Homework 3 Advanced Analytics and Metaheuristics

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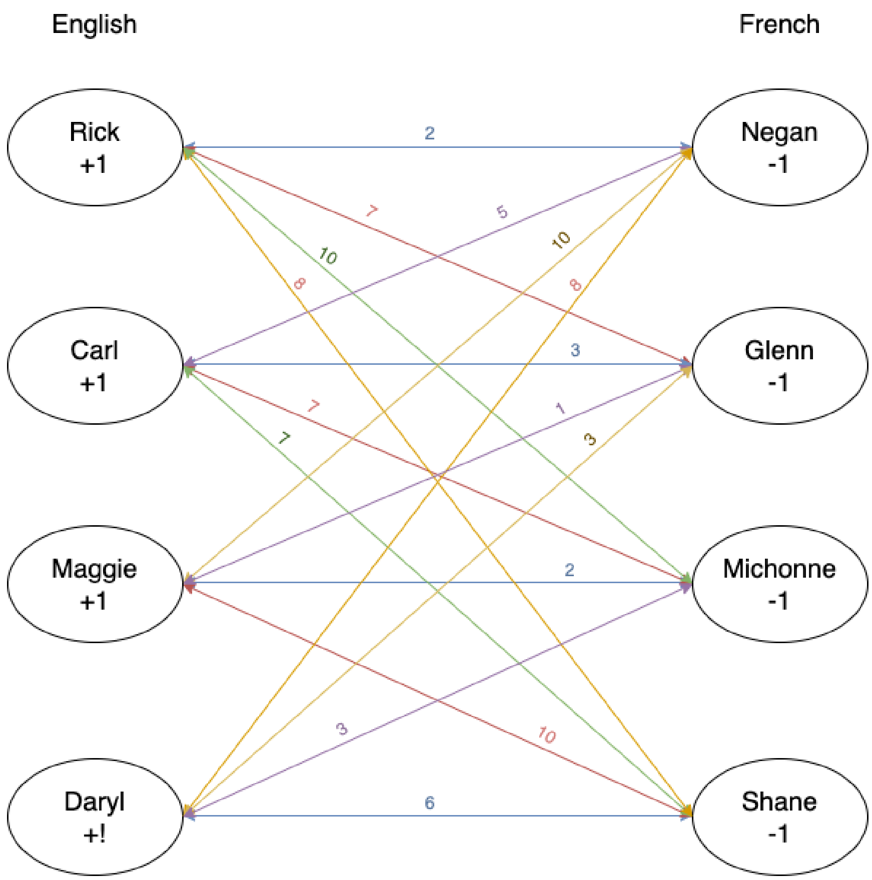
1. Team Building

   (a) Pairing English and French children. Our network flow diagram is below. We give the nodes representing English children a positive weight of one and nodes representing French children a negative weight of one. We have the costs labeled on each arc. Here we slightly changed the model of the MCNFP by introducing a negative to the objective function. We could have simply made the weights negative instead to correct for this, which would have allowed us not to change the mod file at all. This causes the minimizer to return a maximum solution to the objective function, which in this context is the maximum compatibility of the pairs.



Here is our model file:

```
# AMPL model for the Minimum Cost Network Flow Problem
#
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# l[i,j] = 0 and u[i,j] = Infinity.
#
# Parameters not specified in the data file will get their default values.
reset;

options solver cplex;

set NODES;                          # nodes in the network
set ARCS within {NODES, NODES};     # arcs in the network

param b {NODES} default 0;          # supply/demand for node i
param c {ARCS}  default 0;          # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;          # lower bound on flow on arc(i,j)
```

```
param u {ARCS}  default Infinity; # upper bound on flow on arc(i,j)

var x {ARCS};                     # flow on arc (i,j)

minimize cost: sum{(i,j) in ARCS} (-1)* c[i,j] * x[i,j];  #objective: minimize arc

# Flow Out(i) - Flow In(i) = b(i)

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS} x[j,i] = b[i

subject to capacity {(i,j) in ARCS}: l[i,j] <= x[i,j] <= u[i,j];

data group1_HW3_p1anj.dat;

solve;

display x;
```

Here is our data file:

```
data;

set NODES := 1, 2, 3, 4, 5, 6, 7, 8;

/*
1: Rick english
2: Negan french
3: Carl english
4: Glenn french
5: Maggie english
6: Michonne french
7: Daryl english
8: Shane french
*/

set ARCS :=
        (1,*)         2        4        6        8
        (3,*)        2        4        6        8
        (5,*)        2        4        6        8
        (7,*)        2        4        6        8
    ;


param c:  1        2        3        4       5       6       7       8  :=
    1     .        2        .        7       .       10      .       9
```

```
2     .        .        .        .       .      .      .       .
3     .        5        .        3       .      7      .       7
4     .        .        .        .       .      .      .       .
5     .        10       .        1       .      2      .       10
6     .       .      .        .       .      .      .       .
7     .        8        .        3       .      3      .       6
8     .        .        .        .       .      .      .       .;

param b:=
        1        1
        2        -1
        3        1
        4       -1
        5       1
        6       -1
        7       1
        8       -1;
```

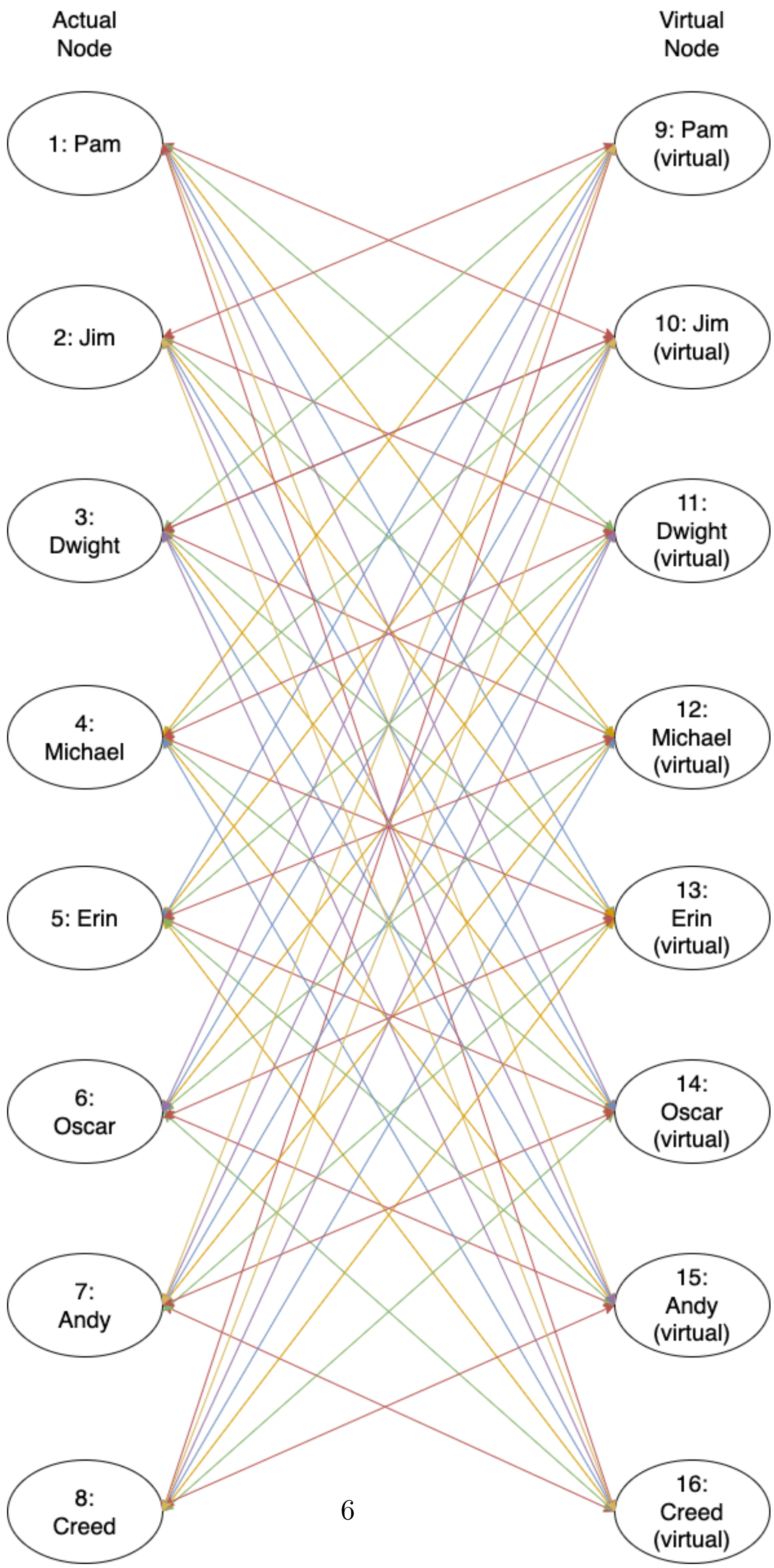
Here is our output:

```
ampl: model group1_HW3_p1anj.mod
CPLEX 20.1.0.0: optimal solution; objective -32
2 dual simplex iterations (0 in phase I)
x :=
1 2   0
1 4   1
1 6   0
1 8   0
3 2   0
3 4   0
3 6   1
3 8   0
5 2   0
5 4   0
5 6   0
5 8   1
7 2   1
7 4   0
7 6   0
7 8   0
;
```

For our example, we see persons 1 and 4 paired, 3 and 6 paired, etc. We see this answer as fully generalizable. This MCNFP is always balanced - we have an equal number of English and French, so we always have even pairs with no leftovers in this model when solved.

(b) Office Team Building

We attempt to build a model with the same idea as above. Our network flow diagram is below. We assign scores of 2 as not compatible, 7 with compatible and 10 with highly compatible. We chose these values to represent the spread between not being compatible and being compatible as being 'large'. We attempted to create a virtual copy of each person and attach them to everyone else (not attached to yourself) and put those costs on the flow. This is represented in our document below. We end up running this with 8 people and getting a working solution but when we expanded to 16 we got some weirdness.


Actual Node
Virtual Node
1: Pam
2: Jim
3: Dwight
4: Michael
5: Erin
6: Oscar
7: Andy
8: Creed
9: Pam (virtual)
10: Jim (virtual)
11: Dwight (virtual)
12: Michael (virtual)
13: Erin (virtual)
14: Oscar (virtual)
15: Andy (virtual)
16: Creed (virtual)

Here is our model file:

```
# AMPL model for the Minimum Cost Network Flow Problem
#
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# l[i,j] = 0 and u[i,j] = Infinity.
#
# Parameters not specified in the data file will get their default values.
reset;

options solver cplex;

set NODES;                        # nodes in the network
set ARCS within {NODES, NODES};   # arcs in the network

param b {NODES} default 0;        # supply/demand for node i
param c {ARCS}  default 0;        # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;        # lower bound on flow on arc(i,j)
param u {ARCS}  default Infinity; # upper bound on flow on arc(i,j)

var x {ARCS};                     # flow on arc (i,j)

minimize cost: sum{(i,j) in ARCS} (-1)* c[i,j] * x[i,j];  #objective: minimize arc flow cost

# Flow Out(i) - Flow In(i) = b(i)

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS} x[j,i] = b[i];

subject to capacity {(i,j) in ARCS}: l[i,j] <= x[i,j] <= u[i,j];

data group1_HW3_p1bnj2.dat;

solve;

display x;

printf "Total Compatibility Score: %f\n", cost;
printf "Pairs:\n";
for {(i, j) in ARCS: x[i,j] > 0} {
    printf "%s - %s\n", i, j;
}
```

Here is our data file:

```
data;

set NODES := 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
                        v1 v2 v3 v4 v5 v6 v7 v8 v9 v10 v11 v12 v13 v14 v15 v16;

/*
1: Pam
2: Jim
3: Dwight
4: Michael
5: Erin
6: Oscar
7: Andy
8: Creed
9: Kevin
10: Angela
11: Phyllis
12: Holly
13: Kelly
14: Stanley
15: Ryan
16: Meredith
*/

/*
2 = Not Compatible
7 = Compatible
10 = Highly Compatible
*/


set ARCS :=
```

```
(1,*)           v2      v3      v4      v5      v6      v7      v8      v9      v10      v11      v12
(2,*)      v1           v3      v4      v5      v6      v7      v8      v9      v10      v11      v12
(3,*)      v1      v2           v4      v5      v6      v7      v8      v9      v10      v11      v12
(4,*)      v1      v2      v3           v5      v6      v7      v8      v9      v10      v11      v12
(5,*)      v1      v2      v3      v4           v6      v7      v8      v9      v10      v11      v12
(6,*)      v1      v2      v3      v4      v5           v7      v8      v9      v10      v11      v12
(7,*)      v1      v2      v3      v4      v5      v6           v8      v9      v10      v11      v12
(8,*)      v1      v2      v3      v4      v5      v6      v7           v9      v10      v11      v12
(9,*)      v1      v2      v3      v4      v5      v6      v7      v8           v10      v11      v12
(10,*)     v1      v2      v3      v4      v5      v6      v7      v8      v9            v11      v12
(11,*)     v1      v2      v3      v4      v5      v6      v7      v8      v9      v10            v12
(12,*)     v1      v2      v3      v4      v5      v6      v7      v8      v9      v10      v11
(13,*)     v1      v2      v3      v4      v5      v6      v7      v8      v9      v10      v11      v12
(14,*)     v1      v2      v3      v4      v5      v6      v7      v8      v9      v10      v11      v12
(15,*)     v1      v2      v3      v4      v5      v6      v7      v8      v9      v10      v11      v12
(16,*)     v1      v2      v3      v4      v5      v6      v7      v8      v9      v10      v11      v12
 ;

param c:      v1     v2    v3    v4     v5    v6    v7    v8    v9    v10   v11   v12   v13   v14   v15   v16 :=
   1        .     10     7     7     7     7     7     2     7     2     7     7     7     2     2     2
   2       10      .     7     7     7     7     7     2     7     2     7     7     2     7     7     2
   3        7      7     .     7     2     2     2     2     2    10     2     2     2     2     7     2
   4        7      7     7     .     7     2    10     2     2     2     2    10     2     2     7     2
   5        7      7     2     7     .     7    10     2     7     7     7     7     7     2     7     2
   6        7      7     2     2     7     .     2     2     2     7     7     7     2     2     2     2
   7        7      7     2    10    10     2     .     2     7     2     2     7     2     2     7     2
   8        2      2     2     2     2     2     2     .     2     2     2     2     2     2     2    10
   9        7      7     2     2     7     2     7     2     .     2     7     2     2     7     2     7
   10       2      2    10     2     7     7     2     2     2     .     2     2     2     2     2     2
   11       7      7     2     2     7     7     2     2     7     2     .     7     7    10     2     7
   12       7      7     2    10     7     7     7     2     2     2     7     .     7     2     2     2
   13       7      2     2     2     7     2     2     2     2     2     7     7     .     2    10     2
   14       2      7     2     2     2     2     2     2     7     2    10     2     2     .     2     2
   15       2      7     7     7     7     2     7     2     2     2     2     2    10     2     .     2
   16       2      2     2     2     2     2     2    10     7     2     7     2     2     2     2     .;

param b:=
     1      1
     2      1
     3      1
     4      1
     5      1
     6      1
     7      1
     8      1
     9      1
     10     1
     11     1
     12     1
     13     1
     14     1
     15     1
     16     1
     v1     -1
     v2     -1
     v3     -1
     v4     -1
     v5     -1
     v6     -1
     v7     -1
     v8     -1
     v9     -1
     v10    -1
     v11    -1
     v12    -1
     v13    -1
     v14    -1
     v15    -1
     v16    -1;
```

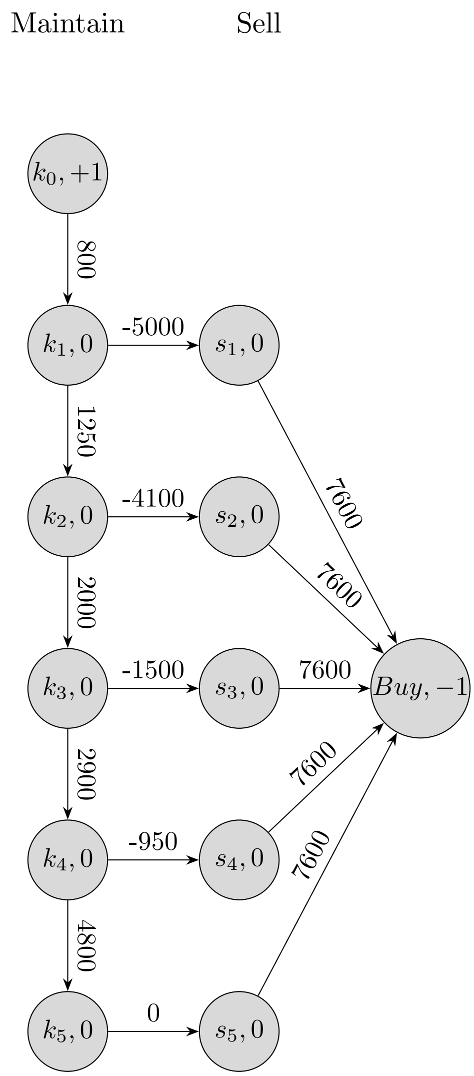
Here is our output:

```
ampl: model group1_HW3_p1bnj.mod
CPLEX 20.1.0.0: optimal solution; objective -148
126 dual simplex iterations (40 in phase I)
x [*,*]
:   v1 v10 v11 v12 v13 v14 v15 v16  v2  v3  v4  v5  v6  v7  v8  v9    :=
1    .   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
2    1   0   0   0   0   0   0   0   .   0   0   0   0   0   0   0
3    0   1   0   0   0   0   0   0   0   .   0   0   0   0   0   0
4    0   0   0   0   0   0   0   0   0   0   .   0   0   1   0   0
5    0   0   0   0   0   0   0   0   0   0   0   .   1   0   0   0
6    0   0   0   1   0   0   0   0   0   0   0   0   .   0   0   0
7    0   0   0   0   0   0   0   0   0   0   0   1   0   .   0   0
8    0   0   0   0   0   0   0   1   0   0   0   0   0   0   .   0
9    0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   .
10   0   .   0   0   0   0   0   0   0   1   0   0   0   0   0   0
11   0   0   .   0   0   1   0   0   0   0   0   0   0   0   0   0
12   0   0   0   .   0   0   0   0   0   0   1   0   0   0   0   0
13   0   0   0   0   .   0   1   0   0   0   0   0   0   0   0   0
14   0   0   1   0   0   .   0   0   0   0   0   0   0   0   0   0
15   0   0   0   0   1   0   .   0   0   0   0   0   0   0   0   0
16   0   0   0   0   0   0   0   .   0   0   0   0   0   0   1   0
;

Total Compatibility Score: -148.000000
Pairs:
1 - v9
2 - v1
3 - v10
4 - v7
5 - v6
6 - v12
7 - v5
8 - v16
9 - v2
10 - v3
11 - v14
12 - v4
13 - v15
14 - v11
15 - v13
16 - v8
ampl:
```

We see weirdness in our output here. There are groups of three formed as well as quintuplet room assignments. We see 1 paired with 9 but 9 paired with 2 and then completing the loop with 2 paired with 1. There is another group of 5 assigned to rooms that will not work. We find this solution to not be generalizable even though in theory it might have worked. The optimizer did too good of job without being able to distinguish that we needed the pairings to coincide. If 1 was matched with v9 we needed 9 to be matched with v1. This was not necessarily the case.

2. Outdoor Grilling

Below is our model diagram for this problem.



Here is our model file:

```
# AMPL model for the Minimum Cost Network Flow Problem
#
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# l[i,j] = 0 and u[i,j] = Infinity.
#
# Parameters not specified in the data file will get their default values.
reset;
```

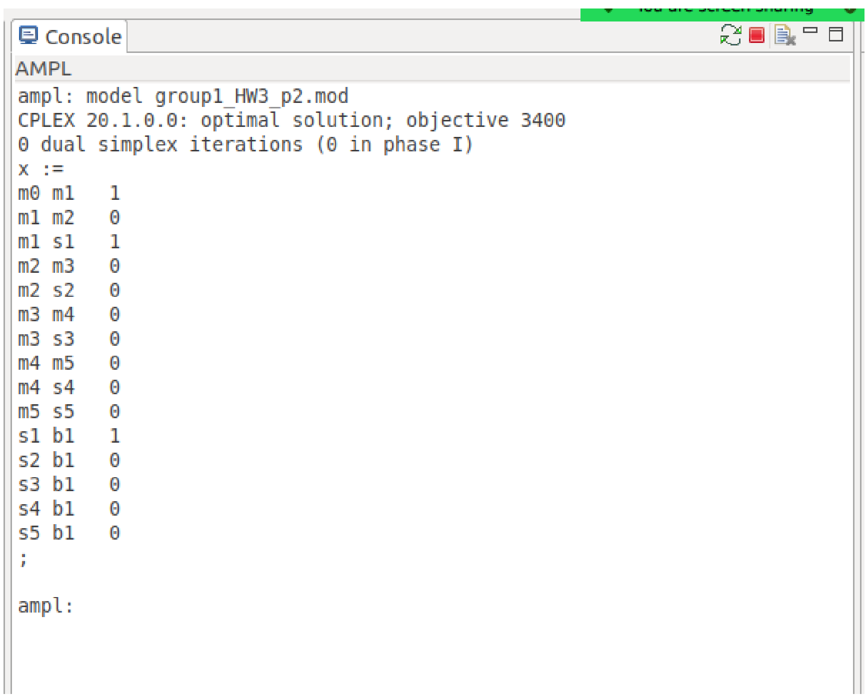
```
options solver cplex;

set NODES;                          # nodes in the network
set ARCS within {NODES, NODES};     # arcs in the network

param b {NODES} default 0;          # supply/demand for node i
param c {ARCS}  default 0;          # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;          # lower bound on flow on arc(i,j)
param u {ARCS}  default Infinity;   # upper bound on flow on arc(i,j)

var x {ARCS};                       # flow on arc (i,j)

minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j];  #objective: minimize arc flow cost

# Flow Out(i) - Flow In(i) = b(i)

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS}  x[j,i] = b[i];

subject to capacity {(i,j) in ARCS}: l[i,j] <= x[i,j] <= u[i,j];

data group1_HW3_p2.dat;

solve;

display x;
```

Here is our data file:

```
set NODES := m0, m1, m2, m3, m4, m5,
                        s1, s2, s3, s4, s5,
                        b1;


set ARCS := (m0, m1), (m1, m2), (m2, m3), (m3, m4), (m4, m5),
                        (m1, s1), (m2, s2), (m3, s3), (m4, s4), (m5, s5),
                        (s1, b1), (s2, b1), (s3, b1), (s4, b1), (s5, b1);

param: b:=
        m0 1
        b1 -1;

param:         c l u:=
                [m0, m1]        800         .         .
```

[m1, m2]	1250	.	.
[m2, m3]	2000	.	.
[m3, m4]	2900	.	.
[m4, m5]	4800	.	.
[m1, s1]	-5000	.	.
[m2, s2]	-4100	.	.
[m3, s3]	-1500	.	.
[m4, s4]	-950	.	.
[m5, s5]	0	.	.
[s1, b1]	7600	.	.
[s2, b1]	7600	.	.
[s3, b1]	7600	.	.
[s4, b1]	7600	.	.
[s5, b1]	7600	.	.

Here is our output:

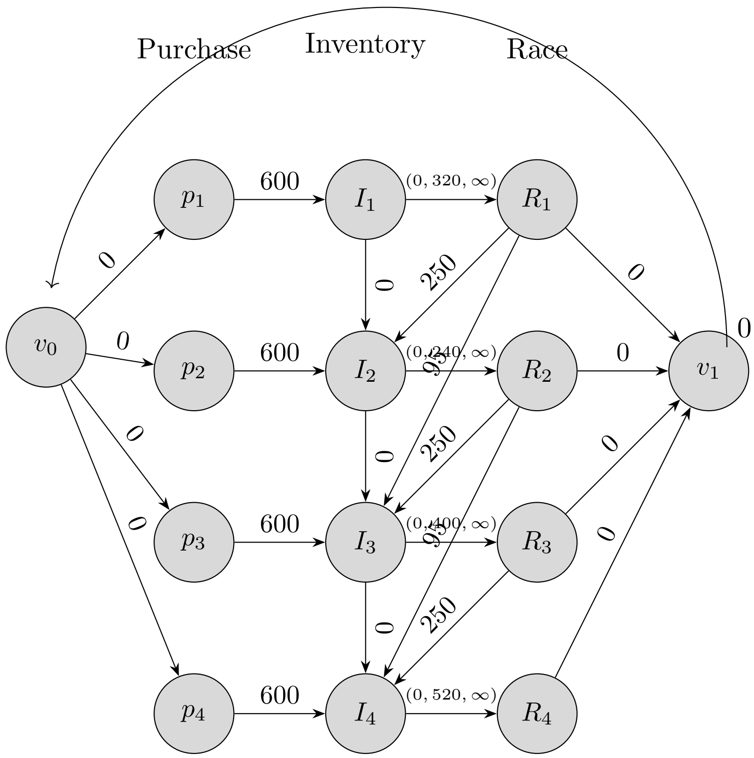


```
Console
AMPL
ampl: model group1_HW3_p2.mod
CPLEX 20.1.0.0: optimal solution; objective 3400
0 dual simplex iterations (0 in phase I)
x :=
m0 m1   1
m1 m2   0
m1 s1   1
m2 m3   0
m2 s2   0
m3 m4   0
m3 s3   0
m4 m5   0
m4 s4   0
m5 s5   0
s1 b1   1
s2 b1   0
s3 b1   0
s4 b1   0
s5 b1   0
;

ampl:
```

Buying a new grill every year seems to be our most cost-effective method, as evidenced by our AMPL output. It would cost us \$3400 each year to be a grill master.

3. Race Car Tires

Here is my flow model: All nodes have zero b costs are displayed on arcs. If minimums are needed, the ordered triple represents $(cost, lowerLimit, upperLimit)$. The v nodes are virtual to balance the flow.



Here is our model file:

```
# AMPL model for the Minimum Cost Network Flow Problem
#
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# l[i,j] = 0 and u[i,j] = Infinity.
#
# Parameters not specified in the data file will get their default values.
reset;

options solver cplex;

set NODES;                          # nodes in the network
set ARCS within {NODES, NODES};     # arcs in the network

param b {NODES} default 0;          # supply/demand for node i
param c {ARCS}  default 0;          # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;          # lower bound on flow on arc(i,j)
param u {ARCS}  default Infinity;   # upper bound on flow on arc(i,j)
```

```
var x {ARCS};                      # flow on arc (i,j)

minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j];  #objective: minimize arc flow cost

# Flow Out(i) - Flow In(i) = b(i)

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS} x[j,i] = b[i];

subject to capacity {(i,j) in ARCS}: l[i,j] <= x[i,j] <= u[i,j];

data group1_HW3_p3.dat;

solve;

display x;
```

Here is our data file:

```
#MCNFP Problem - data file for problem instance
#Charles Nicholson, ISE 5113, 2015

#use with MCNFP.txt model
#note: default arc costs and lower bounds are 0
#      default arc upper bounds are infinity
#      default node requirements are 0


set NODES :=          v0, p1, p2,p3,p4, i1,i2,i3,i4,r1,r2,r3,r4,v1 ;

set ARCS := (v0,p1),(v0,p2),(v0,p3),(v0,p4), #start the flow
                        (p1,i1),(p2,i2),(p3,i3),(p4,i4), #purchase new tires each race
                        (i1,r1),(i2,r2),(i3,r3),(i4,r4), #move inventory to race
                        (r1,v1),(r2,v1),(r3,v1),(r4,v1), #move spent tires not fixed to
                        (i1,i2),(i2,i3),(i3,i4), #move unused inventory
                        (r1,i2),(r1,i3), #race 1 quick and slow fix
                        (r2,i3),(r2,i4), #race 2 quick and slow fix
                        (r3,i4), #race 3 quick fix
                        (v1,v0) #move from virtual to virtual to complete flow
                        ;



param:        c  l u :=
```

```
                   [p1,i1] 600   . . #purchase new tires each race
                   [p2,i2] 600   . .
                   [p3,i3] 600   . .
                   [p4,i4] 600   . .
                   [i1,r1] .         320 . #minimum tires needed each race
                   [i2,r2] .         240 .
                   [i3,r3] .         400 .
                   [i4,r4] .         520 .
                   [r1,i2] 250 .          . #quick fix
                   [r2,i3] 250 .         .
                   [r3,i4] 250 .         .
                   [r1,i3] 95  .         .#slowfix
                   [r2,i4] 95        .         .
;
```

Here is our output:

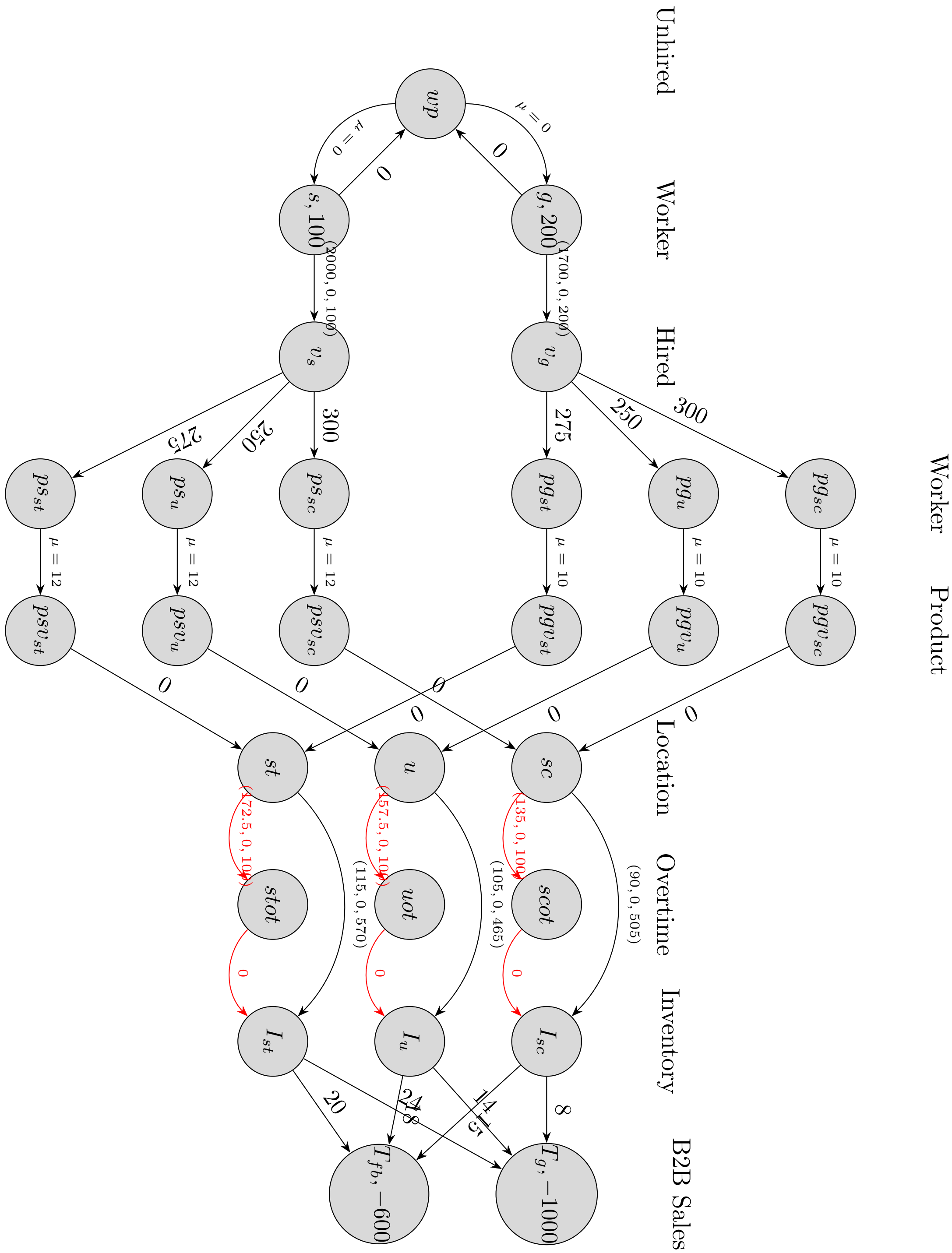
```
Console
AMPL
ampl: model group1_HW3_p3.mod
CPLEX 20.1.0.0: optimal solution; objective 490000
6 dual simplex iterations (0 in phase I)
x [*,*]
:    i1    i2    i3    i4    p1    p2   p3  p4   r1    r2    r3    r4    v0  :=
i1    .     0     .     .     .     .    .   .   320    .     .     .     .
i2    .     .     0     .     .     .    .   .    .    240    .     .     .
i3    .     .     .     0     .     .    .   .    .     .    400    .     .
i4    .     .     .     .     .     .    .   .    .     .     .    520    .
p1   320    .     .     .     .     .    .   .    .     .     .     .     .
p2    .    200    .     .     .     .    .   .    .     .     .     .     .
p3    .     .     0     .     .     .    .   .    .     .     .     .     .
p4    .     .     .     0     .     .    .   .    .     .     .     .     .
r1    .    40    280    .     .     .    .   .    .     .     .     .     .
r2    .     .    120   120    .     .    .   .    .     .     .     .     .
r3    .     .     .    400    .     .    .   .    .     .     .     .     .
v0    .     .     .     .    320   200   0   0    .     .     .     .     .
v1    .     .     .     .     .     .    .   .    .     .     .     .    520

:    v1    :=
r1     0
r2     0
r3     0
r4   520
;

ampl: |
```

We look to be purchasing new tires for both the needs of the first two races, 320 and 200 respectively. This is the maximum number of tires needed. We use the normal service on 280 tires from the first race and quick fix on the other 40. In the second race, we used the normal service on 120 but quick fix on 120. For the third race we quick fix all 400 tires used. We end up with exactly the number of tires needed in the fourth race. Total cost is $490 000.

4. Dunder Mifflin

   This problem presents by far our most complicated model. The network flow diagram is presented below and is described as follows: unhired workers will flow to the worker pool, *wp* and flow back out to the Workers with a weight of zero to maintain the flow. *g* and *s* represent the available generalist and specialists. If they are hired, they flow to the hired pool of each type. They are then transported to each plant at the respective cost. We then convert each worker into the number of products they create with the weight. Finally, we combine the number of possible products they can create in each location. Inventory is created with the regular cost but with limits on the maximums. Overtime is represented in red and uses a different virtual node for each location. Inventory is then shipped from each of the factories to the different businesses we serve, meeting the demand as represented inside the node. We attempted to make the flow circular, but due to the difference in weights based on employment categories, we could never get it to create a balanced flow.

   This model provides labor for overtime but does not account for a premium wage for the employee working outside of regular hours. We see this as an assumption that there is not a bump in pay for working second shift, but there is a 50% increase in our overhead costs of production.


Unhired
Worker
Hired
Worker
Product
Location
Overtime
Inventory
B2B Sales

Here is our model file:

```
# AMPL model for the Minimum Cost Network Flow Problem
#
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# l[i,j] = 0 and u[i,j] = Infinity.
#
# Parameters not specified in the data file will get their default values.
reset;

options solver cplex;

set NODES;                        # nodes in the network
set ARCS within {NODES, NODES};   # arcs in the network

param b {NODES} default 0;        # supply/demand for node i
param c {ARCS}  default 0;        # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;        # lower bound on flow on arc(i,j)
param u {ARCS}  default Infinity; # upper bound on flow on arc(i,j)
param mu {ARCS} default 1;       # multiplier on arc(i,j) -- if one unit leaves i, mu[i,j] units arrive

var x {ARCS};                     # flow on arc (i,j)

minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j];  #objective: minimize arc flow cost

# Flow Out(i) - Flow In(i) = b(i)

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS} mu[j,i] * x[j,i] = b[i];

subject to capacity {(i,j) in ARCS}: l[i,j] <= x[i,j] <= u[i,j];

data group1_HW3_p4.dat;

solve;

display x;
```

Here is our data file:

```
#MCNFP Problem - data file for problem instance
#Charles Nicholson, ISE 5113, 2015

#use with MCNFP.txt model
#note: default arc costs and lower bounds are 0
#      default arc upper bounds are infinity
#      default node requirements are 0


set NODES :=          #v0, v1, #virtual nodes at begining and end to get the flow going
                                g, s, #general and specialist
                                vg, vs, #virtual to get the cost of general and specialist
                                pgsc, pgu, pgst, pssc, psu, psst, #shipping cost of each employee
                                pgvsc, pgvu, pgvst, psvsc, psvu, psvst, #convert each employee to items
                                sc, u, st, #workers (as items) now at the plants
                                scot, uot, stot # overtime possible
                                isc, iu, ist, #inventory at each plant
                                tg, tfb, #transport goods to location
                                wp; #unhired worker pool

set ARCS := (s,vs),(g,vg), #hire the workers
                        (s,wp), (g,wp), #unhired workers
                        (wp,s), (wp,g), #flow the unhired workers back to keep the balance
                        (vg,pgsc),(vg,pgu),(vg,pgst),(vs,pssc),(vs,psu),(vs,psst), #move different workers to factories
                        (pgsc,pgvsc),(pgu,pgvu),(pgst,pgvst),(pssc,psvsc),(psu,psvu),(psst,psvst), #convert the workers into items
                        (pgvsc,sc),(psvsc,sc),(pgvu,u),(psvu,u),(pgvst,st),(psvst,st), #more production capacity to each factory
                        (sc,isc),(u,iu),(st,ist), #create the products
                        (sc, scot), (u, uot), (st,stot), #overtime hours making products
                        (scot, isc), (uot,iu), (stot, ist), #overtime products created go to inventory for free
                        (isc,tg), (isc,tfb), (iu, tg), (iu,tfb), (ist,tg), (ist,tfb), #move the product from inventory to customer
                        ;

param: b:=
        g 200
```

```
        s 100
        tg -1000
        tfb -600;

param:        c  l u mu:=
              [s,vs]                        2000          .         100         . #recruit workers
              [g,vg]                        1700          .         200         .
              [vg,pgsc]                300           .          .                 . #move workers to factories
              [vg,pgu]                250           .          .                 .
              [vg,pgst]               275           .          .                 .
              [vs,pssc]                300           .         .                 .
              [vs,psu]                250          .          .                 .
              [vs,psst]                275           .          .                 .
              [pgsc,pgvsc]         .                  .          .                 10 #convert workers to items
              [pgu,pgvu]                 .                  .          .                 10
              [pgst,pgvst]         .                  .          .                 10
              [pssc,psvsc]         .                  .          .                 12
              [psu,psvu]                 .                  .          .                 12
              [psst,psvst]         .                  .          .                 12
              [sc,isc]                90                   .         505         . #create the products
              [u,iu]                     105          .          465          .
              [st, ist]                115          .          570          .
              [sc, scot]                135          .          100          . #overtime possible
              [u,uot]                157.5          .          100          .
              [st,stot]                172.5          .          100          .
              [isc, tg]                8                   .          .                 . #move product to customer
              [isc, tfb]                15                   .          .                 . ################################check me
              [iu, tg]                14                   .          .                 .
              [iu, tfb]                18                   .          .                 .
              [ist, tg]                24                   .          .                 .
              [ist,tfb]                20                   .          .                 .
              [wp,s]                        .                  .          .                 0
              [wp,g]                        .                  .          .                 0
;
```

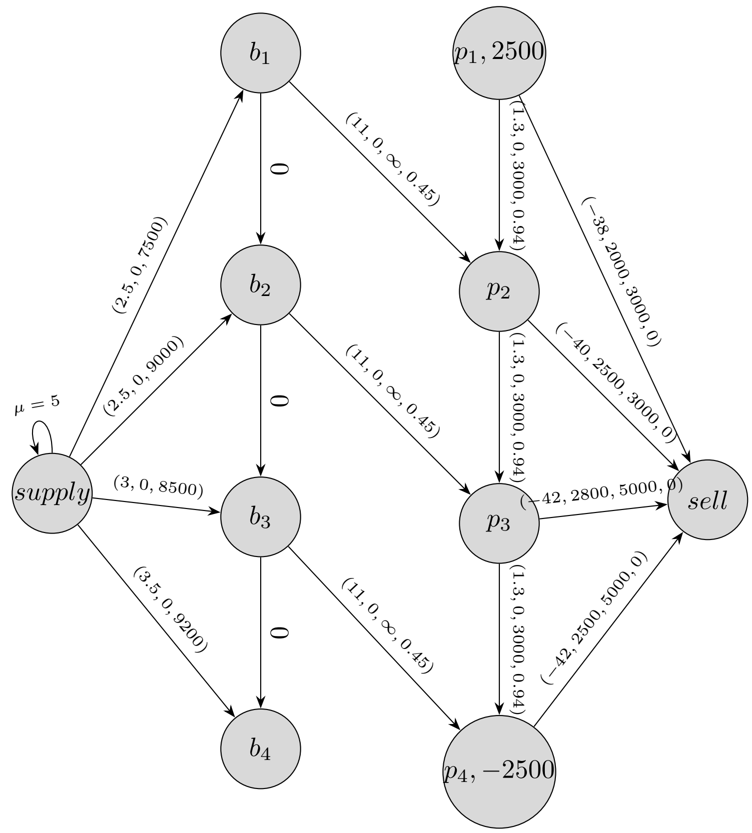
Here is my output:

```
Console
AMPL
ampl: model group1_HW3_p4.mod
CPLEX 20.1.0.0: optimal solution; objective 497016.6667
10 dual simplex iterations (0 in phase I)
x [*,*] (tr)
:        g    isc   ist   iu  pgsc pgst  pgu pgvsc pgvst  pgvu    pssc     psst :=
pgvsc    .     .     .     .    0    .    .    .     .     .       .        .
pgvst    .     .     .     .    .    0    .    .     .     .       .        .
pgvu     .     .     .     .    .    .   40    .     .     .       .        .
psvsc    .     .     .     .    .    .    .    .     .     .    47.0833     .
psvst    .     .     .     .    .    .    .    .     .     .       .      47.5
sc       .     .     .     .    .    .    .    0     .     .       .        .
st       .     .     .     .    .    .    .    .     0     .       .        .
tfb      .     0   570    30    .    .    .    .     .     .       .        .
tg       .   565     0   435    .    .    .    .     .     .       .        .
u        .     .     .     .    .    .    .    .     .    400      .        .
vg      40     .     .     .    .    .    .    .     .     .       .        .
wp     160     .     .     .    .    .    .    .     .     .       .        .

:        psu    psvsc psvst psvu    s    sc  scot   st  stot    u  uot   vg   :=
isc      .        .     .     .     .   505   60    .    .     .    .    .
ist      .        .     .     .     .    .     .   570   0     .    .    .
iu       .        .     .     .     .    .     .    .    .    465   0    .
pgsc     .        .     .     .     .    .     .    .    .     .    .    0
pgst     .        .     .     .     .    .     .    .    .     .    .    0
pgu      .        .     .     .     .    .     .    .    .     .    .   40
psvu   5.41667    .     .     .     .    .     .    .    .     .    .    .
sc       .      565     .     .     .    .     .    .    .     .    .    .
scot     .        .     .     .     .   60     .    .    .     .    .    .
st       .        .   570     .     .    .     .    .    .     .    .    .
stot     .        .     .     .     .    .     .    0    .     .    .    .
u        .        .     .    65     .    .     .    .    .     .    .    .
uot      .        .     .     .     .    .     .    .    .     0    .    .
vs       .        .     .     .   100    .     .    .    .     .    .    .
wp       .        .     .     .     0    .     .    .    .     .    .    .

:         vs      wp     :=
g         .        0
pssc   47.0833     .
psst   47.5        .
psu     5.41667    .
s         .      160
;
```

Solution is non-integer in employees which is unfortunate but can be the case with the generalized network flow problems. All 100 specialists are hired, 40 generalists. Scranton does use 60 products of OT, none of the other plants do but they max out production at each location.

5. Mud b Gone

   Our network flow diagram for this problem is below. This model has the coolest feature of the infinite supply house. We used a multiplicative factor to create new product out of thin air. We also reduce the product to 0 weight as we send it to the sell node to make the flow balance. The weights on p_1 and p_4 were because of the statement in the problem about starting with product. b_i is connected with p_{i+1} due to the base not being immediately ready for production. We also have several μ values for creating the product and the product going bad as it sits.



Here is our model file:

```
# AMPL model for the Minimum Cost Network Flow Problem
#
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# l[i,j] = 0 and u[i,j] = Infinity.
#
# Parameters not specified in the data file will get their default values.
reset;

options solver cplex;
option cplex_options 'sensitivity';

set NODES;                        # nodes in the network
set ARCS within {NODES, NODES};   # arcs in the network

param b {NODES} default 0;        # supply/demand for node i
param c {ARCS}  default 0;        # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;        # lower bound on flow on arc(i,j)
param u {ARCS}  default Infinity; # upper bound on flow on arc(i,j)
param mu {ARCS} default 1;       # multiplier on arc(i,j) -- if one unit leaves i, mu[i,j] units arrive

var x {ARCS};                     # flow on arc (i,j)

minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j];  #objective: minimize arc flow cost

# Flow Out(i) - Flow In(i) = b(i)

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS} mu[j,i] * x[j,i] = b[i];
```

```
subject to upcapacity {(i,j) in ARCS}: x[i,j] <= u[i,j];

subject to lowcapacity {(i,j) in ARCS}: l[i,j] <= x[i,j];

data group1_HW3_p5.dat;

solve;

display x;

display upcapacity,upcapacity.up, upcapacity.down;

display x.current, x.up, x.down;
```

Here is our data file:

```
#MCNFP Problem - data file for problem instance
#Charles Nicholson, ISE 5113, 2015

#use with MCNFP.txt model
#note: default arc costs and lower bounds are 0
#      default arc upper bounds are infinity
#      default node requirements are 0


set NODES :=  supply, #suppliers
                          b1,b2,b3,b4, #base for production
                          p1, p2, p3, p4, #product
                          sold #sold product
;

set ARCS := (supply,*) b1 b2 b3 b4 #base purchased from supplier
                        (*,sold)   p1 p2 p3 p4 #product sold
                        (b1,p2),(b2,p3),(b3,p4), #base converted to product
                        (b1,b2),(b2,b3),(b3,b4),
                        (p1,p2),(p2,p3),(p3,p4)
                        (supply,supply)
                        ;

param: b:=
        p1 2500
        p4 -2500;

param:        c  l u mu:=
                [supply,b1] 2.5        .        7500        . #buy new base
                [supply,b2] 2.5 .         9000        .
                [supply, b3] 3 .         8500        .
                [supply,b4]        3.5        .        9200        .

                [p1,sold] -38 2000        3000        0 #sell product
                [p2,sold] -40 2500        3000        0
                [p3,sold] -42 2800        5000        0
                [p4,sold] -42 2500        5000        0

                [p1,p2] 1.3        .        3000        .94 #store product till next month
                [p2,p3] 1.3        .        3000        .94
                [p3,p4] 1.3        .        3000        .94

                [b1,p2] 11        .        .        .45 #convert base into product.  Assumption not too worry about max storage
                [b2,p3] 11        .        .        .45
                [b3,p4] 11        .        .        .45

                [supply,supply] .        .        .        5
;
```

(a) We see a solution for our flow. We sell 2000, 2500, 4220 and 2500 in each of the respective periods. We remark on the infinite supply house obtained by creating a loop with a μ factor set to 5 but could be any value greater than 1.

```
Console
AMPL
ampl: model group1_HW3_p5.mod
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective -115840
4 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
x :=
b1      b2        2988.89
b1      p2        4511.11
b2      b3        2611.11
b2      p3        9377.78
b3      b4           0
b3      p4       11111.1
p1      p2         500
p1      sold      2000
p2      p3           0
p2      sold      2500
p3      p4           0
p3      sold      4220
p4      sold      2500
supply b1         7500
supply b2         9000
supply b3         8500
supply b4            0
supply supply     6250
;
```

(b) We next look at the sensitivity of the capacity. We see a value of -$5.40 in the report, supply to b2. We interpret this as having additional gallons of base will increase (recall minimizing) our income by $5.40. Both the up and the down are reported as 0. We are unsure of how to interpret this as we clearly would use more of the base if it was available but are unsure why these values are all zero. We even broke apart the upper and lower limit thinking this was the issue but did not change the up and down on the shadow price. We do clearly see this as the bottle neck in our model. We also note that when we moved the cap of 9000 to 9001 we earned another $5.40. We wonder if since the number is negative, we are getting 0 as an upper limit.

```
:               upcapacity upcapacity.up upcapacity.down    :=
b1      b2          0            0               0
b1      p2          0            0               0
b2      b3          0            0               0
b2      p3          0            0               0
b3      b4          0            0               0
b3      p4          0            0               0
p1      p2          0            0               0
p1      sold        0            0               0
p2      p3          0            0               0
p2      sold        0            0               0
p3      p4          0            0               0
p3      sold        0            0               0
p4      sold        0            0               0
supply b1        -5.4            0               0
supply b2        -5.4            0               0
supply b3        -4.9            0               0
supply b4           0            0               0
supply supply       0            0               0
;
```

(c) We see that the contribution to the cost is -\$40 on the flow from p2 to Sold. This value can be modified up to -\$42 and not change the model at all. This means we could raise the price and earn more for our company keeping everything else the same.

```
:               x.current       x.up          x.down          :=
b1      b2          0         0.0861702    -1e+20
b1      p2         11      1e+20             10.9138
b2      b3          0         1.82872        -3.19744e-15
b2      p3         11        11               9.17128
b3      b4          0         0               0
b3      p4         11        12.8287         11
p1      p2          1.3       1.48         -1e+20
p1      sold      -38      1e+20            -38.18
p2      p3          1.3    1e+20             -2.52
p2      sold      -40      1e+20            -42
p3      p4          1.3    1e+20             -2.52
p3      sold      -42        -42           -1e+20
p4      sold      -42      1e+20            -42
supply b1           2.5       7.9          -1e+20
supply b2           2.5       7.9          -1e+20
supply b3           3         7.9          -1e+20
supply b4           0         0               0
supply supply       0        19.6          -1e+20
;
```