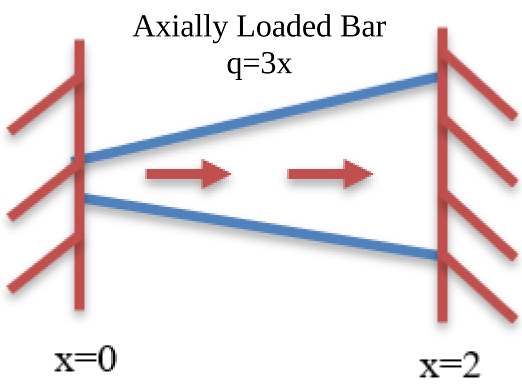

Cross section area of the bar is a function as x. $A(x)=3.0\times 10^4+1.0\times 10^4 x$ (m^2)

E =200 GPa

Length of the bar , L = 2 m

The governing equation for displacement u(x) is given below:

$$\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q(x)=0$$

By multiplying the governing equation with weight function w and integrating 0 to 2, and setting to zero below equation is obtained:

$$\int_0^2 w\left(\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q(x)\right)dx=0$$

Find the approximate solutions of the displacement u(x) by using the collocation method with 1, 2 and 3 collocation points. Also, find N(x). Show that your solutions satisfies boundary conditions at x = 0 and x = 2 m.

<u>Solution :</u>

Boundary Conditions : u(0) = 0 , u(2) = 0

$w_i = \delta(x-x_i)$: weight function

Direc Delta Function Property

$$\int_a^b \delta(x-x_{0)}f(x)=f(x_o)\ where\ a<x_0<b$$

\- For One Collocation Point

$\Phi_0 = 0$

$\Phi_1 = x(2-x)$

$u = \Phi_0 + c_1\,\Phi_1$

$x_1 = 2/2$: collocation point

$w_1 = \delta(x-x_1)$

$$\int_0^2 \delta(\mathrm{x}-\mathrm{x}_1)\left(\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q(x)\right)dx = (\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q(x)\ at\ x_1) = 0$$

- For Two Collocation Point

$\Phi_0 = 0$

$\Phi_1 = x(2-x)$

$\Phi_2 = x^2(2-x)$

$u = \Phi_0 + c_1\,\Phi_1 + c_2\,\Phi_2$

$x_1 = 2/3$: collocation point

$w_1 = \delta(x-x_1)$

$$\int_0^2 \delta(\mathrm{x}-\mathrm{x}_1)\left(\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q(x)\right)dx = (\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q(x)\ at\ x_1) = 0$$

$x_2 = 2*2/3$: collocation point

$w_2 = \delta(x-x_2)$

$$\int_0^2 \delta(\mathrm{x}-\mathrm{x}_2)\left(\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q(x)\right)dx = (\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q(x)\ at\ x_2) = 0$$

- For Three Collocation Point

$\Phi_0 = 0$

$\Phi_1 = x(2-x)$

$\Phi_2 = x^2(2-x)$

$\Phi_3 = x^3(2-x)$

$u = \Phi_0 + c_1\,\Phi_1 + c_2\,\Phi_2 + c_3\,\Phi_3$

$x_1 = 2/4$: collocation point

$w_1 = \delta(x-x_1)$

$$\int_0^2 \delta(\mathrm{x}-\mathrm{x}_1)\left(\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q(x)\right)dx = (\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q(x)\ at\ x_1) = 0$$

$x_2 = 2*2/4$: collocation point

$w_2 = \delta(x-x_2)$

$$\int_0^2 \delta(\mathrm{x}-\mathrm{x}_2)\left(\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q(x)\right)dx = (\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q(x)\ at\ x_2) = 0$$

$x_3 = 3*2/4$: collocation point

$w_3 = \delta(x-x_3)$

$$\int_0^2 \delta(\mathrm{x}-\mathrm{x}_3)\left(\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q(x)\right)dx = (\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q(x)\ at\ x_3)=0$$

$$\sigma(x) = E\varepsilon(x) = E\frac{du}{dx}\ , \qquad N(x) = \ \sigma(x)A$$

$$N(x) = EA\frac{du}{dx}$$