



UTM
UNIVERSITI TEKNOLOGI MALAYSIA

FACULTY OF COMPUTING

SEMESTER 1

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SECI 1013 - DISCRETE STRUCTURE

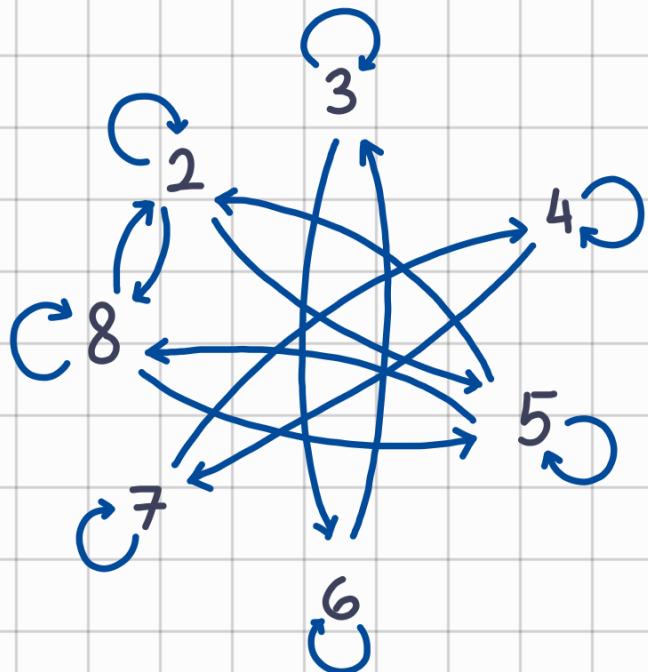
SECTION 2

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RELATION

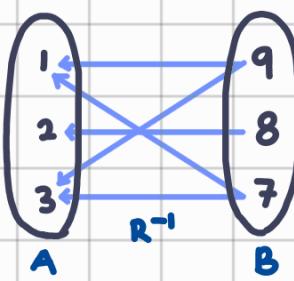
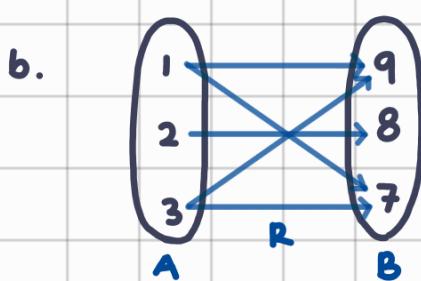
1. $R = \{(2,2), (2,5), (2,8), (3,3), (3,6), (4,4), (4,7), (5,1), (5,5), (5,8), (6,3), (6,6), (7,4), (7,7), (8,2), (8,5), (8,8)\}$



R is reflexive, symmetric and transitive.
 R is equivalence relation on A .

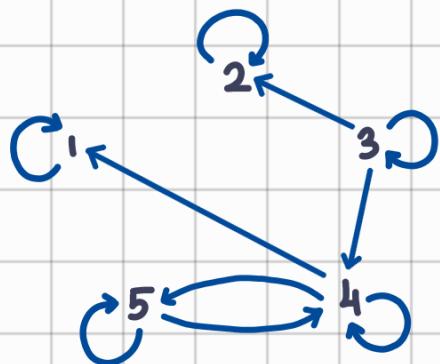
2. a. $R = \{(1,9), (1,7), (2,8), (3,9), (3,7)\}$

$$R^{-1} = \{(9,1), (9,3), (8,2), (7,1), (7,3)\}$$



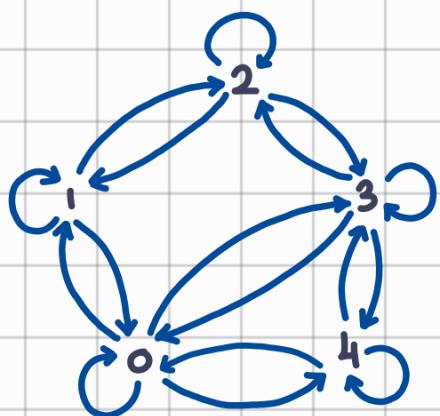
- c. R^{-1} is the opposite of R .
 If $R : A$ to B , then $R^{-1} : B$ to A .

3.



	1	2	3	4	5
In-degree	2	2	1	3	2
Out-degree	1	1	3	3	2

4.

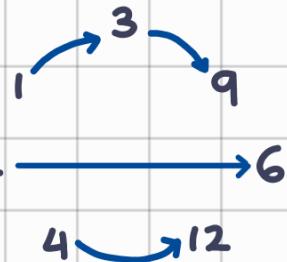


$\therefore R$ is reflexive, symmetric
and not transitive.

$1R0, 0R3, 1 \not R 3$

5. $R = \{(1,3), (2,6), (3,9), (4,12)\}$

a. R is irreflexive because $(1,1) \notin R$
all $x \not R x$. There is no loop. $(2,2) \notin R$



b. R is antisymmetric.
 R is asymmetric.

$$(2,6) \in R$$

$$(6,2) \notin R$$

c. R is not transitive. $(1,3)$ and $(3,9) \in R$, but $(1,9) \notin R$.

$$6. \text{ a.} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\text{b.} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

FUNCTION

7. Function : all $x R y \nRightarrow (x,y), (x,y') \in f$, $y = y'$ (only one output)

Relation : some $x R y \nRightarrow (x,y), (x,y') \in R$, $y \neq y'$ (can have multiple output)

8. (i) Function. The domain $R, \{2,3,4,5\}$ is equal to A.

(ii) Function. The domain $R, \{2,3,4,5\}$ is equal to A.

(iii) Not function. The domain $R, \{2,5\}$ is not equal to A.

$(2,3), (2,4) \in R, 3 \neq 4$

(iv) Not function. The domain $R, \{2,3,4\}$ is not equal to A.

(v) Not function. The domain $R, \{2,4\}$ is not equal to A.

$(2,2), (2,3) \in R, 2 \neq 3 \nRightarrow (4,4), (4,5) \in R, 4 \neq 5$

9. $R = \{(1,6), (2,7), (3,8), (4,9), (5,10)\}$

Domain = $\{1,2,3,4,5\}$

Range = $\{6,7,8,9,10\}$

$$\text{v) } f(x_1) = f(x_2)$$

$$1 - 2x_1 = 1 - 2x_2 \quad (-1)$$

$$-2x_1 = -2x_2 \quad (\div -2)$$

$$x_1 = x_2$$

$$y = 1 - 2x$$

$$x = -\frac{y-1}{2}$$

$$f(x) = 1 - 2\left(-\frac{y-1}{2}\right)$$

$\therefore f(x)$ is one-to-one and onto. $f(x)$ is bijective

$$y = 1 + y - 1$$

$$y = y$$

$$\text{vi) } f(x_1) = f(x_2)$$

$$5x_1^2 - 1 = 5x_2^2 - 1 \quad (+1)$$

$$5x_1^2 = 5x_2^2 \quad (\div 5)$$

$$x_1^2 = x_2^2$$

$$x_1 = x_2 \text{ or } x_1 = -x_2$$

$$y = 5x^2 - 1$$

$$x = \sqrt{\frac{y+1}{5}}$$

$$f(x) = 5\left(\sqrt{\frac{y+1}{5}}\right)^2 - 1$$

$\therefore f(x)$ is not one-one.
 $f(x)$ is onto.

$$y = y + 1 - 1$$

$$y = y$$

$$\text{vii) } f(x_1) = f(x_2)$$

$$x_1^4 = x_2^4$$

$$x_1 = x_2 \text{ or } x_1 = -x_2$$

$$y = x^4$$

$$x = \sqrt[4]{y}$$

$$f(x) = (\sqrt[4]{y})^4$$

$$y = y$$

$\therefore f(x)$ is not one-one.
 $f(x)$ is onto

$$viii) f(x_1) = f(x_2)$$

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$x_1 x_2 - 3x_1 - 2x_2 + 6 = x_1 x_2 - 3x_2 - 2x_1 + 6$$

$$(-x_1 x_2 - 6)$$

$$-3x_1 - 2x_2 = -3x_2 - 2x_1$$

$$-3x_1 + 2x_1 = -3x_2 + 2x_2$$

$$x_1 = x_2$$

$$y = \frac{x-2}{x-3}$$

$\therefore f(x)$ is one-one and onto. $f(x)$ is bijective

$$y(x-3) = x-2$$

$$xy - x = -2 + 3y$$

$$x(y-1) = 3y-2$$

$$x = \frac{3y-2}{y-1}$$

$$f(x) = \frac{\left(\frac{3y-2}{y-1}\right) - 2}{\left(\frac{3y-2}{y-1}\right) - 3}$$

$$y = \frac{3y-2-2(y-1)}{\cancel{y-1}} \times \frac{\cancel{y-1}}{3y-2-3(y-1)}$$

$$y = \frac{3y-2-2y+2}{3y-2-3y+3}$$

$$y = \frac{y}{1}$$

$$y = y$$

$$\text{ii) ix) } f(g(x)) = f(x^2 - 1)$$

$$= 3(x^2 - 1) - 1$$

$$= 3x^2 - 4$$

$x=0$	$x=1$	$x=2$	$x=3$
$= 3(0)^2 - 4$	$= 3(1)^2 - 4$	$= 3(2)^2 - 4$	$= 3(3)^2 - 4$
$= -4$	$= -1$	$= 8$	$= 23$

$$\text{x) } f(g(x)) = f(5x - 6)$$

$$= (5x - 6)^2$$

$$= 25x^2 - 60x + 36$$

$x=0$	$x=1$
$= 25(0)^2 - 60(0) + 36$	$= 25(1)^2 - 60(1) + 36$
$= 36$	$= 1$
$x=2$	$x=3$
$= 25(2)^2 - 60(2) + 36$	$= 25(3)^2 - 60(3) + 36$
$= 16$	$= 81$

$$\text{xi) } f(g(x)) = f(x^3 + 1)$$

$$= (x^3 + 1) - 1$$

$$= x^3$$

$x=0$	$x=1$	$x=2$	$x=3$
$= (0)^3$	$= (1)^3$	$= (2)^3$	$= (3)^3$
$= 0$	$= 1$	$= 8$	$= 27$

RECURRANCE RELATION

12. xii) $a_0 = 1$

$$a_1 = 6$$

$$a_2 = 6(a_1) - 9(a_0) = 6(6) - 9(1) = 27$$

$$a_3 = 6(a_2) - 9(a_1) = 6(27) - 9(6) = 108$$

$$a_4 = 6(a_3) - 9(a_2) = 6(108) - 9(27) = 405$$

$$a_5 = 6(a_4) - 9(a_3) = 6(405) - 9(108) = 1458$$

1, 6, 27, 108, 405, 1458, ...

xiii) $a_0 = 2$

$$a_1 = 5$$

$$a_2 = 15$$

$$a_3 = 6(a_2) - 11(a_1) + 6(a_0) = 6(15) - 11(5) + 6(2) = 47$$

$$a_4 = 6(a_3) - 11(a_2) + 6(a_1) = 6(47) - 11(15) + 6(5) = 147$$

$$a_5 = 6(a_4) - 11(a_3) + 6(a_2) = 6(147) - 11(47) + 6(15) = 455$$

$$a_6 = 6(a_5) - 11(a_4) + 6(a_3) = 6(455) - 11(147) + 6(47) = 1395$$

2, 5, 15, 47, 147, 455, 1395, ...

xiv) $a_0 = 1$

$$a_1 = -2$$

$$a_2 = -1$$

$$a_3 = -3(a_2) - 3(a_1) + a_0 = -3(-1) - 3(-2) + 1 = 10$$

$$a_4 = -3(a_3) - 3(a_2) + a_1 = -3(10) - 3(-1) - 2 = -29$$

$$a_5 = -3(a_4) - 3(a_3) + a_2 = -3(-29) - 3(10) - 1 = 56$$

$$a_6 = -3(a_5) - 3(a_4) + a_3 = -3(56) - 3(-29) + 10 = -71$$

1, -2, -1, 10, -29, 56, -71, ...

$$13. \quad a_1 = k$$

$$a_2 = 5(a_1) - 3 = 5k - 3$$

$$a_3 = 5(a_2) - 3 = 5(5k - 3) - 3 = 25k - 18$$

$$a_4 = 5(a_3) - 3 = 5(25k - 18) - 3 = 125k - 93$$

$$k, 5k - 3, 25k - 18, 125k - 93, \dots$$

$$\text{i) } a_4 = 125k - 93$$

$$\text{ii) } a_4 = 7$$

$$125k - 93 = 7$$

$$125k = 100$$

$$k = \frac{100}{125}$$

$$k = 0.8$$