Project Week06, by Nurken Abeuov (NETID: na233)

Answer to Problem 1:

First, to calculate the time to maturity in years using calendar days, I need to find the number of days between the current date and the expiration date and then divide by the total number of days in a year (typically, it is 365 days).

Given:
- Current Date: 03/03/2023
- Expiration Date: 03/17/2023

Let's calculate it:

1. **Days to Maturity**: Calculate the number of days from March 3 to March 17, 2023.
2. **Convert to Years**: Divide the number of days by 365 to express this as a fraction of a year.

days_to_maturity = (expiration_date - current_date).days

T = days_to_maturity / 365

The time to maturity is **14 calendar days**, which translates to approximately **T = 0.0384** years when expressed as a fraction of a year.

Second, I will calculate the values of the call and put options across a range of implied volatilities (from 10% to 80%) using the Black-Scholes formula. Then, I will generate plots to show how the option values change with implied volatility.

Our parameters:

- Current Stock Price S = 165
- Risk-free rate r = 5.25%
- Continuously compounding coupon q = 0.53%
- Time to Maturity T = 0.0384

Also, I will assume K = S, assuming the strike price is the same as the stock price.

Black-Scholes Formula:

- For a call option:

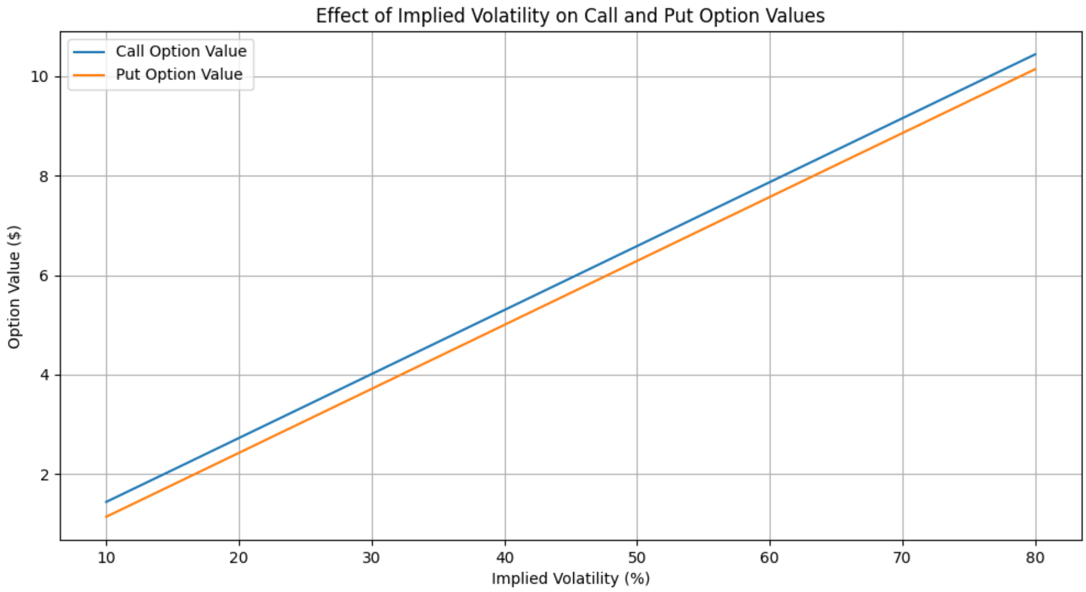
$$C = Se^{-qT}\,\Phi(d_1) - Ke^{-rT}\,\Phi(d_2)$$

- For a put option:

$$P = Ke^{-rT}\,\Phi(-d_2) - Se^{-qT}\,\Phi(-d_1)$$

Where:
- $d_1 = (\ln(S/K) + (r - q + \sigma^2/2) \times T) / (\sigma \times \sqrt{T})$
- $d_2 = d_1 - \sigma \times \sqrt{T}$

As a result, I will get a graph:



And, finally, let’s discuss how supply and demand influence implied volatility.

1. Impact of implied volatility on option values:

- **Rising Option Values:** As implied volatility increases from 10% to 80%, both the call and put option values grow. This increase reflects greater expected price movement of the underlying asset, which raises the likelihood that the options may finish in-the-money.
- **Risk Compensation:** Higher implied volatility suggests greater risk, which buyers compensate for by paying a premium. This premium makes options more valuable, as there is a broader range of potential future stock prices that could lead to profitable outcomes.

2. Supply and demand's influence on implied volatility:

- When there is high demand for options, such as in times of expected market volatility or economic uncertainty, investors are willing to pay higher prices for options as protection or speculation tools.
- The demand drives option prices up, which in turn raises the implied volatility because implied volatility is calculated based on the option’s current market price.
- If more investors are selling or writing options, such as in a calm market with low expected price movement, the increased supply of options pushes prices down. Lower option prices lead to lower implied volatility, as the market doesn’t anticipate major price changes.
- Lower implied volatility aligns with a perception of stability in the underlying asset price, reducing the premium that investors are willing to pay for options.

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Answer to Problem 2:

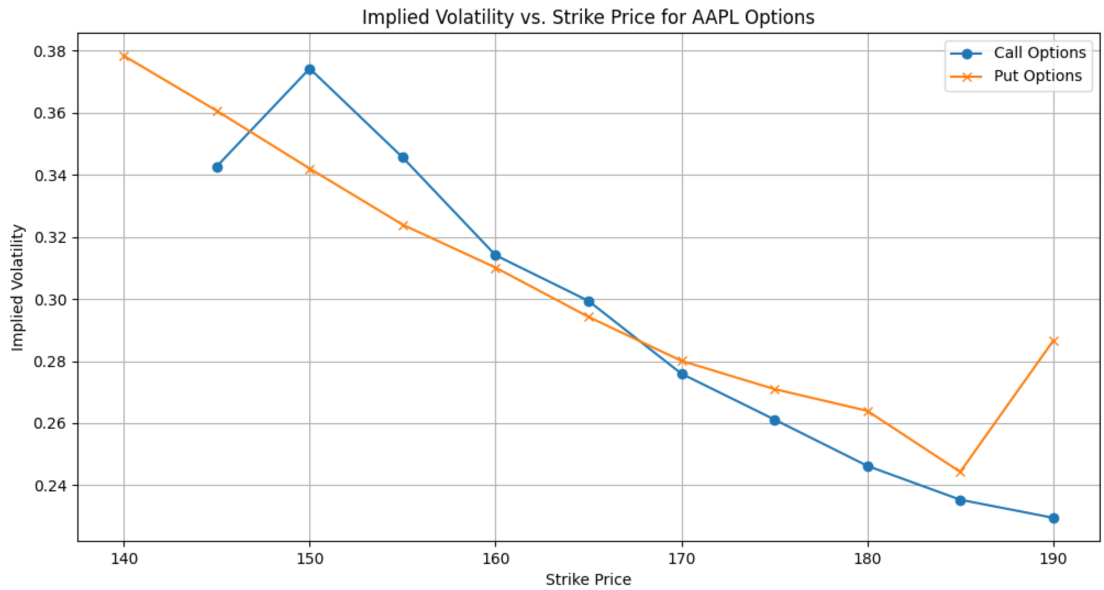
Initially, to solve for the implied volatility of each option for AAPL, I will begin by using data from the AAPL options CSV file, which contains details such as the expiration date, strike price, option type (call or put), and market price for each option. Additionally, I know the current stock price of AAPL, the risk-free rate, and the dividend yield, which are essential inputs for calculating option prices using the Black-Scholes model.

Implied volatility is the volatility level that aligns the theoretical price of an option—calculated via the Black-Scholes formula—with the observed market price of that option. Since I don't directly know this volatility, I need to iteratively adjust it to find the value that makes the Black-Scholes price match the market price.

The Black-Scholes model computes the theoretical option price using factors like the stock price (AAPL's current price of $170.15), strike price, time to expiration (derived from the expiration date in the CSV file), the risk-free rate of 5.25%, and the dividend yield of 0.57%. However, volatility (σ) isn't directly observable; instead, it's implied by the option's market price. Therefore, to find implied volatility, I will use a numerical method, adjusting σ until the Black-Scholes price equals the market price provided in the CSV file.

For each option, I will start by assuming an initial volatility and plug it into the Black-Scholes formula to compute the theoretical price. If this price doesn't match the market price, I will adjust the volatility and recalculate until I reach the value where the two prices converge. This iterative process yields the implied volatility for that specific option.

As a result, after plotting implied volatility and strike price, I can get a graph:



Discussion on the Shape of the Graphs

Typically, implied volatility curves exhibit shapes such as the volatility smile or volatility skew, depending on market dynamics and investor sentiment:

1. **Volatility Smile:** When both deep in-the-money (ITM) and out-of-the-money (OTM) options have higher implied volatilities compared to at-the-money (ATM) options, a smile shape appears.

This happens due to higher demand for protection in extreme scenarios, reflecting the market's expectations of larger price swings (or tail risks) for the stock.

2. **Volatility Skew (Smirk):** A common phenomenon in equity markets, where implied volatility increases as the strike price decreases, often producing an upward slope for put options. This reflects the higher perceived risk of a large downward move, driven by demand for put options as hedges against significant drops in stock prices.

Market Dynamics Contributing to the Graph Shapes

Implied volatility reflects the market's expectations for future volatility. Therefore, demand for options at different strikes impacts the implied volatility curve:

- **Risk Aversion:** Investors tend to hedge against extreme scenarios, leading to higher demand and thus higher implied volatilities for far OTM and ITM options, shaping the smile.
- **Bearish Sentiment:** Increased demand for put options (common in equity markets) can lead to a skew, where lower strike (put) options exhibit higher implied volatilities due to their use as downside protection.
- **Arbitrage-Free Pricing:** The theoretical no-arbitrage conditions, as discussed in options theory, ensure that implied volatilities remain consistent across strikes, supporting stable pricing models.

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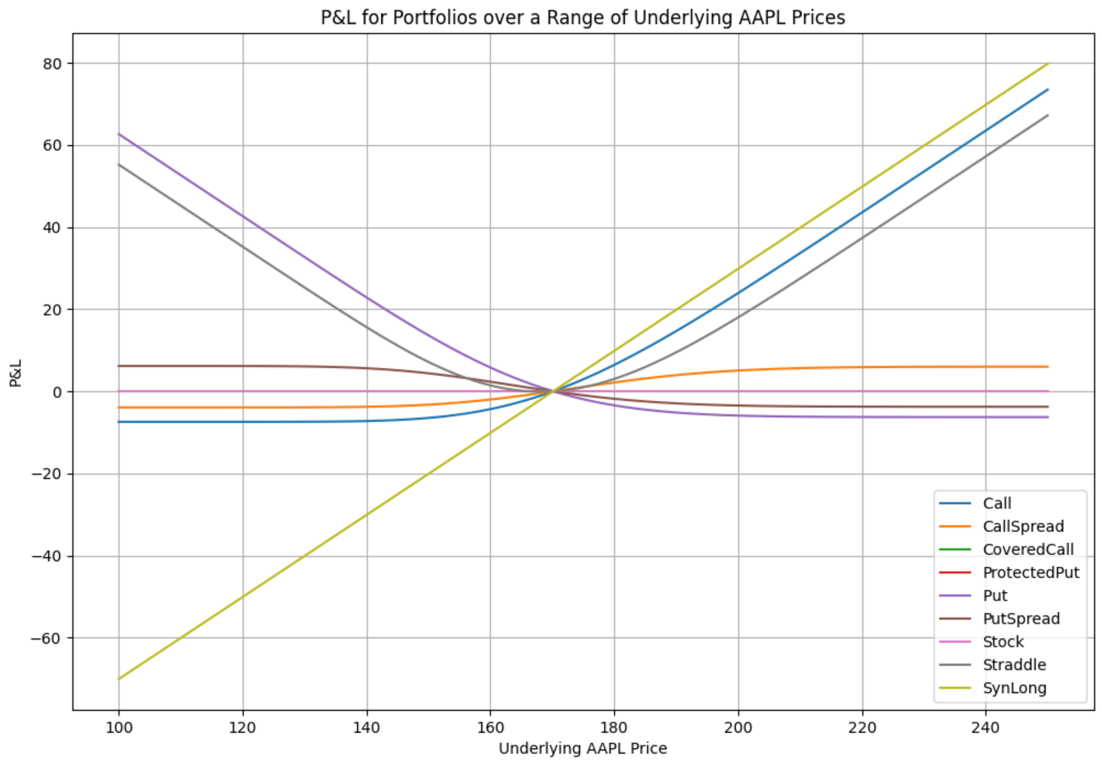
Answer to Problem 3:

To analyze the portfolio values and forecast AAPL stock performance, I approached the task in two main parts.

1. Portfolio Value Analysis Across Underlying Prices: I started with several option portfolios defined in `problem3.csv`, which provided data on each option's type (e.g., call or put), strike prices, holding amounts, and expiration details. Using this information, I calculated the value of each portfolio across a spectrum of potential AAPL prices. This allowed me to assess how each portfolio's value changes with fluctuations in AAPL's underlying stock price.

For example, each portfolio's payoff structure, whether it involved a combination of calls and puts or other option setups, yielded distinct value curves. By plotting these, I observed how certain portfolios profit from stock price increases, decreases, or volatility in either direction. And, this explained to me the observed shapes of each portfolio's payoff curve using the concept of put-call parity, which links the value of calls and puts under no-arbitrage conditions.

As a result, each portfolio in the graph below, based on its Profit and Loss (P&L) curve across a range of AAPL prices:



Analysis of the Portfolio Shapes Using Put-Call Parity

1. Call Portfolio: this portfolio consists of long call options. The P&L curve is upward sloping, starting from a slight negative value (due to the premium paid for the call).

As the AAPL price rises above the strike price, the portfolio's profit increases significantly, reflecting the unlimited upside potential of a long call option.

According to put-call parity, a long call position can be replicated by holding a put, the stock, and borrowing cash. Here, the curve's upward slope indicates a bet on the stock price rising above the strike.

2. Call Spread Portfolio: this is a combination of a long call and a short call at a higher strike. The P&L curve is capped on the upside.

The call spread profits when AAPL rises, but gains are limited by the short call. The curve flattens as the price goes above the higher strike price.

The call spread benefits from moderate increases in the stock price, and put-call parity indicates that a similar position could be replicated by using puts to cap the gains.

3. Covered Call Portfolio: this portfolio consists of holding the stock and selling a call option against it. The P&L curve rises initially but then levels off.

The covered call benefits from the stock rising to the call strike but limits profit as the stock price increases beyond this strike.

Selling a call against stock ownership creates a payoff similar to a short put, as the maximum upside is limited once the call option is exercised.

4. Protected Put Portfolio: it consists of holding the stock and a long put option for downside protection. The P&L curve flattens below a certain level.

The put protects against losses below the strike, providing a floor to the portfolio's value.

This portfolio is akin to a synthetic call since holding a stock and a put together replicates a long call. It hedges the downside, maintaining value as the stock price falls.

5. Put Portfolio: this is a portfolio of long put options. The P&L curve slopes downward as the AAPL price rises.

It profits if AAPL's price falls below the strike, with a maximum loss equal to the put premium if the price rises.

A long put position can be replicated by holding a call, shorting the stock, and lending cash. The downward slope indicates a bearish view.

6. Put Spread Portfolio: this portfolio combines a long put and a short put at a lower strike, creating a bear spread.

The portfolio benefits from a moderate decline in the stock price, with limited gains as the price falls below the lower strike.

The put spread limits gains on a bearish outlook, similar to a call spread limiting gains on a bullish position.

7. Stock Portfolio: this portfolio consists solely of AAPL stock. The P&L line is linear and slopes upward as the stock price increases.

The portfolio gains directly with AAPL's price without any caps or floors.

Stock ownership is straightforward and doesn't involve options, so put-call parity does not alter its payoff.

8. Straddle Portfolio: It consists of a long call and a long put at the same strike price. The P&L curve is "V"-shaped.

The straddle benefits from significant price movements in either direction, profiting if the stock moves far from the strike price.

The straddle reflects the combined payoff of calls and puts, maximizing value with high volatility.

9. Synthetic Long (SynLong) Portfolio: This is a synthetic position replicating stock ownership by combining options (typically long call and short put at the same strike).

The P&L curve for SynLong mimics the stock, showing a linear upward trend.

A synthetic long position replicates the stock by combining options, reflecting a similar payoff.

Finally, each portfolio's P&L curve is shaped by its option structure, and put-call parity helps in understanding how similar payoffs can be replicated using different combinations of options and the underlying asset. This concept of equivalency ensures no arbitrage opportunities between options and underlying assets in efficient markets. The graph effectively visualizes how put-call parity helps in structuring portfolios to achieve desired payoff profiles.

2. AAPL Stock Price Forecast Using an AR(1) Model: Using `DailyPrices.csv` data for historical AAPL prices, I calculated log returns and adjusted them to have a zero mean. I applied an AR(1) model, which uses past returns to predict future returns, to simulate AAPL prices over the next 10 days.

- **Mean Price**: The average of simulated prices, $186.20, suggests an upward bias over the period.
- **Value at Risk (VaR 95%)**: The 5th percentile price of $173.40 represents a conservative estimate for the minimum expected price.
- **Expected Shortfall (ES 95%)**: At $170.92, this metric provides the average of the worst-case simulated prices, indicating the risk of adverse price movements.

Between October 30 and November 9, 2023, Apple Inc. (AAPL) stock experienced notable movements. On October 30, 2023, AAPL closed at $170.15. By November 9, 2023, the stock had risen to approximately $175.00, reflecting a gain of about 2.85% over this period.

Comparison:

- **Mean Price:** The simulated mean price of $186.20 overestimates the actual closing price of $175.00 on November 9, 2023. This discrepancy suggests that the AR(1) model may have overpredicted the upward momentum of AAPL during this period.
- **Value at Risk (VaR 95%):** The VaR at 95% confidence is $173.40, indicating that there was a 5% chance the stock would fall below this price. Since the actual price remained above this threshold, the VaR estimate aligns with the observed price movements.
- **Expected Shortfall (ES 95%):** The ES of $170.92 represents the average of the worst 5% of simulated outcomes. Given that the actual price stayed above this level, the ES provides a conservative estimate of potential downside risk.

Conclusion:

While the AR(1) model's mean price forecast was higher than the actual outcome, the risk measures (VaR and ES) offered reasonable estimates of potential downside risk. This highlights the importance of using multiple metrics to assess stock performance and the inherent uncertainties in predictive modeling.