Sublabel-accurate α-expansion

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Start **simple**, improve **later**:

on multi-label energy minimization problems

BACKGROUND:

The goal of multi-label energy minimization is to find a labeling u, which is both **piece-wise smooth** and **consistent with the observed data**, i.e.

$$\underset{u:\Omega\to\mathcal{L}}{\operatorname{argmin}} E_{data}(u) + E_{smooth}(u), \text{ where}$$
$$E_{data}(u) = \sum_{i\in\Omega} E_i(u^i),$$
$$E_{smooth}(u) = \sum_{(i,j)\in\mathcal{E}} E_{ij}(u^i, u^j).$$

The given energy functions E_i, E_{ij} are **defined on a discrete set of labels.** However, we aim to find solutions in-between labels, i.e. **sublabel-accurate.**

***α*-expansion [1]** is a fast approximate discrete solver based on graph cuts.

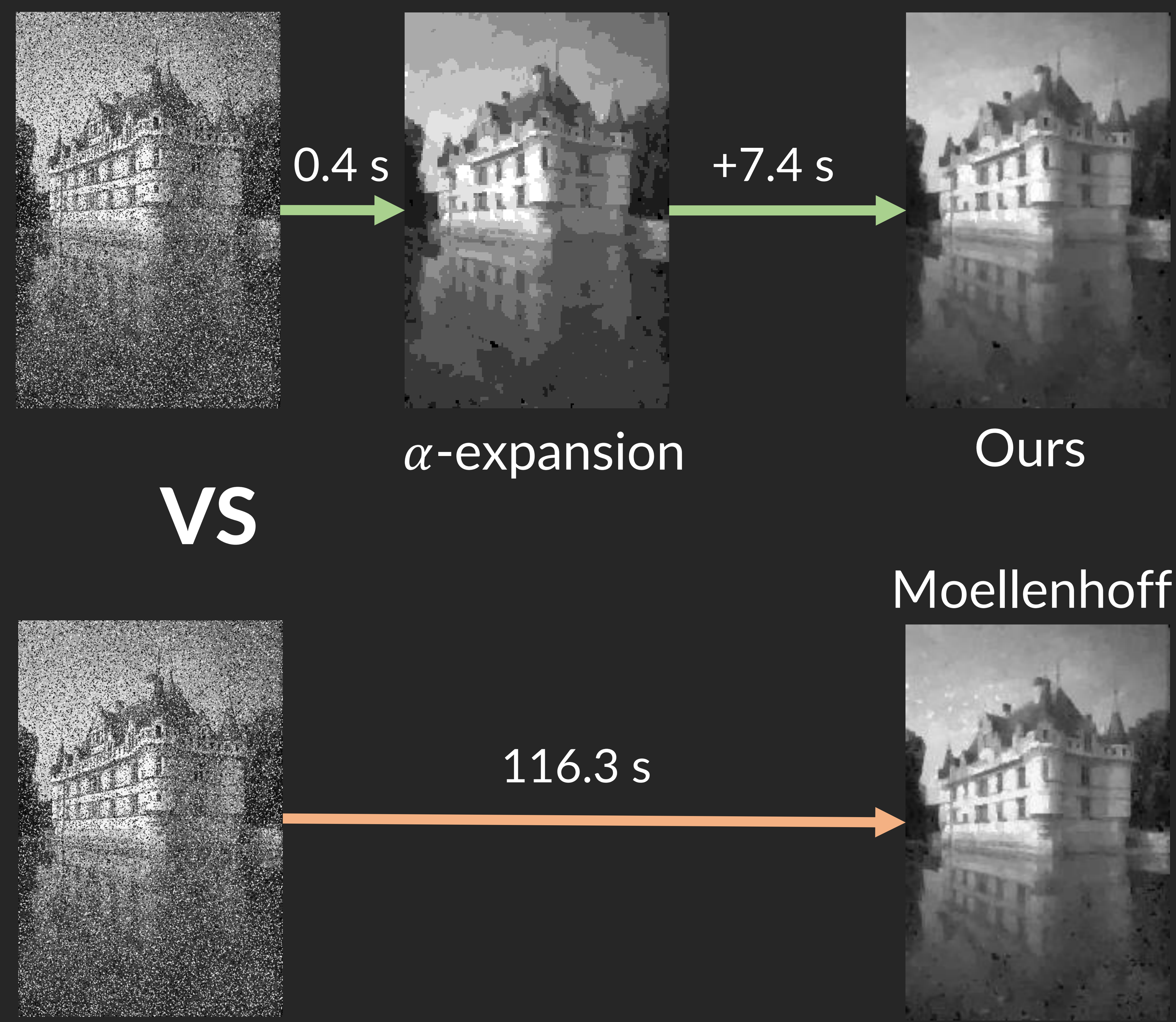
METHODS

1. Initialize with discrete solution $\boldsymbol{u}_d$,
2. Define its neighborhood $\Gamma(\boldsymbol{u}_d)$ in continuous label space,
3. Approximate energies E_i, E_{ij} on $\Gamma(\boldsymbol{u}_d)$ with convex functions $\hat{E}_i, \hat{E}_{ij}$,
4. **Solve the convex problem** $\to \boldsymbol{u}^*$.

Theorem (Optimality preservation)

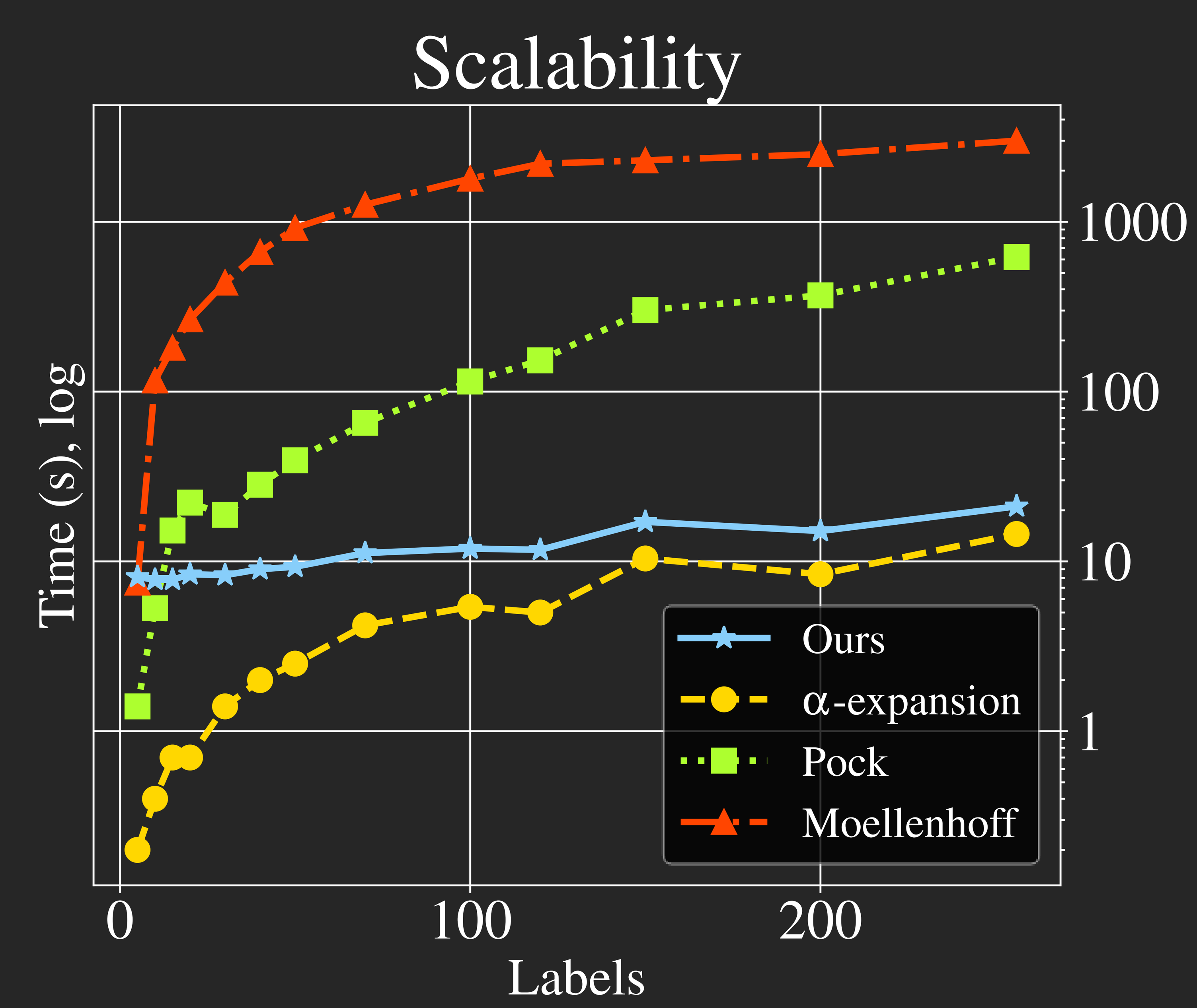
If the local convex approximations $\hat{E}_i, \hat{E}_{ij}$ are continuations of the discrete functions E_i, E_{ij} on continuous label range $\Gamma(\boldsymbol{u}_d)$, then

$$E(\boldsymbol{u}^*) \le E(\boldsymbol{u}_d).$$



- ✓ Sublabel-accurate
- ✓ Efficient
- ✓ Optimality guarantees

- ✓ Sublabel-accurate
- ▪ Expensive
- ▪ No guarantees



Full paper

Continuous Refinement

Assuming the discretized label space $\mathcal{L}_d = \{l_1, \ldots, l_L\}$, we define the *marginalization set* $\mathcal{M}$:

$$\mathcal{M} = \{\boldsymbol{\phi} \mid \Sigma_l \phi_i(l) = 1, \; \Sigma_m \phi_{ij}(l,m) = \phi_i(l), \; \Sigma_l \phi_{ij}(l,m) = \phi_{j(m)}\}$$

(LM): local *linear* data with *marginalization* constraint:

$$\min_{\boldsymbol{\phi}} \sum E_i(l)\phi_i(l) + \sum E_{ij}(l,m)\phi_{ij}(l,m),$$
$$s.t.\, \boldsymbol{\phi} \in [0,1]^N \cap \mathcal{M} \cap \mathcal{S}(\Gamma(\boldsymbol{u}_d))$$

(QM): local *quadratic* data with *marginalization*:

$$\min_{\boldsymbol{\phi},X} \sum Q_i(X_i) + \sum E_{ij}(l,m)\phi_{ij}(l,m),$$
$$s.t.\, \boldsymbol{\phi} \in [0,1]^N \cap \mathcal{M} \cap \mathcal{S}(\Gamma(\boldsymbol{u}_d)),$$
$$\Sigma_l \phi_i(l) \cdot l = X_i \in \Gamma_i(\boldsymbol{u}_d)$$

(QL): local *quadratic* data with *convex* kernel:

$$\min_{X} \sum Q_i(X_i) + \sum \kappa_{ij}(|X_i - X_j|),$$
$$s.t.\, X_i \in \Gamma_i(\boldsymbol{u}_d)$$

Image Denoising

$$E_i(u^i) = \frac{\beta}{2}\min\left\{\left(u^i - f(x_i)\right)^2, \nu\right\},$$
$$E_{ij}(u^i, u^j) = \lambda |u^i - u^j|.$$

Discontinuity Preserving Non-Convex Regularizers

$$E_{ij}(u^i, u^j) = \lambda \min\left(|u^i - u^j|^k, T\right)$$

Method	$k=1$, $T=0.6$ PSNR ↑	$k=2$, $T=0.7$ PSNR ↑
α-expansion [1]	23.93	23.43
Ours	25.13	25.59
Pock [2]	n/a	n/a
Moellenhoff [3]	n/a	n/a

References:

[1] **Boykov** et al., "Fast approximate energy minimization via graph cuts", 2001.

[2] **Pock** et al., "Global solutions of variational models with convex regularization", 2010.

[3] **Moellenhoff** et al., "Sublabel-accurate relaxation of nonconvex energies", 2016.