

Spring2021
MAT120
ASSIGNMENT-04

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Section:07
Set:11

$$\begin{aligned}
 1. \int_0^4 \int_0^x \sqrt{1+x^2} dy dx \\
 = \int_0^4 x \sqrt{x^2+1} dx
 \end{aligned}$$

Let,

$$\begin{aligned}
 z &= \sqrt{1+x^2} \\
 \Rightarrow z^2 &= 1+x^2 \\
 \Rightarrow dz \cdot 2z &= 2x dx \\
 \Rightarrow x dx &= z dz
 \end{aligned}$$

changing the limit,

x	0	4
z	1	$\sqrt{17}$

$$\begin{aligned}
 &\text{By substituting, } \int_0^{\sqrt{17}} u^2 \cdot du \\
 &= \left[\frac{u^3}{3} \right]_0^{\sqrt{17}} \\
 &= \frac{17\sqrt{17}-1}{3}
 \end{aligned}$$

$$\begin{aligned}
& 2. \text{ Given that, } x^2 + y^2 = a^2 \\
& \Rightarrow y = \sqrt{a^2 - x^2} \\
& \text{Area of the circle, } A \\
& = 4 \int_0^a \int_0^{\sqrt{a^2 - x^2}} dy dx \\
& = 4 \int_0^a [y]_0^{\sqrt{a^2 - x^2}} dx \\
& = 4 \int_0^a \sqrt{a^2 - x^2} dx \\
& = 4 \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a \\
& = \pi a^2
\end{aligned}$$

$$\begin{aligned}
& 3. \int_0^1 \int_0^1 (x+y) dx dy \\
&= \int_0^1 \left[\frac{x^2}{2} + yx \right]_0^1 dy \\
&= \int_0^1 \left(\frac{1}{2} + y \right) dy \\
&= \left[\frac{y}{2} + \frac{y^2}{2} \right]_0^1 \\
&= 1
\end{aligned}$$

4. *Given that,*

$$y = 4 - x^2$$

$$y = x^2 - 4$$

$$x^2 - 4 = 4 - x^2$$

$$\Rightarrow 2x^2 = 8$$

$$x = \pm 2$$

$$A = \int_{-2}^{+2} \int_{4-x^2}^{x^2-4} dy dx$$

$$= \int_{-2}^2 [y]_{4-x^2}^{x^2-4} dx$$

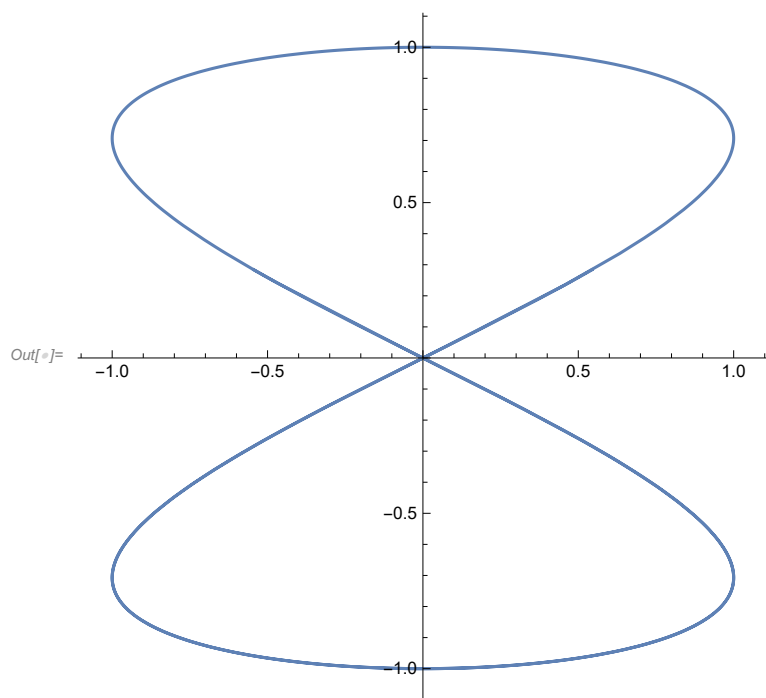
$$= \int_{-2}^2 (2x^2 - 8) dx$$

$$= \left[2 \cdot \frac{x^3}{3} - 8x \right]_{-2}^{+2}$$

$$= \frac{-64}{3}$$

5.a)

`In[]:= ParametricPlot[{Sin[2 t], Cos[t]}, {t, -5, 5}]`



b)

`In[]:= D[Sin[2 t], t]`

`D[Cos[t], t]`

`Out[]:= 2 Cos[2 t]`

`Out[]:= -Sin[t]`

`In[]:= L = $\int_0^4 \sqrt{(2 \cos[2 t])^2 + (-\sin[t])^2} dt$ // N`

`Out[]:= 5.8711`

c)

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In[ ]:= ParametricPlot3D[{Cos[t], Sin[t], Cos[2 t]}, {t, 4, 12}]
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