Conditional Random Fields

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Содержание

- Conditional Random Field, что это такое?
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Определение CRFs

Х – наблюдаемые величины, Ү –ответы.

Пусть G = (V, E) – граф, такой что $Y = (Y_v)_{v \in V}$, то есть Y индексируется вершинами G.

Тогда (X, Y) — условное случайное поле (Conditional Random Field), если случайные величины Y_v , обусловленные X, удовлетворяют марковскому свойству относительно графа G:

 $p(Y_v|X,Y_w,w\neq v)=p(Y_v|X,Y_w,w\sim v)$, где $w\sim v$ означает, что w и v соседние вершины в G.

Linear-chain CRFs

Definition 2.2. Let Y, X be random vectors, $\theta = \{\theta_k\} \in \mathbb{R}^K$ be a parameter vector, and $\mathcal{F} = \{f_k(y, y', \mathbf{x}_t)\}_{k=1}^K$ be a set of real-valued feature functions. Then a linear-chain conditional random field is a distribution $p(\mathbf{y}|\mathbf{x})$ that takes the form:

$$p(\mathbf{y}|\mathbf{x}) = \frac{1}{Z(\mathbf{x})} \prod_{t=1}^{T} \exp \left\{ \sum_{k=1}^{K} \theta_k f_k(y_t, y_{t-1}, \mathbf{x}_t) \right\}, \qquad (2.18)$$

where $Z(\mathbf{x})$ is an input-dependent normalization function

$$Z(\mathbf{x}) = \sum_{\mathbf{y}} \prod_{t=1}^{T} \exp \left\{ \sum_{k=1}^{K} \theta_k f_k(y_t, y_{t-1}, \mathbf{x}_t) \right\}.$$
 (2.19)

Linear-chain CRFs

Notice that a linear chain CRF can be described as a factor graph over \mathbf{x} and \mathbf{y} , i.e.,

$$p(\mathbf{y}|\mathbf{x}) = \frac{1}{Z(\mathbf{x})} \prod_{t=1}^{T} \Psi_t(y_t, y_{t-1}, \mathbf{x}_t)$$
 (2.20)

where each local function Ψ_t has the special log-linear form:

$$\Psi_t(y_t, y_{t-1}, \mathbf{x}_t) = \exp\left\{\sum_{k=1}^K \theta_k f_k(y_t, y_{t-1}, \mathbf{x}_t)\right\}.$$
 (2.21)

Parameter Learning

Параметр heta находится максимизацией правдоподобия $p(Y_i|X_i;\; heta)$.

Если все вершины G из экспоненциального семейства распределений и все вершины доступны во время обучения, то данная задача выпукла. Она может быть решена с помощью метода градиентного спуска, Квази-Ньютоновских методов.

Parameter Learning

$$\ell(\theta) = \sum_{i=1}^{N} \log p(\mathbf{y}^{(i)}|\mathbf{x}^{(i)};\theta). \tag{5.1}$$

To compute the maximum likelihood estimate, we maximize $\ell(\theta)$, that is, the estimate is $\hat{\theta}_{\text{\tiny ML}} = \sup_{\theta} \ell(\theta)$.

After substituting in the CRF model (2.18) into the likelihood (5.1), we get the following expression:

$$\ell(\theta) = \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{k=1}^{K} \theta_k f_k(y_t^{(i)}, y_{t-1}^{(i)}, \mathbf{x}_t^{(i)}) - \sum_{i=1}^{N} \log Z(\mathbf{x}^{(i)}),$$
 (5.3)

Parameter Learning

Regularization

$$\ell(\theta) = \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{k=1}^{K} \theta_k f_k(y_t^{(i)}, y_{t-1}^{(i)}, \mathbf{x}_t^{(i)}) - \sum_{i=1}^{N} \log Z(\mathbf{x}^{(i)}) - \sum_{k=1}^{K} \frac{\theta_k^2}{2\sigma^2}.$$
(5.4)

Области применения

- Text processing (NER, finding semantic roles in text, morphological analysis)
- Computer Vision (gesture recognition, semantic segmentation)
- Bioinformatics (RNA structural alignment, protein structure prediction)

- The New York Times, the White House.
- U.N. official Ekeus heads for Baghdad. U.N. organization, Ekeus person, Baghdad location.
- BIO notation: B first word of a mention, I any subsequent words in the mention, O words that do not reference any named entity.

CoNLL 2003 data set

 $\mathcal{Y} = \{B-PER, I-PER, B-Loc, I-Loc, B-ORG, I-ORG, B-MISC, I-MISC, O\}$

With this labeling, our example sentence looks like:

t	y_t	\mathbf{x}_t
0	B-ORG	U.N.
1	O	official
2	B-PER	Ekeus
3	O	heads
4	O	for
5	B-LOC	Baghdad

• set F of feature function $f_k(y_t, y_{t-1}, x_t)$

$$f_{ij}^{LL}(y_t, y_{t-1}, \mathbf{x}_t) = \mathbf{1}_{\{y_t = i\}} \mathbf{1}_{\{y_{t-1} = j\}} \forall i, j \in \mathcal{Y}.$$
 (2.29)

For this problem, there are 9 different labels, so there are 81 label—label features. The second kind are *label-word* features, which are

$$f_{iv}^{\text{LW}}(y_t, y_{t-1}, \mathbf{x}_t) = \mathbf{1}_{\{y_t = i\}} \mathbf{1}_{\{\mathbf{x}_t = v\}} \forall i \in \mathcal{Y}, v \in \mathcal{V},$$
 (2.30)

where V is the set of all unique words that appear in the corpus. For the CoNLL 2003 English data set, there are 21,249 such words, so there are 191,241 label-word features. Most of these features will not be very

• set F of feature function $f_k(y_t, y_{t-1}, x_t)$

t	y_t	\mathbf{x}_t
0	B-ORG	((START), U.N., official)
1	O	(U.N., official, Ekeus)
2	B-PER	(official, Ekeus, heads)
3	O	(Ekeus, heads, for)
4	O	(heads, for, Baghdad)
5	B-LOC	(for, Baghdad, (END))

• set F of feature function $f_k(y_t, y_{t-1}, x_t)$

t	y_t	\mathbf{x}_t
0	B-ORG	((START), U.N., official)
1	O	(U.N., official, Ekeus)
2	B-PER	(official, Ekeus, heads)
3	O	(Ekeus, heads, for)
4	O	(heads, for, Baghdad)
5	B-LOC	(for, Baghdad, (END))

• set F of feature function $f_k(y_t, y_{t-1}, x_t)$

$$f_{iv}^{\text{LW0}}(y_t, y_{t-1}, \mathbf{x}_t) = \mathbf{1}_{\{y_t = i\}} \mathbf{1}_{\{x_{t0} = v\}} \quad \forall i \in \mathcal{Y}, v \in \mathcal{V}$$

$$f_{iv}^{\text{LW1}}(y_t, y_{t-1}, \mathbf{x}_t) = \mathbf{1}_{\{y_t = i\}} \mathbf{1}_{\{x_{t1} = v\}} \quad \forall i \in \mathcal{Y}, v \in \mathcal{V}$$

$$f_{iv}^{\text{LW2}}(y_t, y_{t-1}, \mathbf{x}_t) = \mathbf{1}_{\{y_t = i\}} \mathbf{1}_{\{x_{t2} = v\}} \quad \forall i \in \mathcal{Y}, v \in \mathcal{V}.$$

However, we wish to use still more features than this, which will depend mostly on the word x_t . So we will add label-observation features, as decribed in Section 2.5. To define these, we will define a series of observation functions $q_b(x)$ that take a single word x as input. For each observation function q_b , the corresponding label-observation features f_{ib}^{LO} have the form:

$$f_{ib}^{\text{LO}}(y_t, y_{t-1}, \mathbf{x}_t) = \mathbf{1}_{\{y_t = i\}} q_b(\mathbf{x}_t) \quad \forall i \in \mathcal{Y}. \tag{2.31}$$

Table 2.2. A subset of observation functions $q_s(\mathbf{x},t)$ for the CoNLL 2003 Engilsh named-entity data, used by Mccallum and Li [86].

```
W=v
                                                                              \forall v \in \mathcal{V}
                    w_t = v
T=j
                    part-of-speech tag for w_t is j (as determined by an
                                                                              ∀POS tags j
                      automatic tagger)
P=I-j
                    w_t is part of a phrase with syntactic type j (as
                      determined by an automatic chunker)
Capitalized
                    w_t matches [A-Z] [a-z]+
Allcaps
                    w_t matches [A-Z] [A-Z] +
EndsInDot
                    w_t matches [^\.]+.*\.
                    w_t contains a dash
                    w_t matches [A-Z]+[a-z]+[A-Z]+[a-z]
                    w_t matches [A-Z] [A-Z\\.]*\\. [A-Z\\.]*
Acro
Stopword
                    w_t appears in a hand-built list of stop words
CountryCapital
                    w_t appears in list of capitals of countries
                    many other lexicons and regular expressions
q_k(\mathbf{x}, t + \delta) for all k and \delta \in [-1, 1]
```

Примеры: Image Labelling

- Пусть вектор $x=(x_1,x_2,...,x_T)$ представляет изображение размера $\sqrt{T} \times \sqrt{T}$. Пусть изображение серое, то есть каждый пиксель принимает значения от 0 до 255, означающих яркость.
- Задача: предсказать вектор $y = (y_1, y_2, ..., y_T)$, где y_i равно метке класса. Пусть y_T может быть 0 или 1, означающие передний и задний план на изображении.

Примеры: Image Labelling

• Let $q(x_i)$ be a vector of features based on a region of the image around x_i , for example, using color histograms or image gradients.

$$f_{m}(y_{i}, x_{i}) = \mathbf{1}_{\{y_{i} = m\}} q(x_{i}) \quad \forall m \in \{0, 1\}$$

$$g_{m,m'}(y_{i}, y_{j}, x_{i}, x_{j}) = \mathbf{1}_{\{y_{i} = m\}} \mathbf{1}_{\{y_{j} = m'\}} \nu(x_{i}, x_{j}) \quad \forall m, m' \in \{0, 1\}$$

$$f(y_{i}, x_{i}) = \begin{pmatrix} f_{0}(y_{i}, x_{i}) \\ f_{1}(y_{i}, x_{i}) \end{pmatrix}$$

$$g(y_{i}, y_{j}, x_{i}, x_{j}) = \begin{pmatrix} g_{00}(y_{i}, y_{j}, x_{i}, x_{j}) \\ g_{01}(y_{i}, y_{j}, x_{i}, x_{j}) \\ g_{10}(y_{i}, y_{j}, x_{i}, x_{j}) \\ g_{11}(y_{i}, y_{j}, x_{i}, x_{j}) \end{pmatrix}$$

Примеры: Image Labelling

• N define the neighborhood relationship among pixels.

$$\nu(x_i, x_j) = \exp\left\{-\beta(x_i - x_j)^2\right\}$$

$$g(y_i, y_j, x_i, x_j) = \mathbf{1}_{\{y_i \neq y_j\}} \nu(x_i, x_j). \tag{2.33}$$

Putting this all together, the CRF model is

$$p(\mathbf{y}|\mathbf{x}) = \frac{1}{Z(\mathbf{x})} \exp \left\{ \sum_{i=1}^{T} \theta^{\top} f(y_i, x_i) + \sum_{(i,j) \in \mathcal{N}} \lambda^{\top} g(y_i, y_j, x_i, x_j) \right\},$$
(2.34)