Forward kinematic

Joint	θ	α	a	d
1	θ_1	90	0	d_1
2	θ_2	0	a_2	0
3	θ_3	0	a_3	0
4	θ_4	90	0	0
5	θ_5	0	0	0
6	θ_6	0	0	d_6

$$^0T_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^1T_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^3T_4 = \begin{bmatrix} C_4 & 0 & S_4 & a_4C_4 \\ S_4 & 0 & -C_4 & a_4S_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^4T_5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^5T_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$n_x = C_1[C_{234}C_5C_6 - S_{234}S_6] - S_1S_5f_6$$
$$n_y = S_1[C_{234}C_5C_6 + S_{234}S_6] + S_1S_5f_6$$
$$n_z = -S_{234}C_5C_6 - C_{234}S_6$$
$$o_x = -C_1[C_{234}C_5C_6 + S_{234}f_6] - S_1S_5S_6$$
$$o_y = -S_1[C_{234}C_5C_6 + S_{234}S_6] + S_1S_5S_6$$
$$o_z = S_{234}C_5C_6 - C_{234}S_6$$
$$a_x = C_1C_{234}S_5 - S_1C_5$$
$$a_y = S_1C_{234}S_5 + C_1C_5$$
$$a_z = S_{234}S_5$$
$$P_x = C_1[d_6S_5C_{234} - a_4C_{234} - a_2C_2 - a_3C_{23}] - S_1C_5d_6$$
$$P_y = S_1[d_6S_5C_{234} - a_4C_{234} - a_2C_2 - a_3C_{23}] + S_1C_5d_6$$
$$P_z = d_1 + d_6S_{234}S_5 - a_3S_{23} - a_2S_2 - a_4S_{234}$$

Reverse kinematic

$C_1 {}^0P_{y6} - S_1 {}^0P_{x6} = d_6 C_5$

$C_5 = C_1 a_y - S_1 a_x$

$C_1 {}^0P_{y6} - S_1 {}^0P_{x6} = d_1 C_1 a_y - d_6 a_x S_1$

$\theta_1 = a\tan 2\left[C P_y - d_6 a_y), (P_x - d_6 a_x) \right]$

$S_5 = \pm \sqrt{1 - C_5^2}$

$\theta_5 = a\tan 2 (S_5, C_5]$

$S_6 C_{234} = -(a_x C_1 + a_y S_1)$

$\theta_{234} = a\tan 2 (-a_z, [a_x C_1 + a_y S_1)], \theta_5 > 0$

$\theta_{234} = \theta_{234} + \pi$

$P_1 = C_1 P_{x6} + S_1 P_{y6} + d_6 S_5 C_{234} - a_4 C_{234}$

$P_1 = a_2 C_2 + a_3 C_{23}$

$P_2 = -P_{z6} - d_1 + d_6 S_5 S_{234} - a_4 S_{234}$

$P_2 = a_3 S_{23} - a_2 S_2$

$a_3 C_{23} = P_1 - a_2 C_2$

$a_3 S_{23} = P_1 - a_2 S_2$

$a_3^2 = (P_1 - a_2 C_2)^2 + (P_2 - a_2 S_2)^2$

$P_1 C_2 + P_2 S_2 = \dfrac{P_1^2 + P_2^2 + a_2^2 - a_3^2}{2 a_2} = N$

$\theta_2 = a\tan 2 (P_2, P_1) + a\tan 2 \left(\sqrt{P_1^2 + P_2^2 - N}, N \right)$

$\theta_{23} = a\tan 2 (P_2 - a_2 S_2, P_1 - a_2 C_2)$

$\theta_3 = \theta_{23} - \theta_2$

$\theta_4 = \theta_{234} - \theta_{23}$

$S_6 C_6 = n_y C_1 - n_x S_1$

$S_6 S_6 = (O_x S_1 - O_y C_1)$

$$\theta_6 = a\tan 2\left[(o_x s_1 - o_y c_1), (n_y c_1 - n_x s_1)\right]$$
$$\theta_6 = \theta_6 + \pi$$

$$\theta_1 = a\tan 2\left[(P_y - d_6 a_y), (P_x - d_6 a_x)\right]$$
$$\theta_2 = a\tan 2\left(P_2, P_1\right) + a\tan 2\left(\sqrt{P_1^2 + P_2^2 - N}, N\right)$$

$$\theta_{23} = a\tan 2\left(P_3 - a_2 s_2, P_1 - a_2 c_2\right)$$
$$\theta_3 = \theta_{23} - \theta_2$$

$$\theta_4 = \theta_{234} - \theta_{23}$$
$$\theta_5 = 0$$