

Computational Companion to “Flexible 3×3 Nets of Equimodular Elliptic Type” — Example 3

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Tested on: Mathematica 14.0

```
In[1]:= (*=====*)
(*====*)
(*=====*)
(*====*)
(*=====*)
(*====*)
(*Quit*)
(*All angle sets in degrees*)
anglesDeg = {
  {91.32487959870187, 27.53122212644406,
   103.21844931187813, 120}, (*Vertex 1*)
  {115.75047063536742,
   29.335366103921295, 109.7807695499394, 80}, (*Vertex 2*)
  {31.19200181228523,
   113.66642350596918, 89.61760260813426, 85}, (*Vertex 3*)
  {28.19551791700786, 107.67391515450669,
   121.75654544610931, 75} (*Vertex 4*)};

(*-----*)
(*Function to compute sigma from 4 angles*)
computeSigma[{ $\alpha$ _,  $\beta$ _,  $\gamma$ _,  $\delta$ _}] := ( $\alpha$  +  $\beta$  +  $\gamma$  +  $\delta$ ) / 2;

(*-----*)
(*Function to compute a,b,c,d from angles*)
computeABCD[{ $\alpha$ _,  $\beta$ _,  $\gamma$ _,  $\delta$ _}] :=
Module[{alpha =  $\alpha$  Degree, beta =  $\beta$  Degree, gamma =  $\gamma$  Degree,
  delta =  $\delta$  Degree, sigma}, sigma = computeSigma[{ $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ ] Degree];
{Sin[alpha] / Sin[sigma - alpha], Sin[beta] / Sin[sigma - beta],
 Sin[gamma] / Sin[sigma - gamma], Sin[delta] / Sin[sigma - delta]};

(*-----*)
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(*Compute all  $\sigma$  values*)
sigmas = computeSigma /@ anglesDeg;

(*Compute all {a,b,c,d} values*)
results = computeABCD /@ anglesDeg;

(*Optional:extract individual values*)
{a1, b1, c1, d1} = results[[1]];
{a2, b2, c2, d2} = results[[2]];
{a3, b3, c3, d3} = results[[3]];
{a4, b4, c4, d4} = results[[4]];
{ $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$ ,  $\sigma_4$ } = FullSimplify[sigmas];

(*=====
====*)
(*=====
CONDITION (N.0) =====*)
(*=====
====*)
(*uniqueCombos={{1,1,1,1},{1,1,1,-1},{1,1,-1,-1},
{1,1,-1,1},{1,-1,1,1},{1,-1,-1,1},{1,-1,1,-1},{1,-1,-1,-1}};

checkConditionN0Degrees[{ $\alpha$ _, $\beta$ _, $\gamma$ _, $\delta$ _}]:=Module[
{angles={ $\alpha$ , $\beta$ , $\gamma$ , $\delta$ },results},results=Mod[uniqueCombos.angles,360]//Chop;
!MemberQ[results,0]];

conditionsN0=checkConditionN0Degrees/@anglesDeg;
allVerticesPass=And@@conditionsN0;

Column[{TextCell[Style["===== CONDITION (N.0) =====",
Darker[Green],Bold, 16],"Text"],
If[allVerticesPass,
Style["✓ All vertices satisfy (N.0).",Darker[Green],Bold],
Style["✗ Some vertices fail (N.0).",Red,Bold]]]*),
uniqueCombos = {{1, 1, 1, 1}, {1, 1, 1, -1}, {1, 1, -1, -1}, {1, 1, -1, 1},
{1, -1, 1, 1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, -1, -1, -1}};

checkConditionN0Degrees[{ $\alpha$ _,  $\beta$ _,  $\gamma$ _,  $\delta$ _}] :=
Module[{angles = { $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ }, results},
results = Mod[uniqueCombos.angles, 360] // Chop;
results];

(*apply to all vertices*)
resultsPerVertex = checkConditionN0Degrees /@ anglesDeg;

(*check pass/fail*)

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conditionsN0 = Map[! MemberQ[#, 0] &, resultsPerVertex];
allVerticesPass = And@@conditionsN0;

Column[{TextCell[Style["===== CONDITION (N.0) =====",
  Darker[Green], Bold, 16], "Text"], (*summary line*)If[allVerticesPass,
  Style["✓ All vertices satisfy (N.0).", Darker[Green], Bold],
  Style["✗ Some vertices fail (N.0).", Red, Bold]],
  (*detailed results for each vertex*)
  Grid[Prepend[Table[{"Vertex " <> ToString[i], resultsPerVertex[[i],
    If[conditionsN0[[i], "✓ Pass", "✗ Fail"]}, {i, Length[anglesDeg]}],
    {"Vertex", "Combinations (mod 360)", "Status"}], Frame → All]]]

(*=====*)
(*=====
  CONDITION (N.3)=====*)
(*=====*)
Ms = FullSimplify[Times@@@results];
allEqualQ = Simplify[Equal@@Ms];

Column[{TextCell[Style["===== CONDITION (N.3) =====",
  Darker[Blue], Bold, 16], "Text"], If[allEqualQ,
  Row[{Style["✓ M1 = M2 = M3 = M4 = ", Bold], Ms[[1]]},
  Style["✗ M_i are not all equal.", Red, Bold]]]}]

(*=====*)
(*=====CONDITION (N.4)=====*)
(*=====*)
aList = results[[All, 1]];
cList = results[[All, 3]];
dList = results[[All, 4]];

rList = FullSimplify /@ (aList * dList); {r1, r2, r3, r4} = rList;
sList = FullSimplify /@ (cList * dList); {s1, s2, s3, s4} = sList;

Column[{TextCell[Style["===== CONDITION (N.4) =====",
  Darker[Purple], Bold, 16], "Text"],
  If[r1 == r2 && r3 == r4 && s1 == s4 && s2 == s3, Column[
    {Row[{Style["✓ r1 = r2 = ", Bold], r1, Style["; ✓ r3 = r4 = ", Bold], r3}],
    Row[{Style["✓ s1 = s4 = ", Bold], s1, Style["; ✓ s2 = s3 = ", Bold],
    s2}]]], Style["✗ Condition (N.4) fails.", Red, Bold]]
}]

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(*=====
====*)
(*=====
CONDITION (N.5)=====*)
(*=====
====*)
fList = FullSimplify /@ (aList * cList); {f1, f2, f3, f4} = fList;
M1 = Ms[[1]];

m[M1_] := Piecewise[{{1 - M1, M1 < 1}, {(M1 - 1) / M1, M1 > 1}}] // N;
K = EllipticK[m[M1]] // N;
Kp = EllipticK[1 - m[M1]] // N;

computeTi[i_] :=
Module[{sigma = sigmas[[i]], r = rList[[i]], s = sList[[i]], f = fList[[i]], base},
  base = I * Im[InverseJacobiDN[Piecewise[
    {{Sqrt[f], M1 < 1}, {1 / Sqrt[f], M1 > 1}}], m[M1]]] / Kp;
  Which[sigma < 180, Which[r > 1 && s > 1, base, (r < 1 && s > 1) || (r > 1 && s < 1),
    1 + base, r < 1 && s < 1, 2 + base], sigma > 180, Which[r > 1 && s > 1,
    2 + base, (r < 1 && s > 1) || (r > 1 && s < 1), 3 + base, r < 1 && s < 1, base]]];

tList = computeTi /@ Range[4];
RoundWithTolerance[x_, tol_ : 10^(-15)] :=
Module[{nearest}, nearest = Round[x];
  If[Abs[x - nearest] ≤ tol, nearest, x]];

checkValidCombination[M1_, ε_ : 10^(-15)] :=
Module[{i, combo, dotProd, rePart, imPart, n2, expr, foundQ = False},
  Print[Style["△ Approximate validation using ε-tolerance. For rigorous
    proof, see the referenced paper.", Darker@Orange, Italic]];
  Do[combo = uniqueCombos[[i]];
    dotProd = tList.combo;
    rePart = Abs[Re[dotProd]];
    imPart = Abs[Im[dotProd]];
    If[M1 < 1,
      If[Mod[RoundWithTolerance[rePart], 4] < ε,
        If[Mod[RoundWithTolerance[imPart], 2] < ε, expr =
          tList[[1]] + combo[[2]] × tList[[2]] + combo[[3]] × tList[[3]] + combo[[4]] × tList[[4]];
          Print[Style["✔ Valid Combination Found (M < 1):",
            Darker[Green], Bold], "\n", Style["e1 = ", Bold], combo[[2]],
            Style["", e2 = ", Bold], combo[[3]], Style["", e3 = ", Bold], combo[[4]],
            "\n", Style["t1 = ", Bold], Re[tList[[1]]], "K + ", Im[tList[[1]]], "iK'",
            "\n", Style["t2 = ", Bold], Re[tList[[2]]], "K + ", Im[tList[[2]]], "iK'",
            "\n", Style["t3 = ", Bold], Re[tList[[3]]], "K + ", Im[tList[[3]]], "iK'",

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"\n", Style["t4 = ", Bold], Re[tList[[4]], "K + ", Im[tList[[4]],
"iK'", "\n", Style["t1 + e1*t2 + e2*t3 + e3*t4 = ", Bold],
Re[expr], "K + ", Im[expr], "iK'"];
foundQ = True;
Break[]]]];
If[M1 > 1,
If[Mod[RoundWithTolerance[imPart], 2] < ε,
n2 = Quotient[RoundWithTolerance[imPart], 2];
If[Mod[RoundWithTolerance[Abs[rePart - 2 n2]], 4] < ε, expr =
tList[[1]] + combo[[2]] × tList[[2]] + combo[[3]] × tList[[3]] + combo[[4]] × tList[[4]];
Print[Style["✔ Valid Combination Found (M > 1):",
Darker[Green], Bold], "\n", Style["e1 = ", Bold], combo[[2]],
Style["e2 = ", Bold], combo[[3]], Style["e3 = ", Bold], combo[[4]],
"\n", Style["t1 = ", Bold], Re[tList[[1]], "K + ", Im[tList[[1]], "iK'",
"\n", Style["t2 = ", Bold], Re[tList[[2]], "K + ", Im[tList[[2]], "iK'",
"\n", Style["t3 = ", Bold], Re[tList[[3]], "K + ", Im[tList[[3]], "iK'",
"\n", Style["t4 = ", Bold], Re[tList[[4]], "K + ", Im[tList[[4]],
"iK'", "\n", Style["t1 + e1*t2 + e2*t3 + e3*t4 = ", Bold],
Re[expr], "K + ", Im[expr], "iK'"];
foundQ = True;
Break[]]]], {i, Length[uniqueCombos]}}];
If[! foundQ, Print[Style["✗ No valid combination found.", Red, Bold], "\n",
Style["t1 = ", Bold], Re[tList[[1]], "K + ", Im[tList[[1]], "iK'", "\n",
Style["t2 = ", Bold], Re[tList[[2]], "K + ", Im[tList[[2]], "iK'", "\n",
Style["t3 = ", Bold], Re[tList[[3]], "K + ", Im[tList[[3]], "iK'", "\n",
Style["t4 = ", Bold], Re[tList[[4]], "K + ", Im[tList[[4]], "iK'", "\n"]];];];

Column[{TextCell[Style["===== CONDITION (N.5) =====",
Darker[Red], Bold, 16], "Text"]}]
res = checkValidCombination[M1];

(*=====
====*)
(*=====
OTHER PARAMETER=====*)
(*=====
====*)
cosSigmaAlt[Mi_, fi_, ri_, si_] :=
(Mi (fi + ri + si - 1) + ri fi si - ri fi - ri si - fi si) / (2 Sqrt[ri si fi] (1 - Mi));

Column[
{TextCell[Style["===== OTHER PARAMETERS =====",
Darker[Orange], Bold, 16], "Text"],
Row[{Style["u = ", Bold], 1 - M1}],

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Row[{Style["σ1 = ", Bold], σ1 Degree, Style["", σ2 = ", Bold], σ2 Degree,
  Style["", σ3 = ", Bold], σ3 Degree, Style["", σ4 = ", Bold], σ4 Degree}],
Row[{Style["σ1 ≈ ", Bold], N[σ1], Style["°", Bold], Style["", σ2 ≈ ", Bold],
  N[σ2], Style["°", Bold], Style["", σ3 ≈ ", Bold], N[σ3],
  Style["°", Bold], Style["", σ4 ≈ ", Bold], N[σ4], Style["°", Bold]}],
Row[{Style["cosσ1 = ", Bold], Simplify@cosSigmaAlt[M1, f1, r1, s1],
  Style["", cosσ2 = ", Bold], Simplify@cosSigmaAlt[M1, f2, r2, s2],
  Style["", cosσ3 = ", Bold], Simplify@cosSigmaAlt[M1, f3, r3, s3],
  Style["", cosσ4 = ", Bold], Simplify@cosSigmaAlt[M1, f4, r4, s4]}],
Row[{Style["f1 = ", Bold], f1, Style["", f2 = ", Bold],
  f2, Style["", f3 = ", Bold], f3, Style["", f4 = ", Bold], f4}],
Row[{Style["z1 = ", Bold], FullSimplify[1 / (f1 - 1)], Style["", z2 = ", Bold],
  FullSimplify[1 / (f2 - 1)], Style["", z3 = ", Bold], FullSimplify[1 / (f3 - 1)],
  Style["", z4 = ", Bold], FullSimplify[1 / (f4 - 1)]}],
Row[{Style["x1 = ", Bold], FullSimplify[1 / (r1 - 1)], Style["", x2 = ", Bold],
  FullSimplify[1 / (r2 - 1)], Style["", x3 = ", Bold], FullSimplify[1 / (r3 - 1)],
  Style["", x4 = ", Bold], FullSimplify[1 / (r4 - 1)]}],
Row[{Style["y1 = ", Bold], FullSimplify[1 / (s1 - 1)], Style["", y2 = ", Bold],
  FullSimplify[1 / (s2 - 1)], Style["", y3 = ", Bold], FullSimplify[1 / (s3 - 1)],
  Style["", y4 = ", Bold], FullSimplify[1 / (s4 - 1)]}],
Row[{Style["p1 = ", Bold], Simplify[Sqrt[r1 - 1]],
  Style["", p2 = ", Bold], Simplify[Sqrt[r2 - 1]], Style["", p3 = ", Bold],
  Simplify[Sqrt[r3 - 1]], Style["", p4 = ", Bold], Simplify[Sqrt[r4 - 1]]}],
Row[{Style["q1 = ", Bold], Simplify[Sqrt[s1 - 1]],
  Style["", q2 = ", Bold], Simplify[Sqrt[s2 - 1]], Style["", q3 = ", Bold],
  Simplify[Sqrt[s3 - 1]], Style["", q4 = ", Bold], Simplify[Sqrt[s4 - 1]]}],
Row[{Style["p1.q1 = ", Bold], Simplify[Sqrt[r1 - 1] Sqrt[s1 - 1]],
  Style["", p2.q2 = ", Bold], Simplify[Sqrt[r2 - 1] Sqrt[s2 - 1]],
  Style["", p3.q3 = ", Bold], Simplify[Sqrt[r3 - 1] Sqrt[s3 - 1]],
  Style["", p4.q4 = ", Bold], Simplify[Sqrt[r4 - 1] Sqrt[s4 - 1]]}]
}]

(*=====
====*)
(*=====
BRICARD's EQUATIONS=====*)
(*=====
====*)
(*=====
FLEXION 1=====*)
Z[t_] := t;

W1[t_] := (2.2689737907253456` (10.50393110877767` t - 2.542115267025096`
  Sqrt[(1 - 0.5721009760467486` t^2) (-1 + 7.594436687028424` t^2)]) /
  (14.662902846473798` + 7.594436687028424` t^2);

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U[t_] := (0.20405148639750417` (1.1778865025892307` t + 10.102197432038`
  Sqrt[(1 - 0.5721009760467486` t^2) (-1 + 7.594436687028424` t^2)]) /
  (3.6622434357916323` - 2.099237700754583` t^2);
W2[t_] := (1.3011995890502461` (5.832687748284718` t - 1.6606733509905591`
  Sqrt[(1 - 0.5721009760467486` t^2) (-1 + 7.594436687028424` t^2)]) /
  (3.588495042139688` + 7.594436687028424` t^2);

```

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(*General polynomial constructor*)
BricardsEquation[{α_, β_, γ_, δ_}, σ_, x_, y_] :=
Module[{c22, c20, c02, c11, c00},
  c22 = Sin[σ - δ] Sin[σ - δ - β];
  c20 = Sin[σ - α] Sin[σ - α - β];
  c02 = Sin[σ - γ] Sin[σ - γ - β];
  c11 = -Sin[α] Sin[γ];
  c00 = Sin[σ] Sin[σ - β];
  c22 x^2 y^2 + c20 x^2 + c02 y^2 + 2 c11 x y + c00];

(*Compute and print all P_i for flexion 1*)
TextCell[
  Style["===== FLEXIBILITY (FLEXION 1) =====",
    Darker[Cyan], Bold, 16], "Text"]
funcs = {{Z, W1}, {Z, W2}, {U, W2}, {U, W1}};
Do[angles = anglesDeg[[i]] Degree;
  sigma = sigmas[[i]] Degree;
  {α, β, γ, δ} = angles;
  poly = Which[i == 1, BricardsEquation[{α, β, γ, δ}, sigma, Z[t], W1[t]],
    i == 2, BricardsEquation[{α, β, γ, δ}, sigma, Z[t], W2[t]],
    i == 3, BricardsEquation[{α, β, γ, δ}, sigma, U[t], W2[t]],
    i == 4, BricardsEquation[{α, β, γ, δ}, sigma, U[t], W1[t]]];
  Print[Row[{Subscript["P", i], "[", funcs[[i, 1],
    ", ", funcs[[i, 2]], "]" = ", FullSimplify[poly]]}], {i, 1, 4}];

(*t-range*)
tMin = 0.4;
tMax = 1.25;

(*Build P_i(t) consistently with your printing logic*)
Clear[PiExpr, labels, funcs];
funcs = {{Z, W1}, {Z, W2}, {U, W2}, {U, W1}};

PiExpr = Table[Module[{angles = anglesDeg[[i]] Degree,
  sigma = sigmas[[i]] Degree, α, β, γ, δ, poly}, {α, β, γ, δ} = angles;
  poly = BricardsEquation[{α, β, γ, δ}, sigma, funcs[[i, 1]] [t], funcs[[i, 2]] [t]]];

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FullSimplify[poly]], {i, 1, 4}];

labels = Table[Row[{Subscript["P", i], "["},
ToString@funcs[[i, 1]], ", ", ToString@funcs[[i, 2]], "]" }], {i, 1, 4}];

(*Pretty panel with plots like your style*)
Column[{TextCell[
  Style["===== FLEXIBILITY (FLEXION 1) =====",
    Darker[Cyan], Bold, 16], "Text"], TextCell[
  Style["Polynomials P_i(t) built from Bricard's equations for flexion 1.",
    GrayLevel[0.3]], "Text"], Spacer[12],
  Panel[GraphicsGrid[Partition[Table[Plot[Evaluate@PiExpr[[i],
    {t, tMin, tMax}], PlotLabel -> Style[labels[[i]], Bold, 14],
    PlotRange -> {-10^(-11), 10^(-11)}, AxesLabel -> {"t", None},
    ImageSize -> 250], {i, Length[PiExpr]}], 2], Spacings -> {2, 2}],
  Background -> Lighter[Gray, 0.95], FrameMargins -> 15]]]

(*=====
FLEXION 2=====*)
Z2[t_] := t;
W12[t_] := 
$$\frac{\left(2.2689737907253456 \left(10.50393110877767 t + 2.542115267025096 \sqrt{(1 - 0.5721009760467486 t^2)(-1 + 7.594436687028424 t^2)}\right)\right)}{(14.662902846473798 + 7.594436687028424 t^2)}$$
;
U2[t_] := 
$$\frac{\left(0.20405148639750417 \left(1.1778865025892307 t - 10.102197432038 \sqrt{(1 - 0.5721009760467486 t^2)(-1 + 7.594436687028424 t^2)}\right)\right)}{(3.6622434357916323 - 2.099237700754583 t^2)}$$
;
W22[t_] := 
$$\frac{\left(1.3011995890502461 \left(5.832687748284718 t + 1.6606733509905591 \sqrt{(1 - 0.5721009760467486 t^2)(-1 + 7.594436687028424 t^2)}\right)\right)}{(3.588495042139688 + 7.594436687028424 t^2)}$$
;

(*General polynomial constructor*)
BricardsEquation[{α_, β_, γ_, δ_}, σ_, x_, y_] :=
Module[{c22, c20, c02, c11, c00},
  c22 = Sin[σ - δ] Sin[σ - δ - β];
  c20 = Sin[σ - α] Sin[σ - α - β];
  c02 = Sin[σ - γ] Sin[σ - γ - β];
  c11 = -Sin[α] Sin[γ];
  c00 = Sin[σ] Sin[σ - β];
  c22 x^2 y^2 + c20 x^2 + c02 y^2 + 2 c11 x y + c00];

(*Compute and print all P_i for flexion 2*)
TextCell[

```



```

Style["===== FLEXIBILITY (FLEXION 2) =====",
  Cyan, Bold, 16], "Text"]
funcs = {{Z, W1}, {Z, W2}, {U, W2}, {U, W1}};
Do[angles = anglesDeg[[i]] Degree;
  sigma = sigmas[[i]] Degree;
  {α, β, γ, δ} = angles;
  poly = Which[i == 1, BricardsEquation[{α, β, γ, δ}, sigma, Z2[t], W12[t]],
    i == 2, BricardsEquation[{α, β, γ, δ}, sigma, Z2[t], W22[t]],
    i == 3, BricardsEquation[{α, β, γ, δ}, sigma, U2[t], W22[t]],
    i == 4, BricardsEquation[{α, β, γ, δ}, sigma, U2[t], W12[t]]];
  Print[Row[{Subscript["P", i], "["}, funcs[[i, 1],
    ", ", funcs[[i, 2], "]" = ", FullSimplify[poly]]], {i, 1, 4}];

(*t-range*)
tMin = 0.4;
tMax = 1.25;

(*Build P_i(t) consistently with your printing logic*)
Clear[PiExpr, labels, funcs];
funcs = {{Z2, W12}, {Z2, W22}, {U2, W22}, {U2, W12}};

PiExpr = Table[Module[{angles = anglesDeg[[i]] Degree,
  sigma = sigmas[[i]] Degree, α, β, γ, δ, poly}, {α, β, γ, δ} = angles;
  poly = BricardsEquation[{α, β, γ, δ}, sigma, funcs[[i, 1]][t], funcs[[i, 2]][t]];
  FullSimplify[poly]], {i, 1, 4}];

labels = Table[Row[{Subscript["P", i], "["},
  ToString@funcs[[i, 1], ", ", ToString@funcs[[i, 2], "]" }], {i, 1, 4}];

(*Pretty panel with plots like your style*)
Column[{TextCell[
  Style["===== FLEXIBILITY (FLEXION 2) =====",
    Cyan, Bold, 16], "Text"], TextCell[
  Style["Polynomials P_i(t) built from Bricard's equations for flexion 2.",
    GrayLevel[0.3]], "Text"], Spacer[12],
  Panel[GraphicsGrid[Partition[Table[Plot[Evaluate@PiExpr[[i]],
    {t, tMin, tMax}, PlotLabel → Style[labels[[i]], Bold, 14],
    PlotRange → {-10^(-11), 10^(-11)}, AxesLabel → {"t", None},
    ImageSize → 250], {i, Length[PiExpr]}], 2], Spacings → {2, 2}],
  Background → Lighter[Gray, 0.95], FrameMargins → 15]]]

(*=====
====*)
(*=====
NOT TRIVIAL=====*)

```

```

(*=====
====*)
(*Define domain limits for t*)
tMin = 0.4;
tMax = 1.25;

(*=====
FLEXION 1=====*)
(*List of expressions to plot*)
expressions = {Z[t], W2[t], U[t], W1[t]};
labels = {"Z", "W2", "U", "W1"};

Column[{TextCell[
  Style["===== NOT TRIVIAL (FLEXION 1) =====",
    Darker[Brown], Bold, 16], "Text"],

  (*Explanatory text*)
  TextCell[Style["This configuration does not belong to the trivial class –
    even after switching the boundary strips – since none of the
    functions Z, W2, U, or W1 is constant.", GrayLevel[0.3]], "Text"],
  Spacer[12],

  (*Plots in a light panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
    PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
    AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
    Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]

  ]}]

(*=====
FLEXION 2=====*)
(*List of expressions to plot*)
expressions = {Z2[t], W22[t], U2[t], W12[t]};
labels = {"Z", "W2", "U", "W1"};

Column[{TextCell[
  Style["===== NOT TRIVIAL (FLEXION 2) =====",
    Brown, Bold, 16], "Text"],

  (*Explanatory text*)
  TextCell[Style["This configuration does not belong to the trivial class –
    even after switching the boundary strips – since none of the
    functions Z, W2, U, or W1 is constant.", GrayLevel[0.3]], "Text"],
  Spacer[12],

  (*Plots in a light panel*)

```

```

Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
  PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
  AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
  Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]

]]

(*=====
====*)
(*=====
NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE=====*)
(*=====
====*)
(*Define domain limits for t*)
tMin = 0.4;
tMax = 1.25;

(*=====
FLEXION 1=====*)
(*List of expressions& labels*)
expressions =
  {Z[t] * U[t], Z[t] / U[t], W1[t] * W2[t], W1[t] / W2[t], Z[t] * W2[t], Z[t] / W2[t],
  U[t] * W1[t], U[t] / W1[t], Z[t] * W1[t], Z[t] / W1[t], W2[t] * U[t], W2[t] / U[t]};
labels = {"Z * U", "Z / U", "W1 * W2", "W1 / W2", "Z * W2", "Z / W2",
  "U * W1", "U / W1", "Z * W1", "Z / W1", "W2 * U", "W2 / U"};

Column[
  {TextCell[Style["===== NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE
    (FLEXION 1) =====", Darker[Magenta], Bold, 16], "Text"],

  (*Explanatory text*)TextCell[
    Style["This configuration does not belong to the Linear compound class nor
      to the linear conjugate class – even after switching the boundary
      strips – since none of the functions ZU, Z/U, W1W2, W1/W2, ZW2,
      Z/W2, UW1, and U/W1 is constant. In addition, none of ZW1,
      Z/W1, W2U, W2/U is constant as well.", GrayLevel[0.3]], "Text"],
  Spacer[12],

  (*Plots panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
    PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
    AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
    Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]
  ]
]]

(*=====

```

```

FLEXION 2=====*)
(*List of expressions& labels*)
expressions = {Z2[t] * U2[t], Z2[t] / U2[t], W12[t] * W22[t], W12[t] / W22[t],
  Z2[t] * W22[t], Z2[t] / W22[t], U2[t] * W12[t], U2[t] / W12[t],
  Z2[t] * W12[t], Z2[t] / W12[t], W22[t] * U2[t], W22[t] / U2[t]};
labels = {"Z * U", "Z / U", "W1 * W2", "W1 / W2", "Z * W2", "Z / W2",
  "U * W1", "U / W1", "Z * W1", "Z / W1", "W2 * U", "W2 / U"};

Column[
  {TextCell[Style["===== NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE
    (FLEXION 2) =====", Magenta, Bold, 16], "Text"],

  (*Explanatory text*)TextCell[
    Style["This configuration does not belong to the Linear compound class nor
      to the linear conjugate class – even after switching the boundary
      strips – since none of the functions ZU, Z/U, W1W2, W1/W2, ZW2,
      Z/W2, UW1, and U/W1 is constant. In addition, none of ZW1,
      Z/W1, W2U, W2/U is constant as well.", GrayLevel[0.3]], "Text"],
  Spacer[12],

  (*Plots panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
    PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
    AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
    Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]
  ]}

(*=====
=====*)
(*=====
SWITCHING BOUNDARY STRIPS=====*)
(*=====
=====*)
SwitchingRightBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={1,2},{1,3},{4,2},{4,3}*)
  modified[[1, 2]] = 180 - anglesDeg[[1, 2]]; (*β1*)
  modified[[1, 3]] = 180 - anglesDeg[[1, 3]]; (*γ1*)
  modified[[4, 2]] = 180 - anglesDeg[[4, 2]]; (*β4*)
  modified[[4, 3]] = 180 - anglesDeg[[4, 3]]; (*γ4*)
  modified]

SwitchingLeftBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={2,2},{2,3},{3,2},{3,3}*)
  modified[[2, 2]] = 180 - anglesDeg[[2, 2]]; (*β2*)
  modified[[2, 3]] = 180 - anglesDeg[[2, 3]]; (*γ2*)
  modified[[3, 2]] = 180 - anglesDeg[[3, 2]]; (*β3*)

```

```

modified[[3, 3]] = 180 - anglesDeg[[3, 3]]; (*γ3*)
modified]

SwitchingLowerBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={1,1},{1,2},{2,1},{2,2}*)
  modified[[1, 1]] = 180 - anglesDeg[[1, 1]]; (*α1*)
  modified[[1, 2]] = 180 - anglesDeg[[1, 2]]; (*β1*)
  modified[[2, 1]] = 180 - anglesDeg[[2, 1]]; (*α2*)
  modified[[2, 2]] = 180 - anglesDeg[[2, 2]]; (*β2*)
  modified]

SwitchingUpperBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={3,1},{3,2},{4,1},{4,2}*)
  modified[[3, 1]] = 180 - anglesDeg[[3, 1]]; (*α3*)
  modified[[3, 2]] = 180 - anglesDeg[[3, 2]]; (*β3*)
  modified[[4, 1]] = 180 - anglesDeg[[4, 1]]; (*α4*)
  modified[[4, 2]] = 180 - anglesDeg[[4, 2]]; (*β4*)
  modified]

(*=====
====*)
(*=====
NOT CONIC=====*)
(*=====
====*)
uniqueCombos = {{1, 1, 1, 1}, {1, 1, 1, -1}, {1, 1, -1, -1}, {1, 1, -1, 1},
  {1, -1, 1, 1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, -1, -1, -1}};

checkConditionN0Degrees[{α_, β_, γ_, δ_}] :=
  Module[{angles = {α, β, γ, δ}, results},
    results = Mod[uniqueCombos.angles, 360] // Chop;
    ! MemberQ[results, 0]];

conditionsN0 = checkConditionN0Degrees /@ anglesDeg;
allVerticesPass = And@@ conditionsN0;

Column[{TextCell[Style["===== NOT CONIC =====",
  Pink, Bold, 16], "Text"],
  TextCell[Style["Condition (N.0) is satisfied ⇒
    this configuration is NOT equimodular-conic. Applying
    any boundary-strip switch still preserves (N.0), so
    no conic form emerges.", GrayLevel[0.3]], "Text"]
}]

(*Now the exact same Module for checking all switch combinations...*)
Module[{angles = anglesDeg, switchers, combinations, results},

```

```

(*Define switch functions*)
switchers = <|"Right" → SwitchingRightBoundaryStrip,
  "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
  "Upper" → SwitchingUpperBoundaryStrip|>;
(*Generate all combinations of switches (from size 1 to 4)*)
combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
(*Evaluate condition after each combination of switches*)results = Table[
  Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];
    passQ = And@@ (checkConditionN0Degrees /@ switched);
    (*Print the result after switching*)
    (*Print["\nSwitch combination: ", name];
    Print["Switched anglesDeg:"];
    Print[MatrixForm[switched]];*)
    {name, passQ}], {combo, combinations}];
(*Display results*)
Column[Prepend[Table[Module[{comboName = res[[1]], passQ = res[[2]]},
  Row[{Style[comboName <> ": ", Bold],
    If[passQ,
      Style["Condition (N.0) is still satisfied.", Darker[Green]],
      Style["Condition (N.0) fails.", Red, Bold]
    ]
  }, {res, results}], TextCell[
  Style["CONDITION (N.0) AFTER SWITCHING BOUNDARY STRIPS", 14], "Text"]]]]

(*=====
====*)
(*=====
NOT ORTHODIAGONAL=====*)
(*=====
====*)
Column[
  {TextCell[Style["===== NOT ORTHODIAGONAL =====",
    Purple, Bold, 16], "Text"],
  TextCell[Style[
    "cos(αi) · cos(γi) ≠ cos(βi) · cos(δi) for each i = 1...4 ⇒ NOT orthodiagonal.
    Switching boundary strips does not
    correct this.", GrayLevel[0.3]], "Text"]
  ]}

Module[{angles = anglesDeg, switchers, combinations, results},
  (*Define switch functions*)
  switchers = <|"Right" → SwitchingRightBoundaryStrip,
    "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
    "Upper" → SwitchingUpperBoundaryStrip|>;

```

```

(*Helper function:compute and print difference only*)
formatOrthodiagonalCheck[quad_List] := Module[{vals},
  vals = Table[Module[{a, b, c, d, lhs, rhs, diff}, {a, b, c, d} = quad[[i]];
    lhs = FullSimplify[Cos[a Degree] Cos[c Degree]];
    rhs = FullSimplify[Cos[b Degree] Cos[d Degree]];
    diff = Chop[lhs - rhs];
    Style[Row[{"cos( $\alpha$ " <> ToString[i] <> ") · cos( $\gamma$ " <> ToString[i] <> ") - ",
      "cos( $\beta$ " <> ToString[i] <> ") · cos( $\delta$ " <> ToString[i] <> ") = ", NumberForm[
        diff, {5, 3}]]], If[diff == 0, Red, Black]]], {i, Length[quad]}];
  Column[vals]];
(*Orthodiagonal check for anglesDeg before any switching*)
Print[TextCell[Style["\nInitial anglesDeg (no switches):", Bold]]];
Print[MatrixForm[angles]];
Print[TextCell[Style[
  "Orthodiagonal check: cos( $\alpha_i$ ) · cos( $\gamma_i$ ) - cos( $\beta_i$ ) · cos( $\delta_i$ ) for i = 1..4",
  Italic]]];
Print[formatOrthodiagonalCheck[angles]];
(*Generate all combinations of switches (from size 1 to 4)*)
combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
(*Evaluate condition after each combination of switches*)results = Table[
  Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];
    passQ = And@@ (checkConditionN0Degrees /@switched);
    Print[Style["\nSwitch combination: ", Bold], name];
    Print[Style["Switched anglesDeg:", Italic]];
    Print[MatrixForm[switched]];
    Print[TextCell[Style[
      "Orthodiagonal check: cos( $\alpha_i$ ) · cos( $\gamma_i$ ) - cos( $\beta_i$ ) · cos( $\delta_i$ ) for i = 1..4",
      Italic]]];
    Print[formatOrthodiagonalCheck[switched]];
    {name, passQ}], {combo, combinations}];

(*=====
====*)
(*=====
NOT ISOGONAL=====*)
(*=====
====*)
Column[
{TextCell[Style["===== NOT ISOGONAL =====", Orange,
  Bold, 15], "Text"],
TextCell[
Style["Condition (N.0) holds AND for all i = 1..4:  $\alpha_i \neq \beta_i$ ,  $\alpha_i \neq \gamma_i$ ,  $\alpha_i \neq \delta_i$ ,  $\beta_i \neq \gamma_i$ ,  $\beta_i \neq \delta_i$ ,  $\gamma_i \neq \delta_i$ ,  $\alpha_i + \beta_i \neq \pi \neq \gamma_i + \delta_i$ ,  $\alpha_i + \gamma_i \neq \pi \neq \beta_i + \delta_i$ ,  $\alpha_i + \delta_i \neq \pi \neq \beta_i + \gamma_i \Rightarrow$  NOT isogonal. Switching boundary strips do not change this.", GrayLevel[0.3]], "Text"]}]]

```

```

Module[{angles = anglesDeg, switchers, combinations, results},
  (*Define switch functions*)
  switchers = <|"Right" → SwitchingRightBoundaryStrip,
    "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
    "Upper" → SwitchingUpperBoundaryStrip|>;
  (*Helper function:extended angle relations*)
  formatAngleRelations[quad_List] :=
    Module[{vals}, vals = Table[Module[{a, b, c, d, exprs}, {a, b, c, d} = quad[[i];
      exprs = {Row[{"α" <> ToString[i] <> " - β" <> ToString[i] <> " = ",
        NumberForm[N[a - b], {5, 3}]]], Row[{"α" <> ToString[i] <>
        " - γ" <> ToString[i] <> " = ", NumberForm[N[a - c], {5, 3}]]],
      Row[{"α" <> ToString[i] <> " - δ" <> ToString[i] <> " = ",
        NumberForm[N[a - d], {5, 3}]]], Row[{"β" <> ToString[i] <>
        " - γ" <> ToString[i] <> " = ", NumberForm[N[b - c], {5, 3}]]],
      Row[{"β" <> ToString[i] <> " - δ" <> ToString[i] <> " = ",
        NumberForm[N[b - d], {5, 3}]]], Row[{"γ" <> ToString[i] <>
        " - δ" <> ToString[i] <> " = ", NumberForm[N[c - d], {5, 3}]]],
      Row[{"α" <> ToString[i] <> " + β" <> ToString[i] <> " - 180 = ",
        NumberForm[N[a + b - 180], {5, 3}]]],
      Row[{"γ" <> ToString[i] <> " + δ" <> ToString[i] <> " - 180 = ",
        NumberForm[N[c + d - 180], {5, 3}]]],
      Row[{"α" <> ToString[i] <> " + γ" <> ToString[i] <> " - 180 = ",
        NumberForm[N[a + c - 180], {5, 3}]]],
      Row[{"β" <> ToString[i] <> " + δ" <> ToString[i] <> " - 180 = ",
        NumberForm[N[b + d - 180], {5, 3}]]],
      Row[{"α" <> ToString[i] <> " + δ" <> ToString[i] <> " - 180 = ",
        NumberForm[N[a + d - 180], {5, 3}]]],
      Row[{"β" <> ToString[i] <> " + γ" <> ToString[i] <> " - 180 = ",
        NumberForm[N[b + c - 180], {5, 3}]]],
      Row[{"α" <> ToString[i] <> " + β" <> ToString[i] <> " - γ" <> ToString[i] <>
        " - δ" <> ToString[i] <> " = ", NumberForm[N[a + b - c - d], {5, 3}]]],
      Row[{"α" <> ToString[i] <> " + γ" <> ToString[i] <> " - β" <> ToString[i] <>
        " - δ" <> ToString[i] <> " = ", NumberForm[N[a + c - b - d], {5, 3}]]],
      Row[{"α" <> ToString[i] <> " + δ" <> ToString[i] <> " - β" <> ToString[i] <>
        " - γ" <> ToString[i] <> " = ", NumberForm[N[a + d - b - c], {5, 3}]]]};
      Column[Prepend[exprs, Style["Vertex " <> ToString[i], Bold]]],
      {i, Length[quad]}];
    Column[vals, Spacings → 1.5]];
  (*Angle relation check for anglesDeg before any switching*)
  Print[TextCell[Style["\nInitial anglesDeg (no switches):", Bold]]];
  Print[MatrixForm[angles]];
  Print[TextCell[Style["Angle relation checks for i = 1..4:", Italic]]];
  Print[formatAngleRelations[angles]];
  (*Generate all combinations of switches (from size 1 to 4)*)
  combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
  (*Evaluate condition after each combination of switches*)results = Table[

```



```

Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
Do[switched = switchers[sw][switched], {sw, combo}];
passQ = And@@(checkConditionN0Degrees /@ switched);
Print[Style["\nSwitch combination: ", Bold], name];
Print[Style["Switched anglesDeg:", Italic]];
Print[MatrixForm[switched]];
Print[TextCell[Style["Angle relation checks for i = 1..4:", Italic]]];
Print[formatAngleRelations[switched]];
{name, passQ}], {combo, combinations}];]

(*=====
====*)
(*=====
NOT CONJUGATE-MODULAR=====*)
(*=====
====*)
Column[
{TextCell[Style["===== NOT CONJUGATE-MODULAR =====",
Brown, Bold, 16], "Text"],
TextCell[Style[" $M_i < 1$  and  $\pi_i \in \mathbb{R}$  for all  $i = 1..4 \Rightarrow$  NOT conjugate-modular.
Boundary-strip switches preserve this.", GrayLevel[0.3]], "Text"]
}]

Module[{angles = anglesDeg, switchers, combinations, results,
computeConjugateModularInfo}, (*Define switch functions*)
switchers = <|"Right" → SwitchingRightBoundaryStrip,
"Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
"Upper" → SwitchingUpperBoundaryStrip|>;
(*Computes  $M_i$  and  $\pi_i$  and prints them,
with classification*)computeConjugateModularInfo[quad_List] :=
Module[{abcdList, Ms, summary}, abcdList = computeABCD /@ quad;
Ms = FullSimplify[Times@@@ abcdList];
summary = If[AllTrue[Ms, # < 1 &], Style[" $M_i < 1$  for all  $i = 1, \dots, 4$ ",
Bold], Style[" $M_i \geq 1$  for some  $i = 1, \dots, 4$ ", Red, Bold]];
Column[{Style[" $M_i$  values:", Bold], Row[{"M1 = ", Ms[[1]], ", M2 = ",
Ms[[2]], ", M3 = ", Ms[[3]], ", M4 = ", Ms[[4]]}, summary]}}];
(*Original anglesDeg check*)
Print[
TextCell[Style["\nInitial configuration (no switches applied):", Bold]]];
Print[MatrixForm[angles]];
Print[computeConjugateModularInfo[angles]];
(*Generate all switch combinations (from size 1 to 4)*)
combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
(*Evaluate each switched configuration*)results = Table[
Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
Do[switched = switchers[sw][switched], {sw, combo}];

```

```

passQ = And@@ (checkConditionN0Degrees /@ switched);
Print[Style["\nSwitch combination: ", Bold], name];
Print[Style["Switched anglesDeg:", Italic]];
Print[MatrixForm[switched]];
Print[computeConjugateModularInfo[switched]];
{name, passQ}], {combo, combinations}];]

```

```

(*=====
====*)
(*=====
NOT CHIMERA=====*)
(*=====
====*)
Column[
{TextCell[Style["===== NOT CHIMERA =====", Blue,
  Bold, 16], "Text"],
TextCell[
Style["Fails conic, orthodiagonal & isogonal tests for all i=1, ...,
  4  $\Rightarrow$  NOT chimera. Boundary-strip switches preserve these
  failures as demonstrated in the NOT CONIC, NOT ORTHODIAGONAL,
  and NOT ISOGONAL sections.", GrayLevel[0.3]], "Text"]
}]

```

Out[16]=

```

===== CONDITION (N.0) =====
✓ All vertices satisfy (N.0).

```

Vertex	Combinations (mod 360)	Status
Vertex 1	{342.075, 102.075, 255.638, 135.638, 287.012, 80.5752, 47.0121, 200.575}	✓ Pass
Vertex 2	{334.867, 174.867, 315.305, 115.305, 276.196, 56.6343, 116.196, 256.634}	✓ Pass
Vertex 3	{319.476, 149.476, 330.241, 140.241, 92.1432, 272.908, 282.143, 102.908}	✓ Pass
Vertex 4	{332.626, 182.626, 299.113, 89.1129, 117.278, 233.765, 327.278, 83.7651}	✓ Pass

Out[19]=

```

===== CONDITION (N.3) =====
✓ M1 = M2 = M3 = M4 = 0.924668

```

Out[25]=

```

===== CONDITION (N.4) =====
✓ r1 = r2 = 1.13168; ✓ r3 = r4 = 0.68379
✓ s1 = s4 = 1.17093; ✓ s2 = s3 = 1.09802

```

Out[35]=

```

===== CONDITION (N.5) =====

```

△ *Approximate validation using ε -tolerance. For rigorous proof, see the referenced paper.*

✓ **Valid Combination Found ($M < 1$):**

```
e1 = -1, e2 = 1, e3 = -1
t1 = 0.K + 0.309024iK'
t2 = 0.K + 0.502198iK'
t3 = 1.K + 0.519082iK'
t4 = 1.K + 0.325908iK'
t1 + e1*t2 + e2*t3 + e3*t4 = 0.K + -3.33067 × 10-16iK'
```

Out[38]=

```
===== OTHER PARAMETERS =====
u = 0.0753316
σ1 = 2.98516, σ2 = 2.92226, σ3 = 2.78795, σ4 = 2.90271
σ1 ≈ 171.037°, σ2 ≈ 167.433°, σ3 ≈ 159.738°, σ4 ≈ 166.313°
cosσ1 = -0.98779, cosσ2 = -0.976043, cosσ3 = -0.938119, cosσ4 = -0.971603
f1 = 1.0682, f2 = 1.27867, f3 = 0.70414, f4 = 0.857702
z1 = 14.6629, z2 = 3.5885, z3 = -3.37998, z4 = -7.02752
x1 = 7.59444, x2 = 7.59444, x3 = -3.16246, x4 = -3.16246
y1 = 5.85051, y2 = 10.2019, y3 = 10.2019, y4 = 5.85051
p1 = 0.362871, p2 = 0.362871, p3 = 0. + 0.562325 i, p4 = 0. + 0.562325 i
q1 = 0.413431, q2 = 0.313084, q3 = 0.313084, q4 = 0.413431
p1·q1 = 0.150022, p2·q2 = 0.113609
, p3·q3 = 0. + 0.176055 i, p4·q4 = 0. + 0.232483 i
```

Out[44]=

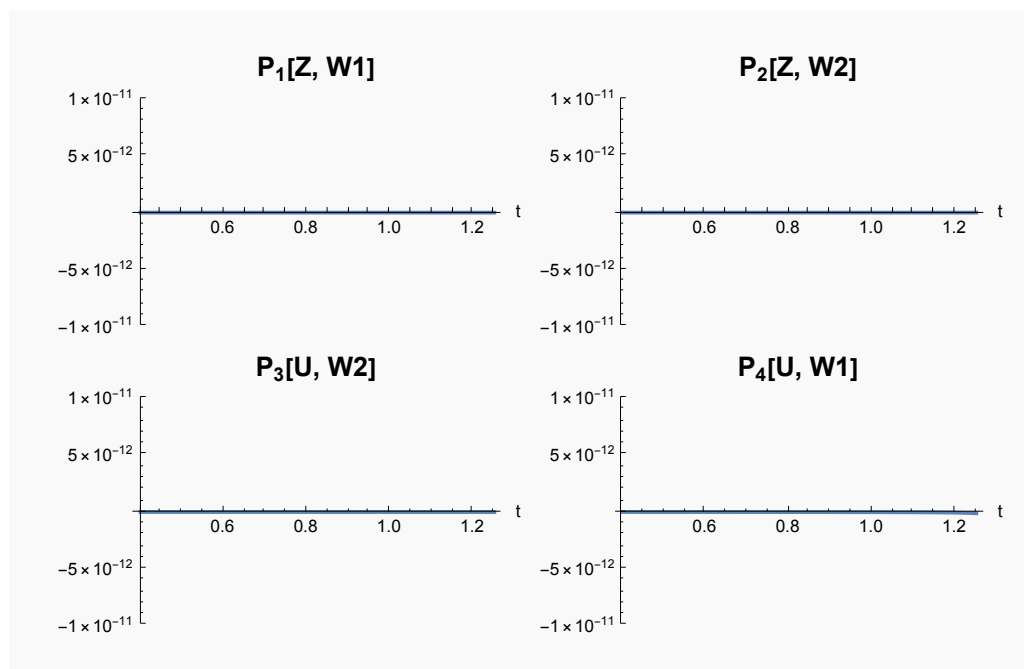
```
===== FLEXIBILITY (FLEXION 1) =====
```

$$\begin{aligned}
P_1[Z, W1] &= \frac{1}{(1.93074 + 1. t^2)^2} \left(-2.41619 \times 10^{-15} + \right. \\
&\quad \left. t \left(-5.77316 \times 10^{-15} \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} + t \left(2.00199 \times 10^{-14} - 1.16063 \times 10^{-14} t^2 + \right. \right. \right. \\
&\quad \left. \left. 2.67507 \times 10^{-15} t^4 + 3.55271 \times 10^{-15} t \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} \right) \right) \Big) \\
P_2[Z, W2] &= \frac{1}{(0.472516 + 1. t^2)^2} \left(1.9458 \times 10^{-16} + \right. \\
&\quad \left. t \left(8.88178 \times 10^{-16} \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} + t \left(-2.18592 \times 10^{-15} - 6.62401 \times 10^{-17} t^2 + \right. \right. \right. \\
&\quad \left. \left. 1.3248 \times 10^{-16} t^4 + 3.88578 \times 10^{-16} t \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} \right) \right) \Big) \\
P_3[U, W2] &= \frac{1}{(0.824332 + 1.27204 t^2 - 1. t^4)^2} \\
&\quad \left(1.68121 \times 10^{-16} + t \left(-3.78271 \times 10^{-15} \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} + \right. \right. \\
&\quad \left. t \left(1.1096 \times 10^{-14} + t \left(-5.37986 \times 10^{-15} \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} + \right. \right. \right. \\
&\quad \left. \left. t \left(8.74227 \times 10^{-15} - 2.03426 \times 10^{-14} t^2 + 6.85092 \times 10^{-15} t^4 + \right. \right. \right. \\
&\quad \left. \left. \left. 4.37114 \times 10^{-15} t \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} \right) \right) \right) \Big) \Big) \Big) \\
P_4[U, W1] &= \frac{1}{(3.36829 - 0.186184 t^2 - 1. t^4)^2} \\
&\quad \left(2.78246 \times 10^{-15} + t \left(-4.47977 \times 10^{-14} \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} + t \left(4.89714 \times 10^{-14} + \right. \right. \right. \\
&\quad \left. t \left(2.22597 \times 10^{-15} \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} + t \left(-7.5683 \times 10^{-14} - 6.67791 \times 10^{-15} \right. \right. \right. \\
&\quad \left. \left. \left. t^2 + 1.81556 \times 10^{-14} t^4 + 1.00169 \times 10^{-14} t \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} \right) \right) \right) \Big) \Big) \Big)
\end{aligned}$$

Out[53]=

===== FLEXIBILITY (FLEXION 1) =====

Polynomials $P_i(t)$ built from Bricard's equations for flexion 1.



Out[59]=

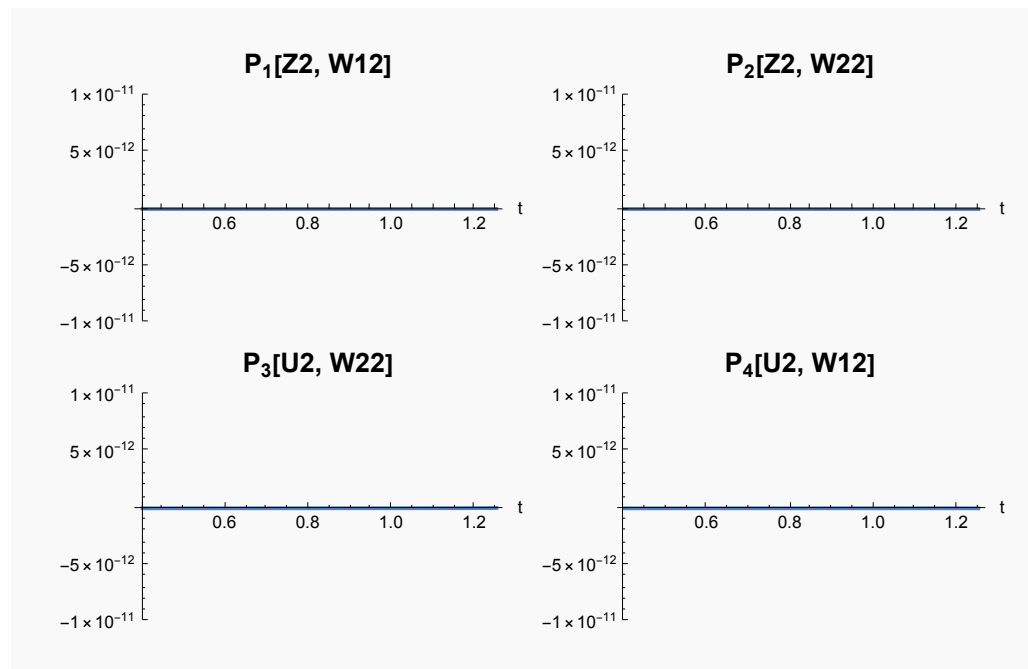
===== FLEXIBILITY (FLEXION 2) =====

$$\begin{aligned}
P_1 [Z , W1] &= \frac{1}{(1.93074 + 1. t^2)^2} \left(-2.41619 \times 10^{-15} + \right. \\
&\quad t \left(5.77316 \times 10^{-15} \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} + t \left(2.00199 \times 10^{-14} - 1.16063 \times 10^{-14} t^2 + \right. \right. \\
&\quad \left. \left. 2.67507 \times 10^{-15} t^4 - 3.55271 \times 10^{-15} t \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} \right) \right) \left. \right) \\
P_2 [Z , W2] &= \frac{1}{(0.472516 + 1. t^2)^2} \left(1.9458 \times 10^{-16} + \right. \\
&\quad t \left(-8.88178 \times 10^{-16} \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} + t \left(-2.18592 \times 10^{-15} - 6.62401 \times 10^{-17} t^2 + \right. \right. \\
&\quad \left. \left. 1.3248 \times 10^{-16} t^4 - 3.88578 \times 10^{-16} t \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} \right) \right) \left. \right) \\
P_3 [U , W2] &= \frac{1}{(0.824332 + 1.27204 t^2 - 1. t^4)^2} \\
&\quad \left(1.68121 \times 10^{-16} + t \left(3.78271 \times 10^{-15} \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} + t \left(1.1096 \times 10^{-14} + \right. \right. \right. \\
&\quad \left. \left. t \left(5.37986 \times 10^{-15} \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} + t \left(8.74227 \times 10^{-15} - 2.03426 \times 10^{-14} \right. \right. \right. \\
&\quad \left. \left. \left. t^2 + 6.85092 \times 10^{-15} t^4 - 4.37114 \times 10^{-15} t \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} \right) \right) \right) \right) \left. \right) \left. \right) \\
P_4 [U , W1] &= \frac{1}{(3.36829 - 0.186184 t^2 - 1. t^4)^2} \\
&\quad \left(2.78246 \times 10^{-15} + t \left(4.47977 \times 10^{-14} \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} + \right. \right. \\
&\quad \left. \left. t \left(4.89714 \times 10^{-14} + t \left(-2.22597 \times 10^{-15} \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} + \right. \right. \right. \\
&\quad \left. \left. \left. t \left(-7.5683 \times 10^{-14} - 6.67791 \times 10^{-15} t^2 + 1.81556 \times 10^{-14} t^4 - \right. \right. \right. \\
&\quad \left. \left. \left. \left. 1.00169 \times 10^{-14} t \sqrt{-1 + 8.16654 t^2 - 4.34478 t^4} \right) \right) \right) \right) \right) \left. \right) \left. \right)
\end{aligned}$$

Out[68]=

===== FLEXIBILITY (FLEXION 2) =====

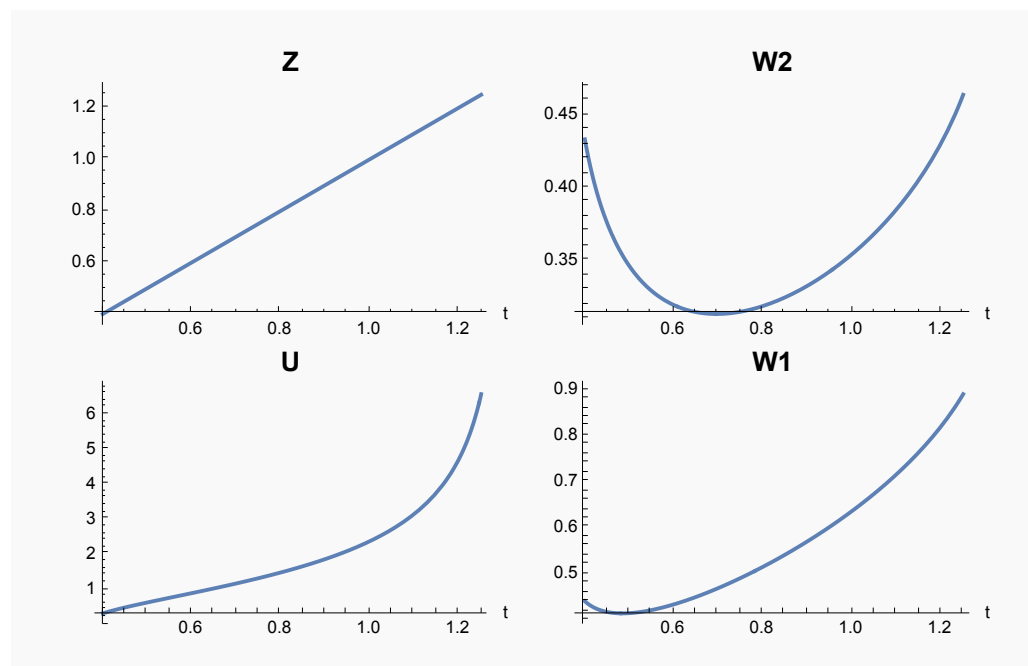
Polynomials $P_i(t)$ built from Bricard's equations for flexion 2.



Out[73]=

===== NOT TRIVIAL (FLEXION 1) =====

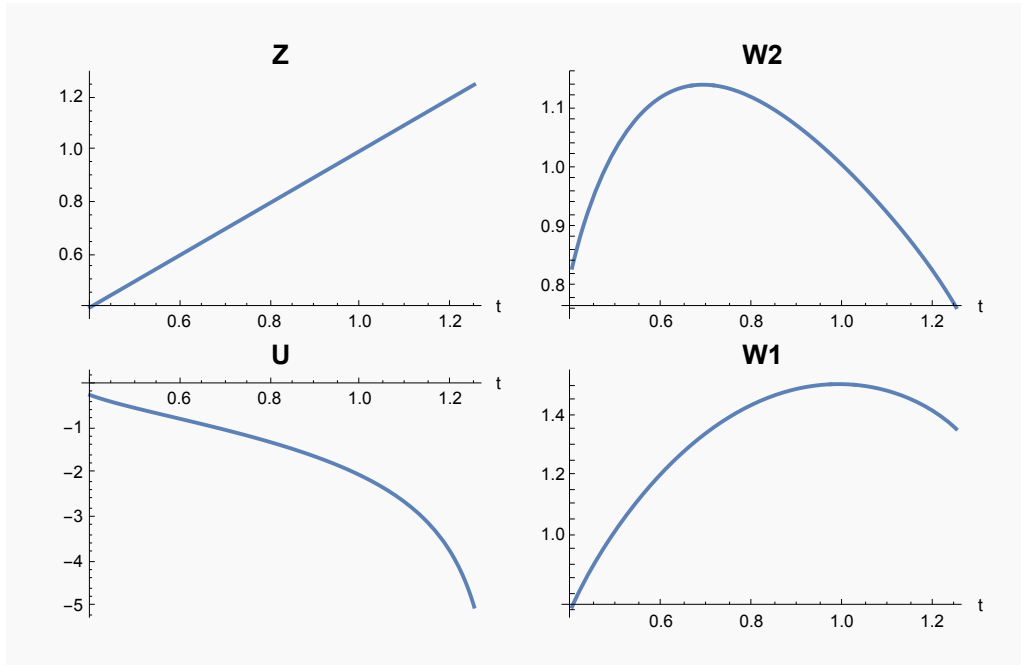
This configuration does not belong to the trivial class – even after switching the boundary strips – since none of the functions Z , $W2$, U , or $W1$ is constant.



Out[76]=

===== NOT TRIVIAL (FLEXION 2) =====

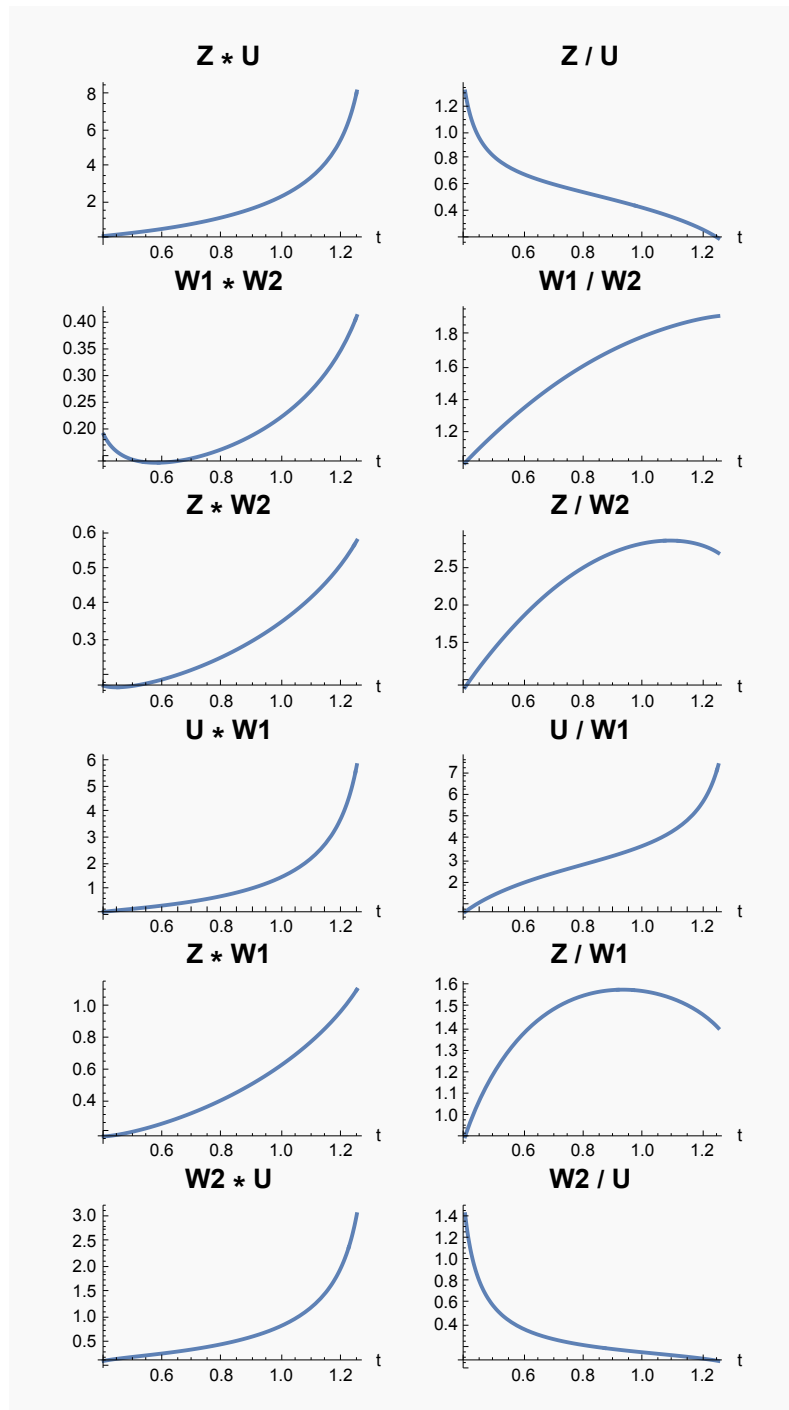
This configuration does not belong to the trivial class – even after switching the boundary strips – since none of the functions Z , $W2$, U , or $W1$ is constant.



Out[81]=

===== NOT LINEAR COMPOUND &
 NOT LINEAR CONJUGATE (FLEXION 1) =====

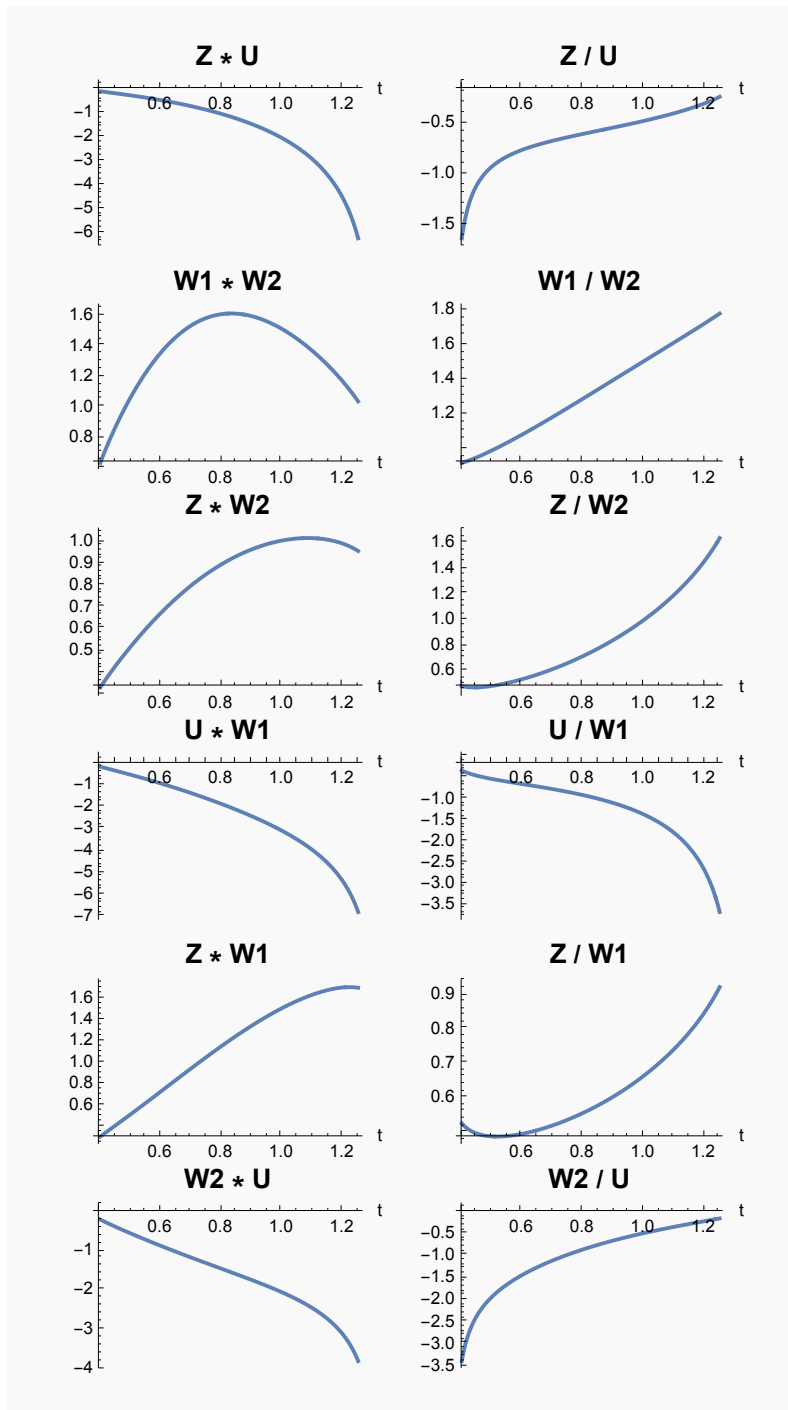
This configuration does not belong to the Linear compound class nor to the linear conjugate class – even after switching the boundary strips – since none of the functions ZU , Z/U , $W1W2$, $W1/W2$, $ZW2$, $Z/W2$, $UW1$, and $U/W1$ is constant. In addition, none of $ZW1$, $Z/W1$, $W2U$, $W2/U$ is constant as well.



Out[84]=

===== NOT LINEAR COMPOUND &
 NOT LINEAR CONJUGATE (FLEXION 2) =====

This configuration does not belong to the Linear compound class nor to the linear conjugate class – even after switching the boundary strips – since none of the functions ZU , Z/U , $W1W2$, $W1/W2$, $ZW2$, $Z/W2$, $UW1$, and $U/W1$ is constant. In addition, none of $ZW1$, $Z/W1$, $W2U$, $W2/U$ is constant as well.



Out[93]=

===== **NOT CONIC** =====

Condition (N.0) is satisfied \Rightarrow this configuration
is NOT equimodular-conic. Applying any boundary-strip
switch still preserves (N.0), so no conic form emerges.

Out[94]=

CONDITION (N.0) AFTER SWITCHING BOUNDARY STRIPS

Right: Condition (N.0) is still satisfied.
Left: Condition (N.0) is still satisfied.
Lower: Condition (N.0) is still satisfied.
Upper: Condition (N.0) is still satisfied.
Right + Left: Condition (N.0) is still satisfied.
Right + Lower: Condition (N.0) is still satisfied.
Right + Upper: Condition (N.0) is still satisfied.
Left + Lower: Condition (N.0) is still satisfied.
Left + Upper: Condition (N.0) is still satisfied.
Lower + Upper: Condition (N.0) is still satisfied.
Right + Left + Lower: Condition (N.0) is still satisfied.
Right + Left + Upper: Condition (N.0) is still satisfied.
Right + Lower + Upper: Condition (N.0) is still satisfied.
Left + Lower + Upper: Condition (N.0) is still satisfied.
Right + Left + Lower + Upper: Condition (N.0) is still satisfied.

Out[95]=

===== **NOT ORTHODIAGONAL** =====

$\cos(\alpha_i) \cdot \cos(\gamma_i) \neq \cos(\beta_i) \cdot \cos(\delta_i)$ for each $i = 1..4 \Rightarrow$ NOT
orthodiagonal. Switching boundary strips does not correct this.

Initial anglesDeg (no switches):

$$\begin{pmatrix} 91.3249 & 27.5312 & 103.218 & 120 \\ 115.75 & 29.3354 & 109.781 & 80 \\ 31.192 & 113.666 & 89.6176 & 85 \\ 28.1955 & 107.674 & 121.757 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = 0.449$
 $\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = -0.004$
 $\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = 0.041$
 $\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = -0.385$

Switch combination: Right

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 152.469 & 76.7816 & 120 \\ 115.75 & 29.3354 & 109.781 & 80 \\ 31.192 & 113.666 & 89.6176 & 85 \\ 28.1955 & 72.3261 & 58.2435 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = -0.449$
 $\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = -0.004$
 $\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = 0.041$
 $\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = 0.385$

Switch combination: Left*Switched anglesDeg:*

$$\begin{pmatrix} 91.3249 & 27.5312 & 103.218 & 120 \\ 115.75 & 150.665 & 70.2192 & 80 \\ 31.192 & 66.3336 & 90.3824 & 85 \\ 28.1955 & 107.674 & 121.757 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= 0.449 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= 0.004 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -0.041 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -0.385 \end{aligned}$$

Switch combination: Lower*Switched anglesDeg:*

$$\begin{pmatrix} 88.6751 & 152.469 & 103.218 & 120 \\ 64.2495 & 150.665 & 109.781 & 80 \\ 31.192 & 113.666 & 89.6176 & 85 \\ 28.1955 & 107.674 & 121.757 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -0.449 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= 0.004 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= 0.041 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -0.385 \end{aligned}$$

Switch combination: Upper*Switched anglesDeg:*

$$\begin{pmatrix} 91.3249 & 27.5312 & 103.218 & 120 \\ 115.75 & 29.3354 & 109.781 & 80 \\ 148.808 & 66.3336 & 89.6176 & 85 \\ 151.804 & 72.3261 & 121.757 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= 0.449 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -0.004 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -0.041 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= 0.385 \end{aligned}$$

Switch combination: Right + Left*Switched anglesDeg:*

$$\begin{pmatrix} 91.3249 & 152.469 & 76.7816 & 120 \\ 115.75 & 150.665 & 70.2192 & 80 \\ 31.192 & 66.3336 & 90.3824 & 85 \\ 28.1955 & 72.3261 & 58.2435 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -0.449 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= 0.004 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -0.041 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= 0.385 \end{aligned}$$

Switch combination: Right + Lower

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 27.5312 & 76.7816 & 120 \\ 64.2495 & 150.665 & 109.781 & 80 \\ 31.192 & 113.666 & 89.6176 & 85 \\ 28.1955 & 72.3261 & 58.2435 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= 0.449 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= 0.004 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= 0.041 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= 0.385 \end{aligned}$$

Switch combination: Right + Upper

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 152.469 & 76.7816 & 120 \\ 115.75 & 29.3354 & 109.781 & 80 \\ 148.808 & 66.3336 & 89.6176 & 85 \\ 151.804 & 107.674 & 58.2435 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -0.449 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -0.004 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -0.041 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -0.385 \end{aligned}$$

Switch combination: Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 152.469 & 103.218 & 120 \\ 64.2495 & 29.3354 & 70.2192 & 80 \\ 31.192 & 66.3336 & 90.3824 & 85 \\ 28.1955 & 107.674 & 121.757 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -0.449 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -0.004 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -0.041 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -0.385 \end{aligned}$$

Switch combination: Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 27.5312 & 103.218 & 120 \\ 115.75 & 150.665 & 70.2192 & 80 \\ 148.808 & 113.666 & 90.3824 & 85 \\ 151.804 & 72.3261 & 121.757 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= 0.449 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= 0.004 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= 0.041 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= 0.385 \end{aligned}$$

Switch combination: Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 152.469 & 103.218 & 120 \\ 64.2495 & 150.665 & 109.781 & 80 \\ 148.808 & 66.3336 & 89.6176 & 85 \\ 151.804 & 72.3261 & 121.757 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -0.449 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= 0.004 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -0.041 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= 0.385 \end{aligned}$$

Switch combination: Right + Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 27.5312 & 76.7816 & 120 \\ 64.2495 & 29.3354 & 70.2192 & 80 \\ 31.192 & 66.3336 & 90.3824 & 85 \\ 28.1955 & 72.3261 & 58.2435 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= 0.449 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -0.004 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -0.041 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= 0.385 \end{aligned}$$

Switch combination: Right + Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 152.469 & 76.7816 & 120 \\ 115.75 & 150.665 & 70.2192 & 80 \\ 148.808 & 113.666 & 90.3824 & 85 \\ 151.804 & 107.674 & 58.2435 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -0.449 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= 0.004 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= 0.041 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -0.385 \end{aligned}$$

Switch combination: Right + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 27.5312 & 76.7816 & 120 \\ 64.2495 & 150.665 & 109.781 & 80 \\ 148.808 & 66.3336 & 89.6176 & 85 \\ 151.804 & 107.674 & 58.2435 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= 0.449 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= 0.004 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -0.041 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -0.385 \end{aligned}$$

Switch combination: Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 152.469 & 103.218 & 120 \\ 64.2495 & 29.3354 & 70.2192 & 80 \\ 148.808 & 113.666 & 90.3824 & 85 \\ 151.804 & 72.3261 & 121.757 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = -0.449$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = -0.004$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = 0.041$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = 0.385$$

Switch combination: Right + Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 27.5312 & 76.7816 & 120 \\ 64.2495 & 29.3354 & 70.2192 & 80 \\ 148.808 & 113.666 & 90.3824 & 85 \\ 151.804 & 107.674 & 58.2435 & 75 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = 0.449$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = -0.004$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = 0.041$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = -0.385$$

Out[97]=

===== NOT ISOGONAL =====

Condition (N.0) holds AND for all $i = 1..4$: $\alpha_i \neq \beta_i$,

$$\alpha_i \neq \gamma_i, \alpha_i \neq \delta_i, \beta_i \neq \gamma_i, \beta_i \neq \delta_i, \gamma_i \neq \delta_i, \alpha_i + \beta_i \neq$$

$$\pi \neq \gamma_i + \delta_i, \alpha_i + \gamma_i \neq \pi \neq \beta_i + \delta_i, \alpha_i + \delta_i \neq \pi \neq \beta_i + \gamma_i \Rightarrow \text{NOT}$$

isogonal. Switching boundary strips do not change this.

Initial anglesDeg (no switches):

$$\begin{pmatrix} 91.3249 & 27.5312 & 103.218 & 120 \\ 115.75 & 29.3354 & 109.781 & 80 \\ 31.192 & 113.666 & 89.6176 & 85 \\ 28.1955 & 107.674 & 121.757 & 75 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\alpha_1 - \beta_1 = 63.794$$

$$\alpha_1 - \gamma_1 = -11.894$$

$$\alpha_1 - \delta_1 = -28.675$$

$$\beta_1 - \gamma_1 = -75.687$$

$$\beta_1 - \delta_1 = -92.469$$

$$\gamma_1 - \delta_1 = -16.782$$

$$\alpha_1 + \beta_1 - 180 = -61.144$$

$$\gamma_1 + \delta_1 - 180 = 43.218$$

$$\alpha_1 + \gamma_1 - 180 = 14.543$$

$$\beta_1 + \delta_1 - 180 = -32.469$$

$$\alpha_1 + \delta_1 - 180 = 31.325$$

$$\beta_1 + \gamma_1 - 180 = -49.250$$

$$\alpha_1 + \beta_1 - \gamma_1 - \delta_1 = -104.360$$

$$\alpha_1 + \gamma_1 - \beta_1 - \delta_1 = 47.012$$

$$\alpha_1 + \delta_1 - \beta_1 - \gamma_1 = 80.575$$

Vertex 2

$$\alpha_2 - \beta_2 = 86.415$$

$$\begin{aligned}
\alpha_2 - \beta_2 &= 55.119 \\
\alpha_2 - \gamma_2 &= 5.970 \\
\alpha_2 - \delta_2 &= 35.750 \\
\beta_2 - \gamma_2 &= -80.445 \\
\beta_2 - \delta_2 &= -50.665 \\
\gamma_2 - \delta_2 &= 29.781 \\
\alpha_2 + \beta_2 - 180 &= -34.914 \\
\gamma_2 + \delta_2 - 180 &= 9.781 \\
\alpha_2 + \gamma_2 - 180 &= 45.531 \\
\beta_2 + \delta_2 - 180 &= -70.665 \\
\alpha_2 + \delta_2 - 180 &= 15.750 \\
\beta_2 + \gamma_2 - 180 &= -40.884 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -44.695 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 116.200 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 56.634
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= -82.474 \\
\alpha_3 - \gamma_3 &= -58.426 \\
\alpha_3 - \delta_3 &= -53.808 \\
\beta_3 - \gamma_3 &= 24.049 \\
\beta_3 - \delta_3 &= 28.666 \\
\gamma_3 - \delta_3 &= 4.618 \\
\alpha_3 + \beta_3 - 180 &= -35.142 \\
\gamma_3 + \delta_3 - 180 &= -5.382 \\
\alpha_3 + \gamma_3 - 180 &= -59.190 \\
\beta_3 + \delta_3 - 180 &= 18.666 \\
\alpha_3 + \delta_3 - 180 &= -63.808 \\
\beta_3 + \gamma_3 - 180 &= 23.284 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -29.759 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -77.857 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -87.092
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= -79.478 \\
\alpha_4 - \gamma_4 &= -93.561 \\
\alpha_4 - \delta_4 &= -46.804 \\
\beta_4 - \gamma_4 &= -14.083 \\
\beta_4 - \delta_4 &= 32.674 \\
\gamma_4 - \delta_4 &= 46.757 \\
\alpha_4 + \beta_4 - 180 &= -44.131 \\
\gamma_4 + \delta_4 - 180 &= 16.757 \\
\alpha_4 + \gamma_4 - 180 &= -30.048 \\
\beta_4 + \delta_4 - 180 &= 2.674 \\
\alpha_4 + \delta_4 - 180 &= -76.804 \\
\beta_4 + \gamma_4 - 180 &= 49.430 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -60.887 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -32.722 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -126.230
\end{aligned}$$

Switch combination: Right*Switched anglesDeg:*

$$\begin{pmatrix} 91.3249 & 152.469 & 76.7816 & 120 \\ 115.75 & 29.3354 & 109.781 & 80 \\ 31.192 & 113.666 & 89.6176 & 85 \\ 28.1955 & 72.3261 & 58.2435 & 75 \end{pmatrix}$$

*Angle relation checks for i = 1..4:***Vertex 1**

$$\begin{aligned}
\alpha_1 - \beta_1 &= -61.144 \\
\alpha_1 - \gamma_1 &= 14.512
\end{aligned}$$

$$\begin{aligned}
\alpha_1 - \gamma_1 &= 14.545 \\
\alpha_1 - \delta_1 &= -28.675 \\
\beta_1 - \gamma_1 &= 75.687 \\
\beta_1 - \delta_1 &= 32.469 \\
\gamma_1 - \delta_1 &= -43.218 \\
\alpha_1 + \beta_1 - 180 &= 63.794 \\
\gamma_1 + \delta_1 - 180 &= 16.782 \\
\alpha_1 + \gamma_1 - 180 &= -11.894 \\
\beta_1 + \delta_1 - 180 &= 92.469 \\
\alpha_1 + \delta_1 - 180 &= 31.325 \\
\beta_1 + \gamma_1 - 180 &= 49.250 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 47.012 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -104.360 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -17.925
\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= 86.415 \\
\alpha_2 - \gamma_2 &= 5.970 \\
\alpha_2 - \delta_2 &= 35.750 \\
\beta_2 - \gamma_2 &= -80.445 \\
\beta_2 - \delta_2 &= -50.665 \\
\gamma_2 - \delta_2 &= 29.781 \\
\alpha_2 + \beta_2 - 180 &= -34.914 \\
\gamma_2 + \delta_2 - 180 &= 9.781 \\
\alpha_2 + \gamma_2 - 180 &= 45.531 \\
\beta_2 + \delta_2 - 180 &= -70.665 \\
\alpha_2 + \delta_2 - 180 &= 15.750 \\
\beta_2 + \gamma_2 - 180 &= -40.884 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -44.695 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 116.200 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 56.634
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= -82.474 \\
\alpha_3 - \gamma_3 &= -58.426 \\
\alpha_3 - \delta_3 &= -53.808 \\
\beta_3 - \gamma_3 &= 24.049 \\
\beta_3 - \delta_3 &= 28.666 \\
\gamma_3 - \delta_3 &= 4.618 \\
\alpha_3 + \beta_3 - 180 &= -35.142 \\
\gamma_3 + \delta_3 - 180 &= -5.382 \\
\alpha_3 + \gamma_3 - 180 &= -59.190 \\
\beta_3 + \delta_3 - 180 &= 18.666 \\
\alpha_3 + \delta_3 - 180 &= -63.808 \\
\beta_3 + \gamma_3 - 180 &= 23.284 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -29.759 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -77.857 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -87.092
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= -44.131 \\
\alpha_4 - \gamma_4 &= -30.048 \\
\alpha_4 - \delta_4 &= -46.804 \\
\beta_4 - \gamma_4 &= 14.083 \\
\beta_4 - \delta_4 &= -2.674 \\
\gamma_4 - \delta_4 &= -16.757 \\
\alpha_4 + \beta_4 - 180 &= -79.478 \\
\gamma_4 + \delta_4 - 180 &= -46.757 \\
\alpha_4 + \gamma_4 - 180 &= -93.561 \\
\beta_4 + \delta_4 - 180 &= -32.674 \\
\alpha_4 + \delta_4 - 180 &= -76.804 \\
\beta_4 + \gamma_4 - 180 &= -49.430
\end{aligned}$$

$$\begin{aligned}\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -32.722 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -60.887 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -27.374\end{aligned}$$

Switch combination: Left

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 27.5312 & 103.218 & 120 \\ 115.75 & 150.665 & 70.2192 & 80 \\ 31.192 & 66.3336 & 90.3824 & 85 \\ 28.1955 & 107.674 & 121.757 & 75 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}\alpha_1 - \beta_1 &= 63.794 \\ \alpha_1 - \gamma_1 &= -11.894 \\ \alpha_1 - \delta_1 &= -28.675 \\ \beta_1 - \gamma_1 &= -75.687 \\ \beta_1 - \delta_1 &= -92.469 \\ \gamma_1 - \delta_1 &= -16.782 \\ \alpha_1 + \beta_1 - 180 &= -61.144 \\ \gamma_1 + \delta_1 - 180 &= 43.218 \\ \alpha_1 + \gamma_1 - 180 &= 14.543 \\ \beta_1 + \delta_1 - 180 &= -32.469 \\ \alpha_1 + \delta_1 - 180 &= 31.325 \\ \beta_1 + \gamma_1 - 180 &= -49.250 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -104.360 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 47.012 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 80.575\end{aligned}$$

Vertex 2

$$\begin{aligned}\alpha_2 - \beta_2 &= -34.914 \\ \alpha_2 - \gamma_2 &= 45.531 \\ \alpha_2 - \delta_2 &= 35.750 \\ \beta_2 - \gamma_2 &= 80.445 \\ \beta_2 - \delta_2 &= 70.665 \\ \gamma_2 - \delta_2 &= -9.781 \\ \alpha_2 + \beta_2 - 180 &= 86.415 \\ \gamma_2 + \delta_2 - 180 &= -29.781 \\ \alpha_2 + \gamma_2 - 180 &= 5.970 \\ \beta_2 + \delta_2 - 180 &= 50.665 \\ \alpha_2 + \delta_2 - 180 &= 15.750 \\ \beta_2 + \gamma_2 - 180 &= 40.884 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 116.200 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -44.695 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -25.133\end{aligned}$$

Vertex 3

$$\begin{aligned}\alpha_3 - \beta_3 &= -35.142 \\ \alpha_3 - \gamma_3 &= -59.190 \\ \alpha_3 - \delta_3 &= -53.808 \\ \beta_3 - \gamma_3 &= -24.049 \\ \beta_3 - \delta_3 &= -18.666 \\ \gamma_3 - \delta_3 &= 5.382 \\ \alpha_3 + \beta_3 - 180 &= -82.474 \\ \gamma_3 + \delta_3 - 180 &= -4.618 \\ \alpha_3 + \gamma_3 - 180 &= -58.426 \\ \beta_3 + \delta_3 - 180 &= -28.666 \\ \alpha_3 + \delta_3 - 180 &= -63.808 \\ \beta_3 + \gamma_3 - 180 &= -23.284 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -77.857\end{aligned}$$

$$\alpha_3 + \gamma_3 - \beta_3 - \delta_3 = -29.759$$

$$\alpha_3 + \delta_3 - \beta_3 - \gamma_3 = -40.524$$

Vertex 4

$$\alpha_4 - \beta_4 = -79.478$$

$$\alpha_4 - \gamma_4 = -93.561$$

$$\alpha_4 - \delta_4 = -46.804$$

$$\beta_4 - \gamma_4 = -14.083$$

$$\beta_4 - \delta_4 = 32.674$$

$$\gamma_4 - \delta_4 = 46.757$$

$$\alpha_4 + \beta_4 - 180 = -44.131$$

$$\gamma_4 + \delta_4 - 180 = 16.757$$

$$\alpha_4 + \gamma_4 - 180 = -30.048$$

$$\beta_4 + \delta_4 - 180 = 2.674$$

$$\alpha_4 + \delta_4 - 180 = -76.804$$

$$\beta_4 + \gamma_4 - 180 = 49.430$$

$$\alpha_4 + \beta_4 - \gamma_4 - \delta_4 = -60.887$$

$$\alpha_4 + \gamma_4 - \beta_4 - \delta_4 = -32.722$$

$$\alpha_4 + \delta_4 - \beta_4 - \gamma_4 = -126.230$$

Switch combination: Lower

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 152.469 & 103.218 & 120 \\ 64.2495 & 150.665 & 109.781 & 80 \\ 31.192 & 113.666 & 89.6176 & 85 \\ 28.1955 & 107.674 & 121.757 & 75 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\alpha_1 - \beta_1 = -63.794$$

$$\alpha_1 - \gamma_1 = -14.543$$

$$\alpha_1 - \delta_1 = -31.325$$

$$\beta_1 - \gamma_1 = 49.250$$

$$\beta_1 - \delta_1 = 32.469$$

$$\gamma_1 - \delta_1 = -16.782$$

$$\alpha_1 + \beta_1 - 180 = 61.144$$

$$\gamma_1 + \delta_1 - 180 = 43.218$$

$$\alpha_1 + \gamma_1 - 180 = 11.894$$

$$\beta_1 + \delta_1 - 180 = 92.469$$

$$\alpha_1 + \delta_1 - 180 = 28.675$$

$$\beta_1 + \gamma_1 - 180 = 75.687$$

$$\alpha_1 + \beta_1 - \gamma_1 - \delta_1 = 17.925$$

$$\alpha_1 + \gamma_1 - \beta_1 - \delta_1 = -80.575$$

$$\alpha_1 + \delta_1 - \beta_1 - \gamma_1 = -47.012$$

Vertex 2

$$\alpha_2 - \beta_2 = -86.415$$

$$\alpha_2 - \gamma_2 = -45.531$$

$$\alpha_2 - \delta_2 = -15.750$$

$$\beta_2 - \gamma_2 = 40.884$$

$$\beta_2 - \delta_2 = 70.665$$

$$\gamma_2 - \delta_2 = 29.781$$

$$\alpha_2 + \beta_2 - 180 = 34.914$$

$$\gamma_2 + \delta_2 - 180 = 9.781$$

$$\alpha_2 + \gamma_2 - 180 = -5.970$$

$$\beta_2 + \delta_2 - 180 = 50.665$$

$$\alpha_2 + \delta_2 - 180 = -35.750$$

$$\beta_2 + \gamma_2 - 180 = 80.445$$

$$\alpha_2 + \beta_2 - \gamma_2 - \delta_2 = 25.133$$

$$\alpha_2 + \gamma_2 - \beta_2 - \delta_2 = -56.634$$

$$\alpha_2 + \delta_2 - \beta_2 - \gamma_2 = -116.200$$

Vertex 3

$$\begin{aligned}\alpha_3 - \beta_3 &= -82.474 \\ \alpha_3 - \gamma_3 &= -58.426 \\ \alpha_3 - \delta_3 &= -53.808 \\ \beta_3 - \gamma_3 &= 24.049 \\ \beta_3 - \delta_3 &= 28.666 \\ \gamma_3 - \delta_3 &= 4.618 \\ \alpha_3 + \beta_3 - 180 &= -35.142 \\ \gamma_3 + \delta_3 - 180 &= -5.382 \\ \alpha_3 + \gamma_3 - 180 &= -59.190 \\ \beta_3 + \delta_3 - 180 &= 18.666 \\ \alpha_3 + \delta_3 - 180 &= -63.808 \\ \beta_3 + \gamma_3 - 180 &= 23.284 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -29.759 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -77.857 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -87.092\end{aligned}$$

Vertex 4

$$\begin{aligned}\alpha_4 - \beta_4 &= -79.478 \\ \alpha_4 - \gamma_4 &= -93.561 \\ \alpha_4 - \delta_4 &= -46.804 \\ \beta_4 - \gamma_4 &= -14.083 \\ \beta_4 - \delta_4 &= 32.674 \\ \gamma_4 - \delta_4 &= 46.757 \\ \alpha_4 + \beta_4 - 180 &= -44.131 \\ \gamma_4 + \delta_4 - 180 &= 16.757 \\ \alpha_4 + \gamma_4 - 180 &= -30.048 \\ \beta_4 + \delta_4 - 180 &= 2.674 \\ \alpha_4 + \delta_4 - 180 &= -76.804 \\ \beta_4 + \gamma_4 - 180 &= 49.430 \\ \alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -60.887 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -32.722 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -126.230\end{aligned}$$

Switch combination: Upper

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 27.5312 & 103.218 & 120 \\ 115.75 & 29.3354 & 109.781 & 80 \\ 148.808 & 66.3336 & 89.6176 & 85 \\ 151.804 & 72.3261 & 121.757 & 75 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}\alpha_1 - \beta_1 &= 63.794 \\ \alpha_1 - \gamma_1 &= -11.894 \\ \alpha_1 - \delta_1 &= -28.675 \\ \beta_1 - \gamma_1 &= -75.687 \\ \beta_1 - \delta_1 &= -92.469 \\ \gamma_1 - \delta_1 &= -16.782 \\ \alpha_1 + \beta_1 - 180 &= -61.144 \\ \gamma_1 + \delta_1 - 180 &= 43.218 \\ \alpha_1 + \gamma_1 - 180 &= 14.543 \\ \beta_1 + \delta_1 - 180 &= -32.469 \\ \alpha_1 + \delta_1 - 180 &= 31.325 \\ \beta_1 + \gamma_1 - 180 &= -49.250 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -104.360 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 47.012 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 80.575\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= 86.415 \\
\alpha_2 - \gamma_2 &= 5.970 \\
\alpha_2 - \delta_2 &= 35.750 \\
\beta_2 - \gamma_2 &= -80.445 \\
\beta_2 - \delta_2 &= -50.665 \\
\gamma_2 - \delta_2 &= 29.781 \\
\alpha_2 + \beta_2 - 180 &= -34.914 \\
\gamma_2 + \delta_2 - 180 &= 9.781 \\
\alpha_2 + \gamma_2 - 180 &= 45.531 \\
\beta_2 + \delta_2 - 180 &= -70.665 \\
\alpha_2 + \delta_2 - 180 &= 15.750 \\
\beta_2 + \gamma_2 - 180 &= -40.884 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -44.695 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 116.200 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 56.634
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= 82.474 \\
\alpha_3 - \gamma_3 &= 59.190 \\
\alpha_3 - \delta_3 &= 63.808 \\
\beta_3 - \gamma_3 &= -23.284 \\
\beta_3 - \delta_3 &= -18.666 \\
\gamma_3 - \delta_3 &= 4.618 \\
\alpha_3 + \beta_3 - 180 &= 35.142 \\
\gamma_3 + \delta_3 - 180 &= -5.382 \\
\alpha_3 + \gamma_3 - 180 &= 58.426 \\
\beta_3 + \delta_3 - 180 &= -28.666 \\
\alpha_3 + \delta_3 - 180 &= 53.808 \\
\beta_3 + \gamma_3 - 180 &= -24.049 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 40.524 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 87.092 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 77.857
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= 79.478 \\
\alpha_4 - \gamma_4 &= 30.048 \\
\alpha_4 - \delta_4 &= 76.804 \\
\beta_4 - \gamma_4 &= -49.430 \\
\beta_4 - \delta_4 &= -2.674 \\
\gamma_4 - \delta_4 &= 46.757 \\
\alpha_4 + \beta_4 - 180 &= 44.131 \\
\gamma_4 + \delta_4 - 180 &= 16.757 \\
\alpha_4 + \gamma_4 - 180 &= 93.561 \\
\beta_4 + \delta_4 - 180 &= -32.674 \\
\alpha_4 + \delta_4 - 180 &= 46.804 \\
\beta_4 + \gamma_4 - 180 &= 14.083 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 27.374 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 126.230 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 32.722
\end{aligned}$$

Switch combination: Right + Left*Switched anglesDeg:*

$$\begin{pmatrix} 91.3249 & 152.469 & 76.7816 & 120 \\ 115.75 & 150.665 & 70.2192 & 80 \\ 31.192 & 66.3336 & 90.3824 & 85 \\ 28.1955 & 72.3261 & 58.2435 & 75 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= -61.144 \\
\alpha_1 - \gamma_1 &= 14.543 \\
\alpha_1 - \delta_1 &= -28.675 \\
\beta_1 - \gamma_1 &= 75.687 \\
\beta_1 - \delta_1 &= 32.469 \\
\gamma_1 - \delta_1 &= -43.218 \\
\alpha_1 + \beta_1 - 180 &= 63.794 \\
\gamma_1 + \delta_1 - 180 &= 16.782 \\
\alpha_1 + \gamma_1 - 180 &= -11.894 \\
\beta_1 + \delta_1 - 180 &= 92.469 \\
\alpha_1 + \delta_1 - 180 &= 31.325 \\
\beta_1 + \gamma_1 - 180 &= 49.250 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 47.012 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -104.360 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -17.925
\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= -34.914 \\
\alpha_2 - \gamma_2 &= 45.531 \\
\alpha_2 - \delta_2 &= 35.750 \\
\beta_2 - \gamma_2 &= 80.445 \\
\beta_2 - \delta_2 &= 70.665 \\
\gamma_2 - \delta_2 &= -9.781 \\
\alpha_2 + \beta_2 - 180 &= 86.415 \\
\gamma_2 + \delta_2 - 180 &= -29.781 \\
\alpha_2 + \gamma_2 - 180 &= 5.970 \\
\beta_2 + \delta_2 - 180 &= 50.665 \\
\alpha_2 + \delta_2 - 180 &= 15.750 \\
\beta_2 + \gamma_2 - 180 &= 40.884 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 116.200 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -44.695 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -25.133
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= -35.142 \\
\alpha_3 - \gamma_3 &= -59.190 \\
\alpha_3 - \delta_3 &= -53.808 \\
\beta_3 - \gamma_3 &= -24.049 \\
\beta_3 - \delta_3 &= -18.666 \\
\gamma_3 - \delta_3 &= 5.382 \\
\alpha_3 + \beta_3 - 180 &= -82.474 \\
\gamma_3 + \delta_3 - 180 &= -4.618 \\
\alpha_3 + \gamma_3 - 180 &= -58.426 \\
\beta_3 + \delta_3 - 180 &= -28.666 \\
\alpha_3 + \delta_3 - 180 &= -63.808 \\
\beta_3 + \gamma_3 - 180 &= -23.284 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -77.857 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -29.759 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -40.524
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= -44.131 \\
\alpha_4 - \gamma_4 &= -30.048 \\
\alpha_4 - \delta_4 &= -46.804 \\
\beta_4 - \gamma_4 &= 14.083 \\
\beta_4 - \delta_4 &= -2.674 \\
\gamma_4 - \delta_4 &= -16.757 \\
\alpha_4 + \beta_4 - 180 &= -79.478 \\
\gamma_4 + \delta_4 - 180 &= -46.757 \\
\alpha_4 + \gamma_4 - 180 &= -93.561 \\
\alpha_4 + \delta_4 - 180 &= -27.674
\end{aligned}$$

$$\begin{aligned}\beta_4 + \gamma_4 - 180 &= -52.017 \\ \alpha_4 + \delta_4 - 180 &= -76.804 \\ \beta_4 + \gamma_4 - 180 &= -49.430 \\ \alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -32.722 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -60.887 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -27.374\end{aligned}$$

Switch combination: Right + Lower

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 27.5312 & 76.7816 & 120 \\ 64.2495 & 150.665 & 109.781 & 80 \\ 31.192 & 113.666 & 89.6176 & 85 \\ 28.1955 & 72.3261 & 58.2435 & 75 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}\alpha_1 - \beta_1 &= 61.144 \\ \alpha_1 - \gamma_1 &= 11.894 \\ \alpha_1 - \delta_1 &= -31.325 \\ \beta_1 - \gamma_1 &= -49.250 \\ \beta_1 - \delta_1 &= -92.469 \\ \gamma_1 - \delta_1 &= -43.218 \\ \alpha_1 + \beta_1 - 180 &= -63.794 \\ \gamma_1 + \delta_1 - 180 &= 16.782 \\ \alpha_1 + \gamma_1 - 180 &= -14.543 \\ \beta_1 + \delta_1 - 180 &= -32.469 \\ \alpha_1 + \delta_1 - 180 &= 28.675 \\ \beta_1 + \gamma_1 - 180 &= -75.687 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -80.575 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 17.925 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 104.360\end{aligned}$$

Vertex 2

$$\begin{aligned}\alpha_2 - \beta_2 &= -86.415 \\ \alpha_2 - \gamma_2 &= -45.531 \\ \alpha_2 - \delta_2 &= -15.750 \\ \beta_2 - \gamma_2 &= 40.884 \\ \beta_2 - \delta_2 &= 70.665 \\ \gamma_2 - \delta_2 &= 29.781 \\ \alpha_2 + \beta_2 - 180 &= 34.914 \\ \gamma_2 + \delta_2 - 180 &= 9.781 \\ \alpha_2 + \gamma_2 - 180 &= -5.970 \\ \beta_2 + \delta_2 - 180 &= 50.665 \\ \alpha_2 + \delta_2 - 180 &= -35.750 \\ \beta_2 + \gamma_2 - 180 &= 80.445 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 25.133 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -56.634 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -116.200\end{aligned}$$

Vertex 3

$$\begin{aligned}\alpha_3 - \beta_3 &= -82.474 \\ \alpha_3 - \gamma_3 &= -58.426 \\ \alpha_3 - \delta_3 &= -53.808 \\ \beta_3 - \gamma_3 &= 24.049 \\ \beta_3 - \delta_3 &= 28.666 \\ \gamma_3 - \delta_3 &= 4.618 \\ \alpha_3 + \beta_3 - 180 &= -35.142 \\ \gamma_3 + \delta_3 - 180 &= -5.382 \\ \alpha_3 + \gamma_3 - 180 &= -59.190 \\ \beta_3 + \delta_3 - 180 &= 18.666 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 63.880\end{aligned}$$

$$\begin{aligned}\alpha_3 + \gamma_3 - 180 &= -65.808 \\ \beta_3 + \gamma_3 - 180 &= 23.284 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -29.759 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -77.857 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -87.092\end{aligned}$$

Vertex 4

$$\begin{aligned}\alpha_4 - \beta_4 &= -44.131 \\ \alpha_4 - \gamma_4 &= -30.048 \\ \alpha_4 - \delta_4 &= -46.804 \\ \beta_4 - \gamma_4 &= 14.083 \\ \beta_4 - \delta_4 &= -2.674 \\ \gamma_4 - \delta_4 &= -16.757 \\ \alpha_4 + \beta_4 - 180 &= -79.478 \\ \gamma_4 + \delta_4 - 180 &= -46.757 \\ \alpha_4 + \gamma_4 - 180 &= -93.561 \\ \beta_4 + \delta_4 - 180 &= -32.674 \\ \alpha_4 + \delta_4 - 180 &= -76.804 \\ \beta_4 + \gamma_4 - 180 &= -49.430 \\ \alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -32.722 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -60.887 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -27.374\end{aligned}$$

Switch combination: Right + Upper*Switched anglesDeg:*

$$\begin{pmatrix} 91.3249 & 152.469 & 76.7816 & 120 \\ 115.75 & 29.3354 & 109.781 & 80 \\ 148.808 & 66.3336 & 89.6176 & 85 \\ 151.804 & 107.674 & 58.2435 & 75 \end{pmatrix}$$

*Angle relation checks for $i = 1..4$:***Vertex 1**

$$\begin{aligned}\alpha_1 - \beta_1 &= -61.144 \\ \alpha_1 - \gamma_1 &= 14.543 \\ \alpha_1 - \delta_1 &= -28.675 \\ \beta_1 - \gamma_1 &= 75.687 \\ \beta_1 - \delta_1 &= 32.469 \\ \gamma_1 - \delta_1 &= -43.218 \\ \alpha_1 + \beta_1 - 180 &= 63.794 \\ \gamma_1 + \delta_1 - 180 &= 16.782 \\ \alpha_1 + \gamma_1 - 180 &= -11.894 \\ \beta_1 + \delta_1 - 180 &= 92.469 \\ \alpha_1 + \delta_1 - 180 &= 31.325 \\ \beta_1 + \gamma_1 - 180 &= 49.250 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 47.012 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -104.360 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -17.925\end{aligned}$$

Vertex 2

$$\begin{aligned}\alpha_2 - \beta_2 &= 86.415 \\ \alpha_2 - \gamma_2 &= 5.970 \\ \alpha_2 - \delta_2 &= 35.750 \\ \beta_2 - \gamma_2 &= -80.445 \\ \beta_2 - \delta_2 &= -50.665 \\ \gamma_2 - \delta_2 &= 29.781 \\ \alpha_2 + \beta_2 - 180 &= -34.914 \\ \gamma_2 + \delta_2 - 180 &= 9.781 \\ \alpha_2 + \gamma_2 - 180 &= 45.531 \\ \beta_2 + \delta_2 - 180 &= -70.665 \\ \alpha_2 + \delta_2 - 180 &= 15.750 \\ \beta_2 + \gamma_2 - 180 &= -74.475\end{aligned}$$

$$\begin{aligned}\beta_2 + \gamma_2 - 180 &= -40.884 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -44.695 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 116.200 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 56.634\end{aligned}$$

Vertex 3

$$\begin{aligned}\alpha_3 - \beta_3 &= 82.474 \\ \alpha_3 - \gamma_3 &= 59.190 \\ \alpha_3 - \delta_3 &= 63.808 \\ \beta_3 - \gamma_3 &= -23.284 \\ \beta_3 - \delta_3 &= -18.666 \\ \gamma_3 - \delta_3 &= 4.618 \\ \alpha_3 + \beta_3 - 180 &= 35.142 \\ \gamma_3 + \delta_3 - 180 &= -5.382 \\ \alpha_3 + \gamma_3 - 180 &= 58.426 \\ \beta_3 + \delta_3 - 180 &= -28.666 \\ \alpha_3 + \delta_3 - 180 &= 53.808 \\ \beta_3 + \gamma_3 - 180 &= -24.049 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 40.524 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 87.092 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 77.857\end{aligned}$$

Vertex 4

$$\begin{aligned}\alpha_4 - \beta_4 &= 44.131 \\ \alpha_4 - \gamma_4 &= 93.561 \\ \alpha_4 - \delta_4 &= 76.804 \\ \beta_4 - \gamma_4 &= 49.430 \\ \beta_4 - \delta_4 &= 32.674 \\ \gamma_4 - \delta_4 &= -16.757 \\ \alpha_4 + \beta_4 - 180 &= 79.478 \\ \gamma_4 + \delta_4 - 180 &= -46.757 \\ \alpha_4 + \gamma_4 - 180 &= 30.048 \\ \beta_4 + \delta_4 - 180 &= 2.674 \\ \alpha_4 + \delta_4 - 180 &= 46.804 \\ \beta_4 + \gamma_4 - 180 &= -14.083 \\ \alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 126.230 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 27.374 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 60.887\end{aligned}$$

Switch combination: Left + Lower*Switched anglesDeg:*

$$\begin{pmatrix} 88.6751 & 152.469 & 103.218 & 120 \\ 64.2495 & 29.3354 & 70.2192 & 80 \\ 31.192 & 66.3336 & 90.3824 & 85 \\ 28.1955 & 107.674 & 121.757 & 75 \end{pmatrix}$$

*Angle relation checks for $i = 1..4$:***Vertex 1**

$$\begin{aligned}\alpha_1 - \beta_1 &= -63.794 \\ \alpha_1 - \gamma_1 &= -14.543 \\ \alpha_1 - \delta_1 &= -31.325 \\ \beta_1 - \gamma_1 &= 49.250 \\ \beta_1 - \delta_1 &= 32.469 \\ \gamma_1 - \delta_1 &= -16.782 \\ \alpha_1 + \beta_1 - 180 &= 61.144 \\ \gamma_1 + \delta_1 - 180 &= 43.218 \\ \alpha_1 + \gamma_1 - 180 &= 11.894 \\ \beta_1 + \delta_1 - 180 &= 92.469 \\ \alpha_1 + \delta_1 - 180 &= 28.675 \\ \beta_1 + \gamma_1 - 180 &= 75.687\end{aligned}$$

$$\begin{aligned}\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 17.925 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -80.575 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -47.012\end{aligned}$$

Vertex 2

$$\begin{aligned}\alpha_2 - \beta_2 &= 34.914 \\ \alpha_2 - \gamma_2 &= -5.970 \\ \alpha_2 - \delta_2 &= -15.750 \\ \beta_2 - \gamma_2 &= -40.884 \\ \beta_2 - \delta_2 &= -50.665 \\ \gamma_2 - \delta_2 &= -9.781 \\ \alpha_2 + \beta_2 - 180 &= -86.415 \\ \gamma_2 + \delta_2 - 180 &= -29.781 \\ \alpha_2 + \gamma_2 - 180 &= -45.531 \\ \beta_2 + \delta_2 - 180 &= -70.665 \\ \alpha_2 + \delta_2 - 180 &= -35.750 \\ \beta_2 + \gamma_2 - 180 &= -80.445 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -56.634 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 25.133 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 44.695\end{aligned}$$

Vertex 3

$$\begin{aligned}\alpha_3 - \beta_3 &= -35.142 \\ \alpha_3 - \gamma_3 &= -59.190 \\ \alpha_3 - \delta_3 &= -53.808 \\ \beta_3 - \gamma_3 &= -24.049 \\ \beta_3 - \delta_3 &= -18.666 \\ \gamma_3 - \delta_3 &= 5.382 \\ \alpha_3 + \beta_3 - 180 &= -82.474 \\ \gamma_3 + \delta_3 - 180 &= -4.618 \\ \alpha_3 + \gamma_3 - 180 &= -58.426 \\ \beta_3 + \delta_3 - 180 &= -28.666 \\ \alpha_3 + \delta_3 - 180 &= -63.808 \\ \beta_3 + \gamma_3 - 180 &= -23.284 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -77.857 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -29.759 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -40.524\end{aligned}$$

Vertex 4

$$\begin{aligned}\alpha_4 - \beta_4 &= -79.478 \\ \alpha_4 - \gamma_4 &= -93.561 \\ \alpha_4 - \delta_4 &= -46.804 \\ \beta_4 - \gamma_4 &= -14.083 \\ \beta_4 - \delta_4 &= 32.674 \\ \gamma_4 - \delta_4 &= 46.757 \\ \alpha_4 + \beta_4 - 180 &= -44.131 \\ \gamma_4 + \delta_4 - 180 &= 16.757 \\ \alpha_4 + \gamma_4 - 180 &= -30.048 \\ \beta_4 + \delta_4 - 180 &= 2.674 \\ \alpha_4 + \delta_4 - 180 &= -76.804 \\ \beta_4 + \gamma_4 - 180 &= 49.430 \\ \alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -60.887 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -32.722 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -126.230\end{aligned}$$

Switch combination: Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 27.5312 & 103.218 & 120 \\ 115.75 & 150.665 & 70.2192 & 80 \\ 148.808 & 113.666 & 90.3824 & 85 \\ 151.804 & 72.3261 & 121.757 & 75 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned} \alpha_1 - \beta_1 &= 63.794 \\ \alpha_1 - \gamma_1 &= -11.894 \\ \alpha_1 - \delta_1 &= -28.675 \\ \beta_1 - \gamma_1 &= -75.687 \\ \beta_1 - \delta_1 &= -92.469 \\ \gamma_1 - \delta_1 &= -16.782 \\ \alpha_1 + \beta_1 - 180 &= -61.144 \\ \gamma_1 + \delta_1 - 180 &= 43.218 \\ \alpha_1 + \gamma_1 - 180 &= 14.543 \\ \beta_1 + \delta_1 - 180 &= -32.469 \\ \alpha_1 + \delta_1 - 180 &= 31.325 \\ \beta_1 + \gamma_1 - 180 &= -49.250 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -104.360 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 47.012 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 80.575 \end{aligned}$$

Vertex 2

$$\begin{aligned} \alpha_2 - \beta_2 &= -34.914 \\ \alpha_2 - \gamma_2 &= 45.531 \\ \alpha_2 - \delta_2 &= 35.750 \\ \beta_2 - \gamma_2 &= 80.445 \\ \beta_2 - \delta_2 &= 70.665 \\ \gamma_2 - \delta_2 &= -9.781 \\ \alpha_2 + \beta_2 - 180 &= 86.415 \\ \gamma_2 + \delta_2 - 180 &= -29.781 \\ \alpha_2 + \gamma_2 - 180 &= 5.970 \\ \beta_2 + \delta_2 - 180 &= 50.665 \\ \alpha_2 + \delta_2 - 180 &= 15.750 \\ \beta_2 + \gamma_2 - 180 &= 40.884 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 116.200 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -44.695 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -25.133 \end{aligned}$$

Vertex 3

$$\begin{aligned} \alpha_3 - \beta_3 &= 35.142 \\ \alpha_3 - \gamma_3 &= 58.426 \\ \alpha_3 - \delta_3 &= 63.808 \\ \beta_3 - \gamma_3 &= 23.284 \\ \beta_3 - \delta_3 &= 28.666 \\ \gamma_3 - \delta_3 &= 5.382 \\ \alpha_3 + \beta_3 - 180 &= 82.474 \\ \gamma_3 + \delta_3 - 180 &= -4.618 \\ \alpha_3 + \gamma_3 - 180 &= 59.190 \\ \beta_3 + \delta_3 - 180 &= 18.666 \\ \alpha_3 + \delta_3 - 180 &= 53.808 \\ \beta_3 + \gamma_3 - 180 &= 24.049 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 87.092 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 40.524 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 29.759 \end{aligned}$$

Vertex 4

$$\begin{aligned} \alpha_4 - \beta_4 &= 79.478 \\ \alpha_4 - \gamma_4 &= 30.048 \\ \alpha_4 - \delta_4 &= 76.804 \\ \beta_4 - \gamma_4 &= -49.420 \end{aligned}$$

```

 $\beta_4 - \delta_4 = -2.674$ 
 $\gamma_4 - \delta_4 = 46.757$ 
 $\alpha_4 + \beta_4 - 180 = 44.131$ 
 $\gamma_4 + \delta_4 - 180 = 16.757$ 
 $\alpha_4 + \gamma_4 - 180 = 93.561$ 
 $\beta_4 + \delta_4 - 180 = -32.674$ 
 $\alpha_4 + \delta_4 - 180 = 46.804$ 
 $\beta_4 + \gamma_4 - 180 = 14.083$ 
 $\alpha_4 + \beta_4 - \gamma_4 - \delta_4 = 27.374$ 
 $\alpha_4 + \gamma_4 - \beta_4 - \delta_4 = 126.230$ 
 $\alpha_4 + \delta_4 - \beta_4 - \gamma_4 = 32.722$ 

```

Switch combination: Lower + Upper

Switched anglesDeg:

```

( 88.6751 152.469 103.218 120 )
( 64.2495 150.665 109.781 80 )
( 148.808 66.3336 89.6176 85 )
( 151.804 72.3261 121.757 75 )

```

Angle relation checks for i = 1..4:

Vertex 1

```

 $\alpha_1 - \beta_1 = -63.794$ 
 $\alpha_1 - \gamma_1 = -14.543$ 
 $\alpha_1 - \delta_1 = -31.325$ 
 $\beta_1 - \gamma_1 = 49.250$ 
 $\beta_1 - \delta_1 = 32.469$ 
 $\gamma_1 - \delta_1 = -16.782$ 
 $\alpha_1 + \beta_1 - 180 = 61.144$ 
 $\gamma_1 + \delta_1 - 180 = 43.218$ 
 $\alpha_1 + \gamma_1 - 180 = 11.894$ 
 $\beta_1 + \delta_1 - 180 = 92.469$ 
 $\alpha_1 + \delta_1 - 180 = 28.675$ 
 $\beta_1 + \gamma_1 - 180 = 75.687$ 
 $\alpha_1 + \beta_1 - \gamma_1 - \delta_1 = 17.925$ 
 $\alpha_1 + \gamma_1 - \beta_1 - \delta_1 = -80.575$ 
 $\alpha_1 + \delta_1 - \beta_1 - \gamma_1 = -47.012$ 

```

Vertex 2

```

 $\alpha_2 - \beta_2 = -86.415$ 
 $\alpha_2 - \gamma_2 = -45.531$ 
 $\alpha_2 - \delta_2 = -15.750$ 
 $\beta_2 - \gamma_2 = 40.884$ 
 $\beta_2 - \delta_2 = 70.665$ 
 $\gamma_2 - \delta_2 = 29.781$ 
 $\alpha_2 + \beta_2 - 180 = 34.914$ 
 $\gamma_2 + \delta_2 - 180 = 9.781$ 
 $\alpha_2 + \gamma_2 - 180 = -5.970$ 
 $\beta_2 + \delta_2 - 180 = 50.665$ 
 $\alpha_2 + \delta_2 - 180 = -35.750$ 
 $\beta_2 + \gamma_2 - 180 = 80.445$ 
 $\alpha_2 + \beta_2 - \gamma_2 - \delta_2 = 25.133$ 
 $\alpha_2 + \gamma_2 - \beta_2 - \delta_2 = -56.634$ 
 $\alpha_2 + \delta_2 - \beta_2 - \gamma_2 = -116.200$ 

```

Vertex 3

```

 $\alpha_3 - \beta_3 = 82.474$ 
 $\alpha_3 - \gamma_3 = 59.190$ 
 $\alpha_3 - \delta_3 = 63.808$ 
 $\beta_3 - \gamma_3 = -23.284$ 
 $\beta_3 - \delta_3 = 10.666$ 

```

$$\begin{aligned}
\mu_3 - \nu_3 &= -10.000 \\
\gamma_3 - \delta_3 &= 4.618 \\
\alpha_3 + \beta_3 - 180 &= 35.142 \\
\gamma_3 + \delta_3 - 180 &= -5.382 \\
\alpha_3 + \gamma_3 - 180 &= 58.426 \\
\beta_3 + \delta_3 - 180 &= -28.666 \\
\alpha_3 + \delta_3 - 180 &= 53.808 \\
\beta_3 + \gamma_3 - 180 &= -24.049 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 40.524 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 87.092 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 77.857
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= 79.478 \\
\alpha_4 - \gamma_4 &= 30.048 \\
\alpha_4 - \delta_4 &= 76.804 \\
\beta_4 - \gamma_4 &= -49.430 \\
\beta_4 - \delta_4 &= -2.674 \\
\gamma_4 - \delta_4 &= 46.757 \\
\alpha_4 + \beta_4 - 180 &= 44.131 \\
\gamma_4 + \delta_4 - 180 &= 16.757 \\
\alpha_4 + \gamma_4 - 180 &= 93.561 \\
\beta_4 + \delta_4 - 180 &= -32.674 \\
\alpha_4 + \delta_4 - 180 &= 46.804 \\
\beta_4 + \gamma_4 - 180 &= 14.083 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 27.374 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 126.230 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 32.722
\end{aligned}$$

Switch combination: Right + Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 27.5312 & 76.7816 & 120 \\ 64.2495 & 29.3354 & 70.2192 & 80 \\ 31.192 & 66.3336 & 90.3824 & 85 \\ 28.1955 & 72.3261 & 58.2435 & 75 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= 61.144 \\
\alpha_1 - \gamma_1 &= 11.894 \\
\alpha_1 - \delta_1 &= -31.325 \\
\beta_1 - \gamma_1 &= -49.250 \\
\beta_1 - \delta_1 &= -92.469 \\
\gamma_1 - \delta_1 &= -43.218 \\
\alpha_1 + \beta_1 - 180 &= -63.794 \\
\gamma_1 + \delta_1 - 180 &= 16.782 \\
\alpha_1 + \gamma_1 - 180 &= -14.543 \\
\beta_1 + \delta_1 - 180 &= -32.469 \\
\alpha_1 + \delta_1 - 180 &= 28.675 \\
\beta_1 + \gamma_1 - 180 &= -75.687 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -80.575 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 17.925 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 104.360
\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= 34.914 \\
\alpha_2 - \gamma_2 &= -5.970 \\
\alpha_2 - \delta_2 &= -15.750 \\
\beta_2 - \gamma_2 &= -40.884 \\
\beta_2 - \delta_2 &= -50.665 \\
\gamma_2 - \delta_2 &= -8.701
\end{aligned}$$

$$\begin{aligned}
\gamma_2 - \alpha_2 &= -9.781 \\
\alpha_2 + \beta_2 - 180 &= -86.415 \\
\gamma_2 + \delta_2 - 180 &= -29.781 \\
\alpha_2 + \gamma_2 - 180 &= -45.531 \\
\beta_2 + \delta_2 - 180 &= -70.665 \\
\alpha_2 + \delta_2 - 180 &= -35.750 \\
\beta_2 + \gamma_2 - 180 &= -80.445 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -56.634 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 25.133 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 44.695
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= -35.142 \\
\alpha_3 - \gamma_3 &= -59.190 \\
\alpha_3 - \delta_3 &= -53.808 \\
\beta_3 - \gamma_3 &= -24.049 \\
\beta_3 - \delta_3 &= -18.666 \\
\gamma_3 - \delta_3 &= 5.382 \\
\alpha_3 + \beta_3 - 180 &= -82.474 \\
\gamma_3 + \delta_3 - 180 &= -4.618 \\
\alpha_3 + \gamma_3 - 180 &= -58.426 \\
\beta_3 + \delta_3 - 180 &= -28.666 \\
\alpha_3 + \delta_3 - 180 &= -63.808 \\
\beta_3 + \gamma_3 - 180 &= -23.284 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -77.857 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -29.759 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -40.524
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= -44.131 \\
\alpha_4 - \gamma_4 &= -30.048 \\
\alpha_4 - \delta_4 &= -46.804 \\
\beta_4 - \gamma_4 &= 14.083 \\
\beta_4 - \delta_4 &= -2.674 \\
\gamma_4 - \delta_4 &= -16.757 \\
\alpha_4 + \beta_4 - 180 &= -79.478 \\
\gamma_4 + \delta_4 - 180 &= -46.757 \\
\alpha_4 + \gamma_4 - 180 &= -93.561 \\
\beta_4 + \delta_4 - 180 &= -32.674 \\
\alpha_4 + \delta_4 - 180 &= -76.804 \\
\beta_4 + \gamma_4 - 180 &= -49.430 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -32.722 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -60.887 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -27.374
\end{aligned}$$

Switch combination: Right + Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 152.469 & 76.7816 & 120 \\ 115.75 & 150.665 & 70.2192 & 80 \\ 148.808 & 113.666 & 90.3824 & 85 \\ 151.804 & 107.674 & 58.2435 & 75 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= -61.144 \\
\alpha_1 - \gamma_1 &= 14.543 \\
\alpha_1 - \delta_1 &= -28.675 \\
\beta_1 - \gamma_1 &= 75.687 \\
\beta_1 - \delta_1 &= 32.469 \\
\gamma_1 - \delta_1 &= -43.218 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -35.142 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -59.190 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -53.808 \\
\beta_1 + \gamma_1 - \alpha_1 - \delta_1 &= -24.049 \\
\beta_1 + \delta_1 - \alpha_1 - \gamma_1 &= -18.666 \\
\gamma_1 + \delta_1 - \alpha_1 - \beta_1 &= 5.382
\end{aligned}$$

$$\begin{aligned}
\alpha_1 + \beta_1 - 180 &= 63.794 \\
\gamma_1 + \delta_1 - 180 &= 16.782 \\
\alpha_1 + \gamma_1 - 180 &= -11.894 \\
\beta_1 + \delta_1 - 180 &= 92.469 \\
\alpha_1 + \delta_1 - 180 &= 31.325 \\
\beta_1 + \gamma_1 - 180 &= 49.250 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 47.012 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -104.360 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -17.925
\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= -34.914 \\
\alpha_2 - \gamma_2 &= 45.531 \\
\alpha_2 - \delta_2 &= 35.750 \\
\beta_2 - \gamma_2 &= 80.445 \\
\beta_2 - \delta_2 &= 70.665 \\
\gamma_2 - \delta_2 &= -9.781 \\
\alpha_2 + \beta_2 - 180 &= 86.415 \\
\gamma_2 + \delta_2 - 180 &= -29.781 \\
\alpha_2 + \gamma_2 - 180 &= 5.970 \\
\beta_2 + \delta_2 - 180 &= 50.665 \\
\alpha_2 + \delta_2 - 180 &= 15.750 \\
\beta_2 + \gamma_2 - 180 &= 40.884 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 116.200 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -44.695 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -25.133
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= 35.142 \\
\alpha_3 - \gamma_3 &= 58.426 \\
\alpha_3 - \delta_3 &= 63.808 \\
\beta_3 - \gamma_3 &= 23.284 \\
\beta_3 - \delta_3 &= 28.666 \\
\gamma_3 - \delta_3 &= 5.382 \\
\alpha_3 + \beta_3 - 180 &= 82.474 \\
\gamma_3 + \delta_3 - 180 &= -4.618 \\
\alpha_3 + \gamma_3 - 180 &= 59.190 \\
\beta_3 + \delta_3 - 180 &= 18.666 \\
\alpha_3 + \delta_3 - 180 &= 53.808 \\
\beta_3 + \gamma_3 - 180 &= 24.049 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 87.092 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 40.524 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 29.759
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= 44.131 \\
\alpha_4 - \gamma_4 &= 93.561 \\
\alpha_4 - \delta_4 &= 76.804 \\
\beta_4 - \gamma_4 &= 49.430 \\
\beta_4 - \delta_4 &= 32.674 \\
\gamma_4 - \delta_4 &= -16.757 \\
\alpha_4 + \beta_4 - 180 &= 79.478 \\
\gamma_4 + \delta_4 - 180 &= -46.757 \\
\alpha_4 + \gamma_4 - 180 &= 30.048 \\
\beta_4 + \delta_4 - 180 &= 2.674 \\
\alpha_4 + \delta_4 - 180 &= 46.804 \\
\beta_4 + \gamma_4 - 180 &= -14.083 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 126.230 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 27.374 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 60.887
\end{aligned}$$

Switch combination: Right + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 27.5312 & 76.7816 & 120 \\ 64.2495 & 150.665 & 109.781 & 80 \\ 148.808 & 66.3336 & 89.6176 & 85 \\ 151.804 & 107.674 & 58.2435 & 75 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned} \alpha_1 - \beta_1 &= 61.144 \\ \alpha_1 - \gamma_1 &= 11.894 \\ \alpha_1 - \delta_1 &= -31.325 \\ \beta_1 - \gamma_1 &= -49.250 \\ \beta_1 - \delta_1 &= -92.469 \\ \gamma_1 - \delta_1 &= -43.218 \\ \alpha_1 + \beta_1 - 180 &= -63.794 \\ \gamma_1 + \delta_1 - 180 &= 16.782 \\ \alpha_1 + \gamma_1 - 180 &= -14.543 \\ \beta_1 + \delta_1 - 180 &= -32.469 \\ \alpha_1 + \delta_1 - 180 &= 28.675 \\ \beta_1 + \gamma_1 - 180 &= -75.687 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -80.575 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 17.925 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 104.360 \end{aligned}$$

Vertex 2

$$\begin{aligned} \alpha_2 - \beta_2 &= -86.415 \\ \alpha_2 - \gamma_2 &= -45.531 \\ \alpha_2 - \delta_2 &= -15.750 \\ \beta_2 - \gamma_2 &= 40.884 \\ \beta_2 - \delta_2 &= 70.665 \\ \gamma_2 - \delta_2 &= 29.781 \\ \alpha_2 + \beta_2 - 180 &= 34.914 \\ \gamma_2 + \delta_2 - 180 &= 9.781 \\ \alpha_2 + \gamma_2 - 180 &= -5.970 \\ \beta_2 + \delta_2 - 180 &= 50.665 \\ \alpha_2 + \delta_2 - 180 &= -35.750 \\ \beta_2 + \gamma_2 - 180 &= 80.445 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 25.133 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -56.634 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -116.200 \end{aligned}$$

Vertex 3

$$\begin{aligned} \alpha_3 - \beta_3 &= 82.474 \\ \alpha_3 - \gamma_3 &= 59.190 \\ \alpha_3 - \delta_3 &= 63.808 \\ \beta_3 - \gamma_3 &= -23.284 \\ \beta_3 - \delta_3 &= -18.666 \\ \gamma_3 - \delta_3 &= 4.618 \\ \alpha_3 + \beta_3 - 180 &= 35.142 \\ \gamma_3 + \delta_3 - 180 &= -5.382 \\ \alpha_3 + \gamma_3 - 180 &= 58.426 \\ \beta_3 + \delta_3 - 180 &= -28.666 \\ \alpha_3 + \delta_3 - 180 &= 53.808 \\ \beta_3 + \gamma_3 - 180 &= -24.049 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 40.524 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 87.092 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 77.857 \end{aligned}$$

.. . -

Vertex 4

$$\begin{aligned}
\alpha 4 - \beta 4 &= 44.131 \\
\alpha 4 - \gamma 4 &= 93.561 \\
\alpha 4 - \delta 4 &= 76.804 \\
\beta 4 - \gamma 4 &= 49.430 \\
\beta 4 - \delta 4 &= 32.674 \\
\gamma 4 - \delta 4 &= -16.757 \\
\alpha 4 + \beta 4 - 180 &= 79.478 \\
\gamma 4 + \delta 4 - 180 &= -46.757 \\
\alpha 4 + \gamma 4 - 180 &= 30.048 \\
\beta 4 + \delta 4 - 180 &= 2.674 \\
\alpha 4 + \delta 4 - 180 &= 46.804 \\
\beta 4 + \gamma 4 - 180 &= -14.083 \\
\alpha 4 + \beta 4 - \gamma 4 - \delta 4 &= 126.230 \\
\alpha 4 + \gamma 4 - \beta 4 - \delta 4 &= 27.374 \\
\alpha 4 + \delta 4 - \beta 4 - \gamma 4 &= 60.887
\end{aligned}$$

Switch combination: Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 152.469 & 103.218 & 120 \\ 64.2495 & 29.3354 & 70.2192 & 80 \\ 148.808 & 113.666 & 90.3824 & 85 \\ 151.804 & 72.3261 & 121.757 & 75 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}
\alpha 1 - \beta 1 &= -63.794 \\
\alpha 1 - \gamma 1 &= -14.543 \\
\alpha 1 - \delta 1 &= -31.325 \\
\beta 1 - \gamma 1 &= 49.250 \\
\beta 1 - \delta 1 &= 32.469 \\
\gamma 1 - \delta 1 &= -16.782 \\
\alpha 1 + \beta 1 - 180 &= 61.144 \\
\gamma 1 + \delta 1 - 180 &= 43.218 \\
\alpha 1 + \gamma 1 - 180 &= 11.894 \\
\beta 1 + \delta 1 - 180 &= 92.469 \\
\alpha 1 + \delta 1 - 180 &= 28.675 \\
\beta 1 + \gamma 1 - 180 &= 75.687 \\
\alpha 1 + \beta 1 - \gamma 1 - \delta 1 &= 17.925 \\
\alpha 1 + \gamma 1 - \beta 1 - \delta 1 &= -80.575 \\
\alpha 1 + \delta 1 - \beta 1 - \gamma 1 &= -47.012
\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha 2 - \beta 2 &= 34.914 \\
\alpha 2 - \gamma 2 &= -5.970 \\
\alpha 2 - \delta 2 &= -15.750 \\
\beta 2 - \gamma 2 &= -40.884 \\
\beta 2 - \delta 2 &= -50.665 \\
\gamma 2 - \delta 2 &= -9.781 \\
\alpha 2 + \beta 2 - 180 &= -86.415 \\
\gamma 2 + \delta 2 - 180 &= -29.781 \\
\alpha 2 + \gamma 2 - 180 &= -45.531 \\
\beta 2 + \delta 2 - 180 &= -70.665 \\
\alpha 2 + \delta 2 - 180 &= -35.750 \\
\beta 2 + \gamma 2 - 180 &= -80.445 \\
\alpha 2 + \beta 2 - \gamma 2 - \delta 2 &= -56.634 \\
\alpha 2 + \gamma 2 - \beta 2 - \delta 2 &= 25.133 \\
\alpha 2 + \delta 2 - \beta 2 - \gamma 2 &= 44.695
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= 35.142 \\
\alpha_3 - \gamma_3 &= 58.426 \\
\alpha_3 - \delta_3 &= 63.808 \\
\beta_3 - \gamma_3 &= 23.284 \\
\beta_3 - \delta_3 &= 28.666 \\
\gamma_3 - \delta_3 &= 5.382 \\
\alpha_3 + \beta_3 - 180 &= 82.474 \\
\gamma_3 + \delta_3 - 180 &= -4.618 \\
\alpha_3 + \gamma_3 - 180 &= 59.190 \\
\beta_3 + \delta_3 - 180 &= 18.666 \\
\alpha_3 + \delta_3 - 180 &= 53.808 \\
\beta_3 + \gamma_3 - 180 &= 24.049 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 87.092 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 40.524 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 29.759
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= 79.478 \\
\alpha_4 - \gamma_4 &= 30.048 \\
\alpha_4 - \delta_4 &= 76.804 \\
\beta_4 - \gamma_4 &= -49.430 \\
\beta_4 - \delta_4 &= -2.674 \\
\gamma_4 - \delta_4 &= 46.757 \\
\alpha_4 + \beta_4 - 180 &= 44.131 \\
\gamma_4 + \delta_4 - 180 &= 16.757 \\
\alpha_4 + \gamma_4 - 180 &= 93.561 \\
\beta_4 + \delta_4 - 180 &= -32.674 \\
\alpha_4 + \delta_4 - 180 &= 46.804 \\
\beta_4 + \gamma_4 - 180 &= 14.083 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 27.374 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 126.230 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 32.722
\end{aligned}$$

Switch combination: Right + Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 27.5312 & 76.7816 & 120 \\ 64.2495 & 29.3354 & 70.2192 & 80 \\ 148.808 & 113.666 & 90.3824 & 85 \\ 151.804 & 107.674 & 58.2435 & 75 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= 61.144 \\
\alpha_1 - \gamma_1 &= 11.894 \\
\alpha_1 - \delta_1 &= -31.325 \\
\beta_1 - \gamma_1 &= -49.250 \\
\beta_1 - \delta_1 &= -92.469 \\
\gamma_1 - \delta_1 &= -43.218 \\
\alpha_1 + \beta_1 - 180 &= -63.794 \\
\gamma_1 + \delta_1 - 180 &= 16.782 \\
\alpha_1 + \gamma_1 - 180 &= -14.543 \\
\beta_1 + \delta_1 - 180 &= -32.469 \\
\alpha_1 + \delta_1 - 180 &= 28.675 \\
\beta_1 + \gamma_1 - 180 &= -75.687 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -80.575 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 17.925 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 104.360
\end{aligned}$$

Vertex 2

$$\alpha_2 - \beta_2 = 34.914$$

```

 $\alpha_2 - \gamma_2 = -5.970$ 
 $\alpha_2 - \delta_2 = -15.750$ 
 $\beta_2 - \gamma_2 = -40.884$ 
 $\beta_2 - \delta_2 = -50.665$ 
 $\gamma_2 - \delta_2 = -9.781$ 
 $\alpha_2 + \beta_2 - 180 = -86.415$ 
 $\gamma_2 + \delta_2 - 180 = -29.781$ 
 $\alpha_2 + \gamma_2 - 180 = -45.531$ 
 $\beta_2 + \delta_2 - 180 = -70.665$ 
 $\alpha_2 + \delta_2 - 180 = -35.750$ 
 $\beta_2 + \gamma_2 - 180 = -80.445$ 
 $\alpha_2 + \beta_2 - \gamma_2 - \delta_2 = -56.634$ 
 $\alpha_2 + \gamma_2 - \beta_2 - \delta_2 = 25.133$ 
 $\alpha_2 + \delta_2 - \beta_2 - \gamma_2 = 44.695$ 

```

Vertex 3

```

 $\alpha_3 - \beta_3 = 35.142$ 
 $\alpha_3 - \gamma_3 = 58.426$ 
 $\alpha_3 - \delta_3 = 63.808$ 
 $\beta_3 - \gamma_3 = 23.284$ 
 $\beta_3 - \delta_3 = 28.666$ 
 $\gamma_3 - \delta_3 = 5.382$ 
 $\alpha_3 + \beta_3 - 180 = 82.474$ 
 $\gamma_3 + \delta_3 - 180 = -4.618$ 
 $\alpha_3 + \gamma_3 - 180 = 59.190$ 
 $\beta_3 + \delta_3 - 180 = 18.666$ 
 $\alpha_3 + \delta_3 - 180 = 53.808$ 
 $\beta_3 + \gamma_3 - 180 = 24.049$ 
 $\alpha_3 + \beta_3 - \gamma_3 - \delta_3 = 87.092$ 
 $\alpha_3 + \gamma_3 - \beta_3 - \delta_3 = 40.524$ 
 $\alpha_3 + \delta_3 - \beta_3 - \gamma_3 = 29.759$ 

```

Vertex 4

```

 $\alpha_4 - \beta_4 = 44.131$ 
 $\alpha_4 - \gamma_4 = 93.561$ 
 $\alpha_4 - \delta_4 = 76.804$ 
 $\beta_4 - \gamma_4 = 49.430$ 
 $\beta_4 - \delta_4 = 32.674$ 
 $\gamma_4 - \delta_4 = -16.757$ 
 $\alpha_4 + \beta_4 - 180 = 79.478$ 
 $\gamma_4 + \delta_4 - 180 = -46.757$ 
 $\alpha_4 + \gamma_4 - 180 = 30.048$ 
 $\beta_4 + \delta_4 - 180 = 2.674$ 
 $\alpha_4 + \delta_4 - 180 = 46.804$ 
 $\beta_4 + \gamma_4 - 180 = -14.083$ 
 $\alpha_4 + \beta_4 - \gamma_4 - \delta_4 = 126.230$ 
 $\alpha_4 + \gamma_4 - \beta_4 - \delta_4 = 27.374$ 
 $\alpha_4 + \delta_4 - \beta_4 - \gamma_4 = 60.887$ 

```

Out[99]=

===== NOT CONJUGATE-MODULAR =====

$M_i < 1$ and $p_i \in \mathbb{R}$ for all $i = 1 \dots 4 \Rightarrow$ NOT

conjugate-modular. Boundary-strip switches preserve this.

Initial configuration (no switches applied):

```

( 91.3249 27.5312 103.218 120 )
( 115.75 29.3354 109.781 80 )
( 31.192 113.666 89.6176 85 )
( 28.1955 107.674 121.757 75 )

```

M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Right

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 152.469 & 76.7816 & 120 \\ 115.75 & 29.3354 & 109.781 & 80 \\ 31.192 & 113.666 & 89.6176 & 85 \\ 28.1955 & 72.3261 & 58.2435 & 75 \end{pmatrix}$$
M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Left

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 27.5312 & 103.218 & 120 \\ 115.75 & 150.665 & 70.2192 & 80 \\ 31.192 & 66.3336 & 90.3824 & 85 \\ 28.1955 & 107.674 & 121.757 & 75 \end{pmatrix}$$
M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Lower

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 152.469 & 103.218 & 120 \\ 64.2495 & 150.665 & 109.781 & 80 \\ 31.192 & 113.666 & 89.6176 & 85 \\ 28.1955 & 107.674 & 121.757 & 75 \end{pmatrix}$$
M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Upper

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 27.5312 & 103.218 & 120 \\ 115.75 & 29.3354 & 109.781 & 80 \\ 148.808 & 66.3336 & 89.6176 & 85 \\ 151.804 & 72.3261 & 121.757 & 75 \end{pmatrix}$$
M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Right + Left

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 152.469 & 76.7816 & 120 \\ 115.75 & 150.665 & 70.2192 & 80 \\ 31.192 & 66.3336 & 90.3824 & 85 \\ 28.1955 & 72.3261 & 58.2435 & 75 \end{pmatrix}$$

M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Right + Lower

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 27.5312 & 76.7816 & 120 \\ 64.2495 & 150.665 & 109.781 & 80 \\ 31.192 & 113.666 & 89.6176 & 85 \\ 28.1955 & 72.3261 & 58.2435 & 75 \end{pmatrix}$$
M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Right + Upper

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 152.469 & 76.7816 & 120 \\ 115.75 & 29.3354 & 109.781 & 80 \\ 148.808 & 66.3336 & 89.6176 & 85 \\ 151.804 & 107.674 & 58.2435 & 75 \end{pmatrix}$$
M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 152.469 & 103.218 & 120 \\ 64.2495 & 29.3354 & 70.2192 & 80 \\ 31.192 & 66.3336 & 90.3824 & 85 \\ 28.1955 & 107.674 & 121.757 & 75 \end{pmatrix}$$
M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 27.5312 & 103.218 & 120 \\ 115.75 & 150.665 & 70.2192 & 80 \\ 148.808 & 113.666 & 90.3824 & 85 \\ 151.804 & 72.3261 & 121.757 & 75 \end{pmatrix}$$
M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 152.469 & 103.218 & 120 \\ 64.2495 & 150.665 & 109.781 & 80 \\ 148.808 & 66.3336 & 89.6176 & 85 \\ 151.804 & 72.3261 & 121.757 & 75 \end{pmatrix}$$

M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Right + Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 27.5312 & 76.7816 & 120 \\ 64.2495 & 29.3354 & 70.2192 & 80 \\ 31.192 & 66.3336 & 90.3824 & 85 \\ 28.1955 & 72.3261 & 58.2435 & 75 \end{pmatrix}$$
M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Right + Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 91.3249 & 152.469 & 76.7816 & 120 \\ 115.75 & 150.665 & 70.2192 & 80 \\ 148.808 & 113.666 & 90.3824 & 85 \\ 151.804 & 107.674 & 58.2435 & 75 \end{pmatrix}$$
M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Right + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 27.5312 & 76.7816 & 120 \\ 64.2495 & 150.665 & 109.781 & 80 \\ 148.808 & 66.3336 & 89.6176 & 85 \\ 151.804 & 107.674 & 58.2435 & 75 \end{pmatrix}$$
M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 152.469 & 103.218 & 120 \\ 64.2495 & 29.3354 & 70.2192 & 80 \\ 148.808 & 113.666 & 90.3824 & 85 \\ 151.804 & 72.3261 & 121.757 & 75 \end{pmatrix}$$
M_i values:

$M_1 = 0.924668, M_2 = 0.924668, M_3 = 0.924668, M_4 = 0.924668$

$M_i < 1$ for all $i = 1, \dots, 4$

Switch combination: Right + Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 88.6751 & 27.5312 & 76.7816 & 120 \\ 64.2495 & 29.3354 & 70.2192 & 80 \\ 148.808 & 113.666 & 90.3824 & 85 \\ 151.804 & 107.674 & 58.2435 & 75 \end{pmatrix}$$

M_i values:

M1 = 0.924668, M2 = 0.924668, M3 = 0.924668, M4 = 0.924668

M_i < 1 for all i = 1, ..., 4

Out[101]=

===== NOT CHIMERA =====

Fails conic, orthodiagonal & isogonal tests for all
i=1, ..., 4 ⇒ NOT chimera. Boundary-strip switches
preserve these failures as demonstrated in the NOT
CONIC, NOT ORTHODIAGONAL, and NOT ISOGONAL sections.