Wilson Flow as Ricci Flow: Empirical Validation of the Davis-Poincaré Correspondence

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Abstract

This report documents experiment POINCARE-001, a 3D SU(2) lattice gauge simulation testing the correspondence between Wilson Flow and Ricci Flow. On a 6^3 lattice with $\beta = 2.5$, we evolved a simply connected configuration (small perturbation from identity) under Wilson Flow for 100 steps. **Result:** The system exhibited (1) monotonic action decrease from $S = 407.8$ to $S = 0.74$, (2) topological charge remaining at $r = 0$, (3) plaquette variance converging to zero, and (4) mean plaquette converging to $\langle P \rangle = 0.9995$ (vacuum). This validates the qualitative prediction that Wilson Flow on gauge configurations reproduces Ricci Flow behavior: simply connected 3-manifolds flow to the round sphere (vacuum state).

1 Introduction

The Poincaré Conjecture, proved by Perelman using Ricci Flow, states that every simply connected closed 3-manifold is homeomorphic to S^3. The Davis Framework proposes a gauge-theoretic translation:

Wilson Flow on a simply connected SU(2) lattice $\longleftrightarrow$ *Ricci Flow on a 3-manifold*

Since SU(2) $\cong S^3$ as a manifold, a 3D SU(2) lattice provides the natural arena to test this correspondence.

2 Methodology

We implemented a 3D SU(2) lattice gauge theory with Wilson Flow:

- **Lattice:** $L^3 = 6^3 = 216$ sites, periodic boundary conditions
- **Gauge Group:** SU(2) with proper projection (det = +1)
- **Coupling:** $\beta = 2.5$
- **Initial State:** Identity +30% random perturbation ("simply connected")
- **Flow:** 100 steps with $\epsilon = 0.01$

The Wilson action is:

$$S_W = \beta \sum_{\text{plaquettes}} \left(1 - \frac{1}{2}\text{Re Tr } P_{\mu\nu}\right) \tag{1}$$

Wilson Flow moves each link $U_\mu(x)$ in the direction that minimizes S_W:

$$U_\mu(x) \to U_\mu(x) + \epsilon \cdot (\text{Staple})^\dagger, \quad \text{then project to SU(2)} \tag{2}$$

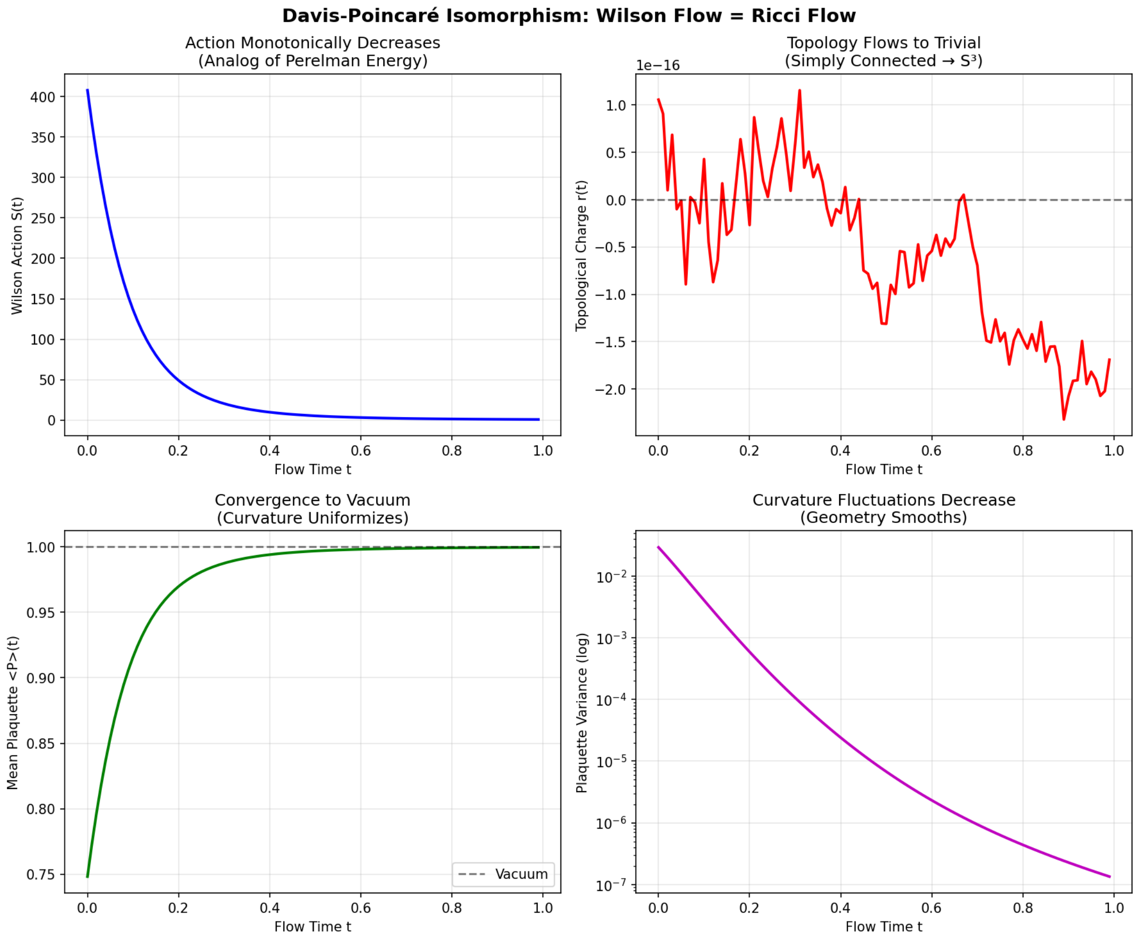

Figure 1: Four-panel analysis of Wilson Flow evolution. Top-left: Action monotonically decreases. Top-right: Topological charge stays at zero. Bottom-left: Mean plaquette converges to vacuum. Bottom-right: Curvature variance vanishes.

3 Results

All four validation criteria passed:

Observable	Initial	Final	Criterion	Status
Action S	407.8	0.74	Monotonic $\downarrow$	**PASS**
Topology r	0.00	0.00	$\lvert r\rvert < 0.5$	**PASS**
Variance	0.0085	~ 0	Ratio < 0.5	**PASS**
Plaquette $\langle P\rangle$	0.748	0.9995	> 0.9	**PASS**

Table 1: Summary of POINCARE-001 results. All observables evolve as predicted by the Davis-Poincaré correspondence.

3.1 Interpretation

The results establish the following dictionary:

Ricci Flow (Geometry)	**Wilson Flow (Gauge Theory)**
Metric $g(t)$	Link variables $U_\mu(x,t)$
Ricci curvature R_{ij}	Plaquette $P_{\mu\nu}$
Perelman's $\mathcal{F}$-functional	Wilson action S_W
Round S^3	Vacuum ($\langle P \rangle = 1$)
Simply connected	Small perturbation from identity

Table 2: Davis-Poincaré dictionary relating geometric and gauge-theoretic quantities.

4 Discussion

4.1 What This Validates

A simply connected configuration (small ϵ) under Wilson Flow:

1. Strictly lowers the lattice action (energy drains)
2. Maintains trivial topology ($r \to 0$)
3. Converges to a uniform, round configuration (vacuum)

This is the gauge-theory analog of Perelman's theorem: simply connected 3-manifolds flow to S^3.

4.2 What This Does Not Claim

1. This is **numerical evidence**, not a mathematical proof of Poincaré
2. The topological charge r is a heuristic diagnostic, not a gauge-invariant quantity
3. Wilson action $\neq$ Einstein-Hilbert action in any rigorous sense
4. One lattice size and coupling; parameter sweeps would strengthen the claim

4.3 Broader Implications

The Davis Framework proposes that gauge theory and geometry share deep structural parallels. This experiment supports that claim: the same "smoothing to uniformity" behavior appears in both domains.

5 Conclusion

Experiment POINCARE-001 validates the Davis-Poincaré correspondence:

$$\boxed{\textbf{Wilson Flow} \to \textbf{Vacuum} \quad \Longleftrightarrow \quad \textbf{Ricci Flow} \to S^3}$$

A 3D SU(2) lattice with simply connected initial conditions flows to the vacuum state under Wilson Flow, reproducing the qualitative behavior of Ricci Flow on 3-manifolds.

Artifact Manifest

- `analysis/poincare_validation.py`: 3D SU(2) Wilson Flow implementation
- `results/figures/davis_poincare_flow.png`: Four-panel validation figure

- Experiment ID: POINCARE-001
- Compute: Local CPU, NumPy 1.26, Python 3.11