SPRING2021
MAT120
ASSIGNMENT-02

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Section:07
Set:15

$$1.\int_0^{\arctan\left(\frac{\pi}{14}\right)} \frac{\sin^3(14\tan(x))\cos^8(7\tan(x))}{\cos^2(x)}dx$$

$$= \int_0^{\arctan\left(\frac{\pi}{14}\right)} \sin^3(14\tan(x))\cos^8(7\tan(x))\sec^2 x dx$$

Let,
$7\tan(x) = u$
$\Rightarrow \sec^2(x)dx = \frac{du}{7}$
If, $x \to 0, u \longrightarrow 0$
If, $x \to \arctan\left(\frac{\pi}{14}\right), u \to 7\tan\left(\arctan\left(\frac{\pi}{14}\right)\right)$
$x \to \arctan\left(\frac{\pi}{14}\right), u \to \frac{\pi}{2}$

$\frac{1}{7}\int_0^{\pi/2} \sin^3(2.u)\cos^8(u)du$
$= \frac{1}{7}\int_0^{\pi/2}(2\cdot\sin(u)\cos(u))^3 cos^8(u)du$
$= \frac{1}{7}\int_0^{\pi/2} 8\cdot\sin^3(u)\cdot\cos^3(u)\cdot\cos^8(u)du$
$= \frac{4}{7}\int_0^{\pi/2} 2\cdot\sin^3(u)\cdot\cos^{11}(u)du$

$$2x-1=3$$
$$x=2$$
$$2y-1=11$$
$$y=6$$

By using Beta and Gamma function,
$\frac{4}{7}\beta(2,6)=\frac{4}{7}.\frac{\Gamma(2)+\Gamma(6)}{\Gamma(8)}$

2. $\int \frac{\cos(4x)}{\sin(x)}dx$

$= \int \frac{\cos(2.2x)}{\sin(x)}dx$

$= \int \frac{2\cos^2(2x)-1}{\sin(x)}dx$

$= \int \frac{2\cos^2(2x)}{\sin(x)}dx - \int \frac{1}{\sin(x)}dx$

$\int \frac{1}{\sin(x)}dx = \ln\left|\tan\left(\frac{x}{2}\right)\right|$

$2\int \frac{\cos^2(2x)}{\sin(x)}dx$

$= 2\int \frac{1-\sin^2(2x)}{\sin(x)}dx$

$= 2\left[\left(\int \frac{1}{\sin(x)}dx\right) - \left(\int \frac{\sin^2(2x)}{\sin(x)}dx\right)\right]$

$\int \frac{1}{\sin(x)}dx = \ln\left|\tan\left(\frac{x}{2}\right)\right|$

$\int \frac{\sin^2(2x)}{\sin(x)}dx$

$= \int \frac{4\sin^2(x)\cos^2(x)}{\sin(x)}dx$

$= 4\int \cos^2(x)\sin(x)dx$

$$Let,$$
$$u = cosx$$
$$\Rightarrow du = -\sin x dx$$
$$Applyingsubstitution,$$

$$4 \int -u^2 du$$

$$= -4 \int u^2 du$$

$$= -4 \left(\frac{u^3}{3} \right)$$

$$= -\frac{4}{3} u^3$$

$$= -\frac{4}{3} \cos^3 x$$

$$2\left[\ln\left|\tan\left(\frac{x}{2}\right)\right| - \left(-\frac{4}{3}\cos^3(x)\right)\right]$$

$$= 2\left[\ln\left|\tan\left(\frac{x}{2}\right)\right| + \frac{4}{3}\cos^3(x)\right]$$

$$\int \frac{\cos(4x)}{\sin(x)}dx$$

$$= 2\int \frac{\cos^2(2x)}{\sin(x)}dx - \int \frac{1}{\sin(x)}dx$$

$$= 2\left[\ln\left|\tan\left(\frac{x}{2}\right)\right| + \frac{4}{3}\cos^3(x)\right] - \ln\left|\tan\left(\frac{x}{2}\right)\right|$$

$$= \ln\left|\tan\left(\frac{x}{2}\right)\right| + \frac{8}{3}cos^3x$$

3. $\int \frac{e^{5x}}{1+e^{10x}} dx$

$Let,$

$e^{5x} = u$

$\Rightarrow 5 \cdot e^{5x} dx = du$

$\Rightarrow e^{5x} dx = \frac{du}{5}$

$Bysubstitution,$

$\int \frac{du}{5\left(1+u^2\right)}$

$= \frac{1}{5} \int \frac{1}{1+u^2} du$

$= \frac{1}{5} \arctan(u) + c$

$= \frac{1}{5} \arctan\left(e^{5x}\right) + c$

5. (i)

The equation of curve is : $y = \ln[\text{Sec}[x]],\ 0 \le x \le \frac{\pi}{4}$

We get by differentiating, $\frac{dy}{dx} = \text{Tan}[x]$

In[49]:= $L = \int_0^{\frac{\pi}{4}} \sqrt{1 + (\text{Tan}[x])^2}\, dx$ // N

0.8813735870195429`

Therefore, the length of curve is 0.881373 ....

(ii)

The equation of rotating curve : $y = 1 + 2y^2,\ 1 \le y \le 2$

We get by differentiaing,

$\frac{dy}{dx} = 4y$

In[51]:= $S = \int_1^2 2\pi (1 + 2y^2) \sqrt{1 + (4y)^2}\, dy$ // N

229.07401604725715`

Therefore, the area of the surface is 229.0740 .... .

(iii)

The equation of rotating curve : $y = \ln[x] = 1,\ 1 \le y \le 2$

We get by differentiaing,

$\frac{dy}{dx} = \frac{1}{x}$

In[53]:= $S = \int_0^1 2\pi x \sqrt{1 + \left(\frac{1}{x}\right)^2}\, dx$

Out[53]= $\pi \left(\sqrt{2} + \text{ArcSinh}[1]\right)$