Homework 8 Advanced Analytics and Metaheuristics

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1. Genetic Algorithm

   (a) Finalize Code

      i. To create the initial chromosomes, we kept it simple, generating a random number between -500 and 500. We used the global variable n as the number of dimensions.

```
def createChromosome(n):
    x = []
    for i in range(n):
      x.append(myPRNG.uniform(-500,500))
    return x
```

      ii. For mutation, we played with this quite a bit. Originally we randomly selected an index and the just reseeded it with a random. This seemed to work but we wanted more control over it so we introduced a new parameter that we could tweak, $mutationFactor$. We used this to change one of the indicies by a random, $r \in [-1, 1]$, with $x[i] = x[i] + r * mutationFactor$. We could then tweak this factor to get more or less mutation. We did check that it was inside the interval and if not, randomly reseed.

```
def mutate(x):

    if mutationRate > myPRNG.random():
      i = myPRNG.randint(0,n-1)
      x[i] += mutationFactor*myPRNG.uniform(-1,1)
      if (x[i] > 500) or (x[i]<-500):
        x[i] = myPRNG.uniform(-500,500)
    return x
```

iii. For cross over, we picked on index to swap start the swap. Everything after that index would be traded between the parents. This happened with a *crossOverRate*. Some times just the parents would be returned.

```
def crossover(x1,x2):
  p = myPRNG.random()
  if crossOverRate>p:
    z = myPRNG.randint(1,n-1)
    offspring1 = x1[:z] + x2[z:]
    offspring2 = x2[:z] + x1[z:]

  else:
    offspring1 = x1[:]
    offspring2 = x2[:]

  return offspring1, offspring2  #two offspring are returned
```

iv. Elitism was achieved by keeping the best numbering to *eliteSolutions* from the parent generation. It was important to resort these as the children were often better. We also made sure to change it so the minimum was first as desired here.

```
def insert(pop,kids):

    newlist = []
    for i in range(eliteSolutions):
      newlist.append(pop[i])
    for kid in kids:
      newlist.append(kid)
    popVals = sorted(newlist, key=lambda newlist: newlist[1], reverse = F
    return popVals
```

v. I want to share the code for computing the function. With *evaluate* we made sure to utilize the numpy speed in dealing with large dimensional data.

```
def evaluate(x):
  x = np.array(x)
  return 418.982887272443*n -sum(x*np.sin(np.sqrt(np.abs(x))))
```

(b) For parameters, I played around with the code a lot. I noticed for the small dimensional and chromosome, it was important to have higher mutation rate. I found very good results with the following parameters

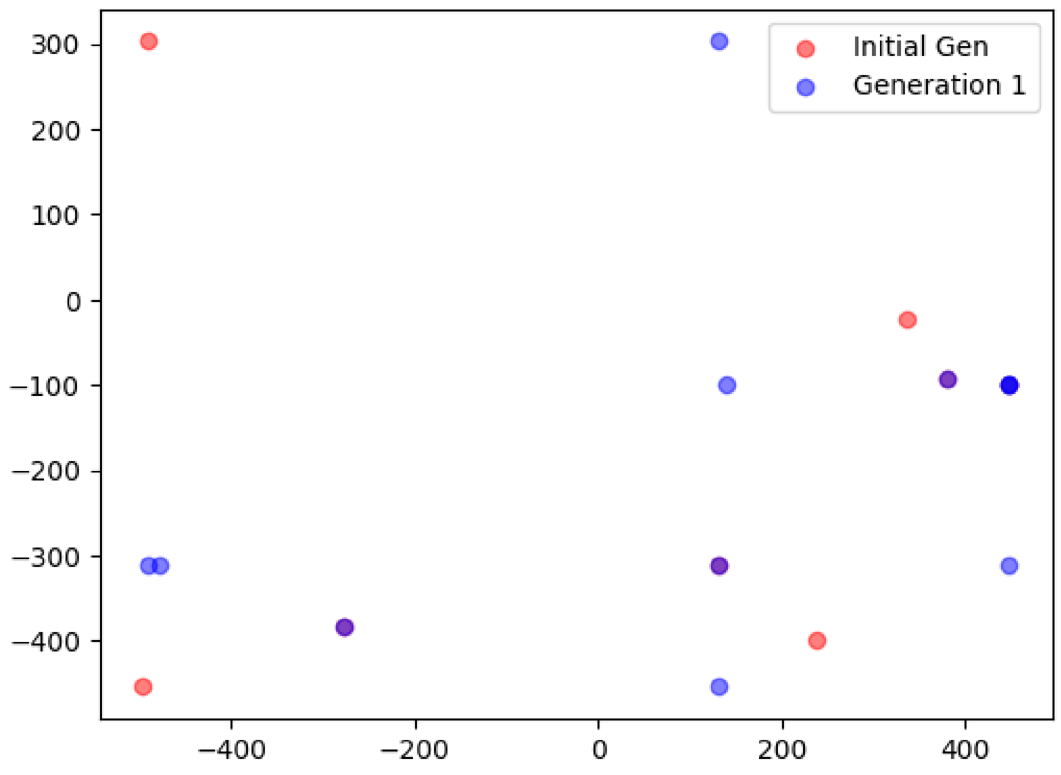
```
#number of dimensions in a solution
n = 2

Generations = 300   #number of GA generations

crossOverRate = 0.8  #whether the couple breeds or just continues into next
mutationRate = 0.25  #rate at which mutation can occur
eliteSolutions = 10  #number of elite solutions form previous generation tha
mutationFactor = 50 # the largest number by which the mutation can change a
populationSize = 80 #size of GA population
```

For the higher dimensions, we used

```
#number of dimensions in a solution
n = 200

Generations = 1000   #number of GA generations

crossOverRate = 0.8  #whether the couple breeds or just continues into next
mutationRate = 0.25  #rate at which mutation can occur
eliteSolutions = 100  #number of elite solutions form previous generation th
mutationFactor = 100 # the largest number by which the mutation can change a
populationSize = 1000 #size of GA population
```

(c) 2D

i. Here is my visualization of 8 chromosomes. We see some genes splicing and we see some repeats. It is also possible we see some mutations.



ii. This had Best solution: [420.9337473832483, 421.0086992961223] as the best solution, accurate to three decimal places, 0.0003559856297670194

(d) This was quite challenging. At 1000 generations, we were still seeing slight improvements. This is with an initial population of 1000 and keeping 100 elite solutions. We saw the value go as low as 18666. While this is nowhere as good as the 2D solution, we do see many entries with a value near the 420 optimal. The execution of this code was slow and with the generations not showing much improvement, do not think expanding beyond 1000 generations will improve the result by much. This problem is hard!

2. Particle Swarm Optimization

(a) Code Basics

i. Particle Tracking:

```
def pbestUpdate(): #update the best lists
  particleUpdate = 0 #how many particles have been updated?
  globalUpdate = 0 #did the global best update
```

```
    global pbestestVal, pbestest #get these variables from global environme
    for i in range(swarmSize): #iterate through swarm
      if curValue[i]< pbestVal[i]: #is the current better than current best
        pbest[i] = pos[i][:] #since yes, change it up
        pbestVal[i] = curValue[i]
        particleUpdate +=1 #increment the counter
        if curValue[i]<pbestestVal: #since this is better than before, chec
          pbestestVal = curValue[i] #since it is, update
          pbestest = pos[i][:]
          globalUpdate +=1
    return particleUpdate, globalUpdate #return counts for stopping criteri
```

ii. Velocity and Position updates

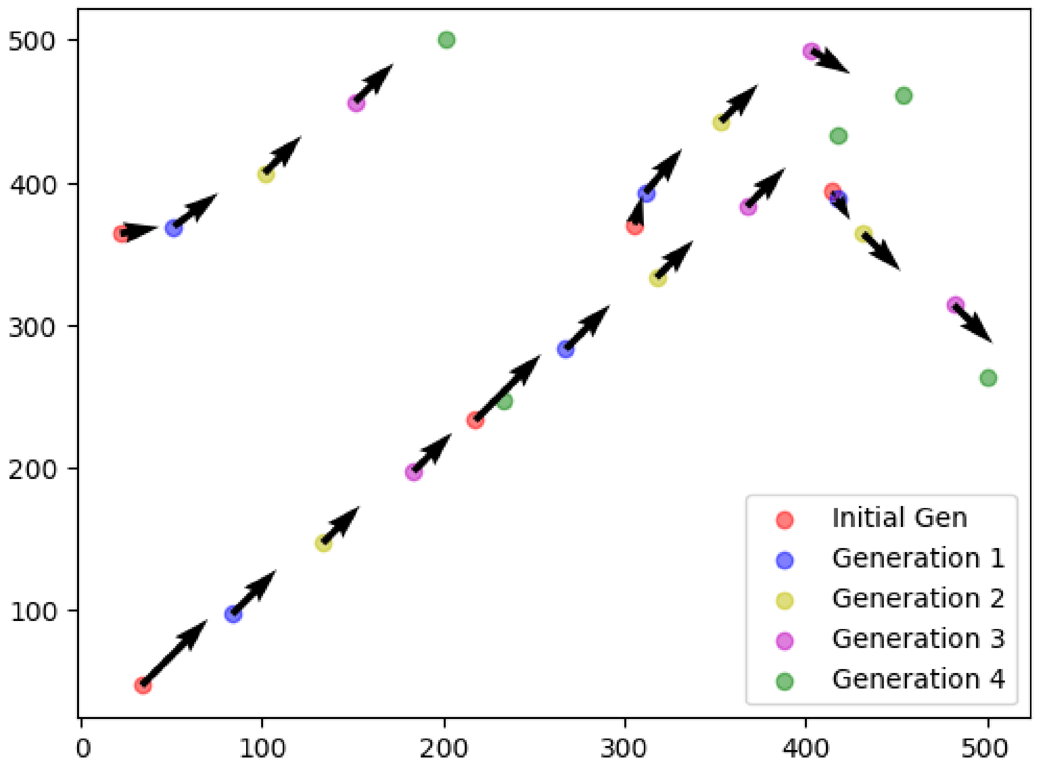
```
  def velocityUpdate(): #update velocities
    for i in range(swarmSize): #iterate through swarm
      for j in range(dimensions): #iterate through dimensions
        vel[i][j] = w*vel[i][j]+phi1*myPRNG.random()*(pbest[i][j]-pos[i][j
        if vel[i][j] >maxVelocity: #check if outside of allowed values
          vel[i][j] = maxVelocity #just return max/min allowed
        if vel[i][j] <-1*maxVelocity:
          vel[i][j] = -1*maxVelocity

  def positionUpdate():#must be called after velocity update
    for i in range(swarmSize): #iterate through swarm
      for j in range(dimensions): #update all dimensions
        pos[i][j] = pos[i][j] + vel[i][j] #follow basic PSO

        if pos[i][j] > upperBound: #check if outside of feasible
          pos[i][j] = upperBound #if so, put on edge
        if pos[i][j] < lowerBound: #outside feasible?
          pos[i][j] = lowerBound
      curValue[i] = evaluate(pos[i]) #update currentvalue too
```

iii. Limits on particle positions are included in the position update. I only allow getting on the boundary

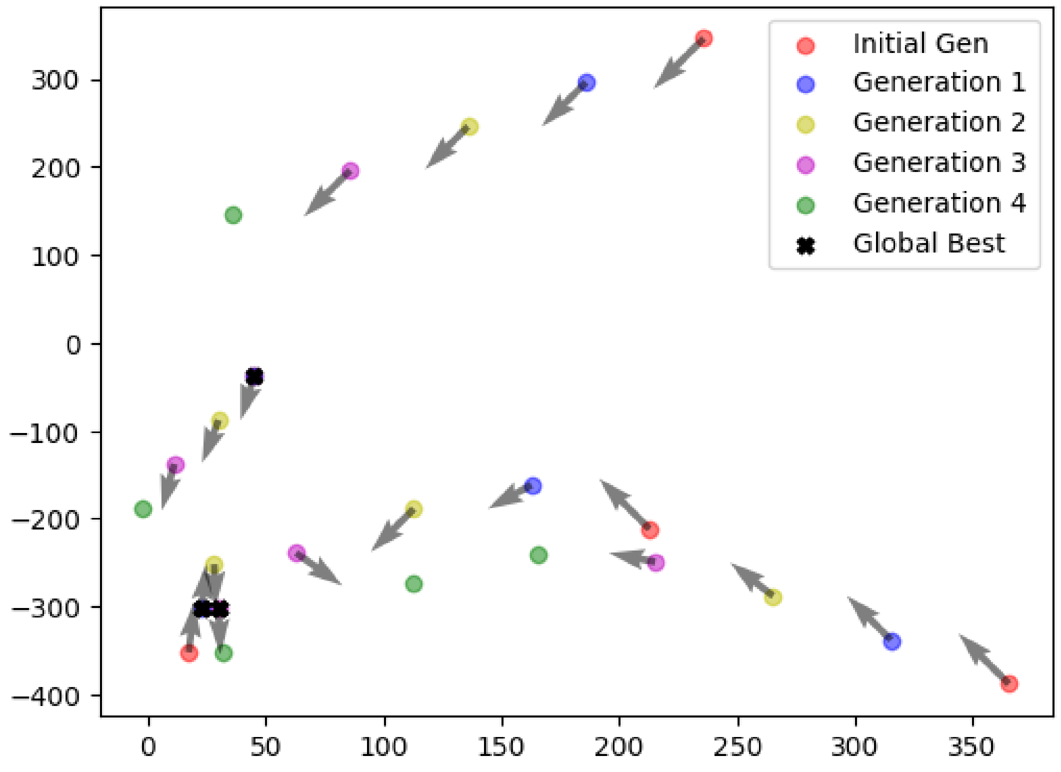
iv. Limits on velocity are included in velocityUpdate. Max velocity is a parameter that can be tweaked later.

(b) 2D Schwefel

i. Here is a plot with just positions and velocities. I don't think those *quiver* arrows are perfect from *matplotlib* but they were what was easily available for graphing.



ii. Here is a plot that includes the global best on each iteration. Some times the global best was repeated, so that is not reported here.



(c) 200D: Results are tabulated below. I really have no theory on what would make this work better. It felt as though I should be just using random numbers for these parameters as I could not suss out a pattern. In any case, they are reported here.

Swarm Size	Time Length	Inertial Weight, w	ϕ_1	ϕ_2	Value
100	1000	1	3	1	44,375
10,000	20	1	3	1	71,645
1000	200	2	2	2	67,307
500	400	0.5	1.5	2.5	48,858
750	400	1	1.25	2.75	67,111
100	1000	0	1.25	2.75	57,403

(d) Local Best: I attempted the star topology with all particles being compared to the first of the list. The modifications to the code base were fairly straight forward. I needed to make the global best a list and update it by comparing both to the particle at hand and the first particle. I only make this comparison when I have updated the bestVal already. The code for doing that is quoted here. There was also minor changes to the initializers and velocity updates.

```
def pbestUpdateLocalStar(): #update the best lists
  particleUpdate = 0 #how many particles have been updated?
  globalUpdate = 0 #did the global best update
  global pbestestVal, pbestest #get these variables from global environment
  for i in range(swarmSize): #iterate through swarm
    if curValue[i]< pbestVal[i]: #is the current better than current best
      pbest[i] = pos[i][:] #since yes, change it up
      pbestVal[i] = curValue[i]
      particleUpdate +=1 #increment the counter
      if min(curValue[i],pbestestVal[0])<pbestestVal[i]: #since this is bett
        if curValue[i] < pbestestVal[i]:
          pbestestVal[i] = curValue[i] #since it is, update
          pbestest[i] = pos[i][:]
          globalUpdate +=1
        else:
          pbestestVal[i] = pbestestVal[0] #since it is, update
          pbestest[i] = pbestest[0][:]
          globalUpdate +=1

  return particleUpdate, globalUpdate #return counts for stopping criteria
```

(e) Test with Local: Lastly I repeat the code a bunch of times and add to the table now with the local star topology. I attempted to reuse the values from before so this table looks a lot like the one above.

Swarm Size	Time Length	Inertial Weight, w	ϕ_1	ϕ_2	Value
100	1000	1	3	1	68,874
10,000	20	1	3	1	72,464
1000	200	2	2	2	73,569
50	400	0.5	1.5	2.5	78,151
750	400	1	1.25	2.75	69,092
100	1000	0	1.25	2.75	77,421

Overall, I do not see marked improvement based on the star topology. Seems like leaderless swarms are better, since this one clearly has a leader and you are always checking only if you did better than them. Also making the inertial weight 0 was a bad choice as it did not explore...

Overall I much preferred the genetic algorithm. It felt more natural for this problem but perchance I just didn't play around enough to get this one tweaked to work well...