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1. Consider a particle of mass *m* in one dimension with a linear potensial *V* = *kx*. The Lagrangian is therefore

$$L = \frac{1}{2m}\dot{x}^2 - kx$$

(a) Write the Hemilton-Jacobi equation for *S*.

(b) Write a separation of variables ansatz for *S* and use this ansatz to find an expression for *S* (you may not want to solve all the integrals occurring in *S*)

(c) Use the result of (b) to find the variables *x* and *p* explicitly as functions of time, with two free constants appearing in your solution.

Solution:

(a) Diketahui persamaan Hemilton-jacoby:

$$H\left(x^i, \frac{\partial S}{\partial x^j}, t\right) + \frac{\partial S}{\partial t} = 0$$

Sehingga:

$$H = \frac{1}{2m}\dot{x}^2 + kx$$

$$\frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2 + kx + \frac{\partial S}{\partial t} = 0 \qquad (1)$$

(b) Solusi separasi variable S adalah:

$$S(x, \alpha, t) = W(x, \alpha) - \alpha t \qquad (2)$$

Subtitusikan nilai S pada persamaan (1) yang didapatkan dari pertanyaan (a), maka:

$$\frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2 + kx + \frac{\partial S}{\partial t} = 0$$

$$\frac{1}{2m}\left(\frac{\partial}{\partial x}[W(x,\alpha) - \alpha t]\right)^2 + kx + \frac{\partial}{\partial t}[W(x,\alpha) - \alpha t] = 0$$

$$\frac{1}{2m}\left(\frac{\partial}{\partial x}[W(x,\alpha) - \alpha t]\right)^2 = -kx - \left(\frac{\partial}{\partial t}[W(x,\alpha) - \alpha t]\right)$$

$$\frac{1}{2m}\left(\frac{\partial(W(x,\alpha))}{\partial x} - \frac{\partial(-\alpha t)}{\partial x}\right)^2 = -kx - \left(\frac{\partial(W(x,\alpha))}{\partial t} + \frac{\partial(-\alpha t)}{\partial t}\right)$$

$$\frac{1}{2m}\left(\frac{\partial(W(x,\alpha))}{\partial x} - 0\right)^2 = -kx - (0 + (-\alpha))$$

$$\left(\frac{\partial(W(x,\alpha))}{\partial x}\right)^2 = 2m(-kx + \alpha)$$

$$\frac{\partial W(x,\alpha)}{\partial x} = \sqrt{2m(-kx + \alpha)}$$

$$\int dW(x,\alpha) = \int \left(\sqrt{2m(-kx + \alpha)}\right) dx$$

Nurkhaliq Futhra Maulana
H032211005

$$W(x,\alpha) = \int \left(\sqrt{2m(-kx+\alpha)}\right) dx \qquad (3)$$

Subtitusikan persamaan (3) ke persamaan (2), sehingga didapatkan nilai S yaitu:

$$S(x,\alpha,t) = \int \left(\sqrt{2m(-kx+\alpha)}\right) dx - \alpha t$$

(c) * Diketahui $Q = \beta = \frac{\partial S(x,\alpha,t)}{\partial \alpha}$ maka,

$$\beta = \int \left(\frac{\partial}{\partial \alpha}\left[\sqrt{2m(-kx+\alpha)}\right]\right) dx - \frac{\partial}{\partial \alpha}[\alpha t]$$

$$= \int \left(\frac{1}{2}(2m)\left(\frac{1}{\sqrt{2m(-kx+\alpha)}}\right)\right) dx - t$$

$$= \int \left(\frac{m}{\sqrt{2m(-kx+\alpha)}}\right) dx - t$$

$$= \frac{1}{k}\sqrt{2m(-kx+\alpha)} - t$$

untuk nilai x(t),

$$\beta = \frac{1}{k}\sqrt{2m(-kx+\alpha)} - t$$

$$\beta + t = \frac{1}{k}\sqrt{2m(-kx+\alpha)}$$

$$k(\beta+t) = \sqrt{2m(-kx+\alpha)}$$

$$[k(\beta+t)]^2 = 2m(-kx+\alpha)$$

$$\frac{1}{2m}[k(\beta+t)]^2 =- kx+\alpha$$

$$-kx = \frac{1}{2m}[k(\beta+t)]^2 - \alpha$$

$$x = \frac{\alpha}{k} - \frac{1}{k}\frac{1}{2m}[k(\beta+t)]^2$$

* Diketahui $p = \frac{\partial S(x,\alpha,t)}{\partial x}$ maka,

$$p = \int \left(\frac{\partial}{\partial x}\left[\sqrt{2m(-kx+\alpha)}\right]\right) dx - \frac{\partial}{\partial x}[\alpha t]$$

$$= \int \left(\frac{\partial}{\partial x}\left[\sqrt{2m(-kx+\alpha)}\right]\right) dx - 0$$

$$p = \sqrt{2m(-kx+\alpha)}$$

2. In the case of a free particle, for which the Hamiltonian is

$$H = \frac{p^2}{2m}$$

Find the Hemilton-Jacobi, Hamilton's characteristic function *W,* and variables *q* and *p* explicitly as functions of time.

Solution:

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H032211005

** Hemilton-jacoby dari $H = \frac{p^2}{2m}$ yaitu:

$$\frac{1}{2m}\left(\frac{\partial S}{\partial q}\right)^2 + \frac{\partial S}{\partial t} = 0$$

$$\frac{1}{2m}\left(\frac{\partial S}{\partial q}\right)^2 =- \frac{\partial S}{\partial t} \tag{4}$$

** Solusi separasi variable S adalah:

$$S(q, \alpha, t) = W(q, \alpha) - \alpha t \tag{5}$$

Subtitusikan nilai S pada persamaan (4) yang didapatkan dari pertanyaan (a), maka:

$$\frac{1}{2m}\left(\frac{\partial S}{\partial q}\right)^2 = -\frac{\partial S}{\partial t}$$

$$\frac{1}{2m}\left(\frac{\partial}{\partial q}[W(q, \alpha) - \alpha t]\right)^2 = -\frac{\partial}{\partial t}[W(1, \alpha) - \alpha t]$$

$$\frac{1}{2m}\left(\frac{\partial(W(q, \alpha))}{\partial q} - \frac{\partial(-\alpha t)}{\partial q}\right)^2 = -\left(\frac{\partial(W(q, \alpha))}{\partial t} + \frac{\partial(-\alpha t)}{\partial t}\right)$$

$$\frac{1}{2m}\left(\frac{\partial(W(q, \alpha))}{\partial q} - 0\right)^2 = (0 + (-\alpha))$$

$$\left(\frac{\partial(W(q, \alpha))}{\partial q}\right)^2 = 2m\alpha$$

$$\frac{\partial W(q, \alpha)}{\partial q} = \sqrt{2m\alpha}$$

$$\int dW(q, \alpha) = \int \left(\sqrt{2m\alpha}\right) dq$$

$$W(q, \alpha) = \sqrt{2m\alpha} q \tag{6}$$

Persamaan (6) merupakan fungsi karakteristik Hamiltonian dari $H = \frac{p^2}{2m}$:

** Subtitusikan persamaan (3) ke persamaan (2), sehingga didapatkan nilai S yaitu:

$$S(q, \alpha, t) = \sqrt{2m\alpha} q \ - \alpha t$$

Diketahui $Q = \beta = \frac{\partial S(q,\alpha,t)}{\partial \alpha}$ maka,

$$\begin{aligned}
\beta &= \left(\frac{\partial}{\partial \alpha}\left[\sqrt{2m\alpha}\right] q + \frac{\partial}{\partial \alpha}[q]\sqrt{2m\alpha}\right) - \frac{\partial}{\partial \alpha}[\alpha t] \\
&= \left(\frac{1}{2}(2m)\left(\frac{1}{\sqrt{2m\alpha}}\right) q + 0\right) - t \\
&= \left(\frac{m}{\sqrt{2m\alpha}}\right) q - t
\end{aligned}$$

Nurkhaliq Futhra Maulana

H032211005

untuk nilai q(t),

$$\beta = \left(\frac{m}{\sqrt{2m\alpha}}\right) q - t$$

$$\beta + t = \left(\frac{m}{\sqrt{2m\alpha}}\right) q$$

$$(\beta + t) = \sqrt{\frac{m}{2\alpha}} q$$

$$q = (\beta + t)\sqrt{\frac{2\alpha}{m}}$$

* Diketahui $p = \frac{\partial S(q,\alpha,t)}{\partial q}$ maka,

$$p = \left(\frac{\partial}{\partial q}\left[\sqrt{2m\alpha}\right] q + \frac{\partial}{\partial q}[q]\sqrt{2m\alpha}\right) - \frac{\partial}{\partial q}[\alpha t]$$

$$= \left(0 + \frac{\partial}{\partial q}[q]\sqrt{2m\alpha}\right) - \frac{\partial}{\partial q}[\alpha t]$$

$$p = \sqrt{2m\alpha}$$

Berikut saya lampirkan beberapa contoh soal yang berkaitan dengan hemilton-jacobi yang menurut saya menarik, tapi saya masih agak kurang faham prof

https://bit.ly/H-J_Example