2+1\right)}\right]\right.$

$= (4x-1)\left(x^2+1\right)\left[\frac{A}{4x-1}\right]$

$+ (4x-1)\left(x^2+1\right)\left[\frac{Bx+C}{x^2+1}\right]$

or, $2x^2-1 = A\left(x^2+1\right)+(Bx+c)(4x-1)$

or, $2x^2-1 = Ax^2+A+4Bx^2-Bx+4Cx-C$

or, $2x^2-1 = \left(Ax^2+4Bx^2\right)+(-Bx+4Cx)+(A-C)$

Or, $2x^2-1 = (A+4B)x^2+(-B+4C)x+(A-C)$

Equate the coefficients to obtain a system of linear equations,

$2x^2-1 = (A+4B)x^2+(-B+4C)x$

$+(A-C)$

$A+4B = 2$

$-B+4C = 0$

$A - C = -1$

After solving the equations,

$A = -\frac{14}{17}$

$B = \frac{12}{17}$

$C = \frac{3}{17}$

$\int \frac{2x^2-1}{(4x-1)\left(x^2+1\right)}$

$= \int \left(\frac{-\frac{14}{17}}{4x-1} + \frac{\frac{12}{17}x+\frac{3}{17}}{x^2+1}\right)$

$= -\frac{14}{17\times4} \int \frac{4}{4x-1} dx + \frac{6}{17} \int \frac{2x}{x^2+1} dx + \frac{3}{17} \int \frac{1}{x^2+1} dx$

By integrate,

$-\frac{7}{34} \ln|4x-1| + \frac{6}{17} \ln\left(x^2+1\right) + \frac{3}{17} \tan^{-1} x$

$+ C$

4.Let, I=$\int_0^\infty x^9 e^{-x^2} dx$

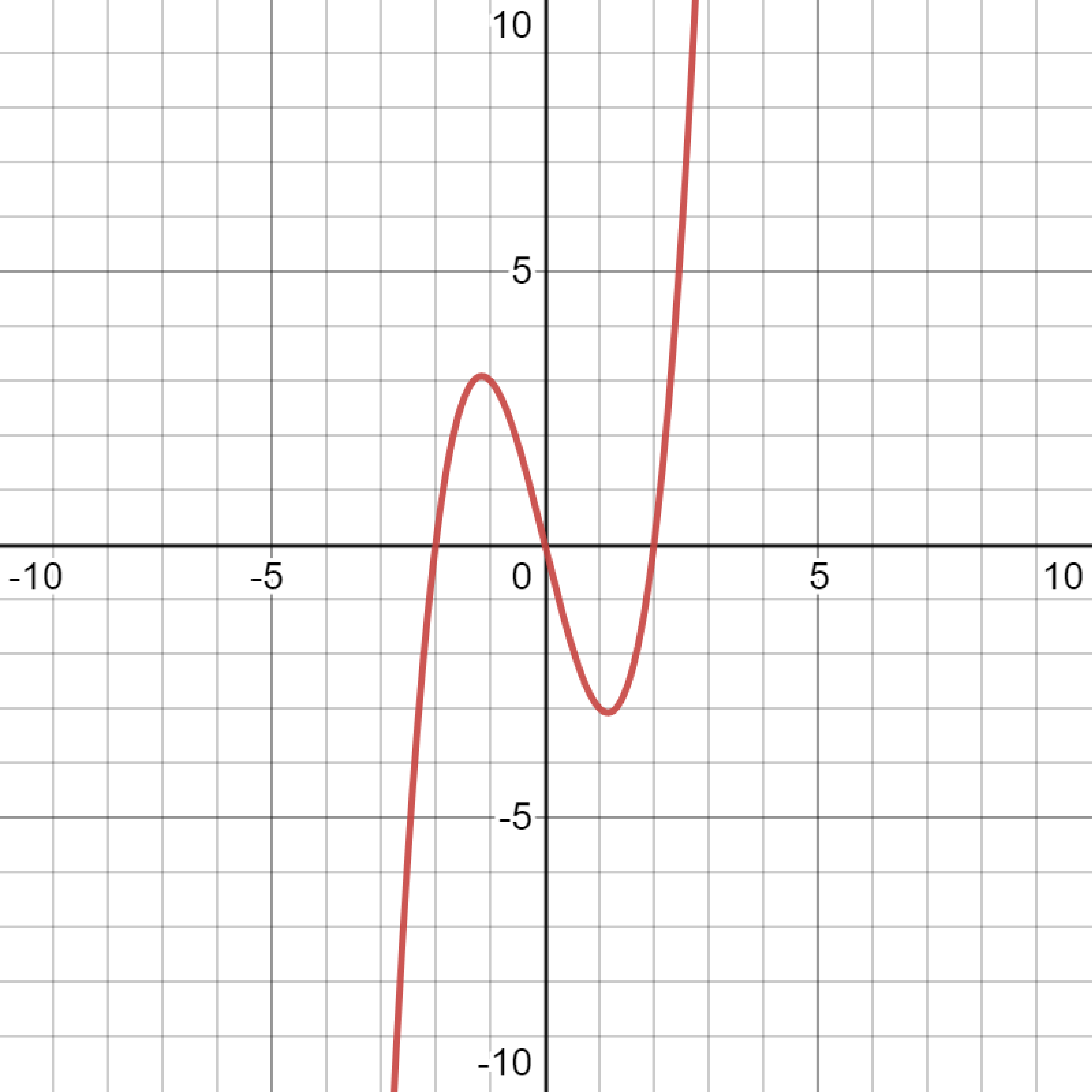
$$t = x^2$$
$$Or, x = t^{1/2}$$
$$Or, dx = \frac{1}{2}t^{-\frac{1}{2}}dt$$

$$I = \int_0^\infty e^{-t}\left(t^{1/2}\right)^9 \cdot \frac{1}{2}t^{-\frac{1}{2}}dt$$

$$= \frac{1}{2}\int_0^\infty e^{-t} \cdot t^4 dt$$

$$= \frac{1}{2} \cdot \Gamma(5)$$

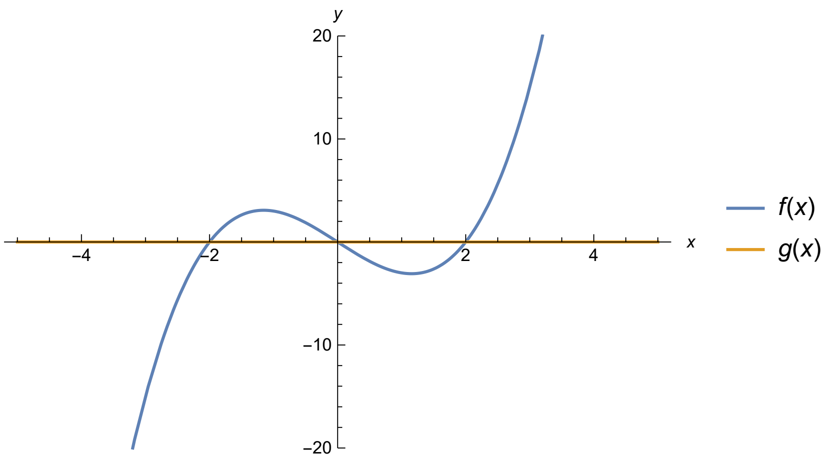
$$= 12$$
$$[note : (n-1) = 4$$
$$or, n = 5$$
$$\Gamma(n) = (n-1)!]$$


10
5
-5
-10
-10
-5
0
5
10

i) : Plotting those function into a single graph

In[47]:=
```
f[x] = x^3 - 4 x;
g[x] = 0;
Plot[{f[x], g[x]}, {x, -5, 5}, PlotRange → {-20, 20},
 PlotLegends → "Expressions", AxesLabel → {x, y}]
```

Out[49]=



In[50]:=
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NSolve[f[x] == g[x]]
```

Out[50]= {{x → -2.}, {x → 0.}, {x → 2.}}

In[51]:= Area = $\int_{-2}^{0} (f[x] - g[x])\, dx + \int_{0}^{2} (g[x] - f[x])\, dx$

··· Set: Symbol Area is Protected.

Out[51]= 8