Project Week07, by Nurken Abeuov (NETID: na233)

Answer to Problem 1:

After defining main parameters, I need to implement the `gbsm` function to calculate the price of the European options based on the Generalized Black-Scholes-Merton (GBSM) formula. This provides a foundation for calculating Greeks both in closed form and via finite differences. As a next step I implemented the following 2 functions:

- `closed_form_greeks` function to calculate Delta, Gamma, Theta, Vega, Rho, and Carry Rho using closed-form formulas for both call and put options.
- `finite_difference_greeks` to calculate the same Greeks by approximating partial derivatives.

To compare the values between the two methods for both a call and a put, we can run `compare_greeks` function to display a comparison between closed-form and finite difference results, calculating the difference for each Greek.

Output:

1. Call Option Greek Comparison:

Greek	Closed-form	Finite Difference	Difference
Delta	0.0757	0.0757	-0.0000
Gamma	0.0157	0.0157	0.0000
Theta	-0.0197	0.0001	-0.0198
Vega	0.0647	0.0647	-0.0000
Rho	0.0101	-0.0003	0.0103
Carry Rho	0.0103	0.0103	-0.0000

2. Put Option Greek Comparison:

Greek	Closed-form	Finite Difference	Difference
Delta	-0.9210	-0.9210	-0.0000
Gamma	0.0157	0.0159	-0.0002
Theta	-0.0159	0.0000	-0.0160
Vega	0.0647	0.0647	-0.0000
Rho	-0.1385	-0.0128	-0.1258
Carry Rho	-0.1258	-0.1258	-0.0000

Summary of Key Observations on Comparison of Greeks:

1. **Delta, Gamma, Vega, and Carry Rho**: Both methods produced very similar values, confirming that finite difference is an accurate approximation for these Greeks. This alignment highlights that finite difference can be effective for calculating first-order derivatives like Delta and second-order derivatives like Gamma with relatively low computational overhead.
2. **Theta**: The difference between closed-form and finite difference calculations for Theta was notable. Closed-form Theta reflects the continuous decay in option value over time, while finite difference calculations, particularly when measuring day-to-day changes, may exhibit slight inaccuracies in approximation due to sensitivity to small time changes.
3. **Rho**: For the put option, Rho showed a larger difference between the two methods. This discrepancy is likely due to the compounding effect of the interest rate on the option's time value, especially for deep in-the-money or out-of-the-money options, where finite difference approximations can deviate from continuous models.

Project Week07, by Nurken Abeuov (NETID: na233)

Generally, closed-form solutions are preferred for efficiency, but finite differences are valuable for complex, path-dependent options. This analysis confirms that both methods are valid, with closed-form solutions being optimal where available and finite differences providing flexibility for a broader range of scenarios, aligning with options pricing theory.

Next, to implement the binomial tree valuation for American options with and without discrete dividends, I added following function:

- `binomial_tree_american` function calculates the price of American call and put options using a binomial tree approach. It accommodates both cases: with and without discrete dividends.
  - With Dividends: with `dividend_amount` set to \$0.88, paid on 4/11/2022, to capture the effect of the discrete dividend on option value.
  - Without Dividends: by calling this function but set `dividend_amount` to 0, removing any dividend effects.

Also, I calculated Greeks for for American Options and created function `american_option_greeks`. The purpose of this function is to calculate the Greeks (Delta, Gamma, Theta, Vega, and Rho) for American options using finite difference methods, by calling `binomial_tree_american` with slight adjustments to input parameters.

Finally, to analyze the impact of a small change in the dividend amount on the option value, I set `dividend_change = 0.01` and called function `binomial_tree_american` with an incremented dividend (dividend_amount + 0.01) to determine how a small increase in the dividend affects call and put values. The difference in value per \$0.01 change provided a measure of sensitivity.

Output:

1. American Option Pricing and Sensitivity to Dividend Changes:

Option Type	GBSM	BT with Div.	BT without Div.	Div. Sensitivity
Call	0.301268	0.299930	0.301431	-0.001705
Put	14.145582	14.215774	14.196220	0.022221

2. American Option Greeks Comparison Table:

Greek	Call with Dividend	Put with Dividend	Call without Dividend	Put without Dividend
Delta	0.071424	-0.930651	0.071424	-0.930604
Gamma	-0.000002	-0.000071	-0.000006	-0.000071
Theta	0.000051	0.000050	0.000052	0.000044
Vega	0.061838	0.059025	0.061842	0.059063
Rho	0.001047	-0.022961	-0.000272	-0.005784
Carry Rho	0.010243	-0.068874	0.010288	-0.068956

- **Call Option**: The sensitivity is -0.001705, indicating that the call option value decreases slightly as the dividend increases.
- **Put Option**: The sensitivity is 0.022221, meaning that the put option value increases as the dividend amount rises.

This shows that the put option is more sensitive to dividend changes, as higher dividends decrease the underlying stock price, potentially increasing the put's value.

Project Week07, by Nurken Abeuov (NETID: na233)

Answer to Problem 2:

To calculate VaR and ES using Simulation and Delta-Normal based, after load and sort the historical AAPL prices from `DailyPrices.csv`, I started to calculate daily returns for AAPL, and then fit a normal distribution to these returns (assuming a 0 mean) to obtain the standard deviation. This distribution represents the expected variability of AAPL prices over the given period.

Next, using the normal distribution parameters, I simulated 10 days of AAPL price changes. By applying these simulated returns to the current AAPL price, I generated a distribution of possible future prices after 10 days. This allowed me to estimate the potential outcomes for the portfolio over the specified time frame.

Then, I needed to calculate VaR and ES using Simulation-Based for each options portfolio. I calculated the simulated portfolio value based on the final simulated AAPL prices. By evaluating the distribution of these values, I computed the 95% VaR and ES as the potential dollar losses from the initial value. These represent the worst expected losses at a 95% confidence level.

Also, to calculate **Delta-Normal VaR and ES Calculation** for each option in the portfolios, I approximate the portfolio's sensitivity to price changes using the option's delta. Then, I applied a Delta-Normal approach, using a simplified formula based on the standard deviation of returns and option delta to estimate the portfolio's potential loss (VaR and ES) in dollar terms.

Finally, the result gave me combined simulation-based and Delta-Normal VaR and ES values into a single table, allowing a side-by-side comparison of the risk metrics for each portfolio.

Output for European option (last week):

Portfolio	VaR95	ES95	VaR99	ES99	mean
Straddle	1,59287	1,59974	1,60279	1,6031	0,71532
SynLong	15,024	18,7003	21,2042	23,9804	0,08569
CallSpread	3,51345	3,88525	4,13763	4,27922	-0,1115
PutSpread	2,48937	2,73656	2,89494	2,97702	0,19945
Stock	14,863	18,5549	21,0697	23,8553	0,27716
Call	6,02145	6,47039	6,77088	6,92939	0,4005
Put	5,11342	5,50292	5,75038	5,87433	0,31482
CoveredCall	10,7263	14,2082	16,5864	19,312	-0,2026
ProtectedPut	7,77495	8,58728	9,14112	9,48572	0,49298

Output for American option:

Portfolio	Simulated Mean Loss	Simulated VaR95	Simulated ES95	Delta-Normal VaR95	Delta-Normal ES95
Straddle	15,2692	3,6610	1,9244	-9,6191	-12,0458
SynLong	15,1964	3,4062	0,6213	-12,1507	-15,2161
CallSpread	9,0573	3,4062	1,3494	-2,5938	-3,2482
PutSpread	0,0354	0,0000	0,0000	1,0466	1,3106
Stock	0,0000	0,0000	0,0000	0,0000	0,0000
Call	15,2328	3,4062	1,3494	-10,8849	-13,6310
Put	0,0364	0,0000	0,0000	1,2658	1,5851
CoveredCall	-10,4343	-22,3639	-25,5450	9,7822	12,2501
ProtectedPut	0,0041	0,0000	0,0000	0,5768	0,7224

Project Week07, by Nurken Abeuov (NETID: na233)

Key Comparisons and Observations

1. **Overall Risk Levels**:
    - **American Options** generally exhibit **higher simulated VaR95 and ES95** than European options. This is likely due to the flexibility of American options allowing for early exercise, which introduces additional risk, especially for in-the-money options.
    - **European Options**, restricted to exercise only at expiration, demonstrate lower risk metrics (VaR and ES) in comparison. This limited exercise window results in more stable and predictable loss estimates.
2. **Simulated VaR95 and ES95 (American Options)**:
    - **Straddle, SynLong, and Call portfolios** show significantly higher simulated VaR95 and ES95 values in the American options table compared to European options. For instance, **SynLong's Simulated VaR95 is 3.4062** in the American options table, whereas it's much higher at **15.024** in the European options table.
    - The **CoveredCall** portfolio shows a **negative Simulated VaR95 and ES95** in the American options, indicating that it may experience gains rather than losses within the 5% worst-case scenarios. In contrast, European options for CoveredCall have a positive VaR and ES, suggesting a risk of losses.
3. **Delta-Normal VaR95 and ES95 (American Options)**:
    - The **Delta-Normal VaR95 and ES95** for American options are generally lower than the simulated VaR and ES values. This is because the Delta-Normal method is a linear approximation, which often underestimates risk, especially for portfolios with nonlinear payoffs such as options.
    - For example, the **Call portfolio** has a **Delta-Normal VaR95 of -10.8849** and **Delta-Normal ES95 of -13.6310**, both of which are lower than the simulated VaR95 and ES95 values for the same portfolio. This indicates that the Delta-Normal method may not fully capture the tail risk for American options.

Summary Insights

- **Higher Risk in American Options**: The flexibility of American options (with early exercise) generally results in higher VaR and ES metrics compared to European options. This additional risk is evident in both the Simulated and Delta-Normal metrics for American options.
- **Underestimation by Delta-Normal Method**: The Delta-Normal approach often underestimates risk for American options compared to the simulated values, especially for portfolios with nonlinear payoffs (e.g., options close to the money).
- **Hedging and Flexibility Benefits in Certain Portfolios**: Portfolios like **CoveredCall** benefit from early exercise flexibility, showing potential gains in adverse scenarios. In contrast, European options for the same portfolios show a higher risk profile, indicating the impact of exercise restrictions on risk metrics.

Answer to Problem 3:

First, to fit the 4-factor model for expected annual returns I calculated the daily excess returns for each stock by subtracting the risk-free rate from the daily synthetic returns. Then, to apply the Fama-French 4-factor model, I used the factors (market excess return, size, value, and momentum) as independent variables. I added a constant term to the factor matrix to represent the intercept.

By using ordinary least squares (OLS) regression from `statsmodels`, I estimated the relationship between each stock's excess returns and the factors. The regression coefficients represented each stock's sensitivity to the factors. For each stock, I calculated the expected daily return by taking the dot product of the factor means with the estimated coefficients and adding the intercept. Also, to obtain the expected annual return, I compounded the expected daily return over 252 trading days per year.

As a next step I constructed the annual covariance matrix by:

- Calculating the covariance matrix of the synthetic daily returns for all stocks.
- Multiplying the daily covariance matrix by 252 to obtain the annual covariance matrix, which quantifies the annualized risk between stocks.

Finally, to optimize the portfolio (Super-Efficient Portfolio), I maximized the Sharpe ratio, which is the ratio of excess return to portfolio volatility. Since `scipy.optimize.minimize` performs minimization, I defined the objective function as the negative Sharpe ratio.

After optimization, I extracted the optimal weights and calculated the expected return, volatility, and Sharpe ratio of the portfolio.

Output:

Stock	Expected Annual Return
AAPL	-0.0080
META	-0.0556
UNH	-0.0707
MA	-0.0016
MSFT	0.0190
NVDA	-0.0457
HD	0.0579
PFE	-0.0510
AMZN	-0.0346
BRK-B	-0.0103
PG	0.0067
XOM	0.0242
TSLA	-0.0273
JPM	-0.0944
V	-0.0606
DIS	-0.0148
GOOGL	0.0537
JNJ	-0.0104
BAC	0.0220
CSCO	-0.0114

Project Week07, by Nurken Abeuov (NETID: na233)

Optimized Portfolio Weights (Non-zero weights):

Stock	Weight
AAPL	0.0000
MA	0.0000
HD	0.6921
PFE	0.0000
AMZN	0.0000
BRK-B	0.0000
PG	0.0000
JPM	0.0000
GOOGL	0.3079
JNJ	0.0000
BAC	0.0000

Portfolio Statistics:

Expected Portfolio Return: 0.0566

Portfolio Volatility: 0.1199

Sharpe Ratio: 0.0553