Project Week04, by Nurken Abeuov (NETID: na233)

Answer to Problem 1:

First, I will simulate 1000 instances of $r_t \sim N$ (0, 0.01) for each of the 3 price return systems: Classical Brownian Motion, Arithmetic Return System, and Log Return (or Geometric Brownian Motion).

For each type of return, I am going to compute P_t with the assumption that P_{t-1} = 100.

Then, I will calculate the mean and standard deviation of Pt from the simulations.

Finally, I will compare these results to the expected values and standard deviations.

Output of Price Return Simulations:

#	Return Type	Expected Value (Mean)	Standard Deviation
0	Classical Brownian	100.004638	0.097197
1	Arithmetic Return	100.463758	9.719694
2	Log Return	100.942037	9.877164

So, after simulating 3 types of price returns systems, let's compare the expected values (means) and standard deviations.

Classical Brownian Motion:

- Expected Value (Mean): 100.004638
- Standard Deviation: 0.097197

Conclusion: In this system, the price as an additive process $P_t = P_{t-1} + r_t$, which means changes in price are simply added directly. This results in low volatility (std. deviation ~0.097) and little deviation from P_{t-1}. However, it is noted that classical Brownian motion does not guarantee positive prices, which limits its use, particularly in financial modeling where prices must stay positive.

Arithmetic Return System:

- Expected Value (Mean): 100.463758
- Standard Deviation: 9.719694

Conclusion: This system models price changes as proportional to the previous price $P_t = P_{t-1} * (1 + r_t)$. It is more commonly used in portfolio management, as arithmetic returns are easier to aggregate across assets in a portfolio. The high std. deviation (~9.719) in our results reflects the greater price volatility due to this multiplicative effect.

Log Return (Geometric Brownian Motion):

- Expected Value (Mean): 100.942037
- Standard Deviation: 9.877164

Conclusion: The log return system uses an exponential function for returns $P_t = P_{t-1}\, e^{rt}$. This system is frequently used in long-term financial modeling, especially when analyzing prices over time since it ensures positive prices and models compounding returns. The higher mean (~100.94) reflects the slight upward drift due to exponential growth, which is typical for models assuming geometric Brownian motion. The standard deviation (~9.877) is slightly higher than arithmetic returns, indicating a higher potential for price volatility due to the compounding effect.

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Answer to Problem 2:

First, I implemented the *return_calculate()* function, which allows for return calculation using two methods: **arithmetic** and **logarithmic**. I used this function to calculate the arithmetic returns for all price data, and the results were saved in a file named *all_calculated_returns.csv*.

Next, I calculated the arithmetic returns for the 'META' stock and adjusted the series by removing the mean, ensuring that the mean of the META returns equals zero.

To compute the Value at Risk (VaR), I implemented separate functions for each method:

- var_normal() for Normal distribution;
- var_ewma() for Normal distribution with an Exponentially Weighted variance (λ = 0. 94);
- var_t_distribution() for MLE-fitted T-distribution;
- var_ar1() for AR(1) model;
- var_historical() for historical simulation.

VaR Results:

VaR (Normal Distribution)	-3.825%
VaR (Normal distribution with EWMA)	-3.099%
VaR (MLE fitted T distribution)	-0.075%
VaR (fitted AR(1) model)	-3.833%
VaR (Historical Simulation)	-2.946%

Conclusion:

The calculated VaR values for each method show some interesting differences:

1. Normal Distribution VaR (-3.825%) and AR(1) VaR (-3.833%) are very close. This shows that assuming normally distributed returns or using an AR(1) model both estimate a similar level of risk for the 'META' stock. Both models estimate a potential loss of around 3.8%, meaning we can expect to lose up to 3.8% in one of the worst 5% scenarios.
2. EWMA VaR (-3.099%) is slightly lower, which can be explained by the exponentially weighted moving average giving more emphasis to recent returns. This suggests that the recent volatility in META has been slightly lower than the long-term average.
3. MLE-fitted T-distribution VaR (-0.075%) is significantly lower than the others. This could indicate that the T-distribution might not be the best fit for this particular dataset, especially if the tails (extreme events) of the return distribution are not properly captured by this model.
4. Historical Simulation VaR (-2.946%) provides a more data-driven approach, and its lower VaR value compared to the normal VaR shows that, based on historical data, the expected worst-case loss is slightly less severe. This method makes no distributional assumptions and simply relies on past observed returns.

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Answer to Problem 3:

The VaR for each portfolio and the total holdings were calculated using an exponentially weighted covariance model with a lambda value of 0.97, and the code was implemented in the *week04_q3_exp_cov.py* file.

Results of the Exponentially Weighted Covariance VaR Calculation (λ = 0.97):

Holding	VaR ($)
Holding A	$6.63
Holding B	$7.72
Holding C	$5.52
Holding Total_VaR	$18.42

I chose the **historical simulation model** as a second approach to calculate VaR. This model is widely used in financial risk management because it does not assume any specific distribution for asset returns (unlike parametric VaR, which assumes normally distributed returns). Instead, it relies on historical return data.

Reasons for Choosing the Historical Simulation Model:

- **No Assumption of Normality:** Unlike parametric models, which assume normally distributed returns, the historical simulation method does not rely on this assumption. In real-world scenarios, returns are often skewed or have fat tails, and these characteristics are better captured by historical data.
- **Non-parametric:** The historical model simply sorts historical returns and chooses VaR based on a chosen quantile (e.g., the 5th percentile for 95% VaR), making it straightforward to understand and implement.
- **Data-Driven:** Since it uses actual historical returns, the model better reflects the true distribution of returns, including extreme events like financial crises.

The results of the historical simulation model were implemented in the *week04_q3_hist.py* file.

Results of the Historical Simulation VaR Calculation:

Holding	VaR ($)
Holding A (Historical Simulation)	$47.82
Holding B (Historical Simulation)	$52.23
Holding C (Historical Simulation)	$37.35
Holding Total_VaR (Historical Simulation)	$132.01

Comparison of VaR Results from Both Models:

The Exponentially Weighted Covariance Model resulted in significantly lower VaR values for each portfolio and the total holdings. For example, the total_VaR was $18.42, compared to $132.01 from the Historical Simulation Model. This discrepancy is primarily due to the assumptions underlying each model. The exponentially weighted covariance approach, which assumes normality in returns, tends to underestimate risk during periods of extreme market volatility. In contrast, the historical simulation method captures real historical market conditions, including extreme events, leading to higher VaR estimates. Consequently, the historical simulation model provides a more conservative and realistic estimate of potential losses, especially in times of market stress.

Effects of the Historical Simulation Model on the Results:

- **Potential Increase in VaR:** The historical simulation often results in a higher VaR because it considers actual extreme events (e.g., financial crises), which may not be captured well by the normal distribution assumed in the parametric approach.
- **Better Tail Risk Representation:** It captures the fat tails of return distributions, where extreme negative returns occur more frequently than predicted by a normal distribution.