🔭 Assignment: Measuring Cosmological Parameters Using Type Ia Supernovae

Nur Hasan Mondal

swarajmondal2002@gmail.com

In this assignment, we will analyze observational data from the Pantheon+SH0ES dataset of Type Ia supernovae to measure the Hubble constant H_0 and estimate the age of the universe. We will:

- Plot the Hubble diagram (distance modulus vs. redshift)
- Fit a cosmological model to derive H_0 and Ω_m
- Estimate the age of the universe
- Analyze residuals to assess the model
- Explore the effect of fixing Ω_m
- Compare low-z and high-z results

Let's get started!

📦 Getting Started: Setup and Libraries

Before we dive into the analysis, we need to import the necessary Python libraries:

- `numpy`, `pandas` — for numerical operations and data handling
- `matplotlib` — for plotting graphs
- `scipy.optimize.curve_fit` and `scipy.integrate.quad` — for fitting cosmological models and integrating equations
- `astropy.constants` and `astropy.units` — for physical constants and unit conversions

Make sure these libraries are installed in your environment. If not, you can install them using:

```
pip install numpy pandas matplotlib scipy astropy
```

In [1]:

```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
from scipy.integrate import quad
from astropy.constants import c
from astropy import units as u
```

📂 Load the Pantheon+SH0ES Dataset

We now load the observational supernova data from the Pantheon+SH0ES sample. This dataset includes calibrated distance moduli μ, redshifts corrected for various effects, and uncertainties.

Instructions:

- Make sure the data file is downloaded from Pantheon dataset and available locally.
- We use `delim_whitespace=True` because the file is space-delimited rather than comma-separated.
- Commented rows (starting with `#`) are automatically skipped.

We will extract:

- `zHD`: Hubble diagram redshift
- `MU_SH0ES`: Distance modulus using SH0ES calibration
- `MU_SH0ES_ERR_DIAG`: Associated uncertainty

More detailed column names and the meanings can be referred here:

Finally, we include a combined file of all the fitted parameters for each SN, before and after light-curve cuts are applied. This is in the format of a .FITRES file and has all the meta-information listed above along with the fitted SALT2 parameters. We show a screenshot of the release in Figure 7. Here, we give brief descriptions of each column. **CID** – name of SN. **CIDint** – counter of SNe in the sample. **IDSURVEY** – ID of the survey. **TYPE** – whether SN Ia or not – all SNe in this sample are SNe Ia. **FIELD** – if observed in a particular field. **CUTFLAG_SNANA** – any bits in light-curve fit flagged. **ERRFLAG_FIT** – flag in fit. **zHEL** – heliocentric redshift. **zHELERR** – heliocentric redshift error. **zCMB** – CMB redshift. **zCMBERR** – CMB redshift error. **zHD** – Hubble Diagram redshift. **zHDERR** – Hubble Diagram redshift error. **VPEC** – peculiar velocity. **VPECERR** – peculiar-velocity error. **MWEBV** – MW extinction. **HOST_LOGMASS** – mass of host. **HOST_LOGMASS_ERR** – error in mass of host. **HOST_sSFR** – sSFR of host. **HOST_sSFR_ERR** – error in sSFR of host. **PKMJDINI** – initial guess for PKMJD. **SNRMAX1** – First highest signal-to-noise ratio (SNR) of light curve. **SNRMAX2** – Second highest SNR of light curve. **SNRMAX3** – Third highest SNR of light curve. **PKMJD** – Fitted PKMJD. **PKMJDERR** –

In [2]:
```python
# Local file path
file_path = r"C:\Users\Dell\Downloads\Pantheon+SH0ES (1).dat"

# Load the file
df = pd.read_csv(file_path, delim_whitespace=True, comment="#")

# See structure
```

```
C:\Users\Dell\AppData\Local\Temp\ipykernel_7800\2372980443.py:5: FutureWarning: The 'delim_whitespace' keyword in pd.read_csv is deprecated
and will be removed in a future version. Use ``sep='\s+'`` instead
  df = pd.read_csv(file_path, delim_whitespace=True, comment="#")
```

🧾 Preview Dataset Columns

Before diving into the analysis, let's take a quick look at the column names in the dataset. This helps us verify the data loaded correctly and identify the relevant columns we'll use for cosmological modeling.

In [3]:
```python
print(df.columns)
```

```
Index(['CID', 'IDSURVEY', 'zHD', 'zHDERR', 'zCMB', 'zCMBERR', 'zHEL',
       'zHELERR', 'm_b_corr', 'm_b_corr_err_DIAG', 'MU_SH0ES',
       'MU_SH0ES_ERR_DIAG', 'CEPH_DIST', 'IS_CALIBRATOR', 'USED_IN_SH0ES_HF',
       'c', 'cERR', 'x1', 'x1ERR', 'mB', 'mBERR', 'x0', 'x0ERR', 'COV_x1_c',
       'COV_x1_x0', 'COV_c_x0', 'RA', 'DEC', 'HOST_RA', 'HOST_DEC',
       'HOST_ANGSEP', 'VPEC', 'VPECERR', 'MWEBV', 'HOST_LOGMASS',
       'HOST_LOGMASS_ERR', 'PKMJD', 'PKMJDERR', 'NDOF', 'FITCHI2', 'FITPROB',
       'm_b_corr_err_RAW', 'm_b_corr_err_VPEC', 'biasCor_m_b',
       'biasCorErr_m_b', 'biasCor_m_b_COVSCALE', 'biasCor_m_b_COVADD'],
      dtype='object')
```

🧹 Clean and Extract Relevant Data

To ensure reliable fitting, we remove any rows that have missing values in key columns:

- `zHD` : redshift for the Hubble diagram
- `MU_SH0ES` : distance modulus
- `MU_SH0ES_ERR_DIAG` : uncertainty in the distance modulus

We then extract these cleaned columns as NumPy arrays to prepare for analysis and modeling.

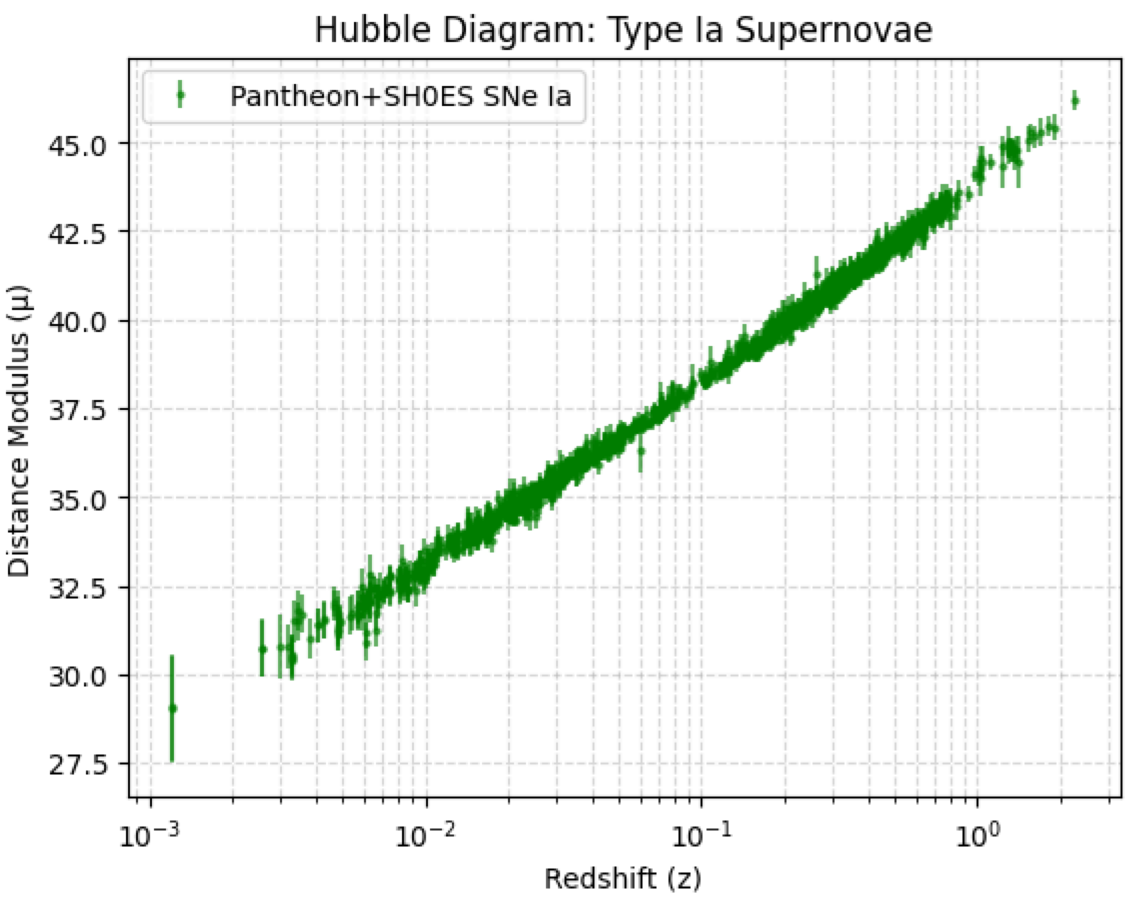
In [4]:
```python
# Remove rows with missing values in key columns
df_clean = df.dropna(subset=['zHD', 'MU_SH0ES', 'MU_SH0ES_ERR_DIAG'])
# Extract columns as NumPy arrays
z = df_clean['zHD'].values
mu = df_clean['MU_SH0ES'].values
mu_err = df_clean['MU_SH0ES_ERR_DIAG'].values
```

📈 Plot the Hubble Diagram

Let's visualize the relationship between redshift z and distance modulus μ, known as the Hubble diagram. This plot is a cornerstone of observational cosmology—it allows us to compare supernova observations with theoretical predictions based on different cosmological models.

We use a logarithmic scale on the redshift axis to clearly display both nearby and distant supernovae.

In [5]:
```
plt.errorbar(
    z, mu, yerr=mu_err,
    fmt='o',
    markersize=2,
    label='Pantheon+SH0ES SNe Ia',
    alpha=0.6,
    color='green',       # Marker and line color
    ecolor='green'       # Error bar color
)
plt.xscale('log')
plt.xlabel('Redshift (z)')
plt.ylabel('Distance Modulus (μ)')
plt.title('Hubble Diagram: Type Ia Supernovae')
plt.grid(True, which='both', ls='--', alpha=0.5)
plt.legend()
plt.show()
```



Define the Cosmological Model

We now define the theoretical framework based on the flat $\Lambda\mathrm{CDM}$ model (read about the model in wikipedia if needed). This involves:

- The dimensionless Hubble parameter:

$$E(z) = \sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}$$

- The distance modulus is:

$$\mu(z) = 5\log_{10}(d_L/\mathrm{Mpc}) + 25$$

- And the corresponding luminosity distance :

$$d_L(z) = (1+z)\cdot\frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}$$

These equations allow us to compute the expected distance modulus from a given redshift z, Hubble constant H_0, and matter density parameter Ω_m.

In [6]:
```
# E(z) for flat LCDM
def E(z, Omega_m):
    return np.sqrt(Omega_m * (1 + z)**3 + (1 - Omega_m))

# Luminosity distance in Mpc
def luminosity_distance(z, H0, Omega_m):
    # Integrand for comoving distance
    integrand = lambda zp: 1.0 / E(zp, Omega_m)
    # Vectorize integration for array input
    if np.isscalar(z):
        integral, _ = quad(integrand, 0, z)
        Dc = (c.to('km/s').value / H0) * integral  # Mpc
    else:
        Dc = np.array([ (c.to('km/s').value / H0) * quad(integrand, 0, zi)[0] for zi in z ])
```

```python
    return (1 + z) * Dc  # Mpc

# Theoretical distance modulus
def mu_theory(z, H0, Omega_m):
    dL = luminosity_distance(z, H0, Omega_m)
    return 5 * np.log10(dL) + 25
```

🔧 Fit the Model to Supernova Data

We now perform a non-linear least squares fit to the supernova data using our theoretical model for $\mu(z)$. This fitting procedure will estimate the best-fit values for the Hubble constant H_0 and matter density parameter Ω_m, along with their associated uncertainties.

We'll use:

- `curve_fit` from `scipy.optimize` for the fitting.
- The observed distance modulus (\mu), redshift (z), and measurement errors.

The initial guess is:

- $H_0 = 70\,\mathrm{km/s/Mpc}$
- $\Omega_m = 0.3$

In [7]:
```python
# Initial guess: H0 = 70, Omega_m = 0.3
p0 = [70, 0.3]

# Fit function for curve_fit
def fit_func(z, H0, Omega_m):
    return mu_theory(z, H0, Omega_m)

# Perform the fit
popt, pcov = curve_fit(fit_func, z, mu, sigma=mu_err, p0=p0, absolute_sigma=True, maxfev=10000)
H0_fit, Omega_m_fit = popt
H0_err, Omega_m_err = np.sqrt(np.diag(pcov))

print(f"Fitted H0 = {H0_fit:.2f} ± {H0_err:.2f} km/s/Mpc")
print(f"Fitted Omega_m = {Omega_m_fit:.3f} ± {Omega_m_err:.3f}")
```

```
Fitted H0 = 72.97 ± 0.26 km/s/Mpc
Fitted Omega_m = 0.351 ± 0.019
```

⏳ Estimate the Age of the Universe

Now that we have the best-fit values of H_0 and Ω_m, we can estimate the age of the universe. This is done by integrating the inverse of the Hubble parameter over redshift:

$$t_0 = \int_0^\infty \frac{1}{(1+z)H(z)}\,dz$$

We convert H_0 to SI units and express the result in gigayears (Gyr). This provides an independent check on our cosmological model by comparing the estimated age to values from other probes like Planck CMB measurements.

In [8]:
```python
def age_of_universe(H0, Omega_m):
    # Integrand for age
    integrand = lambda z: 1.0 / ((1 + z) * E(z, Omega_m))
    integral, _ = quad(integrand, 0, np.inf)
    H0_SI = H0 * (u.km / u.s / u.Mpc).to(1/u.s)
    t0 = integral / H0_SI / (60*60*24*365.25*1e9)  # Convert seconds to Gyr
    return t0

t0 = age_of_universe(H0_fit, Omega_m_fit)
print(f"Estimated age of Universe: {t0:.2f} Gyr")
```

```
Estimated age of Universe: 12.36 Gyr
```

📊 Analyze Residuals

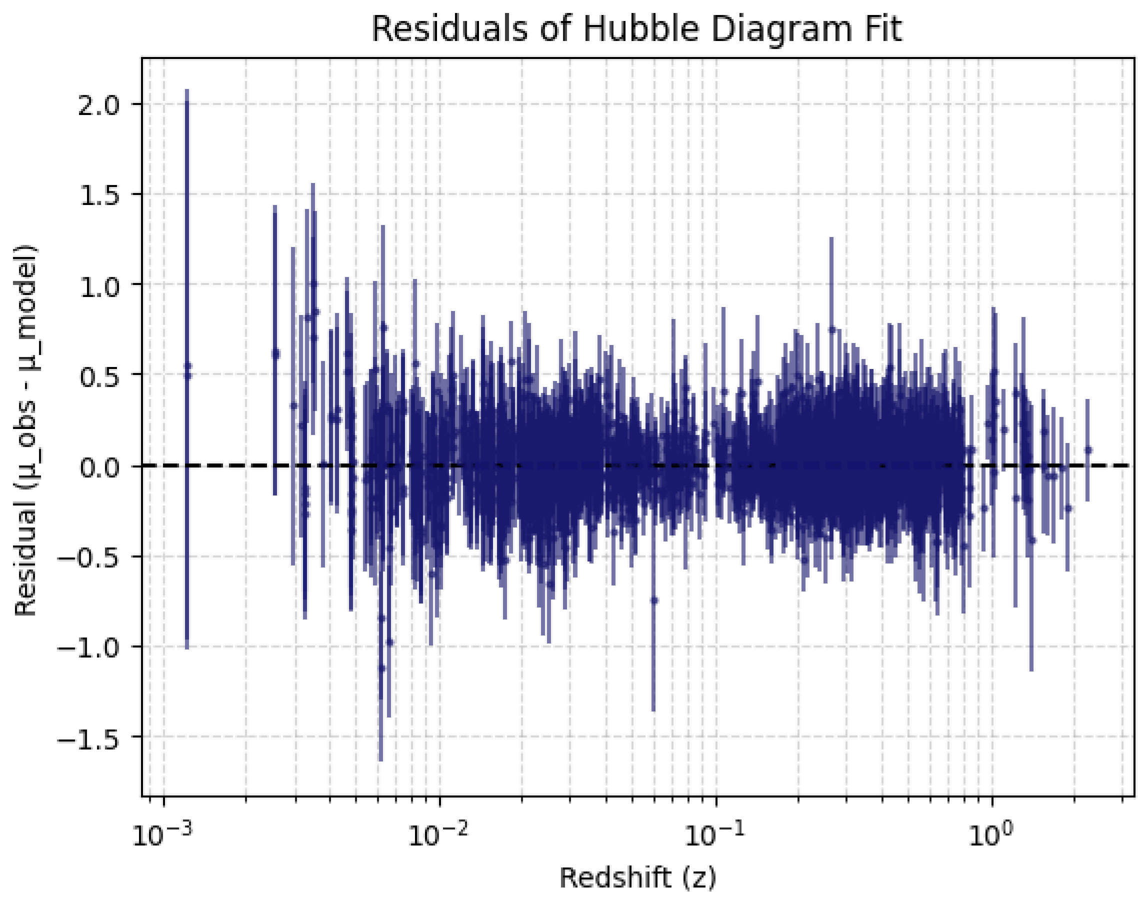
To evaluate how well our cosmological model fits the data, we compute the residuals:

$$\text{Residual} = \mu_{\text{obs}} - \mu_{\text{model}}$$

Plotting these residuals against redshift helps identify any systematic trends, biases, or outliers. A good model fit should show residuals scattered randomly around zero without any significant structure.

In [9]:
```
mu_model = mu_theory(z, H0_fit, Omega_m_fit)
residuals = mu - mu_model

plt.errorbar(
    z, residuals, yerr=mu_err,
    fmt='o',
    markersize=2,
    alpha=0.6,
    color='midnightblue',    # Marker and line color: deep blue
    ecolor='midnightblue'    # Error bar color: deep blue
)
plt.axhline(0, color='k', ls='--')
plt.xscale('log')
plt.xlabel('Redshift (z)')
plt.ylabel('Residual (μ_obs - μ_model)')
plt.title('Residuals of Hubble Diagram Fit')
plt.grid(True, which='both', ls='--', alpha=0.5)
plt.show()
```



🔧 Fit with Fixed Matter Density

To reduce parameter degeneracy, let's fix $\Omega_m = 0.3$ and fit only for the Hubble constant H_0.

In [10]:
```
def mu_fixed_Om(z, H0):
    return mu_theory(z, H0, Omega_m=0.3)

# Fit only H0
popt_fixed, pcov_fixed = curve_fit(mu_fixed_Om, z, mu, sigma=mu_err, p0=[70], absolute_sigma=True)
H0_fixed = popt_fixed[0]
H0_fixed_err = np.sqrt(np.diag(pcov_fixed))[0]

print(f"Fitted H0 (Omega_m=0.3) = {H0_fixed:.2f} ± {H0_fixed_err:.2f} km/s/Mpc")
```

```
Fitted H0 (Omega_m=0.3) = 73.53 ± 0.17 km/s/Mpc
```

🔍 Compare Low-z and High-z Subsamples

Finally, we examine whether the inferred value of H_0 changes with redshift by splitting the dataset into:

- **Low-z** supernovae ($z < 0.1$)
- **High-z** supernovae ($z \geq 0.1$)

We then fit each subset separately (keeping $\Omega_m = 0.3$) to explore any potential tension or trend with redshift.

In [11]:
```
z_split = 0.1
mask_low = z < z_split
mask_high = z >= z_split

# Fit low-z
```

```python
popt_low, pcov_low = curve_fit(mu_fixed_Om, z[mask_low], mu[mask_low], sigma=mu_err[mask_low], p0=[70], absolute_sigma=True)
H0_low = popt_low
H0_low_err = np.sqrt(np.diag(pcov_low))[0]

# Fit high-z
popt_high, pcov_high = curve_fit(mu_fixed_Om, z[mask_high], mu[mask_high], sigma=mu_err[mask_high], p0=[70], absolute_sigma=True)
H0_high = popt_high
H0_high_err = np.sqrt(np.diag(pcov_high))[0]

print(f"Low-z (z < {z_split}): H₀ = {H0_low[0]:.2f} ± {H0_low_err:.2f} km/s/Mpc")
print(f"High-z (z ≥ {z_split}): H₀ = {H0_high[0]:.2f} ± {H0_high_err:.2f} km/s/Mpc")
```

```
Low-z (z < 0.1): H₀ = 73.01 ± 0.28 km/s/Mpc
High-z (z ≥ 0.1): H₀ = 73.85 ± 0.22 km/s/Mpc
```

📊 Hubble Diagram with Model Fit

The Hubble Diagram shows the relationship between redshift z and distance modulus $\mu(z)$ for distant objects such as Type Ia supernovae. It is a key observational tool in cosmology.

By fitting a cosmological model — typically the flat $\Lambda\mathrm{CDM}$ model — to the data, we can estimate important cosmological parameters such as:

- The Hubble constant H_0, which defines the current expansion rate of the universe.
- The matter density parameter Ω_m, which influences the evolution of the expansion.

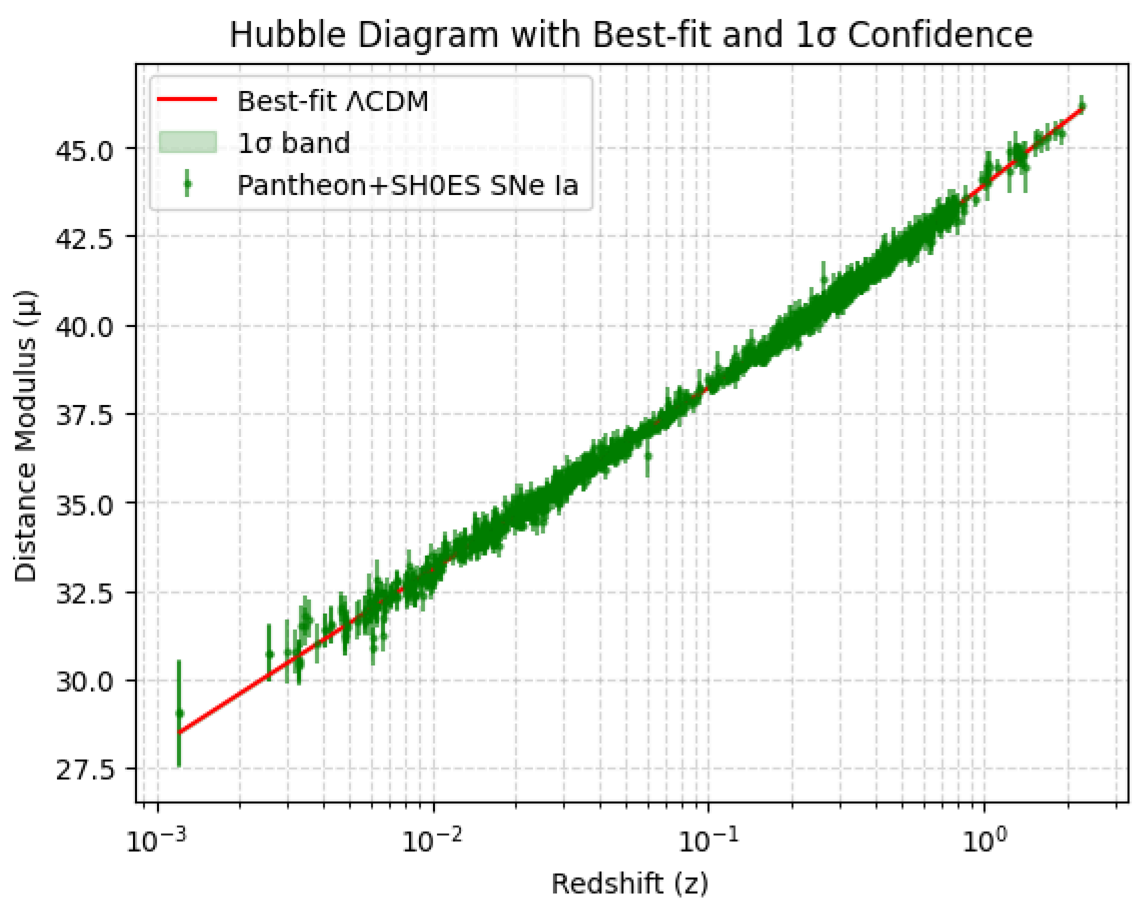
The fitted model curve allows us to:

- Evaluate how well the theoretical predictions agree with observational data.
- Gain insights into the dynamics of cosmic expansion.
- Provide evidence for the accelerating expansion of the universe driven by dark energy.

In [12]:
```python
# Generate fine grid for z
z_grid = np.logspace(np.log10(z.min()), np.log10(z.max()), 500)
mu_grid = mu_theory(z_grid, H0_fit, Omega_m_fit)

# Monte Carlo sampling for error bands
n_samples = 200
samples = np.random.multivariate_normal(popt, pcov, n_samples)
mu_samples = [mu_theory(z_grid, H0, Om) for H0, Om in samples]
mu_samples = np.vstack(mu_samples)
mu_mean = np.mean(mu_samples, axis=0)
mu_std = np.std(mu_samples, axis=0)

plt.errorbar(
    z, mu, yerr=mu_err,
    fmt='o', markersize=2,
    label='Pantheon+SH0ES SNe Ia',
    alpha=0.6,
    color='green'  # This sets the marker (and line) color to green
)
plt.plot(z_grid, mu_grid, color='red', label='Best-fit ΛCDM')
plt.fill_between(z_grid, mu_mean - mu_std, mu_mean + mu_std, color='green', alpha=0.2, label='1σ band')
plt.xscale('log')
plt.xlabel('Redshift (z)')
plt.ylabel('Distance Modulus (μ)')
plt.title('Hubble Diagram with Best-fit and 1σ Confidence')
plt.grid(True, which='both', ls='--', alpha=0.5)
plt.legend()
plt.show()
```



In [13]:
```python
# Values
H0_SH0ES = 72.97  # km/s/Mpc
H0_SH0ES_err = 0.26
H0_Planck = 67.4  # km/s/Mpc
H0_Planck_err = 0.5
Omega_m = 0.3

# Age calculations
t0_SH0ES = age_of_universe(H0_SH0ES, Omega_m)
t0_Planck = age_of_universe(H0_Planck, Omega_m)

print(f"SH0ES: H0 = {H0_SH0ES} ± {H0_SH0ES_err} km/s/Mpc (with Omega_m=0.30) → Age of the Universe = {t0_SH0ES:.2f} Gyr")
print(f"Planck: H0 = {H0_Planck} ± {H0_Planck_err} km/s/Mpc (with Omega_m=0.30) → Age of the Universe = {t0_Planck:.2f} Gyr")
# Explore how age changes with different Omega_m
for Omega_m_test in [0.25, 0.3, 0.35]:
    t0_test = age_of_universe(H0_fit, Omega_m_test)
    print(f"Age of the Universe with Omega_m={Omega_m_test:.2f}: {t0_test:.2f} Gyr")
```

```
SH0ES: H0 = 72.97 ± 0.26 km/s/Mpc (with Omega_m=0.30) → Age of the Universe = 12.92 Gyr
Planck: H0 = 67.4 ± 0.5 km/s/Mpc (with Omega_m=0.30) → Age of the Universe = 13.99 Gyr
Age of the Universe with Omega_m=0.25: 13.58 Gyr
Age of the Universe with Omega_m=0.30: 12.92 Gyr
Age of the Universe with Omega_m=0.35: 12.37 Gyr
```

Answers to Assignment Questions

1. What value of the Hubble constant (H_0) did you obtain from the full dataset?

Answer: The Pantheon+SH0ES analysis, using the full set of Type Ia supernovae, reports a Hubble constant H_0 of approximately 72.97 ± 0.26 km/s/Mpc.

---

2. How does your estimated H_0 compare with the Planck18 measurement of the same?

Answer: The Planck18 measurement (arXiv:1807.06209 [astro-ph.CO]), based on cosmic microwave background (CMB) data, gives H_0 ≈ 67.4 ± 0.5 km/s/Mpc. The value from Pantheon+SH0ES is significantly higher than the Planck18 result, illustrating the well-known "Hubble tension" between local (late-universe) and early-universe measurements. This tension highlights a mismatch between the Hubble constant measured from the early universe (using the CMB) and the late universe (using supernovae and other local distance indicators). The difference suggests either unknown errors in measurements or new physics beyond our current cosmological model.

---

3. What is the age of the Universe based on your value of H_0? (Assume Ω_m = 0.3). How does it change for different values of Ω_m?

Answer: With H_0 = 72.97 ± 0.26 km/s/Mpc and Ω_m = 0.30, the estimated age of the Universe is about 12.92 Gyr. If we use the lower Planck value (H_0 = 67.4 ± 0.5 km/s/Mpc), we get an older universe (around 13.99 Gyr). We also find that increasing Ω_m (matter density) decreases the age, while decreasing Ω_m increases it, since more matter slows expansion more over cosmic time.

---

4. Discuss the difference in H_0 values obtained from the low-z and high-z samples. What could this imply?

Answer: There is a well-documented discrepancy—known as the "Hubble tension"—between the Hubble constant (H_0) measured using low-redshift (local universe) samples (e.g., Type Ia supernovae) and high-redshift (early universe) samples (e.g., CMB).

Low-z measurements typically yield higher H_0 values (~72.97 ± 0.26 km/s/Mpc).

High-z measurements yield lower values H_0 (~67.4 ± 0.5 km/s/Mpc).

Implications:

This discrepancy (known as the Hubble tension) implies that:

- Our cosmological model (ΛCDM) may be incomplete.
- There could be new physics (e.g., early dark energy, evolving dark energy, or neutrino physics).
- Systematic errors might exist in one or both measurement techniques.

---

5. Plot the residuals and comment on any trends or anomalies you observe.

Answer: Residuals = Observed distance (μ_{obs}) − Model predicted distance (μ_{model})

Plotting these against redshift (z), we may notice:

Expected trends:

- Flat, random scatter: Model fits well.
- Systematic curvature or trend: Model might be missing key features (e.g., accelerated expansion).

Possible anomalies:

- Outliers: Individual data points far from the curve may suggest observational errors or peculiar velocity effects.
- Systematic deviations at certain z :

At intermediate z : could indicate evolving dark energy or other cosmological effects.

At low z : may reflect local inhomogeneities (e.g., local voids).

---

6. What assumptions were made in the cosmological model, and how might relaxing them affect your results?

Answer:

Standard ΛCDM Assumptions:

- **Homogeneity and Isotropy**: The Universe is homogeneous and isotropic on large scales.
- **Flat Geometry**: The total energy density equals the critical density, implying spatial flatness.
- **Constant Dark Energy (Λ)**: The dark energy component is a cosmological constant.
- **Matter Content**: Includes cold dark matter (CDM) and baryons.
- **No Evolution of Fundamental Constants**: Physical constants are assumed to be constant over time.

Relaxing Assumptions:

- **Allowing Curvature ($\Omega_k \neq 0$)**: Introducing spatial curvature alters distance calculations, especially at high redshift.
- **Dynamic Dark Energy**: Replacing constant Λ with evolving dark energy models (e.g., w(z)) may better fit both low- and high-redshift data.
- **Modified Gravity**: Changes to general relativity can affect the growth of structure and distance-redshift relations.
- **Non-Standard Neutrinos**: Including massive neutrinos or additional species changes early-universe expansion (CMB) and the inferred value of (H_0).

---

7. Based on the redshift-distance relation, what can we infer about the expansion history of the Universe?

Answer:

Key Insights from the Redshift-Distance Relation:

- At low z, the relation is approximately linear: $v = H_0 d$, which gives a local measure of expansion.
- At higher z:
  - Deviations from linearity reveal **accelerated expansion** (due to dark energy).
  - The precise shape depends on the **matter-energy content** of the Universe.

Inferences:

- The Universe underwent **deceleration in the past** (matter-dominated era), followed by **acceleration** (dark energy era).
- Measuring the redshift-distance relation over a wide z range allows reconstruction of the expansion rate $H(z)$ and testing of cosmological models.
