Binomial Trees on a Risk-Neutral Word

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d}, \quad p^* = \frac{1}{1 + e^{\sigma\sqrt{h}}}$$

$$V_{i,j} = e^{-rh}\left(p^*(\text{payoff if } u) + (1-p^*)\text{payoff if } d\right).$$

$$V_0 = e^{-rT}\sum \binom{n}{k}(p^*)^k(1-p^*)^{n-k}(\text{payoff})$$

$$C(K,T) - P(K,T) = S_0 e^{-\delta T} - K e^{-rT}$$

$$\text{An increase is: } e^{\delta h}\Delta S_u + e^{rh}B = C_u$$

$$\text{A decrease is: } e^{\delta h}\Delta S_d + e^{rh}B = C_d$$

$$\Delta = e^{-\delta h}\tfrac{C_u - C_d}{S_u - S_d}$$

$$B = e^{-rh}(C_u - e^{\delta h}\Delta S_u)$$

$$C_0 = \Delta S_0 + B$$

Let h be the time step $h = \frac{T}{n}$, if the Time is half a year and 4 steps, then $h = \frac{1/2}{4} = \frac{1}{8}$.
Let v be log-return over one time step h.
Let σ be the annualized volatility. $\sigma^2 = \frac{v^2}{h}$ remember need of: $d \leq e^{(r-\delta)h} \leq u$ Var de Ber is $p(1-p)$. Hence, for $u = d = 0.5$, the variance:

$$v^2 = 0.25(\ln u - \ln d)^2 \Rightarrow u = de^{2\sigma\sqrt{h}}$$

$$u = e^{(r-\delta)h + \sigma\sqrt{h}}, \quad d = e^{(r-\delta)h - \sigma\sqrt{h}}$$