

UNIDAD 6: Práctica 27 – Modelos de Regresión Lineal.

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1 REGRESIÓN LINEAL SIMPLE

Los modelos de regresión lineal son modelos probabilísticos basados en una función lineal, expresamos el valor de nuestra variable de estudio (interés), a la que también llamamos variable dependiente, en función de una o más variables a quienes llamamos variables independientes o explicativas, y las cuales suponemos tienen un efecto sobre nuestra variable de estudio. Los pasos básicos a seguir en el estudio de un modelo lineal son:

- Escribir el modelo matemático con todas sus hipótesis.
- Estimación de los parámetros del modelo.
- Inferencias sobre los parámetros.
- Diagnóstico del modelo.

En R la función a utilizar para realizar o ajustar un modelo de regresión es `lm()` (de lineal model). Esta función no nos ofrece ninguna salida en pantalla si no que nos crea un objeto, o mejor dicho, nosotros creamos un objeto que va a ser un modelo de regresión lineal, y el cual podemos referenciarlo posteriormente en nuestro análisis.

La función `lm` tiene la siguiente sintaxis: `lm(formula, data, subset)`

- EJEMPLO 1.

En el archivo “costes.dat” se encuentra la información correspondiente a 34 fábricas de producción en el montaje de placas para ordenador, el archivo contiene la información sobre el costo total (primera columna) y el número de unidades fabricadas (segunda columna). Suponga que deseamos ajustar un modelo de regresión simple a los datos para estimar el costo total en función del número de unidades fabricadas.

Ejecutamos lo siguiente.

```
getwd()
```

```
[1] "C:/Users/abbyc/Desktop/Ciclo II 2022/Análisis estadístico con R/curso-R-2022"
```

```
setwd("C:/Users/abbyc/Desktop/Ciclo II 2022/Análisis estadístico con R/curso-R-2022")
```

```
# lectura de los datos.
```

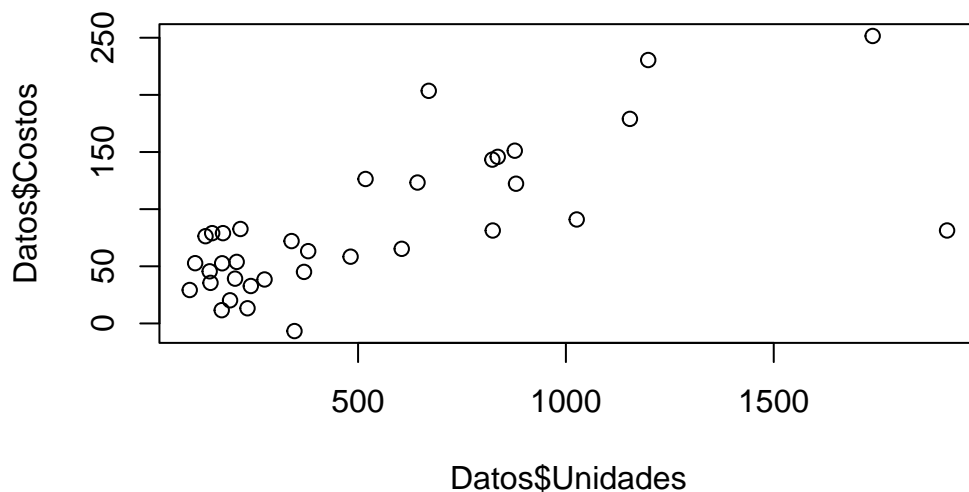
```
Datos=read.table("costes.dat")
```

```
# renombrando a las variables
```

```
names(Datos)<-c("Costos","Unidades")
```

```
# realizando el diagrama de dispersión entres las dos variables
```

```
plot(Datos$Unidades,Datos$Costos)
```



```
# se aprecia una relación entre las variables por lo que se procede a ajustar el modelo de
regresion <- lm(Datos$Costos ~ Datos$Unidades)
summary(regresion)
```

Call:

```
lm(formula = Datos$Costos ~ Datos$Unidades)
```

Residuals:

| Min | 1Q | Median | 3Q | Max |
|----------|---------|--------|--------|---------|
| -137.386 | -24.496 | -0.117 | 29.848 | 105.028 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-----------------|----------|------------|---------|-------------|
| (Intercept) | 33.92200 | 11.57500 | 2.931 | 0.0061 ** |
| Datos\$Unidades | 0.09640 | 0.01665 | 5.789 | 1.8e-06 *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 44.49 on 33 degrees of freedom

Multiple R-squared: 0.5039, Adjusted R-squared: 0.4888

F-statistic: 33.51 on 1 and 33 DF, p-value: 1.796e-06

```
# En este caso el modelo resultante sería:
#Costos = 19.38+0.1345(Unidades)
```

Se observa que el término constante no es significativo porque el p-valor correspondiente a la prueba de hipótesis $H_0 : \beta_0 = 0$; es 0.501 y además no tiene interpretación, pues en teoría si no se fabrican unidades no deberían existir costos asociados a la producción.

Como el término constante no es significativo se quitara del modelo, volvemos a realizar los cálculos con el R

Ejecutar lo siguiente:

```
regresion2 <- lm(Datos$Costos ~ Datos$Unidades -1)
summary(regresion2)
```

Call:

```
lm(formula = Datos$Costos ~ Datos$Unidades - 1)
```

Residuals:

| Min | 1Q | Median | 3Q | Max |
|----------|--------|--------|--------|---------|
| -174.579 | -4.844 | 19.527 | 35.812 | 114.095 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-----------------|----------|------------|---------|--------------|
| Datos\$Unidades | 0.13350 | 0.01197 | 11.16 | 6.59e-13 *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 49.21 on 34 degrees of freedom

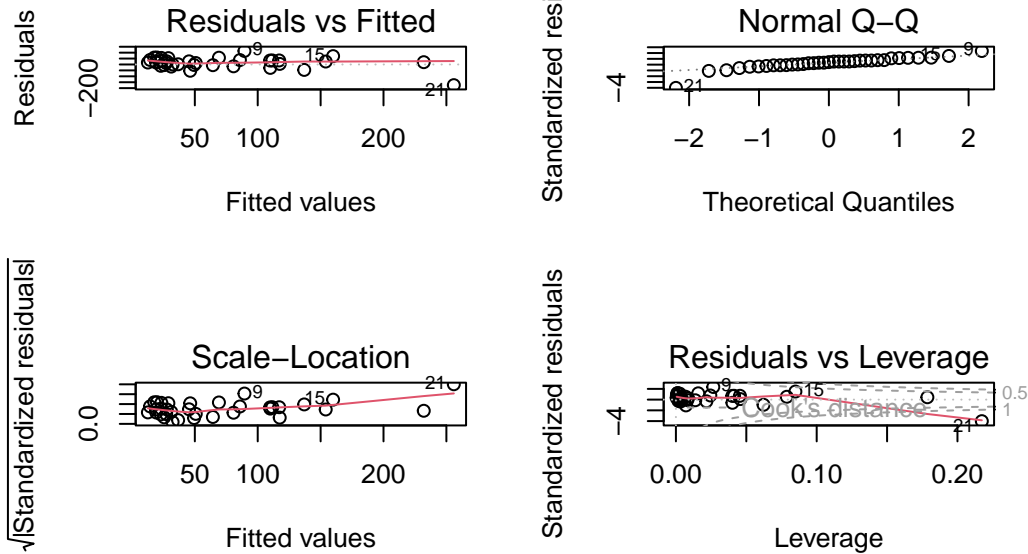
Multiple R-squared: 0.7854, Adjusted R-squared: 0.7791

F-statistic: 124.5 on 1 and 34 DF, p-value: 6.591e-13

Una vez estimados los parámetros del modelo, el siguiente paso es validarlo, es decir verificar si se cumplen las cuatro hipótesis básicas del modelo (nulidad, normalidad, independencia y homocedasticidad de los residuos). Para verificar esto, podríamos realizar los siguientes pasos:

```
# Efectúa un análisis gráfico de bondad de ajuste del modelo
par(mfrow = c(2, 2))
plot(regresion2)
par(oma=c(1,1,1,1), new=T, font=2, cex=0.5)
mtext(outer=T, "Gráficos para validación del modelo: Costos en función
de las unidades",side=3)
```

de las unidades



En los gráficos que se muestra en la parte superior se contrasta los cuatro supuestos. En el de la izquierda se verifican: nulidad, independencia y homocedasticidad; a partir del gráfico mostrado parece existir indicios de falta de homocedasticidad, por su parte los residuos pueden considerarse constante pues no muestran ningún patrón; sin embargo, la media de los residuos no parece ser nula, lo cual indica falta de linealidad en el modelo (es decir, es necesario incorporar más variables o tal vez términos cuadráticos). En la figura de la derecha se contrasta la normalidad, y puede apreciarse que los residuos parecen seguir una distribución normal.

por su parte, también es de mencionar que en el gráfico se muestran puntos que posiblemente sean observaciones atípicas, por lo que habría que estudiarlas.

```
# Información sobre el modelo ajustado que proporciona la función lm()

formula(regresion2) # Extrae la fórmula del modelo.
```

```
Datos$Costos ~ Datos$Unidades - 1
```

```
coef(regresion2) # Extrae el vector de coeficientes de regresión.
```

```
Datos$Unidades
0.1334998
```

```
residuals(regresion2) # Extrae el vector de residuos.
```

| 1 | 2 | 3 | 4 | 5 | 6 |
|-------------|-------------|--------------|-------------|-------------|------------|
| -5.9252241 | 24.9422285 | 57.2435787 | 16.1557654 | 29.5426287 | 55.6391079 |
| 7 | 8 | 9 | 10 | 11 | 12 |
| 37.4643315 | 26.5128603 | 114.0951533 | 16.5660408 | 34.0763027 | 53.6636345 |
| 13 | 14 | 15 | 16 | 17 | 18 |
| 26.6898859 | -15.4751725 | 70.5629667 | 19.5271516 | 11.9533340 | -5.3935511 |
| 19 | 20 | 21 | 22 | 23 | 24 |
| -11.3620767 | 4.7973082 | -174.5790615 | -45.9982846 | -17.9578900 | -4.2938112 |
| 25 | 26 | 27 | 28 | 29 | 30 |
| 34.1602682 | 33.4958317 | 0.4420639 | 38.3225636 | -53.0131662 | 26.0890974 |
| 31 | 32 | 33 | 34 | 35 | |
| 12.5185354 | -28.6938846 | 58.5790516 | 1.7630220 | 59.1208689 | |

```
#modelo2ted.values(regresion2) Extrae un vector con los valores estimados.
```

```
vcov(regresion2) # Extrae la matriz de covarianzas de los parámetros.
```

```
Datos$Unidades
Datos$Unidades 0.0001432011
```

```
ls.diag(regresion2)
```

```
$std.dev
[1] 49.20928
```

```
$hat
[1] 0.0137387236 0.0787524233 0.0158676214 0.0012433354 0.0017698828
[6] 0.0018110415 0.0244497405 0.0012092731 0.0265461726 0.0005337028
[11] 0.0454832506 0.0027846574 0.0068361273 0.0216452725 0.0848722948
[16] 0.1786289532 0.0024610058 0.0021799913 0.0017494809 0.0457949567
[21] 0.2173184179 0.0622511001 0.0032380535 0.0080957252 0.0413299485
[26] 0.0400545613 0.0034632436 0.0006897629 0.0071205126 0.0025584620
[31] 0.0085392456 0.0401519584 0.0010460576 0.0044721637 0.0013128794
```

```
$std.res
[1] -0.12124442 0.52808038 1.17260833 0.32851156 0.60087863 1.13168807
[7] 0.77080784 0.53910368 2.34997086 0.33673450 0.70878358 1.09204005
```

```
[13] 0.54423845 -0.31793642 1.49895619 0.43784678 0.24320756 -0.10972401
[19] -0.23109519 0.09979981 -4.01007557 -0.96527557 -0.36552118 -0.08761148
[25] 0.70898925 0.69473704 0.00899894 0.77903567 -1.08115618 0.53084568
[31] 0.25548695 -0.59517005 1.19102962 0.03590740 1.20220642
```

\$stud.res

```
[1] -0.119473935 0.522403330 1.179328393 0.324159319 0.595144672
[6] 1.136532408 0.766111172 0.533401168 2.529690985 0.332300135
[11] 0.703499199 1.095240031 0.538526063 -0.313692652 1.528103030
[16] 0.432581090 0.239812980 -0.108117520 -0.227850384 0.098335619
[21] -5.441881509 -0.964279034 -0.360815373 -0.086323206 0.703706396
[26] 0.689354550 0.008865626 0.774436698 -1.083933598 0.525161706
[31] 0.251943705 -0.589430743 1.198655383 0.035376083 1.210400702
```

\$cooks

```
[1] 2.047755e-04 2.383898e-02 2.216993e-02 1.343476e-04 6.401583e-04
[6] 2.323641e-03 1.489076e-02 3.518799e-04 1.505953e-01 6.054895e-05
[11] 2.393841e-02 3.330121e-03 2.038767e-03 2.236389e-03 2.083829e-01
[16] 4.169239e-02 1.459274e-04 2.630304e-05 9.359475e-05 4.780083e-04
[21] 4.464949e+00 6.185333e-02 4.340279e-04 6.264812e-05 2.167080e-02
[26] 2.013939e-02 2.814313e-07 4.189037e-04 8.382848e-03 7.228166e-04
[31] 5.621878e-04 1.481789e-02 1.485440e-03 5.792050e-06 1.899999e-03
```

\$dfits

```
[1] -0.0141010149 0.1527389219 0.1497489923 0.0114372818 0.0250599080
[6] 0.0484105087 0.1212841638 0.0185600398 0.4177445106 0.0076788503
[11] 0.1535668861 0.0578762950 0.0446787837 -0.0466592443 0.4653661151
[16] 0.2017315204 0.0119114345 -0.0050535591 -0.0095385991 0.0215426354
[21] -2.8675090121 -0.2484465426 -0.0205651234 -0.0077986703 0.1461131657
[26] 0.1408138463 0.0005226419 0.0203463104 -0.0917931003 0.0265973773
[31] 0.0233816685 -0.1205549113 0.0387881801 0.0023710564 0.0438860875
```

\$correlation

```

          Datos$Unidades
Datos$Unidades          1
```

\$std.err

```

          [,1]
Datos$Unidades 0.01196667
```

\$cov.scaled

```

          Datos$Unidades
Datos$Unidades 0.0001432011
```

```

$cov.unscaled
      Datos$Unidades
Datos$Unidades 5.913605e-08

# Calcula los residuales, errores estándar de los parámetros, distancias Cook.

step(regresion2)

Start: AIC=273.71
Datos$Costos ~ Datos$Unidades - 1

      Df Sum of Sq    RSS   AIC
<none>          82333 273.71
- Datos$Unidades  1   301376 383709 325.58

Call:
lm(formula = Datos$Costos ~ Datos$Unidades - 1)

Coefficients:
Datos$Unidades
      0.1335

# Permite obtener el mejor conjunto de regresión y proporciona la estimación
#de los coeficientes (válido únicamente en modelos de regresión múltiple).

```

De todos los resultados anteriores nos concentraremos en la instrucción: `ls.diag(regresion2)`. Con esta instrucción obtenemos para cada observación en el conjunto de datos, medidas que nos ayudarán a identificar observación atípicas (tienen un impacto únicamente en las medidas resumen del modelo) y observaciones influyentes (tienen un efecto marcado en la estimación de los parámetros). Al digitar la instrucción anterior en R se mostrará los siguientes resultados (cada uno de ellos en un vector).

2 REGRESIÓN LINEAL MÚLTIPLE

Al igual que en el modelo de regresión simple, el modelo de regresión múltiple trata de ajustar una ecuación matemática en la que se relacione a una única variable dependiente en función de dos o más variables independientes. La forma general del modelo es la siguiente:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \dots + \beta_k x_{ki} + u$$

- EJEMPLO 2.

En el archivo “preciocasas.dat” tienen la información sobre 100 datos de precios de viviendas y sus características, el archivo se encuentra estructurado de la siguiente forma:

- Primera columna: precios de viviendas en euros.
- Segunda columna: superficie en metros cuadrados.
- Tercera: numero de cuartos de baño.
- Cuarta: número de dormitorios.
- Quinta: número de plazas de garaje.
- Sexta: edad de la vivienda .
- Séptima: 1 =buenas vistas y 0 =vistas corrientes

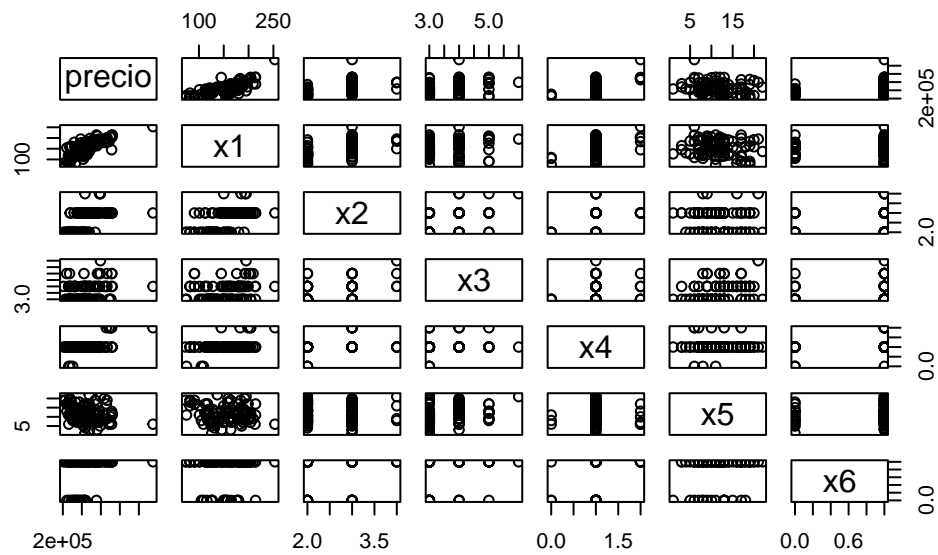
Suponga que deseamos estimar un modelo de regresión en el cual relacionemos el precio de una vivienda en función de sus características.

Ejecutar lo siguiente:

```
# leyendo los datos
datos = read.table(file="preciocasas.txt")

# nombrando a las columnas
names(datos)<-c("precio","x1","x2","x3","x4","x5","x6")

# haciendo la matriz de diagramas de dispersión
plot(datos)
```



```
# se observa gráficamente que las variables independientes parecen influir en el comportam

# ajustamos el modelo de regresión

modelo1<-lm(precio~x1+x2+x3+x4+x5+x6,data = datos)

#resumen del modelo
summary(modelo1)
```

Call:

```
lm(formula = precio ~ x1 + x2 + x3 + x4 + x5 + x6, data = datos)
```

Residuals:

| Min | 1Q | Median | 3Q | Max |
|---------|--------|--------|-------|--------|
| -101248 | -23050 | -345 | 18036 | 141928 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|----------|------------|---------|--------------|
| (Intercept) | 29844.7 | 26365.3 | 1.132 | 0.26056 |
| x1 | 1159.3 | 142.9 | 8.112 | 1.98e-12 *** |
| x2 | 13284.5 | 9286.2 | 1.431 | 0.15591 |
| x3 | 8695.2 | 6708.7 | 1.296 | 0.19814 |

```

x4          59777.1      14604.0    4.093 9.06e-05 ***
x5          -3198.4       974.3   -3.283 0.00145 **
x6          34312.9      10963.6    3.130 0.00234 **
---

```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 38920 on 93 degrees of freedom

Multiple R-squared: 0.7505, Adjusted R-squared: 0.7344

F-statistic: 46.61 on 6 and 93 DF, p-value: < 2.2e-16

De los resultados anteriores puede apreciarse que el intercepto, y las variables x2 (número de cuarto de baño) y x3 (número de dormitorios) no parecen influir en la estimación del precio de la vivienda por lo podrían descartarse de la ecuación.

Una forma alternativa y mucho más eficiente para seleccionar el mejor conjunto de variables independientes es utilizar la instrucción `step()`, con la cual se utilizan los algoritmos conocidos para seleccionar variables (selección hacia adelante -“forward”-, hacia atrás -“backward”- o selección por pasos -“both”-).

```
step(modelo1, direction="both")
```

Start: AIC=2120.58

precio ~ x1 + x2 + x3 + x4 + x5 + x6

| | Df | Sum of Sq | RSS | AIC |
|--------|----|------------|------------|--------|
| - x3 | 1 | 2.5444e+09 | 1.4340e+11 | 2120.4 |
| <none> | | | 1.4086e+11 | 2120.6 |
| - x2 | 1 | 3.0996e+09 | 1.4395e+11 | 2120.8 |
| - x6 | 1 | 1.4835e+10 | 1.5569e+11 | 2128.6 |
| - x5 | 1 | 1.6322e+10 | 1.5718e+11 | 2129.6 |
| - x4 | 1 | 2.5376e+10 | 1.6623e+11 | 2135.2 |
| - x1 | 1 | 9.9664e+10 | 2.4052e+11 | 2172.1 |

Step: AIC=2120.37

precio ~ x1 + x2 + x4 + x5 + x6

| | Df | Sum of Sq | RSS | AIC |
|--------|----|------------|------------|--------|
| <none> | | | 1.4340e+11 | 2120.4 |
| + x3 | 1 | 2.5444e+09 | 1.4086e+11 | 2120.6 |
| - x2 | 1 | 5.3488e+09 | 1.4875e+11 | 2122.0 |
| - x5 | 1 | 1.3780e+10 | 1.5718e+11 | 2127.6 |
| - x6 | 1 | 1.6460e+10 | 1.5986e+11 | 2129.2 |

```
- x4    1 2.4664e+10 1.6806e+11 2134.2
- x1    1 1.0510e+11 2.4850e+11 2173.4
```

Call:

```
lm(formula = precio ~ x1 + x2 + x4 + x5 + x6, data = datos)
```

Coefficients:

| (Intercept) | x1 | x2 | x4 | x5 | x6 |
|-------------|------|-------|-------|-------|-------|
| 42224 | 1182 | 16724 | 58864 | -2686 | 35913 |

- EJERCICIO 1.

Se deja como ejercicio al estudiante, elegir el mejor conjunto de variables a incluir en el modelo, y para el modelo resultante (llamarlo modelo2), realizar el diagnóstico de los residuos y el estudio de las observaciones atípicas e influyentes.

```
coefficients(modelo1) # coeficientes del modelo
```

| (Intercept) | x1 | x2 | x3 | x4 | x5 |
|-------------|-----------|-----------|----------|-----------|-----------|
| 29844.731 | 1159.255 | 13284.517 | 8695.202 | 59777.054 | -3198.381 |
| | x6 | | | | |
| | 34312.868 | | | | |

```
confint(modelo1, level=0.95) # Intervalos de confianza para los parámetros
```

| | 2.5 % | 97.5 % |
|-------------|-------------|-----------|
| (Intercept) | -22511.5521 | 82201.014 |
| x1 | 875.4687 | 1443.041 |
| x2 | -5156.1158 | 31725.150 |
| x3 | -4626.8649 | 22017.269 |
| x4 | 30776.4939 | 88777.614 |
| x5 | -5133.1468 | -1263.615 |
| x6 | 12541.3783 | 56084.359 |

```
fitted(modelo1) # valores estimados
```

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|----------|----------|----------|----------|----------|----------|----------|----------|
| 279222.7 | 266544.8 | 279358.4 | 189667.8 | 257849.6 | 296943.2 | 339872.9 | 314353.6 |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |

| | | | | | | | |
|----------|----------|----------|----------|----------|----------|----------|----------|
| 235171.0 | 290392.2 | 178634.1 | 318058.5 | 328027.0 | 263914.7 | 283839.6 | 295909.1 |
| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 294267.0 | 270922.5 | 281152.3 | 374349.5 | 287379.2 | 329024.5 | 271191.4 | 303744.5 |
| 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
| 335401.2 | 315658.1 | 301569.6 | 348909.3 | 302264.1 | 352862.4 | 377290.2 | 353524.7 |
| 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 379091.6 | 393779.0 | 380187.5 | 434665.3 | 411467.3 | 532447.2 | 219574.1 | 200445.6 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 |
| 245099.4 | 229169.3 | 245099.4 | 168574.1 | 243291.9 | 258491.8 | 290025.0 | 222772.5 |
| 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 |
| 287794.3 | 290350.5 | 264479.6 | 274721.4 | 299945.6 | 288891.8 | 275192.3 | 365697.6 |
| 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 |
| 295629.7 | 265042.8 | 325082.0 | 361924.8 | 376089.2 | 411489.0 | 393554.2 | 272961.6 |
| 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 |
| 308219.7 | 301037.1 | 328088.8 | 294749.8 | 328300.4 | 298428.6 | 334567.6 | 325423.2 |
| 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 363896.2 | 356423.6 | 346248.0 | 358762.2 | 349529.8 | 389720.9 | 330018.5 | 386584.3 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 |
| 365655.9 | 327917.5 | 356401.9 | 349269.0 | 367435.7 | 420763.0 | 379008.2 | 387743.5 |
| 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 |
| 397530.2 | 389503.3 | 418697.8 | 388132.6 | 370893.4 | 357178.3 | 351102.6 | 442604.2 |
| 97 | 98 | 99 | 100 | | | | |
| 463750.1 | 454155.0 | 426300.0 | 405371.6 | | | | |

```
residuals(modelo1) # residuos
```

| 1 | 2 | 3 | 4 | 5 |
|--------------|---------------|--------------|--------------|--------------|
| -57889.68926 | -36565.81290 | -49379.40675 | 41695.15707 | -22682.61106 |
| 6 | 7 | 8 | 9 | 10 |
| -60047.16363 | -101247.91267 | -68984.63691 | 10370.98785 | -38999.18751 |
| 11 | 12 | 13 | 14 | 15 |
| 73135.93784 | -57967.51228 | -65194.03290 | -1081.71872 | -15818.63830 |
| 16 | 17 | 18 | 19 | 20 |
| -19588.07789 | -17945.95009 | 9202.48178 | 5197.68004 | -80736.51621 |
| 21 | 22 | 23 | 24 | 25 |
| 6578.80514 | -31607.52031 | 37568.64742 | 7159.52240 | -24151.21283 |
| 26 | 27 | 28 | 29 | 30 |
| 2508.86990 | 18115.36629 | -21592.29001 | 34207.88296 | -13600.37389 |
| 31 | 32 | 33 | 34 | 35 |
| -37858.16771 | 9600.34471 | 1325.36583 | -6791.04628 | 10604.48751 |
| 36 | 37 | 38 | 39 | 40 |
| -5832.26646 | 48490.68128 | 141927.79018 | -15532.13765 | 12242.35614 |

| | | | | |
|--------------|--------------|--------------|--------------|--------------|
| 41 | 42 | 43 | 44 | 45 |
| -25495.39355 | 2365.71917 | -9932.39355 | 68992.85235 | -4666.94695 |
| 46 | 47 | 48 | 49 | 50 |
| -14844.82285 | -30649.96225 | 43173.48129 | -752.29527 | 13982.53546 |
| 51 | 52 | 53 | 54 | 55 |
| 42893.36326 | 34270.61190 | 10266.39228 | 22358.24265 | 38824.68839 |
| 56 | 57 | 58 | 59 | 60 |
| -49259.62111 | 32912.30503 | 73874.20103 | 19022.01500 | 8117.19902 |
| 61 | 62 | 63 | 64 | 65 |
| 17295.80992 | 20803.02791 | 62945.75914 | -10128.64462 | -31898.72900 |
| 66 | 67 | 68 | 69 | 70 |
| -24716.08965 | -34130.82548 | 6671.17677 | -26733.43567 | 18009.35116 |
| 71 | 72 | 73 | 74 | 75 |
| -18129.63248 | -5527.16050 | -40542.15465 | -31340.58382 | -11827.02143 |
| 76 | 77 | 78 | 79 | 80 |
| -21920.16275 | -8883.84843 | -47345.86309 | 12356.54383 | -32451.27461 |
| 81 | 82 | 83 | 84 | 85 |
| -5988.89813 | 42124.46281 | 16752.06955 | 27689.03797 | 12981.28878 |
| 86 | 87 | 88 | 89 | 90 |
| -38617.00938 | 8324.81178 | 2702.47073 | -4663.21304 | 6475.72725 |
| 91 | 92 | 93 | 94 | 95 |
| -22026.83322 | 9575.40941 | 47564.59441 | 61279.71336 | 81189.36959 |
| 96 | 97 | 98 | 99 | 100 |
| 62.81828 | -14167.14854 | -1113.00535 | 30200.03063 | 52857.40711 |

```
influence(modelo1) # puntos de influencia
```

\$\hat{\theta}\$

| | | | | | | |
|------------|------------|------------|------------|------------|------------|------------|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 0.07644959 | 0.04329746 | 0.03101799 | 0.16470221 | 0.04905149 | 0.10054929 | 0.01897510 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 0.06554873 | 0.07650649 | 0.03416588 | 0.15576025 | 0.10836388 | 0.04982851 | 0.07485435 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 0.09354194 | 0.03757592 | 0.08757059 | 0.03427498 | 0.13159339 | 0.05890256 | 0.07085183 |
| 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| 0.07729138 | 0.09235552 | 0.04200637 | 0.12319478 | 0.06365060 | 0.07744857 | 0.04052721 |
| 29 | 30 | 31 | 32 | 33 | 34 | 35 |
| 0.03219950 | 0.03809646 | 0.02541698 | 0.03275262 | 0.07081194 | 0.06528581 | 0.04193972 |
| 36 | 37 | 38 | 39 | 40 | 41 | 42 |
| 0.13289981 | 0.04986938 | 0.16786143 | 0.07359808 | 0.09044749 | 0.11550283 | 0.06575711 |
| 43 | 44 | 45 | 46 | 47 | 48 | 49 |

| | | | | | | |
|------------|------------|------------|------------|------------|------------|------------|
| 0.11550283 | 0.15413191 | 0.10208614 | 0.03911696 | 0.05499194 | 0.06973092 | 0.03927547 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 |
| 0.03295164 | 0.07257119 | 0.03206266 | 0.03181964 | 0.03757695 | 0.05922658 | 0.02390075 |
| 57 | 58 | 59 | 60 | 61 | 62 | 63 |
| 0.04319111 | 0.10588291 | 0.02262057 | 0.09015279 | 0.02702758 | 0.08352555 | 0.20328452 |
| 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 0.03199632 | 0.08829534 | 0.04839577 | 0.04638077 | 0.03773781 | 0.03545056 | 0.15835318 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 |
| 0.12184146 | 0.04217239 | 0.02977390 | 0.02462946 | 0.03748933 | 0.03614752 | 0.06572773 |
| 78 | 79 | 80 | 81 | 82 | 83 | 84 |
| 0.03282231 | 0.03668504 | 0.04376268 | 0.02187867 | 0.04667088 | 0.03649125 | 0.15511531 |
| 85 | 86 | 87 | 88 | 89 | 90 | 91 |
| 0.03576599 | 0.09019921 | 0.02406875 | 0.04441708 | 0.08172344 | 0.04900424 | 0.10789185 |
| 92 | 93 | 94 | 95 | 96 | 97 | 98 |
| 0.16871732 | 0.02092219 | 0.07365580 | 0.05656898 | 0.14285883 | 0.13476353 | 0.13227248 |
| 99 | 100 | | | | | |
| 0.06338635 | 0.07303966 | | | | | |

\$coefficients

| | (Intercept) | x1 | x2 | x3 | x4 |
|----|--------------|--------------|---------------|---------------|--------------|
| 1 | -4227.090617 | -16.01194059 | 1278.2081179 | 508.0538595 | -718.264482 |
| 2 | -332.494918 | 3.98687049 | 872.7083389 | -457.9104431 | -242.979377 |
| 3 | -3908.905468 | 5.28021271 | 795.6853947 | 376.5560717 | -278.410450 |
| 4 | 8976.007651 | 0.28821773 | -303.3697850 | 223.3498552 | -5199.729746 |
| 5 | -1216.589486 | 0.65829628 | 264.2962209 | 423.0197697 | -77.191809 |
| 6 | -1299.786890 | -39.40326282 | 1883.5157231 | 1749.5181453 | 1228.339847 |
| 7 | 2561.333249 | 13.30258539 | -1437.9884477 | -625.6021869 | 761.253504 |
| 8 | 1096.128770 | -37.36592369 | 3106.4306263 | -506.0976325 | 1120.074014 |
| 9 | 942.764318 | -2.88554528 | -36.4772476 | -58.2867279 | 332.353187 |
| 10 | -3384.699795 | 3.39207970 | 752.2125420 | 169.4473776 | -186.340610 |
| 11 | 14961.049526 | -1.19844346 | -265.3043156 | 111.0311345 | -8950.546768 |
| 12 | -899.432006 | -34.41247374 | 2852.4321459 | -1275.2585546 | -141.163489 |
| 13 | -560.062942 | -26.98974686 | 3032.8785222 | -1081.2036829 | 793.605962 |
| 14 | -118.457082 | 0.22734899 | 13.1385971 | -3.1745651 | -31.565079 |
| 15 | -268.862409 | -4.18879688 | 555.5886876 | -313.3694953 | -219.271434 |
| 16 | -1778.493995 | 1.51023181 | 409.8337411 | 52.9185261 | -85.290138 |
| 17 | -764.811605 | 5.58523647 | -730.4544316 | 315.7125092 | -318.225001 |
| 18 | 600.197396 | -0.36818940 | -135.7843200 | -121.1663049 | 32.353534 |
| 19 | 26.402059 | -0.33488035 | 226.0398350 | -173.0608562 | 51.770712 |
| 20 | 370.758161 | -43.26778083 | -707.3502575 | 2442.6161836 | 2583.786005 |
| 21 | 615.213007 | 0.94068318 | -148.7481259 | -5.6584646 | 109.669589 |
| 22 | 837.293817 | -1.75376691 | -478.3271520 | -274.5887413 | -167.059882 |
| 23 | 4779.254733 | -11.29577510 | -502.9110188 | 336.0131435 | 1222.764085 |

| | | | | | |
|----|---------------|--------------|---------------|---------------|--------------|
| 24 | 673.158315 | -0.27973335 | -168.4983576 | -8.6474396 | 21.013987 |
| 25 | 1814.605579 | 1.44565814 | -85.7446243 | -1043.5213258 | -246.593185 |
| 26 | 274.611008 | -0.16600133 | -67.6931907 | 10.5540804 | 9.238930 |
| 27 | 1491.220860 | 6.91744621 | -533.5215263 | -72.6476356 | 153.160246 |
| 28 | 1.551722 | 6.65268041 | -306.3758583 | -343.6647552 | 41.883955 |
| 29 | 2606.650211 | 3.83328604 | -844.3714076 | -245.4265045 | -74.926075 |
| 30 | 817.549220 | -4.15727643 | -80.3133598 | 65.1139889 | 310.475991 |
| 31 | 853.094863 | -5.56392430 | -50.1038242 | -367.8672604 | 669.672775 |
| 32 | 257.988257 | 0.44319746 | 187.6687045 | -191.0461372 | -140.505672 |
| 33 | 15.870421 | -0.23109060 | 6.3130521 | 30.1244078 | -9.840414 |
| 34 | 610.793070 | -2.32689387 | 111.9952416 | -238.1972509 | 166.144315 |
| 35 | 177.084349 | 3.92702358 | 94.4303937 | -232.5818271 | -278.190164 |
| 36 | 860.341166 | -0.49080311 | 7.6718913 | 19.7866902 | -738.461206 |
| 37 | -2348.979556 | 33.14787811 | -678.2769422 | 167.6729744 | -1786.452824 |
| 38 | -17509.859560 | 104.43325943 | -5485.1736125 | 2074.6245717 | 15324.677776 |
| 39 | -252.346540 | 9.08597918 | 97.7894629 | -187.2106487 | -368.866033 |
| 40 | 824.636286 | -7.89507797 | 134.3398235 | -240.3775799 | 311.929220 |
| 41 | 643.027149 | 15.32354165 | 658.8295344 | -1259.3722135 | -630.887687 |
| 42 | 71.753121 | -1.50267197 | -19.3669430 | 41.5678216 | 59.739210 |
| 43 | 250.507947 | 5.96968413 | 256.6641776 | -490.6211950 | -245.778704 |
| 44 | 12859.103660 | -17.91620546 | 202.7578138 | -581.9568627 | -9327.174331 |
| 45 | 145.564223 | 4.53082820 | -244.5106356 | 6.6394321 | -101.126000 |
| 46 | -1305.864335 | 5.35046872 | 115.7619502 | 92.8245612 | -217.121124 |
| 47 | 198.920350 | 22.98590910 | -1081.1019959 | -303.5891837 | -426.371251 |
| 48 | 904.028520 | -25.94718296 | -298.7749563 | 599.4203339 | 1045.655779 |
| 49 | -70.807036 | 0.13201508 | 13.2948856 | 1.6154793 | -5.897439 |
| 50 | 886.780879 | 1.92267393 | -306.1086109 | -174.0800409 | -39.850186 |
| 51 | -881.827993 | -36.46221935 | 1883.1211446 | 148.2263485 | 752.702850 |
| 52 | 2777.349945 | -5.57821899 | -488.9285676 | -250.7575080 | 261.023386 |
| 53 | 791.210187 | 0.86402111 | -243.7434823 | -72.0010396 | -12.356352 |
| 54 | 399.681737 | -0.17375459 | -687.9158038 | 380.6737777 | 62.683887 |
| 55 | -1184.798030 | -25.14941681 | 1481.8704365 | 22.0594298 | 413.406711 |
| 56 | 889.005642 | -0.42348439 | -292.7710852 | -516.6255151 | 629.148112 |
| 57 | 3202.965438 | -4.53429834 | -666.9137207 | -21.3299700 | 211.637547 |
| 58 | 1666.700393 | 27.09217012 | -1410.9857587 | -2314.9260394 | -740.321103 |
| 59 | -484.623970 | -4.79880558 | 370.7542475 | 98.8484886 | -60.989735 |
| 60 | -90.734880 | 7.51204788 | -517.7547060 | 103.0733660 | -244.171601 |
| 61 | -782.505070 | 6.16354378 | -14.2375961 | 30.2835067 | -428.995091 |
| 62 | -1675.769908 | 8.24354240 | -462.8007549 | 851.1603095 | -558.802336 |
| 63 | -10194.781335 | -63.24790612 | 714.8487483 | 2995.5731700 | 11124.379201 |
| 64 | -709.929744 | 0.72497562 | 150.7100521 | 113.9350625 | -46.068046 |
| 65 | -1965.072247 | 11.86595157 | -1198.9229267 | 319.4147321 | -623.605163 |
| 66 | -747.636421 | 10.46105690 | -971.8142915 | 562.5564726 | -49.648958 |

| | | | | | |
|-----|--------------|---------------|---------------|---------------|--------------|
| 67 | -2897.206164 | -8.79506156 | 1146.0406708 | 99.0714794 | 258.072388 |
| 68 | 608.821969 | -0.60791358 | -136.4741485 | -17.4938312 | 32.361160 |
| 69 | -826.498074 | 5.38744373 | -778.4039674 | 541.5818297 | 155.722947 |
| 70 | 2002.766694 | -16.67375545 | 896.8733786 | 70.3450012 | 704.765662 |
| 71 | 1641.382809 | 11.94962102 | -1559.6553919 | 254.0801955 | 110.652321 |
| 72 | -243.244939 | 1.94224636 | -175.9076660 | 89.0221566 | 4.269793 |
| 73 | 2118.718654 | -13.10571793 | -111.3706229 | 59.0223603 | 953.032241 |
| 74 | 454.340658 | 3.19652063 | -302.3818383 | -348.6592668 | 277.902092 |
| 75 | -145.470068 | -1.51277298 | -243.5559963 | 302.9040269 | 205.222094 |
| 76 | -802.432378 | 0.07185263 | -412.7403683 | 356.4117075 | 286.969187 |
| 77 | -582.444543 | 3.11902241 | -231.0628019 | 68.6059122 | 13.591751 |
| 78 | 2129.539109 | -22.07377830 | 267.6009254 | -135.2556784 | 1365.683343 |
| 79 | 229.887446 | -0.84227737 | 335.2663535 | -302.4253466 | -128.525829 |
| 80 | -843.498695 | -10.87520015 | -248.4450250 | 594.3257750 | 816.934374 |
| 81 | 245.581544 | -1.37992322 | -19.9572446 | -14.6582042 | 121.463579 |
| 82 | -2639.114680 | 4.97593939 | 623.4366526 | -303.8937894 | -679.734450 |
| 83 | 237.497649 | 3.24873012 | 287.0345841 | -406.1752877 | -330.947720 |
| 84 | -507.771005 | 22.35065995 | 389.8587108 | -1031.0384107 | -575.042764 |
| 85 | 279.735321 | 2.77954932 | 181.6670702 | -271.5273030 | -267.683679 |
| 86 | 3281.126424 | -19.99358454 | 1018.6029701 | -1565.5639562 | 1206.982587 |
| 87 | -288.825569 | 2.31975508 | -5.6116758 | 46.3473386 | -184.598013 |
| 88 | 69.063954 | 0.94441704 | 19.4282174 | -49.7449892 | -69.428810 |
| 89 | 323.559041 | 1.15285259 | -281.9396702 | 13.9310759 | 93.194749 |
| 90 | 238.434755 | 1.67292249 | 50.8252425 | -93.8551284 | -146.948493 |
| 91 | 2594.208849 | 5.25972213 | -976.5664023 | -694.0141846 | 457.237133 |
| 92 | -2370.692904 | -0.45656606 | 400.4425037 | 402.2675913 | -267.761011 |
| 93 | -1497.161858 | 8.62605653 | 121.4633213 | 290.0465319 | -889.029156 |
| 94 | -5048.827659 | 33.56096586 | 137.7286855 | -712.8092850 | -1936.717903 |
| 95 | -5887.791028 | 32.89919767 | 437.6922346 | -723.8958103 | -2145.274714 |
| 96 | -2.194293 | -0.01867873 | 0.8189008 | -0.8927769 | 8.804631 |
| 97 | 712.157425 | -0.74438811 | -37.7357436 | 254.4893601 | -1794.307591 |
| 98 | 72.828313 | -0.12436032 | -5.2849082 | 26.6692121 | -138.519974 |
| 99 | -756.120135 | 17.74050493 | -511.5506005 | 390.5122026 | -1027.782614 |
| 100 | -5041.585863 | 25.95155590 | -1136.2541923 | 1825.7481625 | -1576.126894 |
| | x5 | x6 | | | |
| 1 | -112.4132787 | 3514.8068219 | | | |
| 2 | -60.9311326 | -423.7930175 | | | |
| 3 | 19.4753790 | -956.9771960 | | | |
| 4 | -141.5620527 | -1831.3785561 | | | |
| 5 | -79.7634794 | -394.9183079 | | | |
| 6 | -396.8977434 | -1453.7171024 | | | |
| 7 | 29.1178563 | -875.8766750 | | | |
| 8 | -219.7113410 | -1273.0043561 | | | |

| | | |
|----|--------------|---------------|
| 9 | 12.8154798 | -692.5395758 |
| 10 | 63.2741991 | -807.9964274 |
| 11 | -143.0361923 | -3285.0110313 |
| 12 | -31.8460010 | 3752.2022530 |
| 13 | 13.1684692 | -1273.1845264 |
| 14 | 2.1090654 | 68.3326815 |
| 15 | -23.2345029 | 1080.0570220 |
| 16 | 43.9507797 | -419.8013988 |
| 17 | -21.1717404 | 1294.6073968 |
| 18 | 14.4389585 | 171.1633782 |
| 19 | 33.5237888 | -400.3785981 |
| 20 | -238.5814513 | -1674.8019070 |
| 21 | -5.5387982 | -392.0983161 |
| 22 | -36.3349657 | 2322.8883766 |
| 23 | -153.9085457 | -2391.2758980 |
| 24 | -20.3093843 | 160.9507127 |
| 25 | 19.9266467 | 2026.5723119 |
| 26 | -12.2274189 | 61.0764985 |
| 27 | 2.0018738 | -1050.3511605 |
| 28 | 79.1899425 | -207.2656486 |
| 29 | -25.3477614 | 763.5726894 |
| 30 | -46.3862783 | -149.3112233 |
| 31 | 67.9971488 | -504.3545467 |
| 32 | -2.2209447 | 164.1512440 |
| 33 | -8.2279044 | 18.1197954 |
| 34 | 9.8994703 | -63.6388056 |
| 35 | 1.8928601 | 220.5353017 |
| 36 | -20.6935779 | 69.9722591 |
| 37 | 5.2972360 | 892.7797500 |
| 38 | -493.0549498 | 184.3314797 |
| 39 | -37.6726504 | -88.7456534 |
| 40 | 51.7164144 | 127.0176326 |
| 41 | 17.3493946 | -51.6576396 |
| 42 | 0.9285681 | 15.8445083 |
| 43 | 6.7589078 | -20.1245768 |
| 44 | -49.5192857 | 2941.3795616 |
| 45 | -15.6270975 | 16.5482210 |
| 46 | 10.6772479 | -249.1828026 |
| 47 | 33.0882768 | -69.9512778 |
| 48 | 75.1930984 | 260.5664722 |
| 49 | 1.7850319 | -15.3444253 |
| 50 | 16.3212300 | 294.1595545 |
| 51 | 60.0329324 | -34.5861986 |

| | | |
|----|--------------|---------------|
| 52 | -15.9618439 | 644.0363022 |
| 53 | -7.9683363 | 225.9498833 |
| 54 | -2.2594154 | 317.6008356 |
| 55 | 87.9331587 | 28.7788092 |
| 56 | 96.9998673 | -580.9385131 |
| 57 | -97.6106500 | 705.3607373 |
| 58 | 563.5726227 | 1478.4579135 |
| 59 | 3.3665566 | 130.2661406 |
| 60 | 8.7794434 | 204.7068904 |
| 61 | 16.4998690 | 234.1200937 |
| 62 | -70.8812255 | 244.0959688 |
| 63 | -166.4756666 | -1780.4460196 |
| 64 | -9.1171521 | -189.7681137 |
| 65 | 49.4522043 | 2252.8593792 |
| 66 | -26.5294862 | -263.0507020 |
| 67 | 84.3622500 | -881.4447198 |
| 68 | -15.0899764 | 141.9766524 |
| 69 | -2.1305767 | -371.9539239 |
| 70 | -119.8919720 | -1415.2840764 |
| 71 | -56.6552769 | 68.8832005 |
| 72 | 7.5821381 | -74.3541828 |
| 73 | -87.5790767 | -492.7314059 |
| 74 | 66.1727186 | -332.0478063 |
| 75 | -20.7762586 | -194.9919234 |
| 76 | 33.9867023 | -387.3888571 |
| 77 | 41.1500436 | -147.5986488 |
| 78 | -22.3185448 | -725.3095029 |
| 79 | 18.6060238 | 177.1166843 |
| 80 | 36.7359623 | -697.9657928 |
| 81 | -4.7373806 | -72.5284163 |
| 82 | 188.2179428 | 334.1514606 |
| 83 | 20.2909967 | 296.8482299 |
| 84 | 194.5198924 | -1911.5797297 |
| 85 | -0.2395637 | 246.4357207 |
| 86 | 126.7148694 | -506.4295173 |
| 87 | -3.2141404 | 114.0597066 |
| 88 | -3.0128437 | 58.5811746 |
| 89 | 5.2928652 | -20.6427892 |
| 90 | -16.2574128 | 141.4172255 |
| 91 | 81.0234177 | -2.3554635 |
| 92 | 36.2930253 | -106.6175722 |
| 93 | -24.1422034 | 599.5361548 |
| 94 | 355.0357185 | 751.6703116 |

| | | |
|-----|-------------|-------------|
| 95 | 396.1132770 | 904.1556496 |
| 96 | -0.1716774 | -0.2752113 |
| 97 | 22.1913599 | 16.1309102 |
| 98 | -0.6733397 | 2.5000967 |
| 99 | -98.2511653 | 611.1052056 |
| 100 | -67.9086896 | 583.4102407 |

\$sigma

| | | | | | | | | |
|----------|----------|----------|----------|----------|----------|----------|----------|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 38621.12 | 38933.81 | 38777.32 | 38838.26 | 39053.19 | 38567.60 | 37649.06 | 38414.54 | |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | |
| 39112.23 | 38909.07 | 38238.28 | 38601.42 | 38502.10 | 39128.24 | 39090.05 | 39073.00 | |
| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | |
| 39079.36 | 39116.23 | 39124.09 | 38154.24 | 39121.94 | 38977.74 | 38911.83 | 39120.98 | |
| 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | |
| 39035.90 | 39127.48 | 39078.97 | 39060.86 | 38960.11 | 39101.69 | 38923.61 | 39115.18 | |
| 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | |
| 39128.15 | 39121.56 | 39112.11 | 39122.96 | 38783.15 | 35607.78 | 39092.23 | 39105.52 | |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | |
| 39026.20 | 39127.58 | 39112.92 | 38338.83 | 39125.04 | 39096.55 | 38990.09 | 38849.11 | |
| 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | |
| 39128.33 | 39100.32 | 38851.89 | 38959.51 | 39113.29 | 39056.20 | 38905.23 | 38781.59 | |
| 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | |
| 38970.85 | 38271.25 | 39076.96 | 39118.35 | 39085.69 | 39062.77 | 38431.46 | 39113.69 | |
| 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | |
| 38973.09 | 39039.15 | 38958.37 | 39121.99 | 39025.36 | 39074.85 | 39076.39 | 39123.98 | |
| 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | |
| 38892.40 | 38988.29 | 39108.22 | 39059.11 | 39116.68 | 38805.16 | 39106.39 | 38975.15 | |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | |
| 39123.32 | 38869.02 | 39087.94 | 39002.17 | 39104.13 | 38900.08 | 39118.55 | 39127.35 | |
| 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | |
| 39125.12 | 39122.29 | 39052.80 | 39113.09 | 38806.13 | 38561.25 | 38145.61 | 39128.41 | |
| 97 | 98 | 99 | 100 | | | | | |
| 39096.18 | 39128.21 | 38992.93 | 38707.51 | | | | | |

\$wt.res

| | | | | | |
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| 96 | 97 | 98 | 99 | 100 |
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