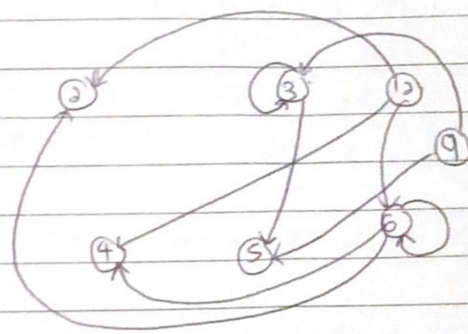
No.: Date:

Assignment 1.2 Discrete Structure

1. $A = \{3, 6, 9, 12\}$ $B = \{2, 3, 4, 5, 6\}$

i) $R = \{(3,3), (3,5), (6,2), (6,4), (6,6), (9,3), (9,5), (12,2), (12,4), (12,6)\}$

ii)



iii) domain $R = \{3, 6, 9, 12\}$
range $R = \{2, 3, 4, 5, 6\}$

2. $D = \{1, 3, 8, 10, 15\}$

$R = \{(1,8), (1,15)\}$

Not equivalence relations, because relation R is not reflexive, not symmetric and not transitive.

3. i)

$$M_R = \begin{array}{c} \\ s \\ t \\ u \\ v \end{array} \begin{array}{c} \begin{array}{cccc} s & t & u & v \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

ii)

	s	t	u	v
in-degree	2	2	3	1
Out-degree	3	3	2	0

iii) Not reflexive because $(u,u) \notin R$
Not antisymmetric because $(s,u) \in R$ and $(u,s) \in R$

$M_R \otimes M_R \neq M_R$

d)

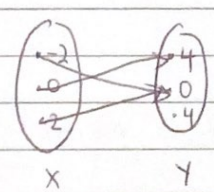
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \neq \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

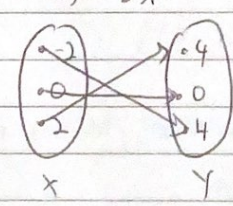
Thus, relation of R is not partial order.

4 $X = \{-2, 0, 2\}$ $\quad Y = \{-4, 0, 4\}$

$V = \{(-2, 0), (0, 4), (2, 0)\}$
$W = \{(-2, -4), (0, 0), (2, 4)\}$

$v(x) = 4 - x^2$ $\quad w(x) = 2x$





V is not one-to-one, and not onto Y, not bijection.
W is one-to-one, onto Y and is a bijection.

5 $f(x) = 7x - 2$ $\quad g(x) = \frac{2}{3}x$

i) $g(x) = \frac{2}{3}x$
$g^{-1}(y) = x$
$y = \frac{2}{3}x$
$x = \frac{3}{2}y$

$g^{-1}(y) = \frac{3}{2}y$

No.: Date.

ii) $(g \circ g \circ f)(x) = g[g(f(x))]$

$= g[g(7x-2)]$

$= g[\frac{2}{3}(7x-2)]$

$= \frac{2}{3}(\frac{2}{3}(7x-2))$

$= \frac{2}{3}(\frac{14}{3}x - \frac{4}{3})$

$= \frac{28}{9}x - \frac{8}{9}$

6. $A+B=C$

$A, F_0 = 5.0$ $t = 0, 1, 2, 3$

$B, F_1 = 4.5$

i) $F_t = F_{t-1} + \frac{1}{5}F_{t-2}, \ t \geq 2$

ii) $F_2 = F_{(2-1)} + \frac{1}{5}F_{(2-2)}$

$= 4.5 + \frac{1}{5}(5.0)$

$= 5.5$

$F_3 = F_2 + \frac{1}{5}F_1$

$= 5.5 + \frac{1}{5}(4.5)$

$= 6.4$

$F_0 = 5.0$

$F_1 = 4.5$

$F_2 = F_1 + \frac{1}{5}F_0$

$= 4.5 + \frac{1}{5}(5.0)$

$= 5.5$

$F_3 = F_2 + \frac{1}{5}F_1$

$= 5.5 + \frac{1}{5}(4.5)$

$= 6.4$

$F_4 = F_3 + \frac{1}{5}F_2$

$= 6.4 + \frac{1}{5}(5.5)$

$= 7.5$

$F_5 = F_4 + \frac{1}{5}F_3$

$= 7.5 + \frac{1}{5}(6.4)$

$= 8.78$

No.:

7. $w_0 = 5$ $\quad w_n = 2w_{n-1} + w_{n-2}, \quad n \geq 2$

$w_1 = 7$

(2). $w_0 = 5$

$w_1 = 7$

$w_2 = 2w_1 + w_0$
$= 2(7) + 5$
$= 19$

$w_3 = 2w_2 + w_1$
$= 2(19) + 7$
$= 45$

$w_4 = 2w_3 + w_2$
$= 2(45) + 19$
$= 109$

(1) recursive algorithm.

input : n
output : w(n)

```
w(n) {
    if (n=0)
        return 5
    else if (n=1)
        return 7
    else return (2w(n-1) + w(n-2))
}
```

(2) trace output n=4

w(4) $\qquad$ w(4) = 109
n=4
n≠1
n≠4
return (2w(3) + w(2))

w(3) $\qquad$ w(3) = 45
n=3
n≠1
n≠4
return (2w(2) + w(1)

w(2) $\qquad$ w(2) = 19
n=2
n≠1
n≠4
return (2w(1) + w(0)

w(1)=7 $\qquad$ w(1)
n=1
return 7

w(0)=5 $\qquad$ w(0)
n=0
return 5

∴ answer w(4) = 109