Homework 4 Advanced Analytics and Metaheuristics

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1. Titan

   (a) Formulate the LP

   Here is the model file for this section

```
reset;

option solver cplex;



set projects;

#param startDate{years,projects} default No;
#param returns{years, projects} default 0;
#param maxAmt{projects} default Infinity;
param risks{projects} default 0;

var amountInvested{a in projects} >=0;

data group1_HW4_p1ai.dat;

maximize totalReturn: amountInvested['B'] + 1.4*amountInvested['E'] + 1.75*amountInvested['D'] + 1.06*amountInvested[3];
minimize risk: sum {a in projects} amountInvested[a]*risks[a];

subject to year1: amountInvested['A']+amountInvested['C']+amountInvested['D'] +amountInvested[1] = 1000000;
subject to year2: 0.3*amountInvested['A'] +1.1*amountInvested['C'] + 1.06*amountInvested[1] = amountInvested['B'] + amountInvested[2];
subject to year3: amountInvested['A'] + 0.3*amountInvested['B'] + 1.06*amountInvested[2] = amountInvested['E'] +amountInvested[3];


problem maxreturn: totalReturn, amountInvested, year1, year2, year3;
problem minrisk:  risk, amountInvested, year1, year2, year3;




#INDEPENDENT OBJECTIVES ------------------------------------------------
printf "\n\nINDEPENDENT OBJECTIVES ------------------------------------------------\n";

printf "\nMaximize Profit...........\n";
solve maxreturn;
display amountInvested, totalReturn, risk;

printf "\nMinimze Risk...........\n";
solve minrisk;
display amountInvested, totalReturn, risk;


 #SCALARIZATION ------------------------------------------------
printf "\n\nSCALARIZATION ------------------------------------------------\n";
param gamma1;
```

```
param gamma2;


#SCALARIZED OBJECTIVE (a.k.a., weighted sum)
maximize objWeightedSum:  (gamma1 * (amountInvested['B'] + 1.4*amountInvested['E'] + 1.75*amountInvested['D'] + 1.06*amountInvested[3])
                        - gamma2 * (sum{i in projects} amountInvested[i]*risks[i]));

problem maxScalarized: objWeightedSum, amountInvested, year1, year2, year3;


printf "\n\nMultiple values for SCALARIZATION -----------------------------------------------\n";
printf "episode, gamma1, gamma2, A, B, C, D, E, 1, 2,3, totalReturn, risk\n" > "titanParetoS.txt";
for {k in 0..4} {
        let gamma1 := k/4;
        let gamma2 := 1 - gamma1;

    solve maxScalarized;

    printf "\n\ngamma1 = %6.2f; gamma2 = %6.2f \n", gamma1, gamma2;
    printf "Optimal solution values: A = %6.2f   B = %6.2f C= %6.2f D= %6.2f E= %6.2f 1= %6.2f 2= %6.2f 3= %6.2f\n",
                    amountInvested['A'], amountInvested['B'], amountInvested['C'], amountInvested['D'], amountInvested['E'], amountInvest
        printf "Return generated: %6.2f\n",  totalReturn;
        printf "Risk Faced: %6.2f \n\n", risk ;
    printf "%d, %3.2f, %3.2f, %7.4f,%7.4f,%7.4f,%7.4f,%7.4f,%7.4f,%7.4f, %7.4f, %7.4f, %7.4f\n", k, gamma1, gamma2,
                    amountInvested['A'], amountInvested['B'], amountInvested['C'], amountInvested['D'], amountInvested['E'],
                    amountInvested[1], amountInvested[2], amountInvested[3], totalReturn, risk > "titanParetoS.txt";
}


#EPSILON-CONSTRAINT
printf "\n\EPSILON-CONSTRAINT METHOD ---------------------------------------------\n";

#get upper and lower bounds for objectives
param upperRisk;
param lowerRisk;

#in this example, put Profits as the objective function and use epsilon contstraints on the labor

#Let's get the lower and upper bounds for labor values by solving the independent problems
solve minrisk;
let lowerRisk:= risk;

solve maxreturn;
let upperRisk:= risk;

param epsilon;
let epsilon := lowerRisk;

s.t. epsilsonRisk: sum {a in projects} amountInvested[a]*risks[a] <= epsilon;
problem epsConst:  totalReturn, amountInvested, year1, year2, year3, epsilsonRisk;

param steps = 20;

printf "\n\nMultiple values for EPSILON-CONSTRAINT -----------------------------------------------\n";
printf "episode, epsilon, A, B, C, D, E, 1, 2,3, totalReturn, risk\n" > "titanParetoEps.txt";
for {eps in 0..steps} {

        let epsilon := lowerRisk + eps*(upperRisk - lowerRisk)/(steps);
        solve epsConst;

        display epsilon, totalReturn, risk;

        printf "%d, %3.2f, %7.4f,%7.4f,%7.4f,%7.4f,%7.4f,%7.4f,%7.4f, %7.4f, %7.4f, %7.4f\n", eps, epsilon,amountInvested['A'], amountInv
                    amountInvested[1], amountInvested[2], amountInvested[3], totalReturn, risk > "titanParetoEps.txt";

}
```

And here is the small data file

```
data;

#set years:= 2021 2022 2023 2024;
set projects:= A B C D E 1        2 3;

param risks:=
        A       0.10
        B       0.12
```

```
        C        0.05
        D        0.20
        E        0.05;

/*param startDate:          A       B        C        D        E:=
2021                                Yes         .       Yes        Yes         .
2022                                .        Yes         .        .        .
2023                                .        .        .        .        Yes;


param returns:                 A        B        C        D               E:=
2021                                -1         .       -1         -1                .
2022                                .3        -1        1.1         .                .
2023                                1        .3         .        .                -1
2024                                .        1        .        1.75         1.4;
/*
param maxAmt:=
A        500000
B        500000
E        750000;

*/
```

i. We see the maximum return as \$1845200. This includes a risk of 165900 units. You will invest in A the entire million. You will take the profits in year 2 and hold in an account. Then in year 3 all monies (million from A and the cash held over from previous year) will be invested in E.

```
INDEPENDENT OBJECTIVES ---------------------------------------------

Maximize Profit...........
CPLEX 20.1.0.0: optimal solution; objective 1845200
0 dual simplex iterations (0 in phase I)
amountInvested [*] :=
1        0
2    3e+05
3        0
A    1e+06
B        0
C        0
D        0
E  1318000
;

totalReturn = 1845200
risk = 165900
```

ii. When minimizing risk, we do indeed see Mr. Lee's assumption come true, all monies are held in the bank. They do gain compound interest... Risk is 0 units and return is \$1191020

```
AMPL

Minimze Risk...........
CPLEX 20.1.0.0: optimal solution; objective 0
2 simplex iterations (0 in phase I)
amountInvested [*] :=
1    1e+06
2  1060000
3  1123600
A        0
B        0
C        0
D        0
E        0
;

totalReturn = 1191020
risk = 0
```

iii. We preform the scalarization with γ_1 effecting the weight of the return and γ_2 effecting the weight of the risk. We see the minimum risk ($\gamma_1 = 0$)at the top and the maximum return ($\gamma_1 = 1$) at the bottom. Interestingly, we see that maximum return is generated at $\gamma_1 = \frac{1}{2}$. This is possibly due to the unbalanced range of the two objectives. We note that return will vary from $[1191016, 1845200]$ while risk varies from $[0, 1659000]$. Risk has about 2.5 times more range than return.

```
AMPL

SCALARIZATION ------------------------------------------------


Multiple values for SCALARIZATION ---------------------------------------------
CPLEX 20.1.0.0: optimal solution; objective 0
0 simplex iterations (0 in phase I)


gamma1 =   0.00; gamma2 =   1.00
Optimal solution values: A =   0.00   B =   0.00 C=   0.00 D=   0.00 E=   0.00 1= 1000000.00 2= 1060000.00 3= 1123600.00
Return generated: 1191016.00
Risk Faced:   0.00

CPLEX 20.1.0.0: optimal solution; objective 351125
1 simplex iterations (0 in phase I)


gamma1 =   0.25; gamma2 =   0.75
Optimal solution values: A =   0.00   B =   0.00 C=   0.00 D=   0.00 E= 1123600.00 1= 1000000.00 2= 1060000.00 3=   0.00
Return generated: 1573040.00
Risk Faced: 56180.00

CPLEX 20.1.0.0: optimal solution; objective 839650
1 simplex iterations (0 in phase I)


gamma1 =   0.50; gamma2 =   0.50
Optimal solution values: A = 1000000.00   B =   0.00 C=   0.00 D=   0.00 E= 1318000.00 1=   0.00 2= 300000.00 3=   0.00
Return generated: 1845200.00
Risk Faced: 165900.00

CPLEX 20.1.0.0: optimal solution; objective 1342425
0 simplex iterations (0 in phase I)


gamma1 =   0.75; gamma2 =   0.25
Optimal solution values: A = 1000000.00   B =   0.00 C=   0.00 D=   0.00 E= 1318000.00 1=   0.00 2= 300000.00 3=   0.00
Return generated: 1845200.00
Risk Faced: 165900.00

CPLEX 20.1.0.0: optimal solution; objective 1845200
0 simplex iterations (0 in phase I)


gamma1 =   1.00; gamma2 =   0.00
Optimal solution values: A = 1000000.00   B =   0.00 C=   0.00 D=   0.00 E= 1318000.00 1=   0.00 2= 300000.00 3=   0.00
Return generated: 1845200.00
Risk Faced: 165900.00
```

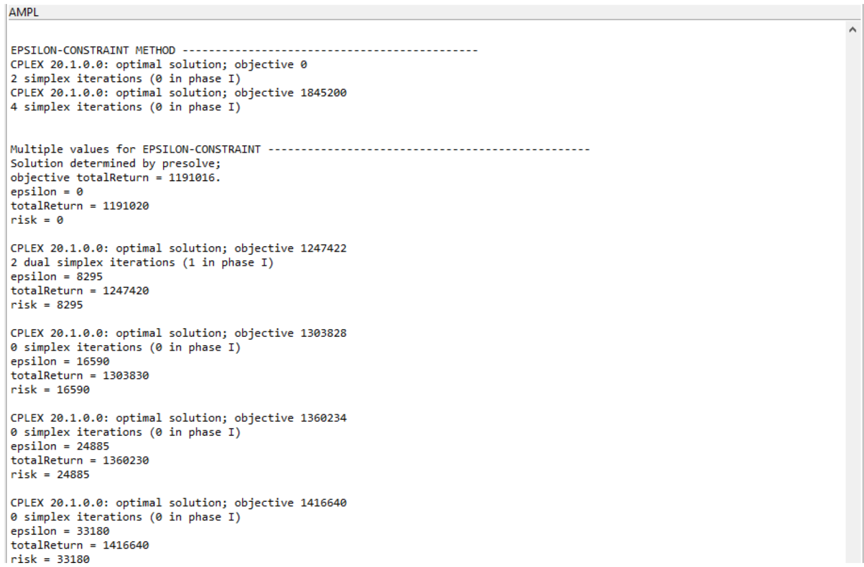
We include the text output generated from this exercise for completeness.

```
episode, gamma1, gamma2, A, B, C, D, E, 1, 2,3, totalReturn, risk
0, 0.00, 1.00,  0.0000, 0.0000, 0.0000, 0.0000, 0.0000,1000000.0000,1060000.0000, 1123600.0000, 1191016.0000,  0.0000
1, 0.25, 0.75,  0.0000, 0.0000, 0.0000, 0.0000,1123600.0000,1000000.0000,1060000.0000,  0.0000, 1573040.0000, 56180.0000
2, 0.50, 0.50, 1000000.0000, 0.0000, 0.0000, 0.0000,1318000.0000, 0.0000,300000.0000,  0.0000, 1845200.0000, 165900.0000
3, 0.75, 0.25, 1000000.0000, 0.0000, 0.0000, 0.0000,1318000.0000, 0.0000,300000.0000,  0.0000, 1845200.0000, 165900.0000
4, 1.00, 0.00, 1000000.0000, 0.0000, 0.0000, 0.0000,1318000.0000, 0.0000,300000.0000,  0.0000, 1845200.0000, 165900.0000
```

iv. Lastly, we preform ϵ-constraint on this problem with 20 steps. We use the risk as the constraint, setting

$$\epsilon = \frac{episode \cdot maxRisk}{20}.$$

We note that the minimum of the risk was zero so not included in the equation above. We do not include all the 21 episodes of the output for brevity's sake but some are included below

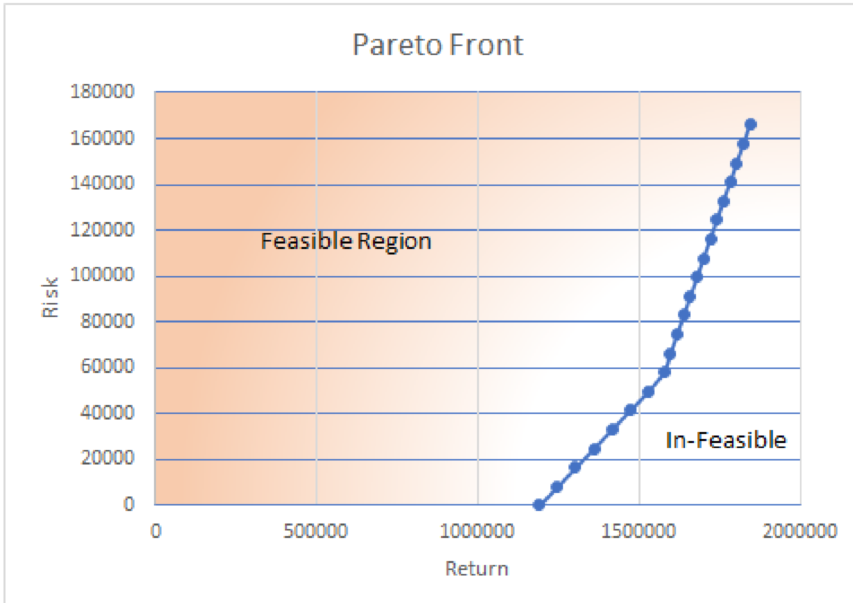


```
AMPL

EPSILON-CONSTRAINT METHOD -------------------------------------------
CPLEX 20.1.0.0: optimal solution; objective 0
2 simplex iterations (0 in phase I)
CPLEX 20.1.0.0: optimal solution; objective 1845200
4 simplex iterations (0 in phase I)


Multiple values for EPSILON-CONSTRAINT -----------------------------------------------
Solution determined by presolve;
objective totalReturn = 1191016.
epsilon = 0
totalReturn = 1191020
risk = 0

CPLEX 20.1.0.0: optimal solution; objective 1247422
2 dual simplex iterations (1 in phase I)
epsilon = 8295
totalReturn = 1247420
risk = 8295

CPLEX 20.1.0.0: optimal solution; objective 1303828
0 simplex iterations (0 in phase I)
epsilon = 16590
totalReturn = 1303830
risk = 16590

CPLEX 20.1.0.0: optimal solution; objective 1360234
0 simplex iterations (0 in phase I)
epsilon = 24885
totalReturn = 1360230
risk = 24885

CPLEX 20.1.0.0: optimal solution; objective 1416640
0 simplex iterations (0 in phase I)
epsilon = 33180
totalReturn = 1416640
risk = 33180
```

We include the text output generated from this exercise for completeness.

```
episode, epsilon, A, B, C, D, E, 1, 2,3, totalReturn, risk
0, 0.00,  0.0000, 0.0000, 0.0000, 0.0000, 0.0000,1000000.0000,1060000.0000, 1123600.0000, 1191016.0000,  0.0000
1, 8295.00,  0.0000, 0.0000, 0.0000, 0.0000,165900.0000,1000000.0000,1060000.0000, 957700.0000, 1247422.0000, 8295.0000
2, 16590.00,  0.0000, 0.0000, 0.0000, 0.0000,331800.0000,1000000.0000,1060000.0000, 791800.0000, 1303828.0000, 16590.0000
3, 24885.00,  0.0000, 0.0000, 0.0000, 0.0000,497700.0000,1000000.0000,1060000.0000, 625900.0000, 1360234.0000, 24885.0000
4, 33180.00,  0.0000, 0.0000, 0.0000, 0.0000,663600.0000,1000000.0000,1060000.0000, 460000.0000, 1416640.0000, 33180.0000
5, 41475.00,  0.0000, 0.0000, 0.0000, 0.0000,829500.0000,1000000.0000,1060000.0000, 294100.0000, 1473046.0000, 41475.0000
6, 49770.00,  0.0000, 0.0000, 0.0000, 0.0000,995400.0000,1000000.0000,1060000.0000, 128200.0000, 1529452.0000, 49770.0000
7, 58065.00, 17180.0948, 0.0000, 0.0000, 0.0000,1126939.8104,982819.9052,1046943.1280,  0.0000, 1577715.7346, 58065.0000
8, 66360.00, 92781.6260, 0.0000, 0.0000, 0.0000,1141636.7481,907218.3740,989485.9643,  0.0000, 1598291.4473, 66360.0000
9, 74655.00, 168383.1571, 0.0000, 0.0000, 0.0000,1156333.6857,831616.8429,932028.8006,  0.0000, 1618867.1600, 74655.0000
10, 82950.00, 243984.6883, 0.0000, 0.0000, 0.0000,1171030.6234,756015.3117,874571.6369,  0.0000, 1639442.8728, 82950.0000
11, 91245.00, 319586.2195, 0.0000, 0.0000, 0.0000,1185727.5611,680413.7805,817114.4732,  0.0000, 1660018.5855, 91245.0000
12, 99540.00, 395187.7506, 0.0000, 0.0000, 0.0000,1200424.4987,604812.2494,759657.3095,  0.0000, 1680594.2982, 99540.0000
13, 107835.00, 470789.2818, 0.0000, 0.0000, 0.0000,1215121.4364,529210.7182,702200.1458,  0.0000, 1701170.0109, 107835.0000
14, 116130.00, 546390.8130, 0.0000, 0.0000, 0.0000,1229818.3740,453609.1870,644742.9821,  0.0000, 1721745.7237, 116130.0000
15, 124425.00, 621992.3441, 0.0000, 0.0000, 0.0000,1244515.3117,378007.6559,587285.8184,  0.0000, 1742321.4364, 124425.0000
16, 132720.00, 697593.8753, 0.0000, 0.0000, 0.0000,1259212.2494,302406.1247,529828.6548,  0.0000, 1762897.1491, 132720.0000
17, 141015.00, 773195.4065, 0.0000, 0.0000, 0.0000,1273909.1870,226804.5935,472371.4911,  0.0000, 1783472.8618, 141015.0000
18, 149310.00, 848796.9377, 0.0000, 0.0000, 0.0000,1288606.1247,151203.0623,414914.3274,  0.0000, 1804048.5746, 149310.0000
19, 157605.00, 924398.4688, 0.0000, 0.0000, 0.0000,1303303.0623,75601.5312,357457.1637,  0.0000, 1824624.2873, 157605.0000
20, 165900.00, 1000000.0000, 0.0000, 0.0000, 0.0000,1318000.0000, 0.0000,300000.0000,  0.0000, 1845200.0000, 165900.0000
```

(b) Pareto Front From ϵ-Constraint

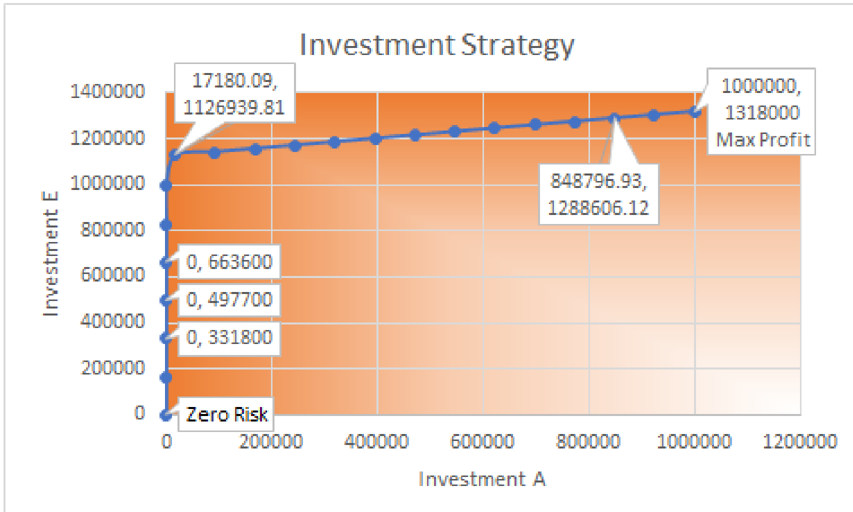


Above we see the expression of the Pareto Front utilizing the data generated from the ϵ- constraint method. As we change the maximum allowed risk, we see the return on investment increase. Anything to the left of our front will belong in the feasible set and everything to the right is not possible to achieve as we trade risk for greater returns. We see an interesting feature of a corner where the risk to return slope becomes much steeper. This shows that we will have to incur more risk to achieve more returns.

(c) To discuss how the portfolio changes across time, we first examine the investments based on the risk/profit trade off. We display the data here again for completeness

```
episode, epsilon, A, B, C, D, E, 1, 2,3, totalReturn, risk
0, 0.00,  0.0000, 0.0000, 0.0000, 0.0000, 0.0000,1000000.0000,1060000.0000, 1123600.0000, 1191016.0000,  0.0000
1, 8295.00,  0.0000, 0.0000, 0.0000, 0.0000,165900.0000,1000000.0000,1060000.0000, 957700.0000, 1247422.0000, 8295.0000
2, 16590.00,  0.0000, 0.0000, 0.0000, 0.0000,331800.0000,1000000.0000,1060000.0000, 791800.0000, 1303828.0000, 16590.0000
3, 24885.00,  0.0000, 0.0000, 0.0000, 0.0000,497700.0000,1000000.0000,1060000.0000, 625900.0000, 1360234.0000, 24885.0000
4, 33180.00,  0.0000, 0.0000, 0.0000, 0.0000,663600.0000,1000000.0000,1060000.0000, 460000.0000, 1416640.0000, 33180.0000
5, 41475.00,  0.0000, 0.0000, 0.0000, 0.0000,829500.0000,1000000.0000,1060000.0000, 294100.0000, 1473046.0000, 41475.0000
6, 49770.00,  0.0000, 0.0000, 0.0000, 0.0000,995400.0000,1000000.0000,1060000.0000, 128200.0000, 1529452.0000, 49770.0000
7, 58065.00, 17180.0948, 0.0000, 0.0000, 0.0000,1126939.8104,982819.9052,1046943.1280,  0.0000, 1577715.7346, 58065.0000
8, 66360.00, 92781.6260, 0.0000, 0.0000, 0.0000,1141636.7481,907218.3740,989485.9643,  0.0000, 1598291.4473, 66360.0000
9, 74655.00, 168383.1571, 0.0000, 0.0000, 0.0000,1156333.6857,831616.8429,932028.8006,  0.0000, 1618867.1600, 74655.0000
10, 82950.00, 243984.6883, 0.0000, 0.0000, 0.0000,1171030.6234,756015.3117,874571.6369,  0.0000, 1639442.8728, 82950.0000
11, 91245.00, 319586.2195, 0.0000, 0.0000, 0.0000,1185727.5611,680413.7805,817114.4732,  0.0000, 1660018.5855, 91245.0000
12, 99540.00, 395187.7506, 0.0000, 0.0000, 0.0000,1200424.4987,604812.2494,759657.3095,  0.0000, 1680594.2982, 99540.0000
13, 107835.00, 470789.2818, 0.0000, 0.0000, 0.0000,1215121.4364,529210.7182,702200.1458,  0.0000, 1701170.0109, 107835.0000
14, 116130.00, 546390.8130, 0.0000, 0.0000, 0.0000,1229818.3740,453609.1870,644742.9821,  0.0000, 1721745.7237, 116130.0000
15, 124425.00, 621992.3441, 0.0000, 0.0000, 0.0000,1244515.3117,378007.6559,587285.8184,  0.0000, 1742321.4364, 124425.0000
16, 132720.00, 697593.8753, 0.0000, 0.0000, 0.0000,1259212.2494,302406.1247,529828.6548,  0.0000, 1762897.1491, 132720.0000
17, 141015.00, 773195.4065, 0.0000, 0.0000, 0.0000,1273909.1870,226804.5935,472371.4911,  0.0000, 1783472.8618, 141015.0000
18, 149310.00, 848796.9377, 0.0000, 0.0000, 0.0000,1288606.1247,151203.0623,414914.3274,  0.0000, 1804048.5746, 149310.0000
19, 157605.00, 924398.4688, 0.0000, 0.0000, 0.0000,1303303.0623,75601.5312,357457.1637,  0.0000, 1824624.2873, 157605.0000
20, 165900.00, 1000000.0000, 0.0000, 0.0000, 0.0000,1318000.0000, 0.0000,300000.0000,  0.0000, 1845200.0000, 165900.0000
```

We notice that the B, C, and D investments are never used. We note that as risk tolerance increases, more money is first invested into account E and then into account A. In the maximum return scenario, everything is invested in each of these accounts. Let's visualize the trade-off by examining these two accounts.



Again we notice the corner. This is not a coincidence as this corner occurs at $\epsilon = 7$ in both graphs. We included a few data call outs to help illustrate the points represented in the graph. We also believe that we did not need to include investments in the bank as banking is boring and provides no risk and little reward even though all values of ϵ use the bank at some point.