# Regiones admisibles para los procesos MA(1) y MA(2)

martes, 8 de febrero de 2022 01:44 p. m.

Considere un proceso MA(1):

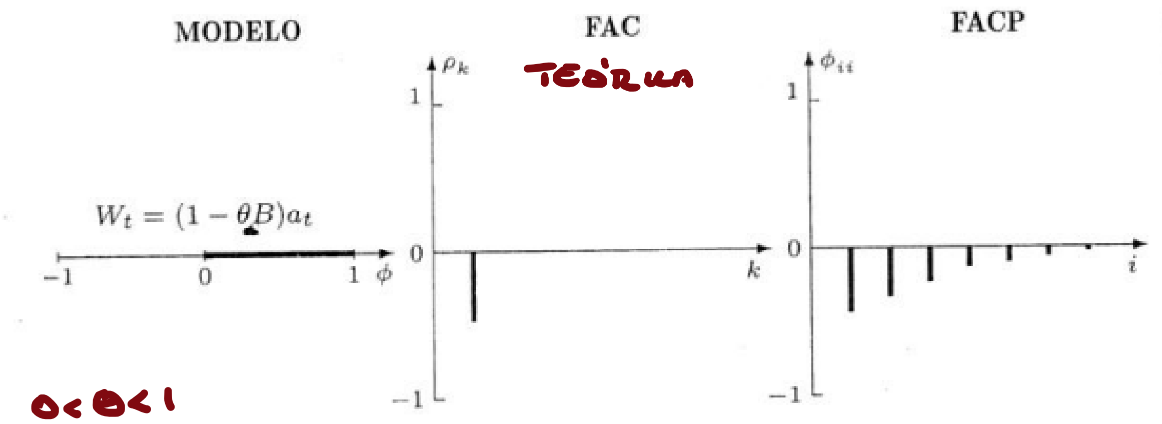
$$(1-\theta B)a_t = W_t \quad ; \quad a_t \sim WN(0,\sigma_a^2)$$

Recuerde que este proceso es estacionario pero no siempre es invertible.

Condición de invertibilidad: $|\theta|<1$

A partir de esta condición identificamos dos regiones admisibles

Región admisible #1: $0<\theta<1$




$0<\theta<1$

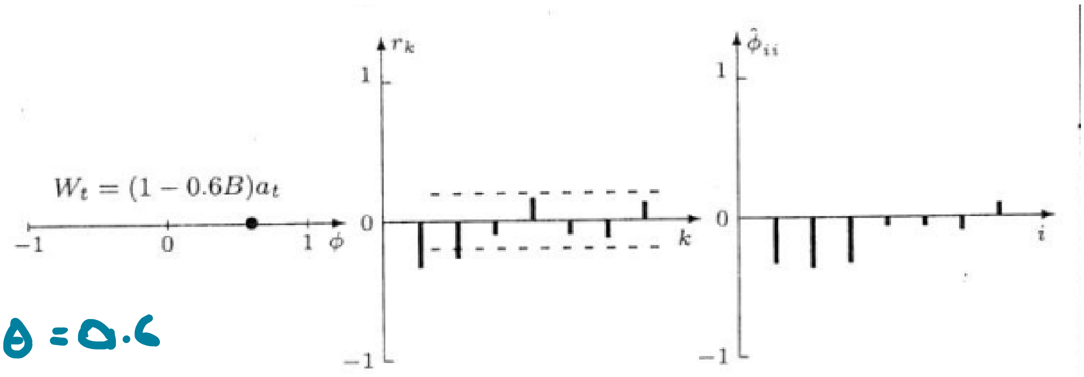
FAC teórica

$$\rho_k = \begin{cases} \frac{-\theta}{1+\theta^2} & \text{si } k=1 \\ 0 & \text{si } k\geq 2 \end{cases}$$

Comportamiento de la FAC

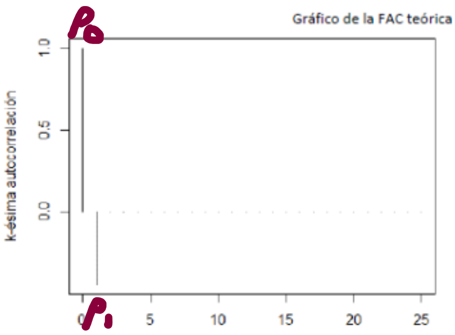
- Sólo la primera autocorrelación será distinta de cero. ($\rho_1<0$)
- $|\rho_1|\leq 0.5$

Ejemplo:

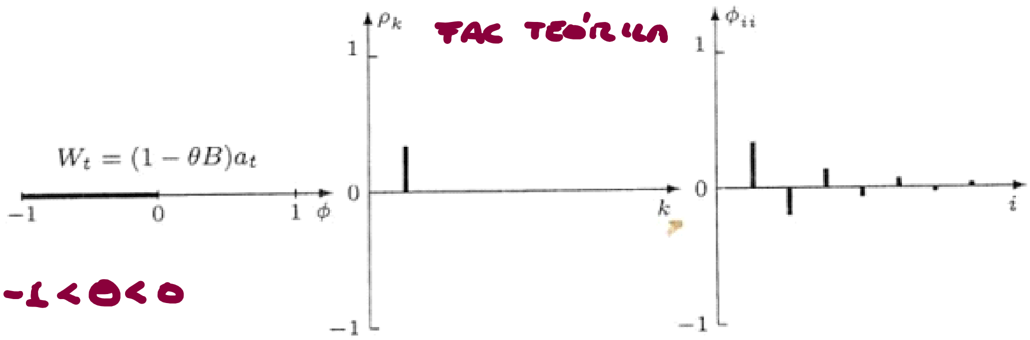



$\theta = 0.6$

$$\rho_k = \begin{cases} \frac{-0.6}{1+(0.6)^2} = -0.44 \end{cases}$$




Región admisible #2 : $-1 < \theta < 0$

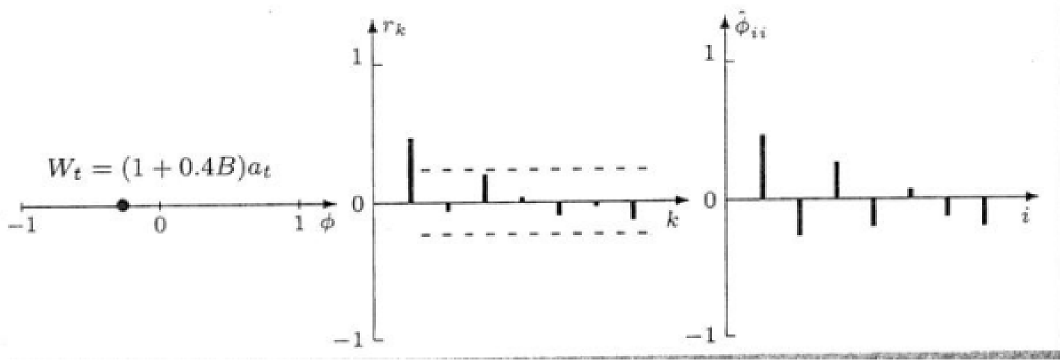



$-1 < \theta < 0$

Comportamiento de la FAC

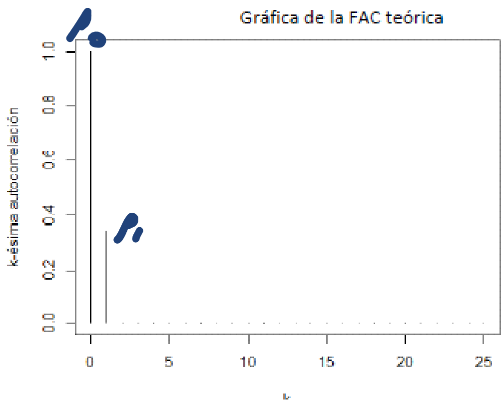
- Sólo la primera autocorrelación será distinta de cero. $(\rho_1 > 0)$
- $|\rho_1| \leq 0.5$

Ejemplo



$\theta = -0.4$

$$\rho_k = \begin{cases} \dfrac{-(-0.4)}{1+(-0.4)^2} = 0.34 & k=1 \\ 0 & |k| \geq 2 \end{cases}$$




Considere un proceso MA(2)

$$(1-\theta_2 B-\theta_2 B^2)\, a_t = W_t \quad ; \quad a_t \sim WN(0,\sigma^2_a)$$

Condiciones para probar invertibilidad

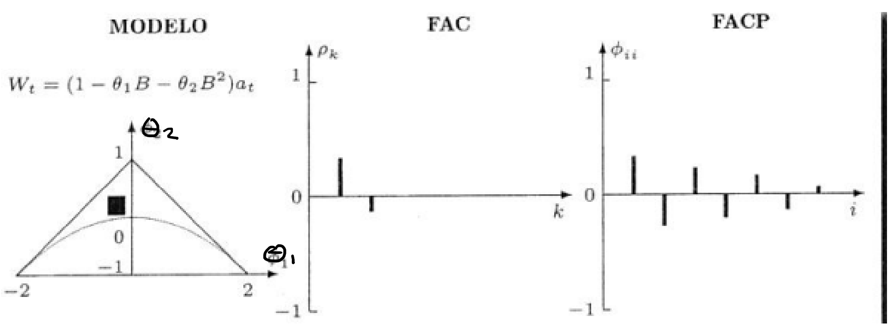
i) $|\theta_2|<1$

ii) $\theta_2+\theta_1<1$

iii) $\theta_2-\theta_1<1$

De aquí surgen las siguientes regiones admisibles

Región admisible #1: Discriminante $>0$ y $\theta_1<0$




$\theta_1^2 + 4\theta_2 > 0$ y $\theta_1 < 0$

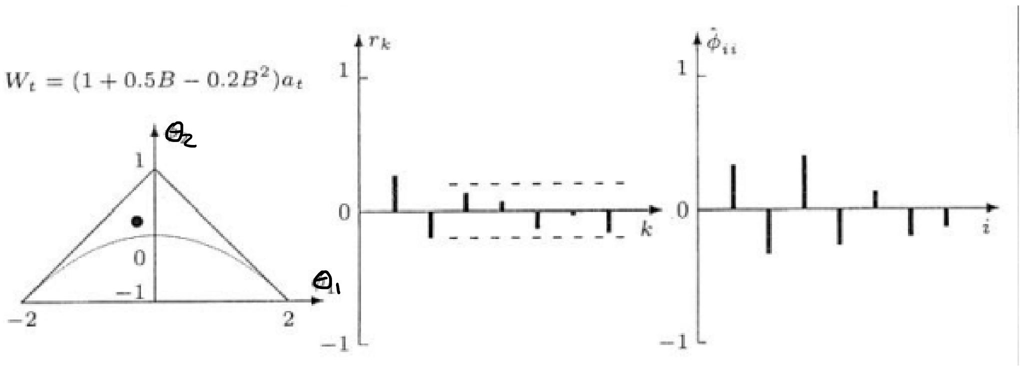
FAC teórica

Comportamiento:

$$\rho_k = \begin{cases} \frac{-\theta_1(1-\theta_2)}{1+\theta_1^2+\theta_2^2} & \text{si } k = 1 \\ \frac{-\theta_2}{1+\theta_1^2+\theta_2^2} & \text{si } k = 2 \\ 0 & \text{si } k \geq 3 \end{cases}$$

Solo $\rho_1$ y $\rho_2$ son las autocorrelaciones que serán distintas de cero, a partir de $\rho_3$ todas serán igual a cero.

Ejemplo:




$\theta_1 = -0.5$ ; $\theta_2 = 0.2$

$\theta_1^2 + 4\theta_2 = (-0.5)^2 + 4(0.2) > 0$

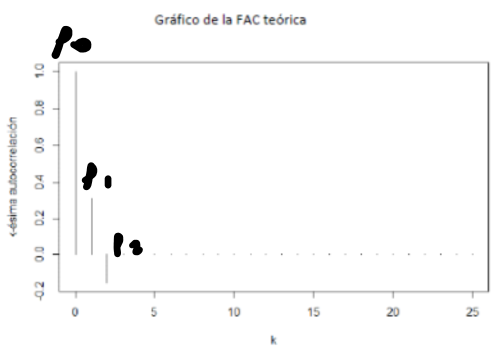
Se cumplen las 3 condiciones de invertibilidad

$|\theta_2| = |0.2| < 1$ ; $\theta_2 + \theta_1 < 1$ ; $\theta_2 - \theta_1 < 1$

FAC teórica

$$\rho_k = \begin{cases} \frac{-\theta_1(1-\theta_2)}{1+\theta_1^2+\theta_2^2} & \text{si } k = 1 \\ \frac{-\theta_2}{1+\theta_1^2+\theta_2^2} & \text{si } k = 2 \\ 0 & \text{si } k \geq 3 \end{cases}$$

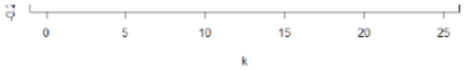
$$\rho_k = \begin{cases} \frac{(0.5)(0.8)}{1+(-0.5)^2+(0.2)^2} = 0.31 & k=1 \\ \frac{-0.2}{1.29} = -0.15 & k=2 \\ 0 & |k| \geq 3 \end{cases}$$




Nota: Se puede probar que

$$\rho_1^2 \leq 0.5 \quad (1.41)$$

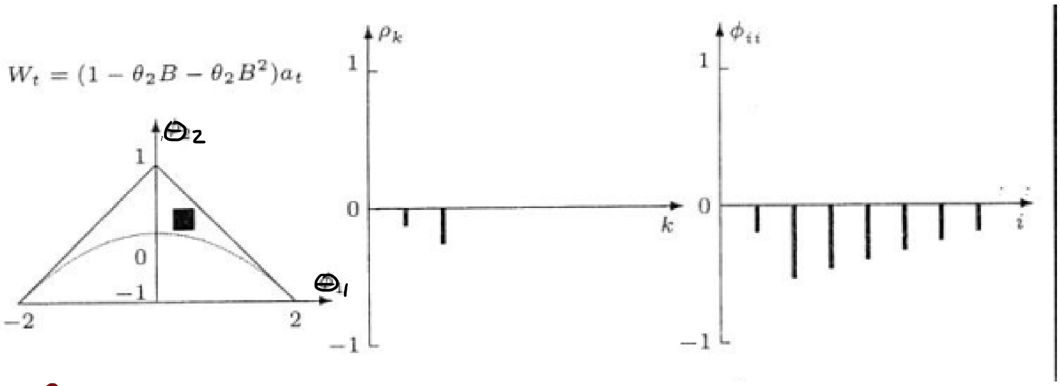
$$|\rho_2| \leq 0.5 \quad (1.42)$$




$$\rho_1^2 \leq 0.5 \tag{1.41}$$

$$|\rho_2| \leq 0.5 \tag{1.42}$$

## Región admisible #2: Discriminante > 0 y $\theta_1 > 0$



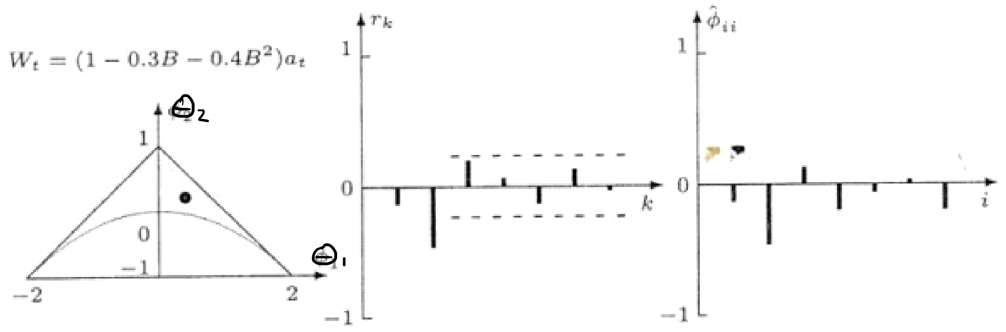

$\theta_1^2 + 4\theta_2 > 0$

y $\theta_1 > 0$

FAC teórica

$$\rho_k = \begin{cases} \frac{-\theta_1(1-\theta_2)}{1+\theta_1^2+\theta_2^2} & \text{si } k = 1 \\ \frac{-\theta_2}{1+\theta_1^2+\theta_2^2} & \text{si } k = 2 \\ 0 & \text{si } k \geq 3 \end{cases}$$

## Ejemplo:




$\theta_1 = 0.3$ ; $\theta_2 = 0.4$

$\theta_1^2 + 4\theta_2 = (0.3)^2 + 4(0.4) > 0$

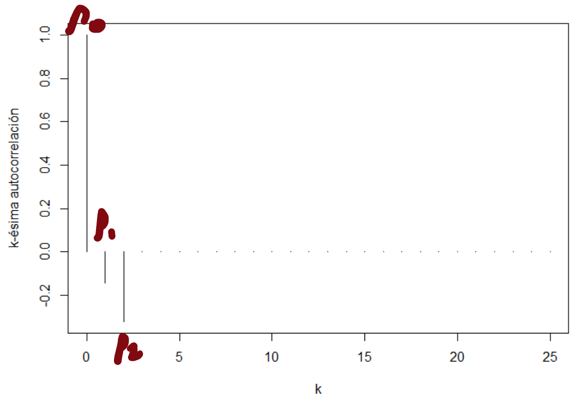
Condiciones de invertibilidad

$|\theta_2| = |0.4| < 1$ ; $\theta_2 + \theta_1 = 0.7 < 1$ ; $\theta_2 - \theta_1 = 0.1 < 1$

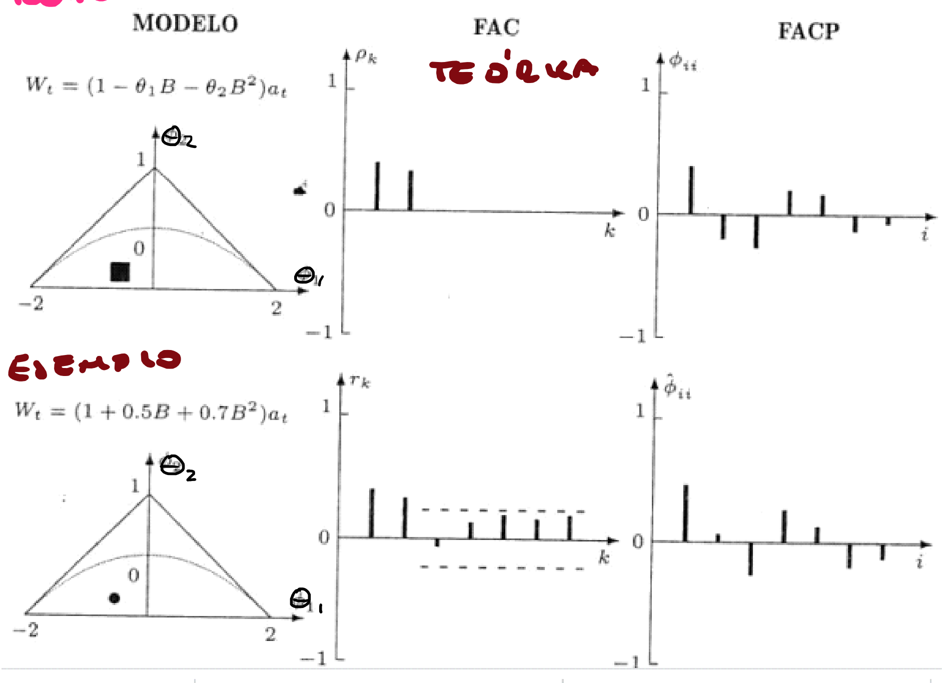
FAC TEÓRICA

$$\rho_k = \begin{cases} \frac{-\theta_1(1-\theta_2)}{1+\theta_1^2+\theta_2^2} & \text{si } k=1 \\ \frac{-\theta_2}{1+\theta_1^2+\theta_2^2} & \text{si } k=2 \\ 0 & \text{si } k \geq 3 \end{cases}$$

$$\rho_K = \begin{cases} -0.14 & K=1 \\ -0.32 & K=2 \\ 0 & |K|\geq 3 \end{cases}$$




REGIÓN ADMISIBLE #3 : DISCRIMINANTE $<0$ ; $\theta_1<0$




$\theta_1 = -0.5$; $\theta_2 = -0.7$

$\theta_1^2 + 4\theta_2 = (-0.5)^2 + 4(-0.7) < 0$

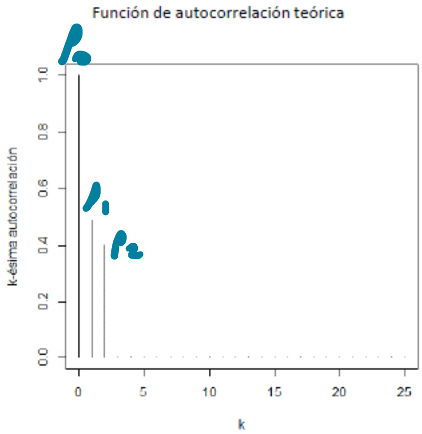
SE CUMPLEN LAS CONDICIONES DE INVERTIBILIDAD:

$|\theta_2|<1$ ; $\theta_2+\theta_1<1$ ; $\theta_2-\theta_1<1$

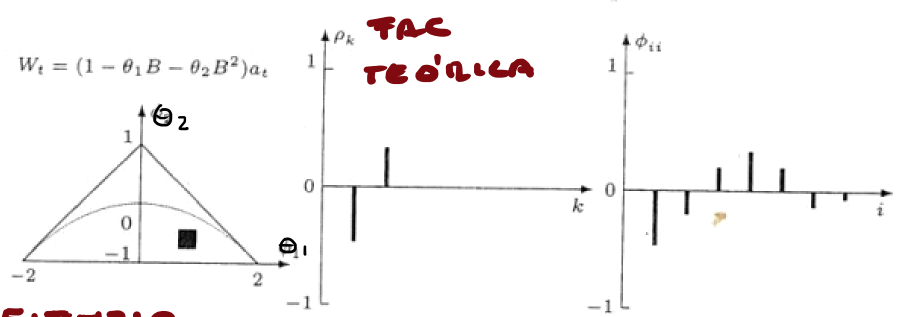
FAC

$$\rho_k = \begin{cases} \frac{-\theta_1(1-\theta_2)}{1+\theta_1^2+\theta_2^2} & \text{si } k=1 \\ \frac{-\theta_2}{1+\theta_1^2+\theta_2^2} & \text{si } k=2 \\ 0 & \text{si } k \geq 3 \end{cases}$$

$$\rho_k = \begin{cases} 0.48 & k=1 \\ 0.40 & k=2 \\ 0 & |k| \geq 3 \end{cases}$$



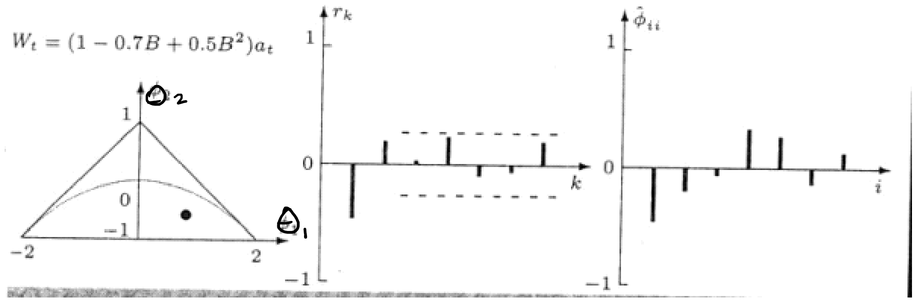

REGIÓN ADMISIBLE #4: DISCRIMINANTE < 0 Y $\theta_1 > 0$




$\theta_1^2 + 4\theta_2 < 0$

RAÍCES COMPLEJAS CONJUGADAS Y $\theta_1 > 0$.

EJEMPLO




$\theta_1 = 0.7$ ; $\theta_2 = -0.5$

$\theta_1^2 + 4\theta_2 = (0.7)^2 + 4(-0.5) < 0$

SE CUMPLEN LAS 3 CONDICIONES DE INVERTIBILIDAD.

$\theta_1 = 0.7$ ; $\theta_2 = -0.5$

FAC TEÓRICA

$$\left\{ \frac{-0.7(1+0.5)}{1+(0.7)^2+(-0.5)^2} = \frac{(-0.7)(1.5)}{1+0.49+0.25} ; k=1 \right.$$

$\rho_k = \Big\{ \frac{-\theta_1(1-\theta_2)}{\dots}$ si $k = 1$

$$\rho_k = \begin{cases} \frac{-\theta_1(1-\theta_2)}{1+\theta_1^2+\theta_2^2} & \text{si } k = 1 \\ \frac{-\theta_2}{1+\theta_1^2+\theta_2^2} & \text{si } k = 2 \\ 0 & \text{si } k \geq 3 \end{cases}$$

$$\rho_k = \begin{cases} \frac{[illegible]}{1+(0.7)^2+(-0.5)^2} = \frac{[illegible]}{1+0.49+0.25} & ; k=1 \\ \frac{0.5}{1.74} & ; k=2 \\ 0 & ; |k| \geq 3 \end{cases}$$

$$\rho_k = \begin{cases} -0.603 & k=1 \\ 0.28 & k=2 \\ 0 & |k| \geq 3 \end{cases}$$