Multiple Linear Regression

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Abstract

This paper reproduces Section 3.2 of *Introduction to Statistical Learning* by Hastie, Tibshirani, James, and Witten. This section discusses the Multiple Linear Regression model and fits one to the *Advertising* dataset. The model uses data on television, newspaper, and radio advertising to predict units of sales.

Introduction

We return to the *Advertising* dataset, where we want to build a model from data on three advertising media to predict Sales. A past paper discusses the Simple Linear Regression model; so, one solution is fit three Simple Linear Regression models - one for each predictor. However, this is not the only option - we can simply tweak the linear model to allow for more predictors, using a **Multiple Linear Regression Model**.

A Multiple Linear Regression model extends the Simple Linear Regression model to accomodate more predictors for a single response variable. Recall the Linear Regression model we have previously encountered:

$$Y = \beta_0 + \beta_1 X + \epsilon$$

,

where Y is our response variable, X is our predictor, ϵ is random noise, and the beta coefficients are the intercept and slope of our linear model, respectively.

The Multiple Linear Regression model is:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_n X_n + \epsilon$$

,

where X_j represents the jth predictor and β_j quantifies the association between that variable and the response.

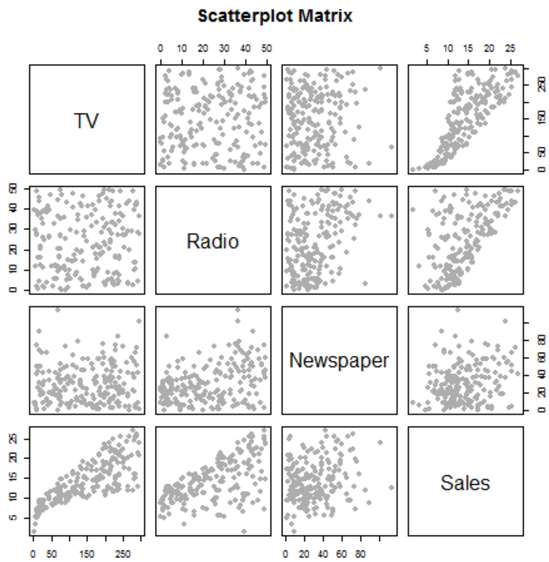
Data

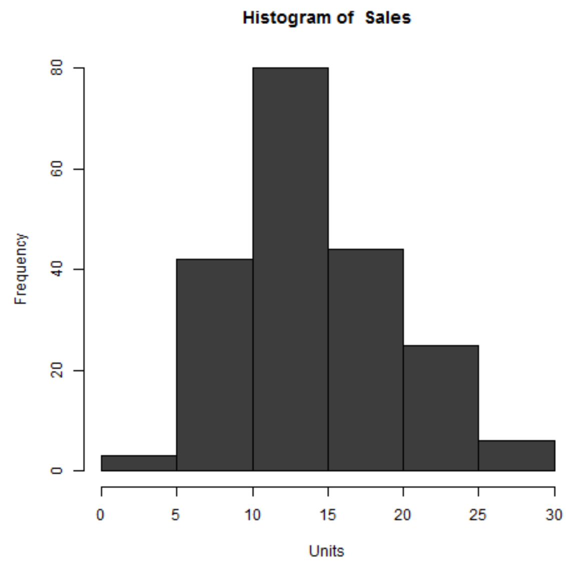
Before fitting any model it is imperative that we discuss our data. The *Advertising* dataset consists of *Sales* (in thousands of units) of a particular product in 200 different markets, along with advertising budgets (in thousands of dollars) for the product in each of those markets for three different media: *TV*, *Radio*, and *Newspaper*.

Below is the first five rows of the Advertising Dataset:

```
##      TV Radio Newspaper Sales
## 1 230.1  37.8      69.2  22.1
## 2  44.5  39.3      45.1  10.4
## 3  17.2  45.9      69.3   9.3
## 4 151.5  41.3      58.5  18.5
## 5 180.8  10.8      58.4  12.9
```

It is also a good idea to look at some data visualizatoins:





Finally, because we are using multiple predictors, it is important to see their level of correlation. Predictors that are too highly correlated may cause problems in making predictions, as one predictor can be easily interchanged with another.

Figure 2 Correlation matrix for TV, radio, newspaper, and sales for the *Advertising* dataset.

	TV	Radio	Newspaper	Sales
TV	1.00	0.05	0.06	0.78
Radio	0.05	1.00	0.35	0.58
Newspaper	0.06	0.35	1.00	0.23
Sales	0.78	0.58	0.23	1.00

Methodology

We discuss how to specifically fit a multiple linear regression model and compare the outcome to those of three simple linear models.

Fitting Simple Linear Regression Models

We use the methodology discussed in the paper *Simple Linear Regression* by Nura Kawa (reproduced from Section 3.1 of *Introduction to Statistical Learning*) to fit a simple linear model for each predictor with Sales. The results are as follows:

Figure 1 Simple Regression Models for the *Advertising* dataset. The three tables below show coefficients of the linear regression models of Sales regressed onto Radio, Newspaper, and TV, respectively.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	9.3116	0.5629	16.54	0.0000
Radio	0.2025	0.0204	9.92	0.0000

Table 1: Simple Regression of Sales on Radio

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	12.3514	0.6214	19.88	0.0000
Newspaper	0.0547	0.0166	3.30	0.0011

Table 2: Simple Regression of Sales on Newspaper

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	7.0326	0.4578	15.36	0.0000
TV	0.0475	0.0027	17.67	0.0000

Table 3: Simple Regression of Sales on TV

This solution to our question of prediction leaves much to be desired, as it becomes difficult to make one estimate using three completely different linear models. Furthermore, each model leaves out a possible influence from a separate predictor. Thus, we will fit a Multiple Linear Regression Model.

Fitting a Multiple Linear Regression Model

Specifically, our Multiple Linear Regression model is the following:

$$\textbf{sales} = \beta_0 + \beta_1 \textbf{TV} + \beta_2 \textbf{radio} + \beta_3 \textbf{newspaper} + \epsilon$$

.

As with simple linear models, we first test hypotheses, namely:

Hypothesis Testing

Our *null hypothesis* H_0 is:

H_0: $\beta_1 = \beta_2 = ... = \beta_p = 0$

Our *alternative hypothesis* H_a is:

H_a: at least one β_j is non-zero

Simply put, we assume the null hypothesis of each predictor having no influence on Sales, and test to see if at least one predictor has an association with Sales. We set our p-value to be 0.05, meaning that anything below this would allow us to reject our null hypothesis.

Computing Coefficients

As with a simple linear model, we will compute the following coefficients as follows:

Residual Sum of Squares (RSS)
Residuals are the difference between the observed value of the dependent variable y and the predicted value, $\hat(y)$. The Residual Sum of Squares is simply the summation of all residuals, squared:

$$RSS = \sum (y_1 - \hat{y_i})^2$$

Total Sum of Squares (TSS)
The total sum of squares explains the variability already present in the dataset: it is the sum of the difference between each value of y (Sales, in our case) and its mean, squared:

$$TSS = \sum (y_1 - \bar{y})^2$$

R-Squared
The R^2 measures the goodness of fit of our model. It measures the proportion of variance explained by our model. Thus it has the range [0,1] and is independent of Y's scale.

$$R^2 = \frac{(TSS - RSS)}{TSS}$$

Residual Standard Error
The RSE is an estimate of the standard deviation of the error in our model. This shows how far our data will deviate from the generated regression line.

$$RSE = \sqrt{\frac{1}{n-2} \sum (y_i - \hat{y_i})^2}$$

F-Statistic
This is our test statistic: Allowing p to be the number of predictors and n to be the number of observations, we have:

$$F = \frac{(TSS - RSS)/p}{RSS/(n - p - 1}$$

Results

Figure 3 Multiple Linear Regression Coefficients___

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	2.9389	0.3119	9.42	0.0000
TV	0.0458	0.0014	32.81	0.0000
Radio	0.1885	0.0086	21.89	0.0000
Newspaper	-0.0010	0.0059	-0.18	0.8599

Table 4: Multiple Linear Regression of Sales onto Radio, TV, and Newspaper

Figure 4 More information about the least squares model for the regression of number of units sold on TV, newspaper, and radio advertising budgets in the *Advertising* dataset.

Quantity	Value
Residual Standard Error	1.68
R-Squared	0.90
F-Statistic	570.27

Table 5: Multiple Linear Regression Coefficients

Conclusion