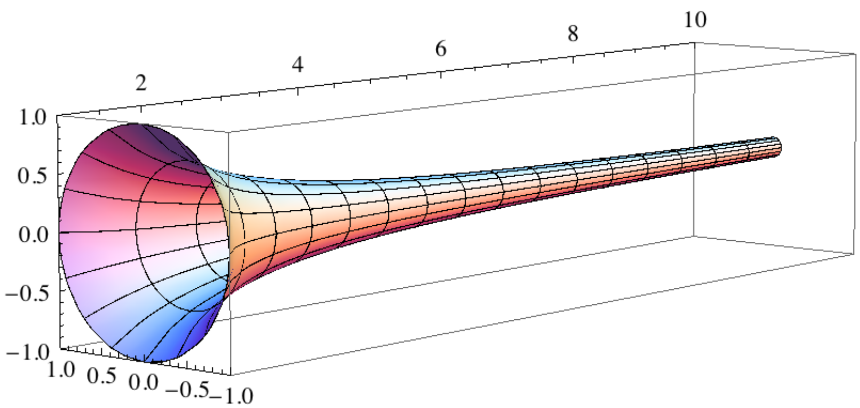
MATH 3025 NAME: ______________________________

Mathematica Lab #3 Due Monday April 3rd 2023 by 11:59 P.M.

All work is to be done in a programming notebook.
Please refer to the blackboard site for commands and examples.
You will be graded on the output that I am able to generate from your commands.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
27	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

1. Consider the function $f(x) = \frac{1}{x}$ for $x > 1$. Revolve this function about the x-axis creates what is referred to as Gabriel's (or Torricelli's) Horn. A picture is provided below.



   (a) Calculate the volume of the horn for $1 < x < \infty$.
   (b) Calculate the anti-derivative that would represent the surface area.
   (c) Use part 1b to compute the surface area for $1 < x < \infty$ by taking the limit as $t \to \infty$.
   (d) Explain in words why this is a paradox. It is best to talk in terms of paint.

2. Consider the function $g(x) = \dfrac{x^5}{x!}$. Let a be the number corresponding to the first initial of your family (last) name.
   (a) Graph the function for $0 \leq x \leq 20$. Note the plot command will not work since $g(x)$ is not defined for $n < x < n+1$ for all $n \in \mathbb{N}$. See template for project 3 for a suggestion on which package to use.
   (b) Guess the limit of $g(x)$ as x approaches infinity.
   (c) Find the smallest value N that correspond to $\epsilon = \frac{1}{a^3}$ in the precise (formal) definition of the limit. See definition 2 on page 692 of your book or the wikipedia page on formal definition of the limit of a sequence.

3. Consider the series.
$$\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^2}$$
   (a) Show that the series is convergent by computing its sum, S.
   (b) Let $S_a = \displaystyle\sum_{n=1}^{a} \frac{(\ln n)^2}{n^2}$, find the error in $S \approx S_a$.
   (c) Find the error in $S \approx S_{a+100}$ and $S \approx S_{a+1000}$.