

Computational Companion to “Flexible 3×3 Nets of Equimodular Elliptic Type” — Example 1

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Tested on: Mathematica 14.0

In[1587]:=

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(*=====*)
=====*)
(*=====*)
=====*)
(*=====*)
=====*)
(*Quit*)
(*All angle sets in degrees*)
anglesDeg = {
  {105, 15, 120, 90}, (*Vertex 1*)
  {90, 120, 15, 105}, (*Vertex 2*)
  {90, 60, 165, 75}, (*Vertex 3*)
  {105, 15, 120, 90} (*Vertex 4*)};
(*anglesDeg={
  {26.20863403213998,82.2407675648952,
  21.949109994264898,59.9999999999997}, (*Vertex 1*)
  {16.166237389600262,130.87095233025335,
  18.85247535405415,114.9999999999991}, (*Vertex 2*)
  {134.65533802039442,34.44439013740831,145.3694664686027,80.0}, (*Vertex 3*)
  {117.95117201340666,49.52829397349284,
  149.0275482144225,105.00000000000001} (*Vertex 4*)});*)

(*-----*)
(*Function to compute sigma from 4 angles*)
computeSigma[{ $\alpha$ _,  $\beta$ _,  $\gamma$ _,  $\delta$ _}] := ( $\alpha + \beta + \gamma + \delta$ ) / 2;

(*-----*)
(*Function to compute a,b,c,d from angles*)
computeABCD[{ $\alpha$ _,  $\beta$ _,  $\gamma$ _,  $\delta$ _}] :=
Module[{alpha =  $\alpha$  Degree, beta =  $\beta$  Degree, gamma =  $\gamma$  Degree,
  delta =  $\delta$  Degree, sigma}, sigma = computeSigma[{ $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ } Degree];
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{Sin[alpha] / Sin[sigma - alpha], Sin[beta] / Sin[sigma - beta],
 Sin[gamma] / Sin[sigma - gamma], Sin[delta] / Sin[sigma - delta]}}];

(*-----*)
(*Compute all  $\sigma$  values*)
sigmas = computeSigma /@ anglesDeg;

(*Compute all {a,b,c,d} values*)
results = computeABCD /@ anglesDeg;

(*Optional:extract individual values*)
{a1, b1, c1, d1} = results[[1]];
{a2, b2, c2, d2} = results[[2]];
{a3, b3, c3, d3} = results[[3]];
{a4, b4, c4, d4} = results[[4]];
{ $\sigma$ 1,  $\sigma$ 2,  $\sigma$ 3,  $\sigma$ 4} = sigmas;

(*=====*)
(*=====*)
CONDITION (N.0)=====*)
(*=====*)
=====*)
uniqueCombos = {{1, 1, 1, 1}, {1, 1, 1, -1}, {1, 1, -1, -1}, {1, 1, -1, 1},
 {1, -1, 1, 1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, -1, -1, -1}};

checkConditionN0Degrees[{ $\alpha$ _,  $\beta$ _,  $\gamma$ _,  $\delta$ _}] :=
Module[{angles = { $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ }, results},
 results = Mod[uniqueCombos.angles, 360] // Chop;
 ! MemberQ[results, 0]];

conditionsN0 = checkConditionN0Degrees /@ anglesDeg;
allVerticesPass = And@@ conditionsN0;

Column[{TextCell[Style["===== CONDITION (N.0) =====",
 Darker[Green], Bold, 16], "Text"],
 If[allVerticesPass,
 Style["✓ All vertices satisfy (N.0).", Darker[Green], Bold],
 Style["✗ Some vertices fail (N.0).", Red, Bold]]]}]

(*=====*)
=====*)
(*=====*)
CONDITION (N.3)=====*)
(*=====*)

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====*)
Ms = FullSimplify[Times@@@results];
allEqualQ = Simplify[Equal@@Ms];

Column[{TextCell[Style["===== CONDITION (N.3) =====",
  Darker[Blue], Bold, 16], "Text"], If[allEqualQ,
  Row[{Style["✓ M1 = M2 = M3 = M4 = ", Bold], Ms[[1]]},
  Style["✗ M_i are not all equal.", Red, Bold]]]}]

(*=====
====*)
(*=====CONDITION (N.4)=====*)
(*=====
====*)
aList = results[[All, 1]];
cList = results[[All, 3]];
dList = results[[All, 4]];

rList = FullSimplify /@ (aList*dList); {r1, r2, r3, r4} = rList;
sList = FullSimplify /@ (cList*dList); {s1, s2, s3, s4} = sList;

Column[{TextCell[Style["===== CONDITION (N.4) =====",
  Darker[Purple], Bold, 16], "Text"],
  If[r1 == r2 && r3 == r4 && s1 == s4 && s2 == s3, Column[
    {Row[{Style["✓ r1 = r2 = ", Bold], r1, Style["; ✓ r3 = r4 = ", Bold], r3}],
    Row[{Style["✓ s1 = s4 = ", Bold], s1, Style["; ✓ s2 = s3 = ", Bold],
    s2}]}], Style["✗ Condition (N.4) fails.", Red, Bold]]
  ]}]

(*=====
====*)
(*=====
CONDITION (N.5)=====*)
(*=====
====*)
fList = FullSimplify /@ (aList*cList); {f1, f2, f3, f4} = fList;
M1 = Ms[[1]];

m[M1_] := Piecewise[{{1 - M1, M1 < 1}, {(M1 - 1) / M1, M1 > 1}}] // N;
K = EllipticK[m[M1]] // N;
Kp = EllipticK[1 - m[M1]] // N;

computeTi[i_] :=
Module[{sigma = sigmas[[i]], r = rList[[i]], s = sList[[i]], f = fList[[i]], base},
  base = I * Im[InverseJacobiDN[Piecewise[

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      {{Sqrt[f], M1 < 1}, {1 / Sqrt[f], M1 > 1}}, m[M1]]] / Kp;
Which[sigma < 180, Which[r > 1 && s > 1, base, (r < 1 && s > 1) || (r > 1 && s < 1),
  1 + base, r < 1 && s < 1, 2 + base], sigma > 180, Which[r > 1 && s > 1,
  2 + base, (r < 1 && s > 1) || (r > 1 && s < 1), 3 + base, r < 1 && s < 1, base]]];

tList = computeTi /@ Range[4];
RoundWithTolerance[x_, tol_ : 10^-6] := Module[{nearest}, nearest = Round[x];
  If[Abs[x - nearest] ≤ tol, nearest, x]];

checkValidCombination[M1_, ε_ : 10^-6] :=
Module[{i, combo, dotProd, rePart, imPart, n2, expr, foundQ = False},
  Print[Style["△ Approximate validation using ε-tolerance. For rigorous
    proof, see the referenced paper.", Darker@Orange, Italic]];
  Do[combo = uniqueCombos[[i]];
    dotProd = tList.combo;
    rePart = Abs[Re[dotProd]];
    imPart = Abs[Im[dotProd]];
    If[M1 < 1,
      If[Mod[RoundWithTolerance[rePart], 4] < ε,
        If[Mod[RoundWithTolerance[imPart], 2] < ε, expr =
          tList[[1]] + combo[[2]] × tList[[2]] + combo[[3]] × tList[[3]] + combo[[4]] × tList[[4]];
          Print[Style["✔ Valid Combination Found (M < 1):",
            Darker[Green], Bold], "\n", Style["e1 = ", Bold], combo[[2]],
            Style["e2 = ", Bold], combo[[3]], Style["e3 = ", Bold], combo[[4]],
            "\n", Style["t1 = ", Bold], Re[tList[[1]], "K + ", Im[tList[[1]], "iK'",
            "\n", Style["t2 = ", Bold], Re[tList[[2]], "K + ", Im[tList[[2]], "iK'",
            "\n", Style["t3 = ", Bold], Re[tList[[3]], "K + ", Im[tList[[3]], "iK'",
            "\n", Style["t4 = ", Bold], Re[tList[[4]], "K + ", Im[tList[[4]],
            "iK'", "\n", Style["t1 + e1*t2 + e2*t3 + e3*t4 = ", Bold],
            Re[expr], "K + ", Im[expr], "iK'"];
          foundQ = True;
          Break[]]]];
    If[M1 > 1,
      If[Mod[RoundWithTolerance[imPart], 2] < ε,
        n2 = Quotient[RoundWithTolerance[imPart], 2];
        If[Mod[RoundWithTolerance[Abs[rePart - 2 n2]], 4] < ε, expr =
          tList[[1]] + combo[[2]] × tList[[2]] + combo[[3]] × tList[[3]] + combo[[4]] × tList[[4]];
          Print[Style["✔ Valid Combination Found (M > 1):",
            Darker[Green], Bold], "\n", Style["e1 = ", Bold], combo[[2]],
            Style["e2 = ", Bold], combo[[3]], Style["e3 = ", Bold], combo[[4]],
            "\n", Style["t1 = ", Bold], Re[tList[[1]], "K + ", Im[tList[[1]], "iK'",
            "\n", Style["t2 = ", Bold], Re[tList[[2]], "K + ", Im[tList[[2]], "iK'",
            "\n", Style["t3 = ", Bold], Re[tList[[3]], "K + ", Im[tList[[3]], "iK'",
            "\n", Style["t4 = ", Bold], Re[tList[[4]], "K + ", Im[tList[[4]],
            "iK'", "\n", Style["t1 + e1*t2 + e2*t3 + e3*t4 = ", Bold],

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      Re[expr], "K + ", Im[expr], "iK'"];
foundQ = True;
Break[]]]], {i, Length[uniqueCombos]}}];
If[! foundQ, Print[Style["✗ No valid combination found.", Red, Bold], "\n",
  Style["t1 = ", Bold], Re[tList[[1]], "K + ", Im[tList[[1]], "iK'", "\n",
  Style["t2 = ", Bold], Re[tList[[2]], "K + ", Im[tList[[2]], "iK'", "\n",
  Style["t3 = ", Bold], Re[tList[[3]], "K + ", Im[tList[[3]], "iK'", "\n",
  Style["t4 = ", Bold], Re[tList[[4]], "K + ", Im[tList[[4]], "iK'", "\n"]]]];];

Column[{TextCell[Style["===== CONDITION (N.5) =====",
  Darker[Red], Bold, 16], "Text"]}]
res = checkValidCombination[M1];

(*=====
====*)
(*=====
OTHER PARAMETER=====*)
(*=====
====*)
Column[
{TextCell[Style["===== OTHER PARAMETERS =====",
  Darker[Orange], Bold, 16], "Text"],
Row[{Style["u = ", Bold], 1 - M1}],
Row[{Style["σ1 = ", Bold], σ1 Degree, Style["", σ2 = ", Bold], σ2 Degree,
  Style["", σ3 = ", Bold], σ3 Degree, Style["", σ4 = ", Bold], σ4 Degree}],
Row[{Style["f1 = ", Bold], f1, Style["", f2 = ", Bold],
  f2, Style["", f3 = ", Bold], f3, Style["", f4 = ", Bold], f4}],
Row[{Style["z1 = ", Bold], FullSimplify[1 / (f1 - 1)], Style["", z2 = ", Bold],
  FullSimplify[1 / (f2 - 1)], Style["", z3 = ", Bold], FullSimplify[1 / (f3 - 1)],
  Style["", z4 = ", Bold], FullSimplify[1 / (f4 - 1)]}],
Row[{Style["x1 = ", Bold], FullSimplify[1 / (r1 - 1)], Style["", x2 = ", Bold],
  FullSimplify[1 / (r2 - 1)], Style["", x3 = ", Bold], FullSimplify[1 / (r3 - 1)],
  Style["", x4 = ", Bold], FullSimplify[1 / (r4 - 1)]}],
Row[{Style["y1 = ", Bold], FullSimplify[1 / (s1 - 1)], Style["", y2 = ", Bold],
  FullSimplify[1 / (s2 - 1)], Style["", y3 = ", Bold], FullSimplify[1 / (s3 - 1)],
  Style["", y4 = ", Bold], FullSimplify[1 / (s4 - 1)]}],
Row[{Style["p1 = ", Bold], Simplify[Sqrt[r1 - 1]],
  Style["", p2 = ", Bold], Simplify[Sqrt[r2 - 1]], Style["", p3 = ", Bold],
  Simplify[Sqrt[r3 - 1]], Style["", p4 = ", Bold], Simplify[Sqrt[r4 - 1]]}],
Row[{Style["q1 = ", Bold], Simplify[Sqrt[s1 - 1]],
  Style["", q2 = ", Bold], Simplify[Sqrt[s2 - 1]], Style["", q3 = ", Bold],
  Simplify[Sqrt[s3 - 1]], Style["", q4 = ", Bold], Simplify[Sqrt[s4 - 1]]}],
Row[{Style["p1.q1 = ", Bold], Simplify[Sqrt[r1 - 1] Sqrt[s1 - 1]],
  Style["", p2.q2 = ", Bold], Simplify[Sqrt[r2 - 1] Sqrt[s2 - 1]],

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Style["", p3.q3 = "", Bold], Simplify[Sqrt[r3 - 1] Sqrt[s3 - 1]],
Style["", p4.q4 = "", Bold], Simplify[Sqrt[r4 - 1] Sqrt[s4 - 1]]}],
Row[{Style["!\(\(*OverscriptBox[\(\alpha\), \(_\)]\) = ", Bold],
 $\sigma_1 - \text{anglesDeg}[1, 1]$ , "°", Style[
", \!\(\(*OverscriptBox[\(\beta\), \(_\)]\) = ", Bold],  $\sigma_1 - \text{anglesDeg}[1, 2]$ ,
"°", Style["", \!\(\(*OverscriptBox[\(\gamma\), \(_\)]\) = ", Bold],
 $\sigma_1 - \text{anglesDeg}[1, 3]$ , "°",
Style["", \!\(\(*OverscriptBox[\(\delta\), \(_\)]\) = ", Bold],
 $\sigma_1 - \text{anglesDeg}[1, 4]$ , "°"}],
Row[Module[{ $\alpha_1 = \sigma_1 - \text{anglesDeg}[1, 1]$ ,  $\beta_1 = \sigma_1 - \text{anglesDeg}[1, 2]$ ,
 $\gamma_1 = \sigma_1 - \text{anglesDeg}[1, 3]$ ,  $\delta_1 = \sigma_1 - \text{anglesDeg}[1, 4]$ , ineq1, ineq2, ineq3},
ineq1 = Sin[anglesDeg[1, 1] Degree] > Sin[ $\alpha_1$  Degree] Sin[ $\delta_1$  Degree];
ineq2 = Sin[anglesDeg[1, 3] Degree] > Sin[ $\gamma_1$  Degree] Sin[ $\delta_1$  Degree];
ineq3 = Sin[anglesDeg[1, 1] Degree] Sin[anglesDeg[1, 2] Degree] <
Sin[ $\alpha_1$  Degree] Sin[ $\beta_1$  Degree];
{Style["Inequalities (vertex 1): ", Bold], TraditionalForm@
(Sin[Subscript[ $\alpha$ , 1]] > Sin[Overscript[Subscript[ $\alpha$ , 1], _]] *
Sin[Overscript[Subscript[ $\delta$ , 1], _]]),
": ", Simplify[ineq1], " ", " ", TraditionalForm@
(Sin[Subscript[ $\gamma$ , 1]] > Sin[Overscript[Subscript[ $\gamma$ , 1], _]] *
Sin[Overscript[Subscript[ $\delta$ , 1], _]]), ": ", Simplify[ineq2],
" ", " ", TraditionalForm@ (Sin[Subscript[ $\alpha$ , 1]] Sin[Subscript[ $\beta$ , 1]] <
Sin[Overscript[Subscript[ $\alpha$ , 1], _]] *
Sin[Overscript[Subscript[ $\beta$ , 1], _]]), ": ", Simplify[ineq3]]}]

}]

(*=====
====*)
(*=====
BRICARD's EQUATIONS=====*)
(*=====
====*)
(*=====
FLEXION 1=====*)
Z[t_] := t;

$$W1[t_] := \frac{(3 + 2\sqrt{3})t - \sqrt{1 + \sqrt{3}} \sqrt{(\sqrt{3}t^2 - 1)(1 - (2\sqrt{3} + 3)t^2)}}{1 + \sqrt{3} + (3 + 2\sqrt{3})t^2};$$


$$U[t_] := \frac{(36 + 17\sqrt{3})t - 2\sqrt{3(1 + \sqrt{3})} \sqrt{(\sqrt{3}t^2 - 1)(1 - (2\sqrt{3} + 3)t^2)}}{\sqrt{3} + 12(9 + 5\sqrt{3})t^2};$$


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$$W2[t_] := \frac{\sqrt{2} \left(-2t + \sqrt{1 + \sqrt{3}} \sqrt{(\sqrt{3} t^2 - 1) (1 - (2\sqrt{3} + 3) t^2)} \right)}{(1 + \sqrt{3}) (-1 + 3 t^2)};$$

(*General polynomial constructor*)

BricardsEquation[{α_, β_, γ_, δ_}, σ_, x_, y_] :=

Module[{c22, c20, c02, c11, c00},

c22 = Sin[σ - δ] Sin[σ - δ - β];

c20 = Sin[σ - α] Sin[σ - α - β];

c02 = Sin[σ - γ] Sin[σ - γ - β];

c11 = -Sin[α] Sin[γ];

c00 = Sin[σ] Sin[σ - β];

c22 x^2 y^2 + c20 x^2 + c02 y^2 + 2 c11 x y + c00];

(*Compute and print all P_i for flexion 1*)

TextCell[

Style["===== FLEXIBILITY (FLEXION 1) =====",

Darker[Cyan], Bold, 16], "Text"]

funcs = {{Z, W1}, {Z, W2}, {U, W2}, {U, W1}};

Do[angles = anglesDeg[[i]] Degree;

sigma = sigmas[[i]] Degree;

{α, β, γ, δ} = angles;

poly = Which[i == 1, BricardsEquation[{α, β, γ, δ}, sigma, Z[t], W1[t]],

i == 2, BricardsEquation[{α, β, γ, δ}, sigma, Z[t], W2[t]],

i == 3, BricardsEquation[{α, β, γ, δ}, sigma, U[t], W2[t]],

i == 4, BricardsEquation[{α, β, γ, δ}, sigma, U[t], W1[t]]];

Print[Row[{Subscript["P", i], "[", funcs[[i, 1],

", ", funcs[[i, 2], "] = ", FullSimplify[poly]]], {i, 1, 4}];

(*=====

FLEXION 2=====*)

Z2[t_] := t;

$$W12[t_] := \frac{(3 + 2\sqrt{3})t + \sqrt{1 + \sqrt{3}} \sqrt{(\sqrt{3} t^2 - 1) (1 - (2\sqrt{3} + 3) t^2)}}{1 + \sqrt{3} + (3 + 2\sqrt{3}) t^2};$$

$$U2[t_] := \frac{(36 + 17\sqrt{3})t + 2\sqrt{3}(1 + \sqrt{3}) \sqrt{(\sqrt{3} t^2 - 1) (1 - (2\sqrt{3} + 3) t^2)}}{\sqrt{3} + 12(9 + 5\sqrt{3}) t^2};$$

$$W22[t_] := \frac{\sqrt{2} \left(-2t - \sqrt{1 + \sqrt{3}} \sqrt{(\sqrt{3} t^2 - 1) (1 - (2\sqrt{3} + 3) t^2)} \right)}{(1 + \sqrt{3}) (-1 + 3 t^2)};$$

(*General polynomial constructor*)

BricardsEquation[{α_, β_, γ_, δ_}, σ_, x_, y_] :=

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Module[{c22, c20, c02, c11, c00},
  c22 = Sin[σ - δ] Sin[σ - δ - β];
  c20 = Sin[σ - α] Sin[σ - α - β];
  c02 = Sin[σ - γ] Sin[σ - γ - β];
  c11 = -Sin[α] Sin[γ];
  c00 = Sin[σ] Sin[σ - β];
  c22 x^2 y^2 + c20 x^2 + c02 y^2 + 2 c11 x y + c00];

(*Compute and print all P_i for flexion 2*)
TextCell[
  Style["===== FLEXIBILITY (FLEXION 2) =====",
    Cyan, Bold, 16], "Text"]
funcs = {{Z, W1}, {Z, W2}, {U, W2}, {U, W1}};
Do[angles = anglesDeg[[i]] Degree;
  sigma = sigmas[[i]] Degree;
  {α, β, γ, δ} = angles;
  poly = Which[i == 1, BricardsEquation[{α, β, γ, δ}, sigma, Z2[t], W12[t]],
    i == 2, BricardsEquation[{α, β, γ, δ}, sigma, Z2[t], W22[t]],
    i == 3, BricardsEquation[{α, β, γ, δ}, sigma, U2[t], W22[t]],
    i == 4, BricardsEquation[{α, β, γ, δ}, sigma, U2[t], W12[t]]];
  Print[Row[{Subscript["P", i], "[", funcs[[i, 1],
    ", ", funcs[[i, 2], "] = ", FullSimplify[poly]]}], {i, 1, 4}];

(*=====
====*)
(*=====
NOT TRIVIAL=====*)
(*=====
====*)
(*Define domain limits for t*)
tMin = 1 / Sqrt[2 Sqrt[3] + 3];
tMax = 1 / 2;

(*=====
FLEXION 1=====*)
(*List of expressions to plot*)
expressions = {Z[t], W2[t], U[t], W1[t]};
labels = {"Z", "W2", "U", "W1"};

Column[{TextCell[
  Style["===== NOT TRIVIAL (FLEXION 1) =====",
    Darker[Brown], Bold, 16], "Text"],

  (*Explanatory text*)
  TextCell[Style["This configuration does not belong to the trivial class –
    even after switching the boundary strips – since none of the

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        functions Z, W2, U, or W1 is constant.", GrayLevel[0.3]], "Text"],
    Spacer[12],

    (*Plots in a light panel*)
    Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
        PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
        AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
        Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]

    ]]

    (*=====
    FLEXION 2=====*)
    (*List of expressions to plot*)
    expressions = {Z2[t], W22[t], U2[t], W12[t]};
    labels = {"Z", "W2", "U", "W1"};

    Column[{TextCell[
        Style["===== NOT TRIVIAL (FLEXION 2) =====",
            Brown, Bold, 16], "Text"],

        (*Explanatory text*)
        TextCell[Style["This configuration does not belong to the trivial class –
            even after switching the boundary strips – since none of the
            functions Z, W2, U, or W1 is constant.", GrayLevel[0.3]], "Text"],
        Spacer[12],

        (*Plots in a light panel*)
        Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
            PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
            AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
            Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]

        ]]

    (*=====
    =====*)
    (*=====
    NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE=====*)
    (*=====
    =====*)
    (*Define domain limits for t*)
    tMin = 1 / Sqrt[2 Sqrt[3] + 3];
    tMax = 1 / 2;

    (*=====

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FLEXION 1=====*)
(*List of expressions& labels*)
expressions =
  {Z[t] * U[t], Z[t] / U[t], W1[t] * W2[t], W1[t] / W2[t], Z[t] * W2[t], Z[t] / W2[t],
   U[t] * W1[t], U[t] / W1[t], Z[t] * W1[t], Z[t] / W1[t], W2[t] * U[t], W2[t] / U[t]};
labels = {"Z * U", "Z / U", "W1 * W2", "W1 / W2", "Z * W2", "Z / W2",
  "U * W1", "U / W1", "Z * W1", "Z / W1", "W2 * U", "W2 / U"};

Column[
  {TextCell[Style["===== NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE
    (FLEXION 1) =====", Darker[Magenta], Bold, 16], "Text"],

  (*Explanatory text*)TextCell[
    Style["This configuration does not belong to the Linear compound class nor
      to the linear conjugate class – even after switching the boundary
      strips – since none of the functions ZU, Z/U, W1W2, W1/W2, ZW2,
      Z/W2, UW1, and U/W1 is constant. In addition, none of ZW1,
      Z/W1, W2U, W2/U is constant as well.", GrayLevel[0.3]], "Text"],
  Spacer[12],

  (*Plots panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
    PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
    AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
    Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]
  ]}

(*=====
FLEXION 2=====*)
(*List of expressions& labels*)
expressions = {Z2[t] * U2[t], Z2[t] / U2[t], W12[t] * W22[t], W12[t] / W22[t],
  Z2[t] * W22[t], Z2[t] / W22[t], U2[t] * W12[t], U2[t] / W12[t],
  Z2[t] * W12[t], Z2[t] / W12[t], W22[t] * U2[t], W22[t] / U2[t]};
labels = {"Z * U", "Z / U", "W1 * W2", "W1 / W2", "Z * W2", "Z / W2",
  "U * W1", "U / W1", "Z * W1", "Z / W1", "W2 * U", "W2 / U"};

Column[
  {TextCell[Style["===== NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE
    (FLEXION 2) =====", Magenta, Bold, 16], "Text"],

  (*Explanatory text*)TextCell[
    Style["This configuration does not belong to the Linear compound class nor
      to the linear conjugate class – even after switching the boundary
      strips – since none of the functions ZU, Z/U, W1W2, W1/W2, ZW2,
      Z/W2, UW1, and U/W1 is constant. In addition, none of ZW1,
      Z/W1, W2U, W2/U is constant as well.", GrayLevel[0.3]], "Text"],
  Spacer[12],

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(*Plots panel*)
Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
    PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
    AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
    Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]
}]

(*=====
====*)
(*=====
SWITCHING BOUNDARY STRIPS=====*)
(*=====
====*)
SwitchingRightBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
    (*Indices:{row,column}={1,2},{1,3},{4,2},{4,3}*)
    modified[[1, 2]] = 180 - anglesDeg[[1, 2]]; (*β1*)
    modified[[1, 3]] = 180 - anglesDeg[[1, 3]]; (*γ1*)
    modified[[4, 2]] = 180 - anglesDeg[[4, 2]]; (*β4*)
    modified[[4, 3]] = 180 - anglesDeg[[4, 3]]; (*γ4*)
    modified]

SwitchingLeftBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
    (*Indices:{row,column}={2,2},{2,3},{3,2},{3,3}*)
    modified[[2, 2]] = 180 - anglesDeg[[2, 2]]; (*β2*)
    modified[[2, 3]] = 180 - anglesDeg[[2, 3]]; (*γ2*)
    modified[[3, 2]] = 180 - anglesDeg[[3, 2]]; (*β3*)
    modified[[3, 3]] = 180 - anglesDeg[[3, 3]]; (*γ3*)
    modified]

SwitchingLowerBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
    (*Indices:{row,column}={1,1},{1,2},{2,1},{2,2}*)
    modified[[1, 1]] = 180 - anglesDeg[[1, 1]]; (*α1*)
    modified[[1, 2]] = 180 - anglesDeg[[1, 2]]; (*β1*)
    modified[[2, 1]] = 180 - anglesDeg[[2, 1]]; (*α2*)
    modified[[2, 2]] = 180 - anglesDeg[[2, 2]]; (*β2*)
    modified]

SwitchingUpperBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
    (*Indices:{row,column}={3,1},{3,2},{4,1},{4,2}*)
    modified[[3, 1]] = 180 - anglesDeg[[3, 1]]; (*α3*)
    modified[[3, 2]] = 180 - anglesDeg[[3, 2]]; (*β3*)
    modified[[4, 1]] = 180 - anglesDeg[[4, 1]]; (*α4*)
    modified[[4, 2]] = 180 - anglesDeg[[4, 2]]; (*β4*)
    modified]

```

```

(*=====
====*)
(*=====
NOT CONIC=====*)
(*=====
====*)
Column[{TextCell[Style["===== NOT CONIC =====",
    Pink, Bold, 16], "Text"],
    TextCell[Style["Condition (N.0) is satisfied ⇒
        this configuration is NOT equimodular-conic. Applying
        any boundary-strip switch still preserves (N.0), so
        no conic form emerges.", GrayLevel[0.3]], "Text"]
    ]}]

(*Now the exact same Module for checking all switch combinations...*)
Module[{angles = anglesDeg, switchers, combinations, results},
    (*Define switch functions*)
    switchers = <|"Right" → SwitchingRightBoundaryStrip,
        "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
        "Upper" → SwitchingUpperBoundaryStrip|>;
    (*Generate all combinations of switches (from size 1 to 4)*)
    combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
    (*Evaluate condition after each combination of switches*) results = Table[
        Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
        Do[switched = switchers[sw][switched], {sw, combo}];
        passQ = And@@ (checkConditionN0Degrees /@ switched);
        (*Print the result after switching*)
        (*Print["\nSwitch combination: ", name];
        Print["Switched anglesDeg:"];
        Print[MatrixForm[switched]];*)
        {name, passQ}], {combo, combinations}];
    (*Display results*)
    Column[Prepend[Table[Module[{comboName = res[[1]], passQ = res[[2]]},
        Row[{Style[comboName <> ": ", Bold],
            If[passQ,
                Style["Condition (N.0) is still satisfied.", Darker[Green]],
                Style["Condition (N.0) fails.", Red, Bold]
            ]
        }, {res, results}], TextCell[
        Style["CONDITION (N.0) AFTER SWITCHING BOUNDARY STRIPS", 14], "Text"]]]]

(*=====
====*)
(*=====

```

```

NOT ORTHODIAGONAL=====*)
(*=====
====*)
Column[
{TextCell[Style["===== NOT ORTHODIAGONAL =====",
  Purple, Bold, 16], "Text"],
TextCell[Style[
"cos( $\alpha_i$ )·cos( $\gamma_i$ )  $\neq$  cos( $\beta_i$ )·cos( $\delta_i$ ) for each i = 1...4  $\Rightarrow$  NOT orthodiagonal.
Switching boundary strips does not
correct this.", GrayLevel[0.3]], "Text"]
}]

Module[{angles = anglesDeg, switchers, combinations, results},
(*Define switch functions*)
switchers = <|"Right" → SwitchingRightBoundaryStrip,
  "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
  "Upper" → SwitchingUpperBoundaryStrip|>;
(*Helper function:compute and print difference only*)
formatOrthodiagonalCheck[quad_List] := Module[{vals},
  vals = Table[Module[{a, b, c, d, lhs, rhs, diff}, {a, b, c, d} = quad[[i];
    lhs = FullSimplify[Cos[a Degree] Cos[c Degree]];
    rhs = FullSimplify[Cos[b Degree] Cos[d Degree]];
    diff = Chop[lhs - rhs];
    Style[Row[{"cos( $\alpha$ " <> ToString[i] <> ")·cos( $\gamma$ " <> ToString[i] <> ") - ",
      "cos( $\beta$ " <> ToString[i] <> ")·cos( $\delta$ " <> ToString[i] <> ") = ", NumberForm[
        diff, {5, 3}]]], If[diff == 0, Red, Black]]], {i, Length[quad]}}];
  Column[vals]];
(*Orthodiagonal check for anglesDeg before any switching*)
Print[TextCell[Style["\nInitial anglesDeg (no switches):", Bold]]];
Print[MatrixForm[angles]];
Print[TextCell[Style[
  "Orthodiagonal check: cos( $\alpha_i$ )·cos( $\gamma_i$ ) - cos( $\beta_i$ )·cos( $\delta_i$ ) for i = 1..4",
  Italic]]];
Print[formatOrthodiagonalCheck[angles]];
(*Generate all combinations of switches (from size 1 to 4)*)
combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
(*Evaluate condition after each combination of switches*)results = Table[
Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
  Do[switched = switchers[sw][switched], {sw, combo}];
  passQ = And@@ (checkConditionN0Degrees /@ switched);
  Print[Style["\nSwitch combination: ", Bold], name];
  Print[Style["Switched anglesDeg:", Italic]];
  Print[MatrixForm[switched]];
  Print[TextCell[Style[
    "Orthodiagonal check: cos( $\alpha_i$ )·cos( $\gamma_i$ ) - cos( $\beta_i$ )·cos( $\delta_i$ ) for i = 1..4",
    Italic]]];
  Print[formatOrthodiagonalCheck[switched]];

```

```

{name, passQ}], {combo, combinations}}];]

(*=====
====*)
(*=====
NOT ISOGONAL=====*)
(*=====
====*)
Column[
{TextCell[Style["===== NOT ISOGONAL =====", Orange,
  Bold, 15], "Text"],
TextCell[
Style["Condition (N.0) holds AND for all i = 1...4:  $\alpha_i \neq \beta_i$ ,  $\alpha_i \neq \gamma_i$ ,  $\alpha_i \neq \delta_i$ ,  $\beta_i \neq \gamma_i$ ,  $\beta_i \neq \delta_i$ ,  $\gamma_i \neq \delta_i$ ,  $\alpha_i + \beta_i \neq \pi \neq \gamma_i + \delta_i$ ,  $\alpha_i + \gamma_i \neq \pi \neq \beta_i + \delta_i$ ,  $\alpha_i + \delta_i \neq \pi \neq \beta_i + \gamma_i \Rightarrow$  NOT isogonal. Switching boundary strips do not change this.", GrayLevel[0.3]], "Text"]}]

Module[{angles = anglesDeg, switchers, combinations, results},
(*Define switch functions*)
switchers = <|"Right" → SwitchingRightBoundaryStrip,
  "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
  "Upper" → SwitchingUpperBoundaryStrip|>;
(*Helper function:extended angle relations*)
formatAngleRelations[quad_List] :=
Module[{vals}, vals = Table[Module[{a, b, c, d, exprs}, {a, b, c, d} = quad[[i]];
  exprs = {Row[{" $\alpha$ " <> ToString[i] <> " -  $\beta$ " <> ToString[i] <> " = ",
    NumberForm[N[a - b], {5, 3}]}], Row[{" $\alpha$ " <> ToString[i] <>
  " -  $\gamma$ " <> ToString[i] <> " = ", NumberForm[N[a - c], {5, 3}]}],
  Row[{" $\alpha$ " <> ToString[i] <> " -  $\delta$ " <> ToString[i] <> " = ",
    NumberForm[N[a - d], {5, 3}]}], Row[{" $\beta$ " <> ToString[i] <>
  " -  $\gamma$ " <> ToString[i] <> " = ", NumberForm[N[b - c], {5, 3}]}],
  Row[{" $\beta$ " <> ToString[i] <> " -  $\delta$ " <> ToString[i] <> " = ",
    NumberForm[N[b - d], {5, 3}]}], Row[{" $\gamma$ " <> ToString[i] <>
  " -  $\delta$ " <> ToString[i] <> " = ", NumberForm[N[c - d], {5, 3}]}],
  Row[{" $\alpha$ " <> ToString[i] <> " +  $\beta$ " <> ToString[i] <> " - 180 = ",
    NumberForm[N[a + b - 180], {5, 3}]}],
  Row[{" $\gamma$ " <> ToString[i] <> " +  $\delta$ " <> ToString[i] <> " - 180 = ",
    NumberForm[N[c + d - 180], {5, 3}]}],
  Row[{" $\alpha$ " <> ToString[i] <> " +  $\gamma$ " <> ToString[i] <> " - 180 = ",
    NumberForm[N[a + c - 180], {5, 3}]}],
  Row[{" $\beta$ " <> ToString[i] <> " +  $\delta$ " <> ToString[i] <> " - 180 = ",
    NumberForm[N[b + d - 180], {5, 3}]}],
  Row[{" $\alpha$ " <> ToString[i] <> " +  $\delta$ " <> ToString[i] <> " - 180 = ",
    NumberForm[N[a + d - 180], {5, 3}]}],
  Row[{" $\beta$ " <> ToString[i] <> " +  $\gamma$ " <> ToString[i] <> " - 180 = ",
    NumberForm[N[b + c - 180], {5, 3}]}],

```

```

Row[{" $\alpha$ " <> ToString[i] <> " +  $\beta$ " <> ToString[i] <> " -  $\gamma$ " <> ToString[i] <>
  " -  $\delta$ " <> ToString[i] <> " = ", NumberForm[N[a + b - c - d], {5, 3}]],
Row[{" $\alpha$ " <> ToString[i] <> " +  $\gamma$ " <> ToString[i] <> " -  $\beta$ " <> ToString[i] <>
  " -  $\delta$ " <> ToString[i] <> " = ", NumberForm[N[a + c - b - d], {5, 3}]],
Row[{" $\alpha$ " <> ToString[i] <> " +  $\delta$ " <> ToString[i] <> " -  $\beta$ " <> ToString[i] <>
  " -  $\gamma$ " <> ToString[i] <> " = ", NumberForm[N[a + d - b - c], {5, 3}]]];
Column[Prepend[exprs, Style["Vertex " <> ToString[i], Bold]]],
{i, Length[quad]}];
Column[vals, Spacings → 1.5]];
(*Angle relation check for anglesDeg before any switching*)
Print[TextCell[Style["\nInitial anglesDeg (no switches):", Bold]]];
Print[MatrixForm[angles]];
Print[TextCell[Style["Angle relation checks for i = 1..4:", Italic]]];
Print[formatAngleRelations[angles]];
(*Generate all combinations of switches (from size 1 to 4)*)
combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
(*Evaluate condition after each combination of switches*)results = Table[
Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
Do[switched = switchers[sw][switched], {sw, combo}];
passQ = And@@ (checkConditionN0Degrees /@ switched);
Print[Style["\nSwitch combination: ", Bold], name];
Print[Style["Switched anglesDeg:", Italic]];
Print[MatrixForm[switched]];
Print[TextCell[Style["Angle relation checks for i = 1..4:", Italic]]];
Print[formatAngleRelations[switched]];
{name, passQ}], {combo, combinations}];

(*=====
====*)
(*=====
NOT CONJUGATE-MODULAR=====*)
(*=====
====*)
Column[
{TextCell[Style["===== NOT CONJUGATE-MODULAR =====",
  Brown, Bold, 16], "Text"],
TextCell[Style[" $M_i < 1$  and  $\pi_i \in \mathbb{R}$  for all  $i = 1..4 \Rightarrow$  NOT conjugate-modular.
Boundary-strip switches preserve this.", GrayLevel[0.3]], "Text"]
}]

Module[{angles = anglesDeg, switchers, combinations, results,
  computeConjugateModularInfo}, (*Define switch functions*)
switchers = <|"Right" → SwitchingRightBoundaryStrip,
  "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
  "Upper" → SwitchingUpperBoundaryStrip|>;

```

```

(*Computes  $M_i$  and  $p_i$  and prints them,
with classification*)computeConjugateModularInfo[quad_List] :=
Module[{abcdList, Ms, aList, dList, rList, pList, summary},
  abcdList = computeABCD /@ quad;
  Ms = FullSimplify[Times@@@ abcdList];
  aList = abcdList[[All, 1]];
  dList = abcdList[[All, 4]];
  rList = FullSimplify /@ (aList * dList);
  pList = FullSimplify[Sqrt[# - 1]] & /@ rList;
  summary = If[AllTrue[Ms, # < 1 &] && AllTrue[pList, Element[#, Reals] &],
    Style[" $M_i < 1$  and  $p_i \in \mathbb{R}$  for all  $i = 1, \dots, 4$ ", Bold],
    Style["Either  $M_i \geq 1$  or  $p_i \notin \mathbb{R}$  for some  $i = 1, \dots, 4$ ", Red, Bold]];
  Column[{Style[" $M_i$  values:", Bold],
    Row[{"M1 = ", Ms[[1]], ", M2 = ", Ms[[2]], ", M3 = ", Ms[[3]], ", M4 = ", Ms[[4]]}],
    Style[" $p_i$  values:", Bold], Row[{"p1 = ", pList[[1]], ", p2 = ",
      pList[[2]], ", p3 = ", pList[[3]], ", p4 = ", pList[[4]]}], summary}]];
(*Original anglesDeg check*)
Print[
  TextCell[Style["\nInitial configuration (no switches applied):", Bold]]];
Print[MatrixForm[angles]];
Print[computeConjugateModularInfo[angles]];
(*Generate all switch combinations (from size 1 to 4)*)
combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
(*Evaluate each switched configuration*)results = Table[
  Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];
    passQ = And@@ (checkConditionN0Degrees /@ switched);
    Print[Style["\nSwitch combination: ", Bold], name];
    Print[Style["Switched anglesDeg:", Italic]];
    Print[MatrixForm[switched]];
    Print[computeConjugateModularInfo[switched]];
    {name, passQ}], {combo, combinations}];

(*=====
====*)
(*=====
NOT CHIMERA=====*)
(*=====
====*)
Column[
  {TextCell[Style["===== NOT CHIMERA =====", Blue,
    Bold, 16], "Text"],
  TextCell[
    Style["Fails conic, orthodiagonal & isogonal tests for all  $i=1, \dots, 4$ 
 $\Rightarrow$  NOT chimera. Boundary-strip switches preserve these
failures as demonstrated in the NOT CONIC, NOT ORTHODIAGONAL,

```



```

and NOT ISOGONAL sections.", GrayLevel[0.3]], "Text"]
}]

(*=====
====*)
(*=====
EXTRA INFO=====*)
(*=====
====*)
{α1, β1, γ1, δ1} = anglesDeg[[1]] * Degree;
{α2, β2, γ2, δ2} = anglesDeg[[2]] * Degree;
{α3, β3, γ3, δ3} = anglesDeg[[3]] * Degree;
{α4, β4, γ4, δ4} = anglesDeg[[4]] * Degree;

(*=====
FLEXION 1=====*)
(*---Link lengths (parameters)---*)
(*A-links:l1=A1A2,l2=A2A3,l3=A1A4,l4=A3A4*)
Clear[l1, l2, l3, l4];
(*B-links:m1=A1B1,m2=A2B2,m3=A3B3,m4=A4B4*)
Clear[m1, m2, m3, m4];
(*C-links:n1=A1C1,n2=A2C2,n3=A3C3,n4=A4C4*)
Clear[n1, n2, n3, n4];

(*---Joint half-angle formulas:cosθi,sinθi---*)
cosθ1[t_] := FullSimplify[((Z[t])^2 - 1) / ((Z[t])^2 + 1)];
sinθ1[t_] := FullSimplify[2 Z[t] / ((Z[t])^2 + 1)];
cosθ2[t_] := FullSimplify[((W2[t])^2 - 1) / ((W2[t])^2 + 1)];
sinθ2[t_] := FullSimplify[2 W2[t] / ((W2[t])^2 + 1)];
cosθ3[t_] := FullSimplify[((U[t])^2 - 1) / ((U[t])^2 + 1)];
sinθ3[t_] := FullSimplify[2 U[t] / ((U[t])^2 + 1)];
cosθ4[t_] := FullSimplify[((W1[t])^2 - 1) / ((W1[t])^2 + 1)];
sinθ4[t_] := FullSimplify[2 W1[t] / ((W1[t])^2 + 1)];

(*=====A-POINTS=====*)
xA2 = 0;
xA1 = l1;
xA3 = l2 Cos[δ2];
xA4 = l1 - l3 Cos[δ1];
yA2 = 0;
yA1 = 0;
yA3 = l2 Sin[δ2];
yA4 = l3 Sin[δ1];
zA2 = 0; zA1 = 0; zA3 = 0; zA4 = 0;

(*=====B-POINTS=====*)

```

```

xB1[t_] := xA1 - m1 Cos[α1];
yB1[t_] := yA1 + m1 Sin[α1] cosθ1[t];
zB1[t_] := zA1 + m1 Sin[α1] sinθ1[t];
xB2[t_] := xA2 + m2 Cos[α2];
yB2[t_] := yA2 + m2 Sin[α2] cosθ1[t];
zB2[t_] := zA2 + m2 Sin[α2] sinθ1[t];
xB3[t_] := xA3 - m3 (Cos[α3] Cos[δ2 + δ3] + Sin[α3] cosθ3[t] Sin[δ2 + δ3]);
yB3[t_] := yA3 - m3 (Cos[α3] Sin[δ2 + δ3] - Sin[α3] cosθ3[t] Cos[δ2 + δ3]);
zB3[t_] := zA3 + m3 Sin[α3] sinθ3[t];
xB4[t_] := xA4 + m4 (Cos[α4] Cos[δ2 + δ3] - Sin[α4] cosθ3[t] Sin[δ2 + δ3]);
yB4[t_] := yA4 + m4 (Cos[α4] Sin[δ2 + δ3] + Sin[α4] cosθ3[t] Cos[δ2 + δ3]);
zB4[t_] := zA4 + m4 Sin[α4] sinθ3[t];

(*=====C-POINTS=====*)
xC1[t_] := xA1 - n1 (Cos[γ1] Cos[δ1] + Sin[γ1] cosθ4[t] Sin[δ1]);
yC1[t_] := yA1 + n1 (Cos[γ1] Sin[δ1] - Sin[γ1] cosθ4[t] Cos[δ1]);
zC1[t_] := zA1 + n1 Sin[γ1] sinθ4[t];
xC2[t_] := xA2 + n2 (Cos[γ2] Cos[δ2] + Sin[γ2] cosθ2[t] Sin[δ2]);
yC2[t_] := yA2 + n2 (Cos[γ2] Sin[δ2] - Sin[γ2] cosθ2[t] Cos[δ2]);
zC2[t_] := zA2 + n2 Sin[γ2] sinθ2[t];
xC3[t_] := xA3 - n3 (Cos[γ3] Cos[δ2] - Sin[γ3] cosθ2[t] Sin[δ2]);
yC3[t_] := yA3 - n3 (Cos[γ3] Sin[δ2] + Sin[γ3] cosθ2[t] Cos[δ2]);
zC3[t_] := zA3 + n3 Sin[γ3] sinθ2[t];
xC4[t_] := xA4 + n4 (Cos[γ4] Cos[δ1] - Sin[γ4] cosθ4[t] Sin[δ1]);
yC4[t_] := yA4 - n4 (Cos[γ4] Sin[δ1] + Sin[γ4] cosθ4[t] Cos[δ1]);
zC4[t_] := zA4 + n4 Sin[γ4] sinθ4[t];

(*COPLANARITY (DET-VALUE) FUNCTIONS*)
detA2A1A2C2A1C1[t_] :=
Module[{vA2A1, vA2C2, vA1C1}, vA2A1 = {xA1, yA1, zA1} - {xA2, yA2, zA2};
vA2C2 = {xC2[t], yC2[t], zC2[t]} - {xA2, yA2, zA2};
vA1C1 = {xC1[t], yC1[t], zC1[t]} - {xA1, yA1, zA1};
Simplify[Det[{vA2A1, vA2C2, vA1C1}]]];

detA3A4A3C3A4C4[t_] :=
Module[{vA3A4, vA3C3, vA4C4}, vA3A4 = {xA4, yA4, zA4} - {xA3, yA3, zA3};
vA3C3 = {xC3[t], yC3[t], zC3[t]} - {xA3, yA3, zA3};
vA4C4 = {xC4[t], yC4[t], zC4[t]} - {xA4, yA4, zA4};
Simplify[Det[{vA3A4, vA3C3, vA4C4}]]];

detA1A4A1B1A4B4[t_] :=
Module[{vA1A4, vA1B1, vA4B4}, vA1A4 = {xA4, yA4, zA4} - {xA1, yA1, zA1};
vA1B1 = {xB1[t], yB1[t], zB1[t]} - {xA1, yA1, zA1};
vA4B4 = {xB4[t], yB4[t], zB4[t]} - {xA4, yA4, zA4};
Simplify[Det[{vA1A4, vA1B1, vA4B4}]]];

detA2A3A2B2A3B3[t_] :=
Module[{vA2A3, vA2B2, vA3B3}, vA2A3 = {xA3, yA3, zA3} - {xA2, yA2, zA2};

```

```

vA2B2 = {xB2[t], yB2[t], zB2[t]} - {xA2, yA2, zA2};
vA3B3 = {xB3[t], yB3[t], zB3[t]} - {xA3, yA3, zA3};
Simplify[Det[{vA2A3, vA2B2, vA3B3}]]];

(*---t-range---*)
tMin = 1 / Sqrt[2 * Sqrt[3] + 3];
tMax = 1 / 2;

Column[{TextCell[
  Style["===== EXTRA INFO (FLEXION 1) =====",
    Darker[Red], Bold, 16], "Text"], (*Explanatory text*)TextCell[
  Style["This example does NOT have planar parameter lines (C2A2A1C1,
    C3A3A4C4, B1A1A4B4, B2A2A3B3)", GrayLevel[0.3]], "Text"],
  Spacer[12],

  TextCell[Style["Coplanarity Check: A2A1-A2C2-A1C1",
    Darker[Green], Bold, 12], "Text"],
  TextCell[Style["Determinant of {A2A1, A2C2, A1C1} as a function of t",
    GrayLevel[0.3]], "Text"],
  Spacer[5],
  detA2A1A2C2A1C1[t],
  Spacer[5],

  TextCell[Style["Coplanarity Check: A3A4-A3C3-A4C4",
    Darker[Blue], Bold, 12], "Text"],
  TextCell[Style["Determinant of {A3A4, A3C3, A4C4} as a function of t",
    GrayLevel[0.3]], "Text"],
  Spacer[5],
  detA3A4A3C3A4C4[t],
  Spacer[5],

  TextCell[Style["Coplanarity Check: A2A3-A2B2-A3B3",
    Darker[Purple], Bold, 12], "Text"],
  TextCell[Style["Determinant of {A2A3, A2B2, A3B3} as a function of t",
    GrayLevel[0.3]], "Text"],
  Spacer[5],
  detA2A3A2B2A3B3[t],
  Spacer[5],

  TextCell[Style["Coplanarity Check: A1A4-A1B1-A4B4",
    Darker[Brown], Bold, 12], "Text"],
  TextCell[Style["Determinant of {A1A4, A1B1, A4B4} as a function of t",
    GrayLevel[0.3]], "Text"],
  Spacer[5],
  detA1A4A1B1A4B4[t],
  Spacer[5],

```

```

(*---Numeric parameter values ---*)
l1 = 4; l2 = 4; l3 = 3.863703305156273; l4 = 5.035276180410083;
m1 = 4.528845917006416; m2 = 5.0554559073559995;
m3 = 6.363904177855777; m4 = 5.700997492662467;
n1 = 6.181925288250036; n2 = 6.954665949281291;
n3 = 3.090962644125018; n4 = 6.181925288250037;

(*Plots in a light panel arranged in a 2x2 grid*)
TextCell[
  Style["Determinant Plots for Our Example", Darker[Cyan], Bold, 14], "Text",
  Panel[GraphicsGrid[{{(*1) A2A1-A2C2-A1C1*)Plot[detA2A1A2C2A1C1[t],
    {t, tMin, tMax}, PlotLabel → Style["det(A2A1, A2C2, A1C1)", Bold, 14],
    AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250],
    (*2) A3A4-A3C3-A4C4*)Plot[detA3A4A3C3A4C4[t], {t, tMin, tMax},
    PlotLabel → Style["det(A3A4, A3C3, A4C4)", Bold, 14],
    AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250]}],
    {{(*3) A1A4-A1B1-A4B4*)Plot[detA1A4A1B1A4B4[t], {t, tMin, tMax},
    PlotLabel → Style["det(A1A4, A1B1, A4B4)", Bold, 14],
    AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250],
    (*4) A2A3-A2B2-A3B3*)Plot[detA2A3A2B2A3B3[t], {t, tMin, tMax},
    PlotLabel → Style["det(A2A3, A2B2, A3B3)", Bold, 14],
    AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250]}]],
  Background → Lighter[Gray, 0.95], FrameMargins → 15]]]

(*=====
  FLEXION 2=====*)
(*---Link lengths (parameters)---*)
(*A-links:l1=A1A2,l2=A2A3,l3=A1A4,l4=A3A4*)
Clear[l1, l2, l3, l4];
(*B-links:m1=A1B1,m2=A2B2,m3=A3B3,m4=A4B4*)
Clear[m1, m2, m3, m4];
(*C-links:n1=A1C1,n2=A2C2,n3=A3C3,n4=A4C4*)
Clear[n1, n2, n3, n4];

(*---Joint half-angle formulas:cosθi,sinθi---*)
cosθ1[t_] := FullSimplify[(Z2[t])^2 - 1 / ((Z2[t])^2 + 1)];
sinθ1[t_] := FullSimplify[2 Z2[t] / ((Z2[t])^2 + 1)];
cosθ2[t_] := FullSimplify[(W22[t])^2 - 1 / ((W22[t])^2 + 1)];
sinθ2[t_] := FullSimplify[2 W22[t] / ((W22[t])^2 + 1)];
cosθ3[t_] := FullSimplify[(U2[t])^2 - 1 / ((U2[t])^2 + 1)];
sinθ3[t_] := FullSimplify[2 U2[t] / ((U2[t])^2 + 1)];
cosθ4[t_] := FullSimplify[(W12[t])^2 - 1 / ((W12[t])^2 + 1)];
sinθ4[t_] := FullSimplify[2 W12[t] / ((W12[t])^2 + 1)];

(*=====A-POINTS=====*)
xA2 = 0;

```

```

xA1 = l1;
xA3 = l2 Cos[δ2];
xA4 = l1 - l3 Cos[δ1];
yA2 = 0;
yA1 = 0;
yA3 = l2 Sin[δ2];
yA4 = l3 Sin[δ1];
zA2 = 0; zA1 = 0; zA3 = 0; zA4 = 0;

(*=====B-POINTS=====*)
xB1[t_] := xA1 - m1 Cos[α1];
yB1[t_] := yA1 + m1 Sin[α1] cosθ1[t];
zB1[t_] := zA1 + m1 Sin[α1] sinθ1[t];
xB2[t_] := xA2 + m2 Cos[α2];
yB2[t_] := yA2 + m2 Sin[α2] cosθ1[t];
zB2[t_] := zA2 + m2 Sin[α2] sinθ1[t];
xB3[t_] := xA3 - m3 (Cos[α3] Cos[δ2 + δ3] + Sin[α3] cosθ3[t] Sin[δ2 + δ3]);
yB3[t_] := yA3 - m3 (Cos[α3] Sin[δ2 + δ3] - Sin[α3] cosθ3[t] Cos[δ2 + δ3]);
zB3[t_] := zA3 + m3 Sin[α3] sinθ3[t];
xB4[t_] := xA4 + m4 (Cos[α4] Cos[δ2 + δ3] - Sin[α4] cosθ3[t] Sin[δ2 + δ3]);
yB4[t_] := yA4 + m4 (Cos[α4] Sin[δ2 + δ3] + Sin[α4] cosθ3[t] Cos[δ2 + δ3]);
zB4[t_] := zA4 + m4 Sin[α4] sinθ3[t];

(*=====C-POINTS=====*)
xC1[t_] := xA1 - n1 (Cos[γ1] Cos[δ1] + Sin[γ1] cosθ4[t] Sin[δ1]);
yC1[t_] := yA1 + n1 (Cos[γ1] Sin[δ1] - Sin[γ1] cosθ4[t] Cos[δ1]);
zC1[t_] := zA1 + n1 Sin[γ1] sinθ4[t];
xC2[t_] := xA2 + n2 (Cos[γ2] Cos[δ2] + Sin[γ2] cosθ2[t] Sin[δ2]);
yC2[t_] := yA2 + n2 (Cos[γ2] Sin[δ2] - Sin[γ2] cosθ2[t] Cos[δ2]);
zC2[t_] := zA2 + n2 Sin[γ2] sinθ2[t];
xC3[t_] := xA3 - n3 (Cos[γ3] Cos[δ2] - Sin[γ3] cosθ2[t] Sin[δ2]);
yC3[t_] := yA3 - n3 (Cos[γ3] Sin[δ2] + Sin[γ3] cosθ2[t] Cos[δ2]);
zC3[t_] := zA3 + n3 Sin[γ3] sinθ2[t];
xC4[t_] := xA4 + n4 (Cos[γ4] Cos[δ1] - Sin[γ4] cosθ4[t] Sin[δ1]);
yC4[t_] := yA4 - n4 (Cos[γ4] Sin[δ1] + Sin[γ4] cosθ4[t] Cos[δ1]);
zC4[t_] := zA4 + n4 Sin[γ4] sinθ4[t];

(*COPLANARITY (DET-VALUE) FUNCTIONS*)
detA2A1A2C2A1C1[t_] :=
Module[{vA2A1, vA2C2, vA1C1}, vA2A1 = {xA1, yA1, zA1} - {xA2, yA2, zA2};
vA2C2 = {xC2[t], yC2[t], zC2[t]} - {xA2, yA2, zA2};
vA1C1 = {xC1[t], yC1[t], zC1[t]} - {xA1, yA1, zA1};
Simplify[Det[{vA2A1, vA2C2, vA1C1}]]];

detA3A4A3C3A4C4[t_] :=
Module[{vA3A4, vA3C3, vA4C4}, vA3A4 = {xA4, yA4, zA4} - {xA3, yA3, zA3};
vA3C3 = {xC3[t], yC3[t], zC3[t]} - {xA3, yA3, zA3};
vA4C4 = {xC4[t], yC4[t], zC4[t]} - {xA4, yA4, zA4};

```

```

Simplify[Det[{vA3A4, vA3C3, vA4C4}]]];

detA1A4A1B1A4B4[t_] :=
Module[{vA1A4, vA1B1, vA4B4}, vA1A4 = {xA4, yA4, zA4} - {xA1, yA1, zA1};
vA1B1 = {xB1[t], yB1[t], zB1[t]} - {xA1, yA1, zA1};
vA4B4 = {xB4[t], yB4[t], zB4[t]} - {xA4, yA4, zA4};
Simplify[Det[{vA1A4, vA1B1, vA4B4}]]];

detA2A3A2B2A3B3[t_] :=
Module[{vA2A3, vA2B2, vA3B3}, vA2A3 = {xA3, yA3, zA3} - {xA2, yA2, zA2};
vA2B2 = {xB2[t], yB2[t], zB2[t]} - {xA2, yA2, zA2};
vA3B3 = {xB3[t], yB3[t], zB3[t]} - {xA3, yA3, zA3};
Simplify[Det[{vA2A3, vA2B2, vA3B3}]]];

(*---t-range---*)
tMin = 1 / Sqrt[2 * Sqrt[3] + 3];
tMax = 1 / 2;

Column[{TextCell[
Style["===== EXTRA INFO (FLEXION 2) =====",
Red, Bold, 16], "Text"], (*Explanatory text*)TextCell[
Style["This example does NOT have planar parameter lines (C2A2A1C1,
C3A3A4C4, B1A1A4B4, B2A2A3B3)", GrayLevel[0.3]], "Text"],
Spacer[12],

TextCell[
Style["Coplanarity Check: A2A1-A2C2-A1C1", Green, Bold, 12], "Text"],
TextCell[Style["Determinant of {A2A1, A2C2, A1C1} as a function of t",
GrayLevel[0.3]], "Text"],
Spacer[5],
detA2A1A2C2A1C1[t],
Spacer[5],

TextCell[
Style["Coplanarity Check: A3A4-A3C3-A4C4", Blue, Bold, 12], "Text"],
TextCell[Style["Determinant of {A3A4, A3C3, A4C4} as a function of t",
GrayLevel[0.3]], "Text"],
Spacer[5],
detA3A4A3C3A4C4[t],
Spacer[5],

TextCell[
Style["Coplanarity Check: A2A3-A2B2-A3B3", Purple, Bold, 12], "Text"],
TextCell[Style["Determinant of {A2A3, A2B2, A3B3} as a function of t",
GrayLevel[0.3]], "Text"],
Spacer[5],
detA2A3A2B2A3B3[t],

```

Spacer[5],

TextCell[

Style["Coplanarity Check: A1A4-A1B1-A4B4", Brown, Bold, 12], "Text"],

TextCell[Style["Determinant of {A1A4, A1B1, A4B4} as a function of t",

GrayLevel[0.3]], "Text"],

Spacer[5],

detA1A4A1B1A4B4[t],

Spacer[5],

(*---Numeric parameter values ---*)

l1 = 4; l2 = 4; l3 = 3.863703305156273; l4 = 5.035276180410083;

m1 = 4.528845917006416; m2 = 5.0554559073559995;

m3 = 6.363904177855777; m4 = 5.700997492662467;

n1 = 6.181925288250036; n2 = 6.954665949281291;

n3 = 3.090962644125018; n4 = 6.181925288250037;

(*Plots in a light panel arranged in a 2x2 grid*)

TextCell[Style["Determinant Plots for Our Example", Cyan, Bold, 14], "Text"],

Panel[GraphicsGrid[{{(*1) A2A1-A2C2-A1C1*)Plot[detA2A1A2C2A1C1[t],
{t, tMin, tMax}, PlotLabel → Style["det(A2A1, A2C2, A1C1)", Bold, 14],
AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250],
(*2) A3A4-A3C3-A4C4*)Plot[detA3A4A3C3A4C4[t], {t, tMin, tMax},
PlotLabel → Style["det(A3A4, A3C3, A4C4)", Bold, 14],
AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250]}],
{(*3) A1A4-A1B1-A4B4*)Plot[detA1A4A1B1A4B4[t], {t, tMin, tMax},
PlotLabel → Style["det(A1A4, A1B1, A4B4)", Bold, 14],
AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250],
(*4) A2A3-A2B2-A3B3*)Plot[detA2A3A2B2A3B3[t], {t, tMin, tMax},
PlotLabel → Style["det(A2A3, A2B2, A3B3)", Bold, 14],
AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250]}]],
Background → Lighter[Gray, 0.95], FrameMargins → 15]]]

Out[1601]=

===== **CONDITION (N.0)** =====
✓ All vertices satisfy (N.0).

Out[1604]=

===== **CONDITION (N.3)** =====
✓ $M1 = M2 = M3 = M4 = -1 + \sqrt{3}$

Out[1610]=

===== **CONDITION (N.4)** =====
✓ $r1 = r2 = \frac{2}{\sqrt{3}}$; ✓ $r3 = r4 = \frac{2}{\sqrt{3}}$
✓ $s1 = s4 = 3 - \sqrt{3}$; ✓ $s2 = s3 = \frac{1}{\sqrt{3}}$

Out[1620]=

===== **CONDITION (N.5)** =====

△ Approximate validation using ε -tolerance. For rigorous proof, see the referenced paper.

✓ Valid Combination Found ($M < 1$):

$e1 = -1, e2 = -1, e3 = 1$
 $t1 = 0.K + 0.446521iK'$
 $t2 = 1.K + 0.446521iK'$
 $t3 = 3.K + 0.446521iK'$
 $t4 = 0.K + 0.446521iK'$
 $t1 + e1*t2 + e2*t3 + e3*t4 = -4.K + 0.iK'$

Out[1622]=

===== OTHER PARAMETERS =====

$u = 2 - \sqrt{3}$
 $\sigma1 = 165^\circ, \sigma2 = 165^\circ, \sigma3 = 195^\circ, \sigma4 = 165^\circ$
 $f1 = \frac{1}{2}(1 + \sqrt{3}), f2 = 4 - 2\sqrt{3}, f3 = 4 - 2\sqrt{3}, f4 = \frac{1}{2}(1 + \sqrt{3})$
 $z1 = 1 + \sqrt{3}, z2 = -1 - \frac{2}{\sqrt{3}}, z3 = -1 - \frac{2}{\sqrt{3}}, z4 = 1 + \sqrt{3}$
 $x1 = 3 + 2\sqrt{3}, x2 = 3 + 2\sqrt{3}, x3 = 3 + 2\sqrt{3}, x4 = 3 + 2\sqrt{3}$
 $y1 = 2 + \sqrt{3}, y2 = \frac{1}{-1 + \frac{1}{\sqrt{3}}}, y3 = \frac{1}{-1 + \frac{1}{\sqrt{3}}}, y4 = 2 + \sqrt{3}$
 $p1 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p2 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p3 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p4 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$
 $q1 = \sqrt{2 - \sqrt{3}}, q2 = i\sqrt{1 - \frac{1}{\sqrt{3}}}, q3 = i\sqrt{1 - \frac{1}{\sqrt{3}}}, q4 = \sqrt{2 - \sqrt{3}}$
 $p1 \cdot q1 = \sqrt{-4 + \frac{7}{\sqrt{3}}}, p2 \cdot q2 = i\sqrt{-\frac{5}{3} + \sqrt{3}}, p3 \cdot q3 = i\sqrt{-\frac{5}{3} + \sqrt{3}}, p4 \cdot q4 = \sqrt{-4 + \frac{7}{\sqrt{3}}}$
 $\overline{\alpha1} = 60^\circ, \overline{\beta1} = 150^\circ, \overline{\gamma1} = 45^\circ, \overline{\delta1} = 75^\circ$
Inequalities (vertex 1): $\sin(2.05864_1) > \sin(2.05864_1) \sin(105^\circ_1) :$
 True , $\sin(2.60102_1) > \sin(2.60102_1) \sin(105^\circ_1) : \text{True}$,
 $\sin(0.864432_1) \sin(2.05864_1) < \sin(0.864432_1) \sin(2.05864_1) : \text{True}$

Out[1628]=

===== FLEXIBILITY (FLEXION 1) =====

$P_1[Z, W1] = 0$
 $P_2[Z, W2] = 0$
 $P_3[U, W2] = 0$
 $P_4[U, W1] = 0$

Out[1636]=

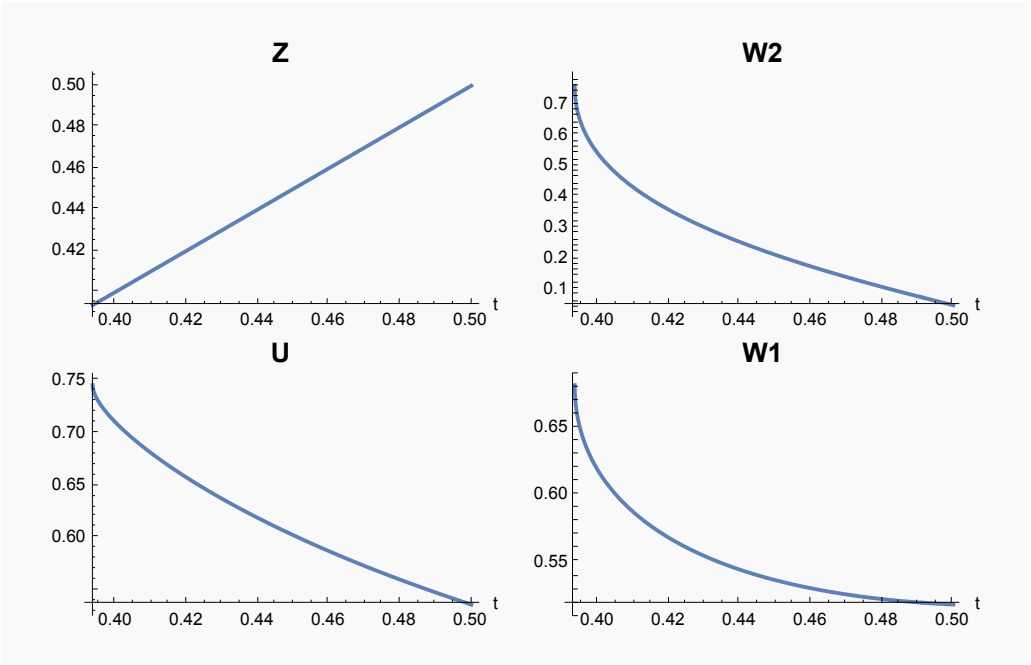
===== FLEXIBILITY (FLEXION 2) =====

$P_1[Z, W1] = 0$
 $P_2[Z, W2] = 0$
 $P_3[U, W2] = 0$
 $P_4[U, W1] = 0$

Out[1643]=

===== NOT TRIVIAL (FLEXION 1) =====

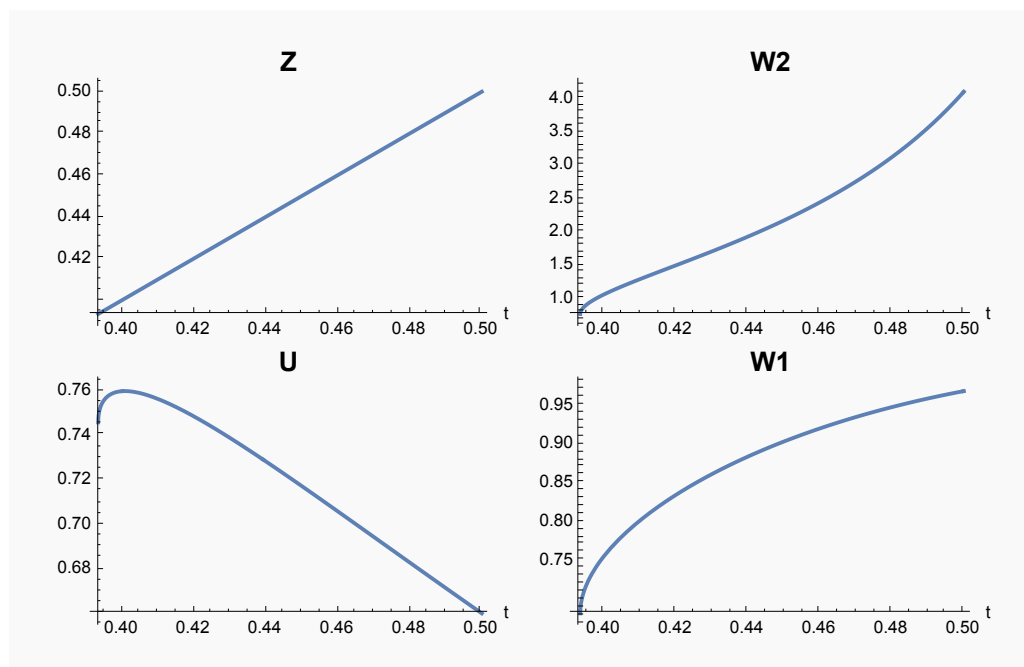
This configuration does not belong to the trivial class – even after switching the boundary strips – since none of the functions Z , $W2$, U , or $W1$ is constant.



Out[1646]=

===== NOT TRIVIAL (FLEXION 2) =====

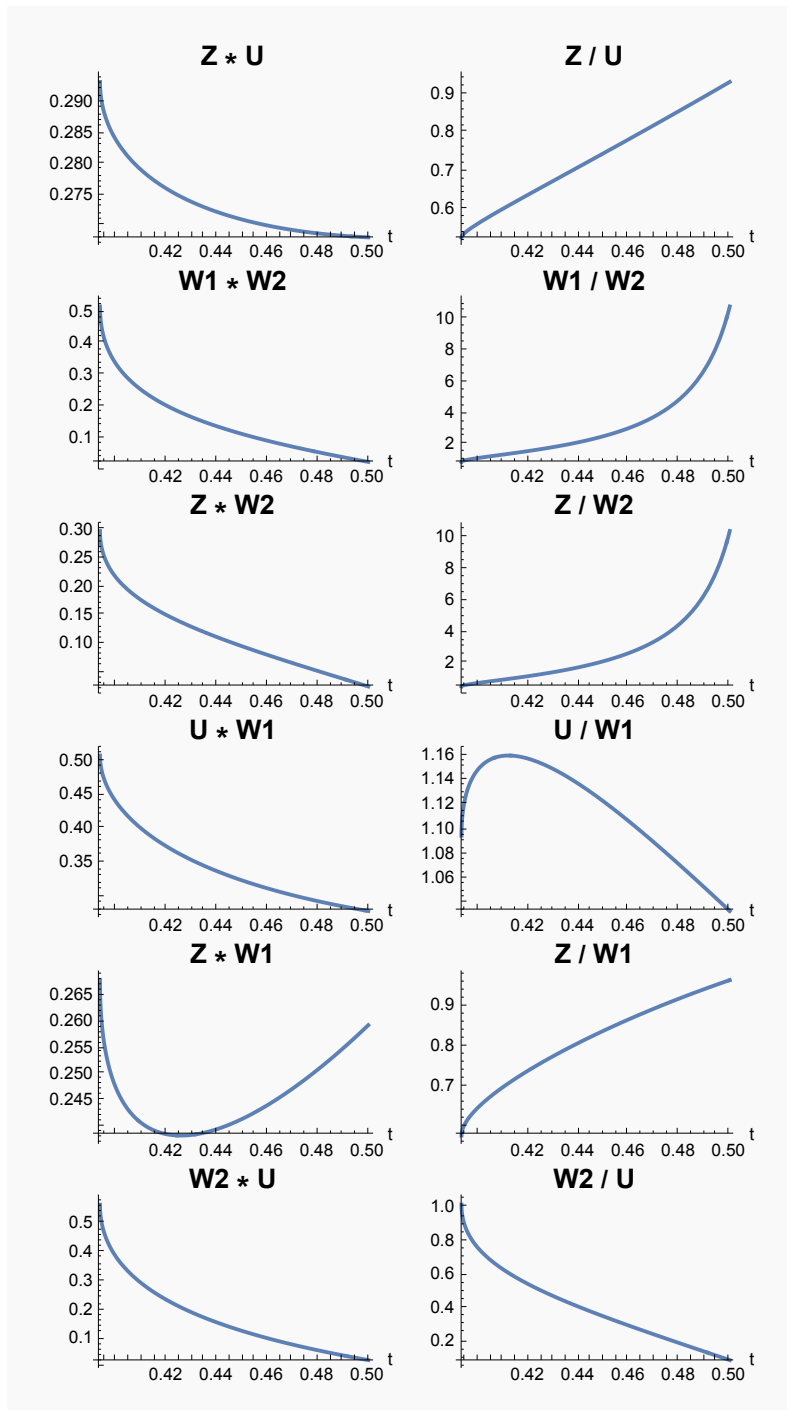
This configuration does not belong to the trivial class – even after switching the boundary strips – since none of the functions Z , $W2$, U , or $W1$ is constant.



Out[1651]=

===== NOT LINEAR COMPOUND &
NOT LINEAR CONJUGATE (FLEXION 1) =====

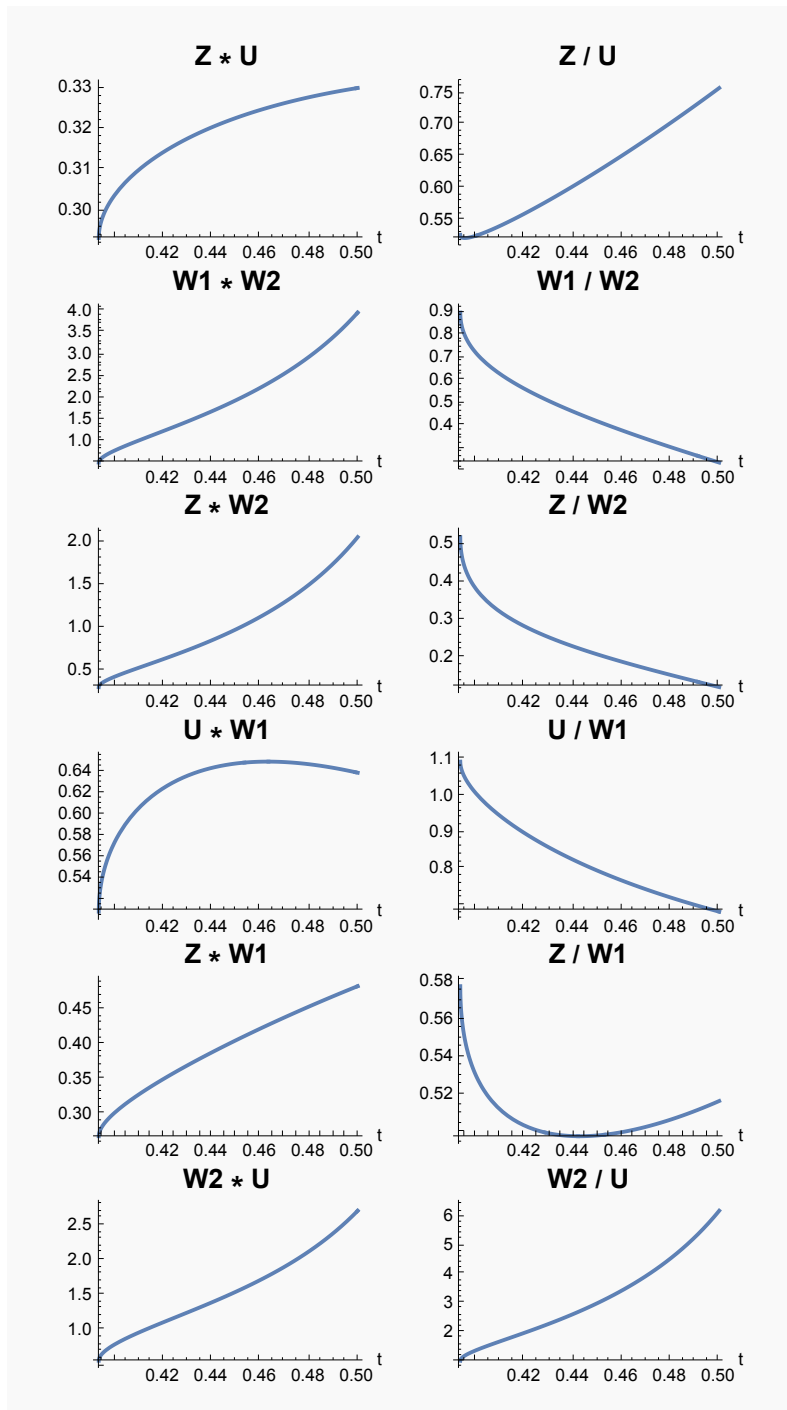
This configuration does not belong to the Linear compound class nor to the linear conjugate class – even after switching the boundary strips – since none of the functions ZU , Z/U , $W1W2$, $W1/W2$, $ZW2$, $Z/W2$, $UW1$, and $U/W1$ is constant. In addition, none of $ZW1$, $Z/W1$, $W2U$, $W2/U$ is constant as well.



Out[1654]=

===== NOT LINEAR COMPOUND &
NOT LINEAR CONJUGATE (FLEXION 2) =====

This configuration does not belong to the Linear compound class nor to the linear conjugate class – even after switching the boundary strips – since none of the functions ZU , Z/U , $W1W2$, $W1/W2$, $ZW2$, $Z/W2$, $UW1$, and $U/W1$ is constant. In addition, none of $ZW1$, $Z/W1$, $W2U$, $W2/U$ is constant as well.



Out[1659]=

===== **NOT CONIC** =====

Condition (N.0) is satisfied \Rightarrow this configuration
is NOT equimodular-conic. Applying any boundary-strip
switch still preserves (N.0), so no conic form emerges.

Out[1660]=

CONDITION (N.0) AFTER SWITCHING BOUNDARY STRIPS

Right: Condition (N.0) is still satisfied.
Left: Condition (N.0) is still satisfied.
Lower: Condition (N.0) is still satisfied.
Upper: Condition (N.0) is still satisfied.
Right + Left: Condition (N.0) is still satisfied.
Right + Lower: Condition (N.0) is still satisfied.
Right + Upper: Condition (N.0) is still satisfied.
Left + Lower: Condition (N.0) is still satisfied.
Left + Upper: Condition (N.0) is still satisfied.
Lower + Upper: Condition (N.0) is still satisfied.
Right + Left + Lower: Condition (N.0) is still satisfied.
Right + Left + Upper: Condition (N.0) is still satisfied.
Right + Lower + Upper: Condition (N.0) is still satisfied.
Left + Lower + Upper: Condition (N.0) is still satisfied.
Right + Left + Lower + Upper: Condition (N.0) is still satisfied.

Out[1661]=

===== **NOT ORTHODIAGONAL** =====

$\cos(\alpha_i) \cdot \cos(\gamma_i) \neq \cos(\beta_i) \cdot \cos(\delta_i)$ for each $i = 1..4 \Rightarrow$ NOT
orthodiagonal. Switching boundary strips does not correct this.

Initial anglesDeg (no switches):

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = \frac{\sqrt{2-\sqrt{3}}}{4}$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = -\frac{1}{4} \sqrt{2-\sqrt{3}}$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = -\frac{1}{4} \sqrt{2-\sqrt{3}}$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = \frac{\sqrt{2-\sqrt{3}}}{4}$$

Switch combination: Right

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned}
\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{1}{8} \left(\sqrt{2} - \sqrt{6} \right) \\
\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -\frac{1}{4} \sqrt{2 - \sqrt{3}} \\
\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -\frac{1}{4} \sqrt{2 - \sqrt{3}} \\
\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{1}{8} \left(\sqrt{2} - \sqrt{6} \right)
\end{aligned}$$

Switch combination: Left

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned}
\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{\sqrt{2 - \sqrt{3}}}{4} \\
\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= \frac{1}{8} \left(-\sqrt{2} + \sqrt{6} \right) \\
\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= \frac{1}{8} \left(-\sqrt{2} + \sqrt{6} \right) \\
\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{\sqrt{2 - \sqrt{3}}}{4}
\end{aligned}$$

Switch combination: Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned}
\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{1}{8} \left(\sqrt{2} - \sqrt{6} \right) \\
\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= \frac{1}{8} \left(-\sqrt{2} + \sqrt{6} \right) \\
\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -\frac{1}{4} \sqrt{2 - \sqrt{3}} \\
\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{\sqrt{2 - \sqrt{3}}}{4}
\end{aligned}$$

Switch combination: Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned}
\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{\sqrt{2 - \sqrt{3}}}{4} \\
\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -\frac{1}{4} \sqrt{2 - \sqrt{3}} \\
\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= \frac{1}{8} \left(-\sqrt{2} + \sqrt{6} \right) \\
\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{1}{8} \left(\sqrt{2} - \sqrt{6} \right)
\end{aligned}$$

Switch combination: Right + Left

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{1}{8} (\sqrt{2} - \sqrt{6}) \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= \frac{1}{8} (-\sqrt{2} + \sqrt{6}) \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= \frac{1}{8} (-\sqrt{2} + \sqrt{6}) \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{1}{8} (\sqrt{2} - \sqrt{6}) \end{aligned}$$

Switch combination: Right + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{\sqrt{2-\sqrt{3}}}{4} \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= \frac{1}{8} (-\sqrt{2} + \sqrt{6}) \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -\frac{1}{4} \sqrt{2-\sqrt{3}} \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{1}{8} (\sqrt{2} - \sqrt{6}) \end{aligned}$$

Switch combination: Right + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{1}{8} (\sqrt{2} - \sqrt{6}) \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -\frac{1}{4} \sqrt{2-\sqrt{3}} \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= \frac{1}{8} (-\sqrt{2} + \sqrt{6}) \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{\sqrt{2-\sqrt{3}}}{4} \end{aligned}$$

Switch combination: Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = \frac{1}{8} (\sqrt{2} - \sqrt{6})$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = -\frac{1}{4} \sqrt{2 - \sqrt{3}}$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = \frac{1}{8} (-\sqrt{2} + \sqrt{6})$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = \frac{\sqrt{2 - \sqrt{3}}}{4}$$

Switch combination: Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = \frac{\sqrt{2 - \sqrt{3}}}{4}$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = \frac{1}{8} (-\sqrt{2} + \sqrt{6})$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = -\frac{1}{4} \sqrt{2 - \sqrt{3}}$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = \frac{1}{8} (\sqrt{2} - \sqrt{6})$$

Switch combination: Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = \frac{1}{8} (\sqrt{2} - \sqrt{6})$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = \frac{1}{8} (-\sqrt{2} + \sqrt{6})$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = \frac{1}{8} (-\sqrt{2} + \sqrt{6})$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = \frac{1}{8} (\sqrt{2} - \sqrt{6})$$

Switch combination: Right + Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = \frac{\sqrt{2 - \sqrt{3}}}{4}$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = -\frac{1}{4} \sqrt{2 - \sqrt{3}}$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = \frac{1}{8} (-\sqrt{2} + \sqrt{6})$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = \frac{1}{8} (\sqrt{2} - \sqrt{6})$$

Switch combination: Right + Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{1}{8} (\sqrt{2} - \sqrt{6}) \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= \frac{1}{8} (-\sqrt{2} + \sqrt{6}) \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -\frac{1}{4} \sqrt{2 - \sqrt{3}} \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{\sqrt{2 - \sqrt{3}}}{4} \end{aligned}$$

Switch combination: Right + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{\sqrt{2 - \sqrt{3}}}{4} \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= \frac{1}{8} (-\sqrt{2} + \sqrt{6}) \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= \frac{1}{8} (-\sqrt{2} + \sqrt{6}) \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{\sqrt{2 - \sqrt{3}}}{4} \end{aligned}$$

Switch combination: Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{1}{8} (\sqrt{2} - \sqrt{6}) \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -\frac{1}{4} \sqrt{2 - \sqrt{3}} \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -\frac{1}{4} \sqrt{2 - \sqrt{3}} \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{1}{8} (\sqrt{2} - \sqrt{6}) \end{aligned}$$

Switch combination: Right + Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = \frac{\sqrt{2-\sqrt{3}}}{4}$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = -\frac{1}{4} \sqrt{2-\sqrt{3}}$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = -\frac{1}{4} \sqrt{2-\sqrt{3}}$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = \frac{\sqrt{2-\sqrt{3}}}{4}$$

Out[1663]=

===== NOT ISOGONAL =====

Condition (N.0) holds AND for all $i = 1..4$: $\alpha_i \neq \beta_i$,

$\alpha_i \neq \gamma_i$, $\alpha_i \neq \delta_i$, $\beta_i \neq \gamma_i$, $\beta_i \neq \delta_i$, $\gamma_i \neq \delta_i$, $\alpha_i + \beta_i \neq$

$\pi \neq \gamma_i + \delta_i$, $\alpha_i + \gamma_i \neq \pi \neq \beta_i + \delta_i$, $\alpha_i + \delta_i \neq \pi \neq \beta_i + \gamma_i \Rightarrow$ NOT

isogonal. Switching boundary strips do not change this.

Initial anglesDeg (no switches):

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\alpha_1 - \beta_1 = 90.000$$

$$\alpha_1 - \gamma_1 = -15.000$$

$$\alpha_1 - \delta_1 = 15.000$$

$$\beta_1 - \gamma_1 = -105.000$$

$$\beta_1 - \delta_1 = -75.000$$

$$\gamma_1 - \delta_1 = 30.000$$

$$\alpha_1 + \beta_1 - 180 = -60.000$$

$$\gamma_1 + \delta_1 - 180 = 30.000$$

$$\alpha_1 + \gamma_1 - 180 = 45.000$$

$$\beta_1 + \delta_1 - 180 = -75.000$$

$$\alpha_1 + \delta_1 - 180 = 15.000$$

$$\beta_1 + \gamma_1 - 180 = -45.000$$

$$\alpha_1 + \beta_1 - \gamma_1 - \delta_1 = -90.000$$

$$\alpha_1 + \gamma_1 - \beta_1 - \delta_1 = 120.000$$

$$\alpha_1 + \delta_1 - \beta_1 - \gamma_1 = 60.000$$

Vertex 2

$$\alpha_2 - \beta_2 = -30.000$$

$$\alpha_2 - \gamma_2 = 75.000$$

$$\alpha_2 - \delta_2 = -15.000$$

$$\beta_2 - \gamma_2 = 105.000$$

$$\beta_2 - \delta_2 = 15.000$$

$$\gamma_2 - \delta_2 = -90.000$$

$$\alpha_2 + \beta_2 - 180 = 30.000$$

$$\gamma_2 + \delta_2 - 180 = -60.000$$

$$\alpha_2 + \gamma_2 - 180 = -75.000$$

$$\beta_2 + \delta_2 - 180 = 45.000$$

$$\alpha_2 + \delta_2 - 180 = 15.000$$

$$\beta_2 + \gamma_2 - 180 = -45.000$$

$$\alpha_2 + \beta_2 - \gamma_2 - \delta_2 = 90.000$$

$$\alpha_2 + \gamma_2 - \beta_2 - \delta_2 = -120.000$$

$$\alpha_2 + \delta_2 - \beta_2 - \gamma_2 = 60.000$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= 30.000 \\
\alpha_3 - \gamma_3 &= -75.000 \\
\alpha_3 - \delta_3 &= 15.000 \\
\beta_3 - \gamma_3 &= -105.000 \\
\beta_3 - \delta_3 &= -15.000 \\
\gamma_3 - \delta_3 &= 90.000 \\
\alpha_3 + \beta_3 - 180 &= -30.000 \\
\gamma_3 + \delta_3 - 180 &= 60.000 \\
\alpha_3 + \gamma_3 - 180 &= 75.000 \\
\beta_3 + \delta_3 - 180 &= -45.000 \\
\alpha_3 + \delta_3 - 180 &= -15.000 \\
\beta_3 + \gamma_3 - 180 &= 45.000 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -90.000 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 120.000 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -60.000
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= 90.000 \\
\alpha_4 - \gamma_4 &= -15.000 \\
\alpha_4 - \delta_4 &= 15.000 \\
\beta_4 - \gamma_4 &= -105.000 \\
\beta_4 - \delta_4 &= -75.000 \\
\gamma_4 - \delta_4 &= 30.000 \\
\alpha_4 + \beta_4 - 180 &= -60.000 \\
\gamma_4 + \delta_4 - 180 &= 30.000 \\
\alpha_4 + \gamma_4 - 180 &= 45.000 \\
\beta_4 + \delta_4 - 180 &= -75.000 \\
\alpha_4 + \delta_4 - 180 &= 15.000 \\
\beta_4 + \gamma_4 - 180 &= -45.000 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -90.000 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 120.000 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 60.000
\end{aligned}$$

Switch combination: Right

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= -60.000 \\
\alpha_1 - \gamma_1 &= 45.000 \\
\alpha_1 - \delta_1 &= 15.000 \\
\beta_1 - \gamma_1 &= 105.000 \\
\beta_1 - \delta_1 &= 75.000 \\
\gamma_1 - \delta_1 &= -30.000 \\
\alpha_1 + \beta_1 - 180 &= 90.000 \\
\gamma_1 + \delta_1 - 180 &= -30.000 \\
\alpha_1 + \gamma_1 - 180 &= -15.000 \\
\beta_1 + \delta_1 - 180 &= 75.000 \\
\alpha_1 + \delta_1 - 180 &= 15.000 \\
\beta_1 + \gamma_1 - 180 &= 45.000 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 120.000 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -90.000 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -30.000
\end{aligned}$$

Vertex 2

$$\alpha_2 - \beta_2 = -30.000$$

$$\begin{aligned}
\alpha_2 - \gamma_2 &= 75.000 \\
\alpha_2 - \delta_2 &= -15.000 \\
\beta_2 - \gamma_2 &= 105.000 \\
\beta_2 - \delta_2 &= 15.000 \\
\gamma_2 - \delta_2 &= -90.000 \\
\alpha_2 + \beta_2 - 180 &= 30.000 \\
\gamma_2 + \delta_2 - 180 &= -60.000 \\
\alpha_2 + \gamma_2 - 180 &= -75.000 \\
\beta_2 + \delta_2 - 180 &= 45.000 \\
\alpha_2 + \delta_2 - 180 &= 15.000 \\
\beta_2 + \gamma_2 - 180 &= -45.000 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 90.000 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -120.000 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 60.000
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= 30.000 \\
\alpha_3 - \gamma_3 &= -75.000 \\
\alpha_3 - \delta_3 &= 15.000 \\
\beta_3 - \gamma_3 &= -105.000 \\
\beta_3 - \delta_3 &= -15.000 \\
\gamma_3 - \delta_3 &= 90.000 \\
\alpha_3 + \beta_3 - 180 &= -30.000 \\
\gamma_3 + \delta_3 - 180 &= 60.000 \\
\alpha_3 + \gamma_3 - 180 &= 75.000 \\
\beta_3 + \delta_3 - 180 &= -45.000 \\
\alpha_3 + \delta_3 - 180 &= -15.000 \\
\beta_3 + \gamma_3 - 180 &= 45.000 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -90.000 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 120.000 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -60.000
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= -60.000 \\
\alpha_4 - \gamma_4 &= 45.000 \\
\alpha_4 - \delta_4 &= 15.000 \\
\beta_4 - \gamma_4 &= 105.000 \\
\beta_4 - \delta_4 &= 75.000 \\
\gamma_4 - \delta_4 &= -30.000 \\
\alpha_4 + \beta_4 - 180 &= 90.000 \\
\gamma_4 + \delta_4 - 180 &= -30.000 \\
\alpha_4 + \gamma_4 - 180 &= -15.000 \\
\beta_4 + \delta_4 - 180 &= 75.000 \\
\alpha_4 + \delta_4 - 180 &= 15.000 \\
\beta_4 + \gamma_4 - 180 &= 45.000 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 120.000 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -90.000 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -30.000
\end{aligned}$$

Switch combination: Left*Switched anglesDeg:*

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

*Angle relation checks for i = 1..4:***Vertex 1**

$$\begin{aligned}
\alpha_1 - \beta_1 &= 90.000 \\
\alpha_1 - \gamma_1 &= -15.000
\end{aligned}$$

$$\begin{aligned}
\alpha_1 - \delta_1 &= -15.000 \\
\alpha_1 - \delta_1 &= 15.000 \\
\beta_1 - \gamma_1 &= -105.000 \\
\beta_1 - \delta_1 &= -75.000 \\
\gamma_1 - \delta_1 &= 30.000 \\
\alpha_1 + \beta_1 - 180 &= -60.000 \\
\gamma_1 + \delta_1 - 180 &= 30.000 \\
\alpha_1 + \gamma_1 - 180 &= 45.000 \\
\beta_1 + \delta_1 - 180 &= -75.000 \\
\alpha_1 + \delta_1 - 180 &= 15.000 \\
\beta_1 + \gamma_1 - 180 &= -45.000 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -90.000 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 120.000 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 60.000
\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= 30.000 \\
\alpha_2 - \gamma_2 &= -75.000 \\
\alpha_2 - \delta_2 &= -15.000 \\
\beta_2 - \gamma_2 &= -105.000 \\
\beta_2 - \delta_2 &= -45.000 \\
\gamma_2 - \delta_2 &= 60.000 \\
\alpha_2 + \beta_2 - 180 &= -30.000 \\
\gamma_2 + \delta_2 - 180 &= 90.000 \\
\alpha_2 + \gamma_2 - 180 &= 75.000 \\
\beta_2 + \delta_2 - 180 &= -15.000 \\
\alpha_2 + \delta_2 - 180 &= 15.000 \\
\beta_2 + \gamma_2 - 180 &= 45.000 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -120.000 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 90.000 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -30.000
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= -30.000 \\
\alpha_3 - \gamma_3 &= 75.000 \\
\alpha_3 - \delta_3 &= 15.000 \\
\beta_3 - \gamma_3 &= 105.000 \\
\beta_3 - \delta_3 &= 45.000 \\
\gamma_3 - \delta_3 &= -60.000 \\
\alpha_3 + \beta_3 - 180 &= 30.000 \\
\gamma_3 + \delta_3 - 180 &= -90.000 \\
\alpha_3 + \gamma_3 - 180 &= -75.000 \\
\beta_3 + \delta_3 - 180 &= 15.000 \\
\alpha_3 + \delta_3 - 180 &= -15.000 \\
\beta_3 + \gamma_3 - 180 &= -45.000 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 120.000 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -90.000 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 30.000
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= 90.000 \\
\alpha_4 - \gamma_4 &= -15.000 \\
\alpha_4 - \delta_4 &= 15.000 \\
\beta_4 - \gamma_4 &= -105.000 \\
\beta_4 - \delta_4 &= -75.000 \\
\gamma_4 - \delta_4 &= 30.000 \\
\alpha_4 + \beta_4 - 180 &= -60.000 \\
\gamma_4 + \delta_4 - 180 &= 30.000 \\
\alpha_4 + \gamma_4 - 180 &= 45.000 \\
\beta_4 + \delta_4 - 180 &= -75.000 \\
\alpha_4 + \delta_4 - 180 &= 15.000 \\
\beta_4 + \gamma_4 - 180 &= -45.000
\end{aligned}$$

$$\begin{aligned}\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -90.000 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 120.000 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 60.000\end{aligned}$$

Switch combination: Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}\alpha_1 - \beta_1 &= -90.000 \\ \alpha_1 - \gamma_1 &= -45.000 \\ \alpha_1 - \delta_1 &= -15.000 \\ \beta_1 - \gamma_1 &= 45.000 \\ \beta_1 - \delta_1 &= 75.000 \\ \gamma_1 - \delta_1 &= 30.000 \\ \alpha_1 + \beta_1 - 180 &= 60.000 \\ \gamma_1 + \delta_1 - 180 &= 30.000 \\ \alpha_1 + \gamma_1 - 180 &= 15.000 \\ \beta_1 + \delta_1 - 180 &= 75.000 \\ \alpha_1 + \delta_1 - 180 &= -15.000 \\ \beta_1 + \gamma_1 - 180 &= 105.000 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 30.000 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -60.000 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -120.000\end{aligned}$$

Vertex 2

$$\begin{aligned}\alpha_2 - \beta_2 &= 30.000 \\ \alpha_2 - \gamma_2 &= 75.000 \\ \alpha_2 - \delta_2 &= -15.000 \\ \beta_2 - \gamma_2 &= 45.000 \\ \beta_2 - \delta_2 &= -45.000 \\ \gamma_2 - \delta_2 &= -90.000 \\ \alpha_2 + \beta_2 - 180 &= -30.000 \\ \gamma_2 + \delta_2 - 180 &= -60.000 \\ \alpha_2 + \gamma_2 - 180 &= -75.000 \\ \beta_2 + \delta_2 - 180 &= -15.000 \\ \alpha_2 + \delta_2 - 180 &= 15.000 \\ \beta_2 + \gamma_2 - 180 &= -105.000 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 30.000 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -60.000 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 120.000\end{aligned}$$

Vertex 3

$$\begin{aligned}\alpha_3 - \beta_3 &= 30.000 \\ \alpha_3 - \gamma_3 &= -75.000 \\ \alpha_3 - \delta_3 &= 15.000 \\ \beta_3 - \gamma_3 &= -105.000 \\ \beta_3 - \delta_3 &= -15.000 \\ \gamma_3 - \delta_3 &= 90.000 \\ \alpha_3 + \beta_3 - 180 &= -30.000 \\ \gamma_3 + \delta_3 - 180 &= 60.000 \\ \alpha_3 + \gamma_3 - 180 &= 75.000 \\ \beta_3 + \delta_3 - 180 &= -45.000 \\ \alpha_3 + \delta_3 - 180 &= -15.000 \\ \beta_3 + \gamma_3 - 180 &= 45.000 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -90.000\end{aligned}$$

$$\alpha 3 + \gamma 3 - \beta 3 - \delta 3 = 120.000$$

$$\alpha 3 + \delta 3 - \beta 3 - \gamma 3 = -60.000$$

Vertex 4

$$\alpha 4 - \beta 4 = 90.000$$

$$\alpha 4 - \gamma 4 = -15.000$$

$$\alpha 4 - \delta 4 = 15.000$$

$$\beta 4 - \gamma 4 = -105.000$$

$$\beta 4 - \delta 4 = -75.000$$

$$\gamma 4 - \delta 4 = 30.000$$

$$\alpha 4 + \beta 4 - 180 = -60.000$$

$$\gamma 4 + \delta 4 - 180 = 30.000$$

$$\alpha 4 + \gamma 4 - 180 = 45.000$$

$$\beta 4 + \delta 4 - 180 = -75.000$$

$$\alpha 4 + \delta 4 - 180 = 15.000$$

$$\beta 4 + \gamma 4 - 180 = -45.000$$

$$\alpha 4 + \beta 4 - \gamma 4 - \delta 4 = -90.000$$

$$\alpha 4 + \gamma 4 - \beta 4 - \delta 4 = 120.000$$

$$\alpha 4 + \delta 4 - \beta 4 - \gamma 4 = 60.000$$

Switch combination: Upper*Switched anglesDeg:*

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

*Angle relation checks for i = 1..4:***Vertex 1**

$$\alpha 1 - \beta 1 = 90.000$$

$$\alpha 1 - \gamma 1 = -15.000$$

$$\alpha 1 - \delta 1 = 15.000$$

$$\beta 1 - \gamma 1 = -105.000$$

$$\beta 1 - \delta 1 = -75.000$$

$$\gamma 1 - \delta 1 = 30.000$$

$$\alpha 1 + \beta 1 - 180 = -60.000$$

$$\gamma 1 + \delta 1 - 180 = 30.000$$

$$\alpha 1 + \gamma 1 - 180 = 45.000$$

$$\beta 1 + \delta 1 - 180 = -75.000$$

$$\alpha 1 + \delta 1 - 180 = 15.000$$

$$\beta 1 + \gamma 1 - 180 = -45.000$$

$$\alpha 1 + \beta 1 - \gamma 1 - \delta 1 = -90.000$$

$$\alpha 1 + \gamma 1 - \beta 1 - \delta 1 = 120.000$$

$$\alpha 1 + \delta 1 - \beta 1 - \gamma 1 = 60.000$$

Vertex 2

$$\alpha 2 - \beta 2 = -30.000$$

$$\alpha 2 - \gamma 2 = 75.000$$

$$\alpha 2 - \delta 2 = -15.000$$

$$\beta 2 - \gamma 2 = 105.000$$

$$\beta 2 - \delta 2 = 15.000$$

$$\gamma 2 - \delta 2 = -90.000$$

$$\alpha 2 + \beta 2 - 180 = 30.000$$

$$\gamma 2 + \delta 2 - 180 = -60.000$$

$$\alpha 2 + \gamma 2 - 180 = -75.000$$

$$\beta 2 + \delta 2 - 180 = 45.000$$

$$\alpha 2 + \delta 2 - 180 = 15.000$$

$$\beta 2 + \gamma 2 - 180 = -45.000$$

$$\alpha 2 + \beta 2 - \gamma 2 - \delta 2 = 90.000$$

$$\alpha 2 + \gamma 2 - \beta 2 - \delta 2 = -120.000$$

$$\alpha_2 + \delta_2 - \beta_2 - \gamma_2 = 60.000$$

Vertex 3

$$\alpha_3 - \beta_3 = -30.000$$

$$\alpha_3 - \gamma_3 = -75.000$$

$$\alpha_3 - \delta_3 = 15.000$$

$$\beta_3 - \gamma_3 = -45.000$$

$$\beta_3 - \delta_3 = 45.000$$

$$\gamma_3 - \delta_3 = 90.000$$

$$\alpha_3 + \beta_3 - 180 = 30.000$$

$$\gamma_3 + \delta_3 - 180 = 60.000$$

$$\alpha_3 + \gamma_3 - 180 = 75.000$$

$$\beta_3 + \delta_3 - 180 = 15.000$$

$$\alpha_3 + \delta_3 - 180 = -15.000$$

$$\beta_3 + \gamma_3 - 180 = 105.000$$

$$\alpha_3 + \beta_3 - \gamma_3 - \delta_3 = -30.000$$

$$\alpha_3 + \gamma_3 - \beta_3 - \delta_3 = 60.000$$

$$\alpha_3 + \delta_3 - \beta_3 - \gamma_3 = -120.000$$

Vertex 4

$$\alpha_4 - \beta_4 = -90.000$$

$$\alpha_4 - \gamma_4 = -45.000$$

$$\alpha_4 - \delta_4 = -15.000$$

$$\beta_4 - \gamma_4 = 45.000$$

$$\beta_4 - \delta_4 = 75.000$$

$$\gamma_4 - \delta_4 = 30.000$$

$$\alpha_4 + \beta_4 - 180 = 60.000$$

$$\gamma_4 + \delta_4 - 180 = 30.000$$

$$\alpha_4 + \gamma_4 - 180 = 15.000$$

$$\beta_4 + \delta_4 - 180 = 75.000$$

$$\alpha_4 + \delta_4 - 180 = -15.000$$

$$\beta_4 + \gamma_4 - 180 = 105.000$$

$$\alpha_4 + \beta_4 - \gamma_4 - \delta_4 = 30.000$$

$$\alpha_4 + \gamma_4 - \beta_4 - \delta_4 = -60.000$$

$$\alpha_4 + \delta_4 - \beta_4 - \gamma_4 = -120.000$$

Switch combination: Right + Left

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\alpha_1 - \beta_1 = -60.000$$

$$\alpha_1 - \gamma_1 = 45.000$$

$$\alpha_1 - \delta_1 = 15.000$$

$$\beta_1 - \gamma_1 = 105.000$$

$$\beta_1 - \delta_1 = 75.000$$

$$\gamma_1 - \delta_1 = -30.000$$

$$\alpha_1 + \beta_1 - 180 = 90.000$$

$$\gamma_1 + \delta_1 - 180 = -30.000$$

$$\alpha_1 + \gamma_1 - 180 = -15.000$$

$$\beta_1 + \delta_1 - 180 = 75.000$$

$$\alpha_1 + \delta_1 - 180 = 15.000$$

$$\beta_1 + \gamma_1 - 180 = 45.000$$

$$\alpha_1 + \beta_1 - \gamma_1 - \delta_1 = 120.000$$

$$\alpha_1 + \gamma_1 - \beta_1 - \delta_1 = -90.000$$

$$\alpha_1 + \delta_1 - \beta_1 - \gamma_1 = -30.000$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= 30.000 \\
\alpha_2 - \gamma_2 &= -75.000 \\
\alpha_2 - \delta_2 &= -15.000 \\
\beta_2 - \gamma_2 &= -105.000 \\
\beta_2 - \delta_2 &= -45.000 \\
\gamma_2 - \delta_2 &= 60.000 \\
\alpha_2 + \beta_2 - 180 &= -30.000 \\
\gamma_2 + \delta_2 - 180 &= 90.000 \\
\alpha_2 + \gamma_2 - 180 &= 75.000 \\
\beta_2 + \delta_2 - 180 &= -15.000 \\
\alpha_2 + \delta_2 - 180 &= 15.000 \\
\beta_2 + \gamma_2 - 180 &= 45.000 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -120.000 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 90.000 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -30.000
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= -30.000 \\
\alpha_3 - \gamma_3 &= 75.000 \\
\alpha_3 - \delta_3 &= 15.000 \\
\beta_3 - \gamma_3 &= 105.000 \\
\beta_3 - \delta_3 &= 45.000 \\
\gamma_3 - \delta_3 &= -60.000 \\
\alpha_3 + \beta_3 - 180 &= 30.000 \\
\gamma_3 + \delta_3 - 180 &= -90.000 \\
\alpha_3 + \gamma_3 - 180 &= -75.000 \\
\beta_3 + \delta_3 - 180 &= 15.000 \\
\alpha_3 + \delta_3 - 180 &= -15.000 \\
\beta_3 + \gamma_3 - 180 &= -45.000 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 120.000 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -90.000 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 30.000
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= -60.000 \\
\alpha_4 - \gamma_4 &= 45.000 \\
\alpha_4 - \delta_4 &= 15.000 \\
\beta_4 - \gamma_4 &= 105.000 \\
\beta_4 - \delta_4 &= 75.000 \\
\gamma_4 - \delta_4 &= -30.000 \\
\alpha_4 + \beta_4 - 180 &= 90.000 \\
\gamma_4 + \delta_4 - 180 &= -30.000 \\
\alpha_4 + \gamma_4 - 180 &= -15.000 \\
\beta_4 + \delta_4 - 180 &= 75.000 \\
\alpha_4 + \delta_4 - 180 &= 15.000 \\
\beta_4 + \gamma_4 - 180 &= 45.000 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 120.000 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -90.000 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -30.000
\end{aligned}$$

Switch combination: Right + Lower*Switched anglesDeg:*

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= 60.000 \\
\alpha_1 - \gamma_1 &= 15.000 \\
\alpha_1 - \delta_1 &= -15.000 \\
\beta_1 - \gamma_1 &= -45.000 \\
\beta_1 - \delta_1 &= -75.000 \\
\gamma_1 - \delta_1 &= -30.000 \\
\alpha_1 + \beta_1 - 180 &= -90.000 \\
\gamma_1 + \delta_1 - 180 &= -30.000 \\
\alpha_1 + \gamma_1 - 180 &= -45.000 \\
\beta_1 + \delta_1 - 180 &= -75.000 \\
\alpha_1 + \delta_1 - 180 &= -15.000 \\
\beta_1 + \gamma_1 - 180 &= -105.000 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -60.000 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 30.000 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 90.000
\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= 30.000 \\
\alpha_2 - \gamma_2 &= 75.000 \\
\alpha_2 - \delta_2 &= -15.000 \\
\beta_2 - \gamma_2 &= 45.000 \\
\beta_2 - \delta_2 &= -45.000 \\
\gamma_2 - \delta_2 &= -90.000 \\
\alpha_2 + \beta_2 - 180 &= -30.000 \\
\gamma_2 + \delta_2 - 180 &= -60.000 \\
\alpha_2 + \gamma_2 - 180 &= -75.000 \\
\beta_2 + \delta_2 - 180 &= -15.000 \\
\alpha_2 + \delta_2 - 180 &= 15.000 \\
\beta_2 + \gamma_2 - 180 &= -105.000 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 30.000 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -60.000 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 120.000
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= 30.000 \\
\alpha_3 - \gamma_3 &= -75.000 \\
\alpha_3 - \delta_3 &= 15.000 \\
\beta_3 - \gamma_3 &= -105.000 \\
\beta_3 - \delta_3 &= -15.000 \\
\gamma_3 - \delta_3 &= 90.000 \\
\alpha_3 + \beta_3 - 180 &= -30.000 \\
\gamma_3 + \delta_3 - 180 &= 60.000 \\
\alpha_3 + \gamma_3 - 180 &= 75.000 \\
\beta_3 + \delta_3 - 180 &= -45.000 \\
\alpha_3 + \delta_3 - 180 &= -15.000 \\
\beta_3 + \gamma_3 - 180 &= 45.000 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -90.000 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 120.000 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -60.000
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= -60.000 \\
\alpha_4 - \gamma_4 &= 45.000 \\
\alpha_4 - \delta_4 &= 15.000 \\
\beta_4 - \gamma_4 &= 105.000 \\
\beta_4 - \delta_4 &= 75.000 \\
\gamma_4 - \delta_4 &= -30.000 \\
\alpha_4 + \beta_4 - 180 &= 90.000 \\
\gamma_4 + \delta_4 - 180 &= -30.000 \\
\alpha_4 + \gamma_4 - 180 &= -15.000 \\
\beta_4 + \delta_4 - 180 &= 75.000
\end{aligned}$$

$$\begin{aligned}
 \alpha 4 + \delta 4 - 180 &= 15.000 \\
 \beta 4 + \gamma 4 - 180 &= 45.000 \\
 \alpha 4 + \beta 4 - \gamma 4 - \delta 4 &= 120.000 \\
 \alpha 4 + \gamma 4 - \beta 4 - \delta 4 &= -90.000 \\
 \alpha 4 + \delta 4 - \beta 4 - \gamma 4 &= -30.000
 \end{aligned}$$

Switch combination: Right + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}
 \alpha 1 - \beta 1 &= -60.000 \\
 \alpha 1 - \gamma 1 &= 45.000 \\
 \alpha 1 - \delta 1 &= 15.000 \\
 \beta 1 - \gamma 1 &= 105.000 \\
 \beta 1 - \delta 1 &= 75.000 \\
 \gamma 1 - \delta 1 &= -30.000 \\
 \alpha 1 + \beta 1 - 180 &= 90.000 \\
 \gamma 1 + \delta 1 - 180 &= -30.000 \\
 \alpha 1 + \gamma 1 - 180 &= -15.000 \\
 \beta 1 + \delta 1 - 180 &= 75.000 \\
 \alpha 1 + \delta 1 - 180 &= 15.000 \\
 \beta 1 + \gamma 1 - 180 &= 45.000 \\
 \alpha 1 + \beta 1 - \gamma 1 - \delta 1 &= 120.000 \\
 \alpha 1 + \gamma 1 - \beta 1 - \delta 1 &= -90.000 \\
 \alpha 1 + \delta 1 - \beta 1 - \gamma 1 &= -30.000
 \end{aligned}$$

Vertex 2

$$\begin{aligned}
 \alpha 2 - \beta 2 &= -30.000 \\
 \alpha 2 - \gamma 2 &= 75.000 \\
 \alpha 2 - \delta 2 &= -15.000 \\
 \beta 2 - \gamma 2 &= 105.000 \\
 \beta 2 - \delta 2 &= 15.000 \\
 \gamma 2 - \delta 2 &= -90.000 \\
 \alpha 2 + \beta 2 - 180 &= 30.000 \\
 \gamma 2 + \delta 2 - 180 &= -60.000 \\
 \alpha 2 + \gamma 2 - 180 &= -75.000 \\
 \beta 2 + \delta 2 - 180 &= 45.000 \\
 \alpha 2 + \delta 2 - 180 &= 15.000 \\
 \beta 2 + \gamma 2 - 180 &= -45.000 \\
 \alpha 2 + \beta 2 - \gamma 2 - \delta 2 &= 90.000 \\
 \alpha 2 + \gamma 2 - \beta 2 - \delta 2 &= -120.000 \\
 \alpha 2 + \delta 2 - \beta 2 - \gamma 2 &= 60.000
 \end{aligned}$$

Vertex 3

$$\begin{aligned}
 \alpha 3 - \beta 3 &= -30.000 \\
 \alpha 3 - \gamma 3 &= -75.000 \\
 \alpha 3 - \delta 3 &= 15.000 \\
 \beta 3 - \gamma 3 &= -45.000 \\
 \beta 3 - \delta 3 &= 45.000 \\
 \gamma 3 - \delta 3 &= 90.000 \\
 \alpha 3 + \beta 3 - 180 &= 30.000 \\
 \gamma 3 + \delta 3 - 180 &= 60.000 \\
 \alpha 3 + \gamma 3 - 180 &= 75.000 \\
 \beta 3 + \delta 3 - 180 &= 15.000 \\
 \alpha 3 + \delta 3 - \beta 3 - \gamma 3 &= 15.000
 \end{aligned}$$

$$\begin{aligned}\alpha 3 + \gamma 3 - 180 &= -15.000 \\ \beta 3 + \gamma 3 - 180 &= 105.000 \\ \alpha 3 + \beta 3 - \gamma 3 - \delta 3 &= -30.000 \\ \alpha 3 + \gamma 3 - \beta 3 - \delta 3 &= 60.000 \\ \alpha 3 + \delta 3 - \beta 3 - \gamma 3 &= -120.000\end{aligned}$$

Vertex 4

$$\begin{aligned}\alpha 4 - \beta 4 &= 60.000 \\ \alpha 4 - \gamma 4 &= 15.000 \\ \alpha 4 - \delta 4 &= -15.000 \\ \beta 4 - \gamma 4 &= -45.000 \\ \beta 4 - \delta 4 &= -75.000 \\ \gamma 4 - \delta 4 &= -30.000 \\ \alpha 4 + \beta 4 - 180 &= -90.000 \\ \gamma 4 + \delta 4 - 180 &= -30.000 \\ \alpha 4 + \gamma 4 - 180 &= -45.000 \\ \beta 4 + \delta 4 - 180 &= -75.000 \\ \alpha 4 + \delta 4 - 180 &= -15.000 \\ \beta 4 + \gamma 4 - 180 &= -105.000 \\ \alpha 4 + \beta 4 - \gamma 4 - \delta 4 &= -60.000 \\ \alpha 4 + \gamma 4 - \beta 4 - \delta 4 &= 30.000 \\ \alpha 4 + \delta 4 - \beta 4 - \gamma 4 &= 90.000\end{aligned}$$

Switch combination: Left + Lower*Switched anglesDeg:*

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

*Angle relation checks for $i = 1..4$:***Vertex 1**

$$\begin{aligned}\alpha 1 - \beta 1 &= -90.000 \\ \alpha 1 - \gamma 1 &= -45.000 \\ \alpha 1 - \delta 1 &= -15.000 \\ \beta 1 - \gamma 1 &= 45.000 \\ \beta 1 - \delta 1 &= 75.000 \\ \gamma 1 - \delta 1 &= 30.000 \\ \alpha 1 + \beta 1 - 180 &= 60.000 \\ \gamma 1 + \delta 1 - 180 &= 30.000 \\ \alpha 1 + \gamma 1 - 180 &= 15.000 \\ \beta 1 + \delta 1 - 180 &= 75.000 \\ \alpha 1 + \delta 1 - 180 &= -15.000 \\ \beta 1 + \gamma 1 - 180 &= 105.000 \\ \alpha 1 + \beta 1 - \gamma 1 - \delta 1 &= 30.000 \\ \alpha 1 + \gamma 1 - \beta 1 - \delta 1 &= -60.000 \\ \alpha 1 + \delta 1 - \beta 1 - \gamma 1 &= -120.000\end{aligned}$$

Vertex 2

$$\begin{aligned}\alpha 2 - \beta 2 &= -30.000 \\ \alpha 2 - \gamma 2 &= -75.000 \\ \alpha 2 - \delta 2 &= -15.000 \\ \beta 2 - \gamma 2 &= -45.000 \\ \beta 2 - \delta 2 &= 15.000 \\ \gamma 2 - \delta 2 &= 60.000 \\ \alpha 2 + \beta 2 - 180 &= 30.000 \\ \gamma 2 + \delta 2 - 180 &= 90.000 \\ \alpha 2 + \gamma 2 - 180 &= 75.000 \\ \beta 2 + \delta 2 - 180 &= 45.000 \\ \alpha 2 + \delta 2 - 180 &= 15.000 \\ \beta 2 + \gamma 2 - 180 &= 105.000\end{aligned}$$

$$\begin{aligned}\beta_2 + \gamma_2 - 180 &= 105.0000 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -60.0000 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 30.0000 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -90.0000\end{aligned}$$

Vertex 3

$$\begin{aligned}\alpha_3 - \beta_3 &= -30.0000 \\ \alpha_3 - \gamma_3 &= 75.0000 \\ \alpha_3 - \delta_3 &= 15.0000 \\ \beta_3 - \gamma_3 &= 105.0000 \\ \beta_3 - \delta_3 &= 45.0000 \\ \gamma_3 - \delta_3 &= -60.0000 \\ \alpha_3 + \beta_3 - 180 &= 30.0000 \\ \gamma_3 + \delta_3 - 180 &= -90.0000 \\ \alpha_3 + \gamma_3 - 180 &= -75.0000 \\ \beta_3 + \delta_3 - 180 &= 15.0000 \\ \alpha_3 + \delta_3 - 180 &= -15.0000 \\ \beta_3 + \gamma_3 - 180 &= -45.0000 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 120.0000 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -90.0000 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 30.0000\end{aligned}$$

Vertex 4

$$\begin{aligned}\alpha_4 - \beta_4 &= 90.0000 \\ \alpha_4 - \gamma_4 &= -15.0000 \\ \alpha_4 - \delta_4 &= 15.0000 \\ \beta_4 - \gamma_4 &= -105.0000 \\ \beta_4 - \delta_4 &= -75.0000 \\ \gamma_4 - \delta_4 &= 30.0000 \\ \alpha_4 + \beta_4 - 180 &= -60.0000 \\ \gamma_4 + \delta_4 - 180 &= 30.0000 \\ \alpha_4 + \gamma_4 - 180 &= 45.0000 \\ \beta_4 + \delta_4 - 180 &= -75.0000 \\ \alpha_4 + \delta_4 - 180 &= 15.0000 \\ \beta_4 + \gamma_4 - 180 &= -45.0000 \\ \alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -90.0000 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 120.0000 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 60.0000\end{aligned}$$

Switch combination: Left + Upper*Switched anglesDeg:*

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

*Angle relation checks for i = 1..4:***Vertex 1**

$$\begin{aligned}\alpha_1 - \beta_1 &= 90.0000 \\ \alpha_1 - \gamma_1 &= -15.0000 \\ \alpha_1 - \delta_1 &= 15.0000 \\ \beta_1 - \gamma_1 &= -105.0000 \\ \beta_1 - \delta_1 &= -75.0000 \\ \gamma_1 - \delta_1 &= 30.0000 \\ \alpha_1 + \beta_1 - 180 &= -60.0000 \\ \gamma_1 + \delta_1 - 180 &= 30.0000 \\ \alpha_1 + \gamma_1 - 180 &= 45.0000 \\ \beta_1 + \delta_1 - 180 &= -75.0000 \\ \alpha_1 + \delta_1 - 180 &= 15.0000 \\ \beta_1 + \gamma_1 - 180 &= -45.0000 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -90.0000 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 120.0000 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 60.0000\end{aligned}$$

$$\begin{aligned}\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -90.000 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 120.000 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 60.000\end{aligned}$$

Vertex 2

$$\begin{aligned}\alpha_2 - \beta_2 &= 30.000 \\ \alpha_2 - \gamma_2 &= -75.000 \\ \alpha_2 - \delta_2 &= -15.000 \\ \beta_2 - \gamma_2 &= -105.000 \\ \beta_2 - \delta_2 &= -45.000 \\ \gamma_2 - \delta_2 &= 60.000 \\ \alpha_2 + \beta_2 - 180 &= -30.000 \\ \gamma_2 + \delta_2 - 180 &= 90.000 \\ \alpha_2 + \gamma_2 - 180 &= 75.000 \\ \beta_2 + \delta_2 - 180 &= -15.000 \\ \alpha_2 + \delta_2 - 180 &= 15.000 \\ \beta_2 + \gamma_2 - 180 &= 45.000 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -120.000 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 90.000 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -30.000\end{aligned}$$

Vertex 3

$$\begin{aligned}\alpha_3 - \beta_3 &= 30.000 \\ \alpha_3 - \gamma_3 &= 75.000 \\ \alpha_3 - \delta_3 &= 15.000 \\ \beta_3 - \gamma_3 &= 45.000 \\ \beta_3 - \delta_3 &= -15.000 \\ \gamma_3 - \delta_3 &= -60.000 \\ \alpha_3 + \beta_3 - 180 &= -30.000 \\ \gamma_3 + \delta_3 - 180 &= -90.000 \\ \alpha_3 + \gamma_3 - 180 &= -75.000 \\ \beta_3 + \delta_3 - 180 &= -45.000 \\ \alpha_3 + \delta_3 - 180 &= -15.000 \\ \beta_3 + \gamma_3 - 180 &= -105.000 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 60.000 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -30.000 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 90.000\end{aligned}$$

Vertex 4

$$\begin{aligned}\alpha_4 - \beta_4 &= -90.000 \\ \alpha_4 - \gamma_4 &= -45.000 \\ \alpha_4 - \delta_4 &= -15.000 \\ \beta_4 - \gamma_4 &= 45.000 \\ \beta_4 - \delta_4 &= 75.000 \\ \gamma_4 - \delta_4 &= 30.000 \\ \alpha_4 + \beta_4 - 180 &= 60.000 \\ \gamma_4 + \delta_4 - 180 &= 30.000 \\ \alpha_4 + \gamma_4 - 180 &= 15.000 \\ \beta_4 + \delta_4 - 180 &= 75.000 \\ \alpha_4 + \delta_4 - 180 &= -15.000 \\ \beta_4 + \gamma_4 - 180 &= 105.000 \\ \alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 30.000 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -60.000 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -120.000\end{aligned}$$

Switch combination: Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned} \alpha_1 - \beta_1 &= -90.000 \\ \alpha_1 - \gamma_1 &= -45.000 \\ \alpha_1 - \delta_1 &= -15.000 \\ \beta_1 - \gamma_1 &= 45.000 \\ \beta_1 - \delta_1 &= 75.000 \\ \gamma_1 - \delta_1 &= 30.000 \\ \alpha_1 + \beta_1 - 180 &= 60.000 \\ \gamma_1 + \delta_1 - 180 &= 30.000 \\ \alpha_1 + \gamma_1 - 180 &= 15.000 \\ \beta_1 + \delta_1 - 180 &= 75.000 \\ \alpha_1 + \delta_1 - 180 &= -15.000 \\ \beta_1 + \gamma_1 - 180 &= 105.000 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 30.000 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -60.000 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -120.000 \end{aligned}$$

Vertex 2

$$\begin{aligned} \alpha_2 - \beta_2 &= 30.000 \\ \alpha_2 - \gamma_2 &= 75.000 \\ \alpha_2 - \delta_2 &= -15.000 \\ \beta_2 - \gamma_2 &= 45.000 \\ \beta_2 - \delta_2 &= -45.000 \\ \gamma_2 - \delta_2 &= -90.000 \\ \alpha_2 + \beta_2 - 180 &= -30.000 \\ \gamma_2 + \delta_2 - 180 &= -60.000 \\ \alpha_2 + \gamma_2 - 180 &= -75.000 \\ \beta_2 + \delta_2 - 180 &= -15.000 \\ \alpha_2 + \delta_2 - 180 &= 15.000 \\ \beta_2 + \gamma_2 - 180 &= -105.000 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 30.000 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -60.000 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 120.000 \end{aligned}$$

Vertex 3

$$\begin{aligned} \alpha_3 - \beta_3 &= -30.000 \\ \alpha_3 - \gamma_3 &= -75.000 \\ \alpha_3 - \delta_3 &= 15.000 \\ \beta_3 - \gamma_3 &= -45.000 \\ \beta_3 - \delta_3 &= 45.000 \\ \gamma_3 - \delta_3 &= 90.000 \\ \alpha_3 + \beta_3 - 180 &= 30.000 \\ \gamma_3 + \delta_3 - 180 &= 60.000 \\ \alpha_3 + \gamma_3 - 180 &= 75.000 \\ \beta_3 + \delta_3 - 180 &= 15.000 \\ \alpha_3 + \delta_3 - 180 &= -15.000 \\ \beta_3 + \gamma_3 - 180 &= 105.000 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -30.000 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 60.000 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -120.000 \end{aligned}$$

Vertex 4

$$\begin{aligned} \alpha_4 - \beta_4 &= -90.000 \\ \alpha_4 - \gamma_4 &= -45.000 \\ \alpha_4 - \delta_4 &= -15.000 \\ \beta_4 - \gamma_4 &= 45.000 \end{aligned}$$

```

 $\beta_4 - \delta_4 = 75.000$ 
 $\gamma_4 - \delta_4 = 30.000$ 
 $\alpha_4 + \beta_4 - 180 = 60.000$ 
 $\gamma_4 + \delta_4 - 180 = 30.000$ 
 $\alpha_4 + \gamma_4 - 180 = 15.000$ 
 $\beta_4 + \delta_4 - 180 = 75.000$ 
 $\alpha_4 + \delta_4 - 180 = -15.000$ 
 $\beta_4 + \gamma_4 - 180 = 105.000$ 
 $\alpha_4 + \beta_4 - \gamma_4 - \delta_4 = 30.000$ 
 $\alpha_4 + \gamma_4 - \beta_4 - \delta_4 = -60.000$ 
 $\alpha_4 + \delta_4 - \beta_4 - \gamma_4 = -120.000$ 

```

Switch combination: Right + Left + Lower

Switched anglesDeg:

```

( 75  15  60  90 )
( 90 120 165 105 )
( 90 120  15  75 )
(105 165  60  90 )

```

Angle relation checks for i = 1..4:

Vertex 1

```

 $\alpha_1 - \beta_1 = 60.000$ 
 $\alpha_1 - \gamma_1 = 15.000$ 
 $\alpha_1 - \delta_1 = -15.000$ 
 $\beta_1 - \gamma_1 = -45.000$ 
 $\beta_1 - \delta_1 = -75.000$ 
 $\gamma_1 - \delta_1 = -30.000$ 
 $\alpha_1 + \beta_1 - 180 = -90.000$ 
 $\gamma_1 + \delta_1 - 180 = -30.000$ 
 $\alpha_1 + \gamma_1 - 180 = -45.000$ 
 $\beta_1 + \delta_1 - 180 = -75.000$ 
 $\alpha_1 + \delta_1 - 180 = -15.000$ 
 $\beta_1 + \gamma_1 - 180 = -105.000$ 
 $\alpha_1 + \beta_1 - \gamma_1 - \delta_1 = -60.000$ 
 $\alpha_1 + \gamma_1 - \beta_1 - \delta_1 = 30.000$ 
 $\alpha_1 + \delta_1 - \beta_1 - \gamma_1 = 90.000$ 

```

Vertex 2

```

 $\alpha_2 - \beta_2 = -30.000$ 
 $\alpha_2 - \gamma_2 = -75.000$ 
 $\alpha_2 - \delta_2 = -15.000$ 
 $\beta_2 - \gamma_2 = -45.000$ 
 $\beta_2 - \delta_2 = 15.000$ 
 $\gamma_2 - \delta_2 = 60.000$ 
 $\alpha_2 + \beta_2 - 180 = 30.000$ 
 $\gamma_2 + \delta_2 - 180 = 90.000$ 
 $\alpha_2 + \gamma_2 - 180 = 75.000$ 
 $\beta_2 + \delta_2 - 180 = 45.000$ 
 $\alpha_2 + \delta_2 - 180 = 15.000$ 
 $\beta_2 + \gamma_2 - 180 = 105.000$ 
 $\alpha_2 + \beta_2 - \gamma_2 - \delta_2 = -60.000$ 
 $\alpha_2 + \gamma_2 - \beta_2 - \delta_2 = 30.000$ 
 $\alpha_2 + \delta_2 - \beta_2 - \gamma_2 = -90.000$ 

```

Vertex 3

```

 $\alpha_3 - \beta_3 = -30.000$ 
 $\alpha_3 - \gamma_3 = 75.000$ 
 $\alpha_3 - \delta_3 = 15.000$ 
 $\beta_3 - \gamma_3 = 105.000$ 
 $\beta_3 - \delta_3 = 15.000$ 

```


$$\begin{aligned}
\beta_3 - \delta_3 &= 45.000 \\
\gamma_3 - \delta_3 &= -60.000 \\
\alpha_3 + \beta_3 - 180 &= 30.000 \\
\gamma_3 + \delta_3 - 180 &= -90.000 \\
\alpha_3 + \gamma_3 - 180 &= -75.000 \\
\beta_3 + \delta_3 - 180 &= 15.000 \\
\alpha_3 + \delta_3 - 180 &= -15.000 \\
\beta_3 + \gamma_3 - 180 &= -45.000 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 120.000 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -90.000 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 30.000
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= -60.000 \\
\alpha_4 - \gamma_4 &= 45.000 \\
\alpha_4 - \delta_4 &= 15.000 \\
\beta_4 - \gamma_4 &= 105.000 \\
\beta_4 - \delta_4 &= 75.000 \\
\gamma_4 - \delta_4 &= -30.000 \\
\alpha_4 + \beta_4 - 180 &= 90.000 \\
\gamma_4 + \delta_4 - 180 &= -30.000 \\
\alpha_4 + \gamma_4 - 180 &= -15.000 \\
\beta_4 + \delta_4 - 180 &= 75.000 \\
\alpha_4 + \delta_4 - 180 &= 15.000 \\
\beta_4 + \gamma_4 - 180 &= 45.000 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 120.000 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -90.000 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -30.000
\end{aligned}$$

Switch combination: Right + Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= -60.000 \\
\alpha_1 - \gamma_1 &= 45.000 \\
\alpha_1 - \delta_1 &= 15.000 \\
\beta_1 - \gamma_1 &= 105.000 \\
\beta_1 - \delta_1 &= 75.000 \\
\gamma_1 - \delta_1 &= -30.000 \\
\alpha_1 + \beta_1 - 180 &= 90.000 \\
\gamma_1 + \delta_1 - 180 &= -30.000 \\
\alpha_1 + \gamma_1 - 180 &= -15.000 \\
\beta_1 + \delta_1 - 180 &= 75.000 \\
\alpha_1 + \delta_1 - 180 &= 15.000 \\
\beta_1 + \gamma_1 - 180 &= 45.000 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 120.000 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -90.000 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -30.000
\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= 30.000 \\
\alpha_2 - \gamma_2 &= -75.000 \\
\alpha_2 - \delta_2 &= -15.000 \\
\beta_2 - \gamma_2 &= -105.000 \\
\beta_2 - \delta_2 &= -45.000 \\
\gamma_2 - \delta_2 &= 60.000
\end{aligned}$$

$$\begin{aligned}
\gamma_2 - \alpha_2 &= 60.000 \\
\alpha_2 + \beta_2 - 180 &= -30.000 \\
\gamma_2 + \delta_2 - 180 &= 90.000 \\
\alpha_2 + \gamma_2 - 180 &= 75.000 \\
\beta_2 + \delta_2 - 180 &= -15.000 \\
\alpha_2 + \delta_2 - 180 &= 15.000 \\
\beta_2 + \gamma_2 - 180 &= 45.000 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -120.000 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 90.000 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -30.000
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= 30.000 \\
\alpha_3 - \gamma_3 &= 75.000 \\
\alpha_3 - \delta_3 &= 15.000 \\
\beta_3 - \gamma_3 &= 45.000 \\
\beta_3 - \delta_3 &= -15.000 \\
\gamma_3 - \delta_3 &= -60.000 \\
\alpha_3 + \beta_3 - 180 &= -30.000 \\
\gamma_3 + \delta_3 - 180 &= -90.000 \\
\alpha_3 + \gamma_3 - 180 &= -75.000 \\
\beta_3 + \delta_3 - 180 &= -45.000 \\
\alpha_3 + \delta_3 - 180 &= -15.000 \\
\beta_3 + \gamma_3 - 180 &= -105.000 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 60.000 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -30.000 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 90.000
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= 60.000 \\
\alpha_4 - \gamma_4 &= 15.000 \\
\alpha_4 - \delta_4 &= -15.000 \\
\beta_4 - \gamma_4 &= -45.000 \\
\beta_4 - \delta_4 &= -75.000 \\
\gamma_4 - \delta_4 &= -30.000 \\
\alpha_4 + \beta_4 - 180 &= -90.000 \\
\gamma_4 + \delta_4 - 180 &= -30.000 \\
\alpha_4 + \gamma_4 - 180 &= -45.000 \\
\beta_4 + \delta_4 - 180 &= -75.000 \\
\alpha_4 + \delta_4 - 180 &= -15.000 \\
\beta_4 + \gamma_4 - 180 &= -105.000 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -60.000 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 30.000 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 90.000
\end{aligned}$$

Switch combination: Right + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= 60.000 \\
\alpha_1 - \gamma_1 &= 15.000 \\
\alpha_1 - \delta_1 &= -15.000 \\
\beta_1 - \gamma_1 &= -45.000 \\
\beta_1 - \delta_1 &= -75.000 \\
\gamma_1 - \delta_1 &= -30.000 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -60.000 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 30.000 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 90.000
\end{aligned}$$

$$\begin{aligned}
\alpha_1 + \beta_1 - 180 &= -90.000 \\
\gamma_1 + \delta_1 - 180 &= -30.000 \\
\alpha_1 + \gamma_1 - 180 &= -45.000 \\
\beta_1 + \delta_1 - 180 &= -75.000 \\
\alpha_1 + \delta_1 - 180 &= -15.000 \\
\beta_1 + \gamma_1 - 180 &= -105.000 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -60.000 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 30.000 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 90.000
\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= 30.000 \\
\alpha_2 - \gamma_2 &= 75.000 \\
\alpha_2 - \delta_2 &= -15.000 \\
\beta_2 - \gamma_2 &= 45.000 \\
\beta_2 - \delta_2 &= -45.000 \\
\gamma_2 - \delta_2 &= -90.000 \\
\alpha_2 + \beta_2 - 180 &= -30.000 \\
\gamma_2 + \delta_2 - 180 &= -60.000 \\
\alpha_2 + \gamma_2 - 180 &= -75.000 \\
\beta_2 + \delta_2 - 180 &= -15.000 \\
\alpha_2 + \delta_2 - 180 &= 15.000 \\
\beta_2 + \gamma_2 - 180 &= -105.000 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 30.000 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -60.000 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 120.000
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= -30.000 \\
\alpha_3 - \gamma_3 &= -75.000 \\
\alpha_3 - \delta_3 &= 15.000 \\
\beta_3 - \gamma_3 &= -45.000 \\
\beta_3 - \delta_3 &= 45.000 \\
\gamma_3 - \delta_3 &= 90.000 \\
\alpha_3 + \beta_3 - 180 &= 30.000 \\
\gamma_3 + \delta_3 - 180 &= 60.000 \\
\alpha_3 + \gamma_3 - 180 &= 75.000 \\
\beta_3 + \delta_3 - 180 &= 15.000 \\
\alpha_3 + \delta_3 - 180 &= -15.000 \\
\beta_3 + \gamma_3 - 180 &= 105.000 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -30.000 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 60.000 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -120.000
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= 60.000 \\
\alpha_4 - \gamma_4 &= 15.000 \\
\alpha_4 - \delta_4 &= -15.000 \\
\beta_4 - \gamma_4 &= -45.000 \\
\beta_4 - \delta_4 &= -75.000 \\
\gamma_4 - \delta_4 &= -30.000 \\
\alpha_4 + \beta_4 - 180 &= -90.000 \\
\gamma_4 + \delta_4 - 180 &= -30.000 \\
\alpha_4 + \gamma_4 - 180 &= -45.000 \\
\beta_4 + \delta_4 - 180 &= -75.000 \\
\alpha_4 + \delta_4 - 180 &= -15.000 \\
\beta_4 + \gamma_4 - 180 &= -105.000 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -60.000 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 30.000 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 90.000
\end{aligned}$$

Switch combination: Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned} \alpha_1 - \beta_1 &= -90.000 \\ \alpha_1 - \gamma_1 &= -45.000 \\ \alpha_1 - \delta_1 &= -15.000 \\ \beta_1 - \gamma_1 &= 45.000 \\ \beta_1 - \delta_1 &= 75.000 \\ \gamma_1 - \delta_1 &= 30.000 \\ \alpha_1 + \beta_1 - 180 &= 60.000 \\ \gamma_1 + \delta_1 - 180 &= 30.000 \\ \alpha_1 + \gamma_1 - 180 &= 15.000 \\ \beta_1 + \delta_1 - 180 &= 75.000 \\ \alpha_1 + \delta_1 - 180 &= -15.000 \\ \beta_1 + \gamma_1 - 180 &= 105.000 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 30.000 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -60.000 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -120.000 \end{aligned}$$

Vertex 2

$$\begin{aligned} \alpha_2 - \beta_2 &= -30.000 \\ \alpha_2 - \gamma_2 &= -75.000 \\ \alpha_2 - \delta_2 &= -15.000 \\ \beta_2 - \gamma_2 &= -45.000 \\ \beta_2 - \delta_2 &= 15.000 \\ \gamma_2 - \delta_2 &= 60.000 \\ \alpha_2 + \beta_2 - 180 &= 30.000 \\ \gamma_2 + \delta_2 - 180 &= 90.000 \\ \alpha_2 + \gamma_2 - 180 &= 75.000 \\ \beta_2 + \delta_2 - 180 &= 45.000 \\ \alpha_2 + \delta_2 - 180 &= 15.000 \\ \beta_2 + \gamma_2 - 180 &= 105.000 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -60.000 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 30.000 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -90.000 \end{aligned}$$

Vertex 3

$$\begin{aligned} \alpha_3 - \beta_3 &= 30.000 \\ \alpha_3 - \gamma_3 &= 75.000 \\ \alpha_3 - \delta_3 &= 15.000 \\ \beta_3 - \gamma_3 &= 45.000 \\ \beta_3 - \delta_3 &= -15.000 \\ \gamma_3 - \delta_3 &= -60.000 \\ \alpha_3 + \beta_3 - 180 &= -30.000 \\ \gamma_3 + \delta_3 - 180 &= -90.000 \\ \alpha_3 + \gamma_3 - 180 &= -75.000 \\ \beta_3 + \delta_3 - 180 &= -45.000 \\ \alpha_3 + \delta_3 - 180 &= -15.000 \\ \beta_3 + \gamma_3 - 180 &= -105.000 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 60.000 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -30.000 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 90.000 \end{aligned}$$

.. . -

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= -90.000 \\
\alpha_4 - \gamma_4 &= -45.000 \\
\alpha_4 - \delta_4 &= -15.000 \\
\beta_4 - \gamma_4 &= 45.000 \\
\beta_4 - \delta_4 &= 75.000 \\
\gamma_4 - \delta_4 &= 30.000 \\
\alpha_4 + \beta_4 - 180 &= 60.000 \\
\gamma_4 + \delta_4 - 180 &= 30.000 \\
\alpha_4 + \gamma_4 - 180 &= 15.000 \\
\beta_4 + \delta_4 - 180 &= 75.000 \\
\alpha_4 + \delta_4 - 180 &= -15.000 \\
\beta_4 + \gamma_4 - 180 &= 105.000 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 30.000 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -60.000 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -120.000
\end{aligned}$$

Switch combination: Right + Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= 60.000 \\
\alpha_1 - \gamma_1 &= 15.000 \\
\alpha_1 - \delta_1 &= -15.000 \\
\beta_1 - \gamma_1 &= -45.000 \\
\beta_1 - \delta_1 &= -75.000 \\
\gamma_1 - \delta_1 &= -30.000 \\
\alpha_1 + \beta_1 - 180 &= -90.000 \\
\gamma_1 + \delta_1 - 180 &= -30.000 \\
\alpha_1 + \gamma_1 - 180 &= -45.000 \\
\beta_1 + \delta_1 - 180 &= -75.000 \\
\alpha_1 + \delta_1 - 180 &= -15.000 \\
\beta_1 + \gamma_1 - 180 &= -105.000 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -60.000 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 30.000 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 90.000
\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= -30.000 \\
\alpha_2 - \gamma_2 &= -75.000 \\
\alpha_2 - \delta_2 &= -15.000 \\
\beta_2 - \gamma_2 &= -45.000 \\
\beta_2 - \delta_2 &= 15.000 \\
\gamma_2 - \delta_2 &= 60.000 \\
\alpha_2 + \beta_2 - 180 &= 30.000 \\
\gamma_2 + \delta_2 - 180 &= 90.000 \\
\alpha_2 + \gamma_2 - 180 &= 75.000 \\
\beta_2 + \delta_2 - 180 &= 45.000 \\
\alpha_2 + \delta_2 - 180 &= 15.000 \\
\beta_2 + \gamma_2 - 180 &= 105.000 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -60.000 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 30.000 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -90.000
\end{aligned}$$

Vertex 3

```

 $\alpha_3 - \beta_3 = 30.000$ 
 $\alpha_3 - \gamma_3 = 75.000$ 
 $\alpha_3 - \delta_3 = 15.000$ 
 $\beta_3 - \gamma_3 = 45.000$ 
 $\beta_3 - \delta_3 = -15.000$ 
 $\gamma_3 - \delta_3 = -60.000$ 
 $\alpha_3 + \beta_3 - 180 = -30.000$ 
 $\gamma_3 + \delta_3 - 180 = -90.000$ 
 $\alpha_3 + \gamma_3 - 180 = -75.000$ 
 $\beta_3 + \delta_3 - 180 = -45.000$ 
 $\alpha_3 + \delta_3 - 180 = -15.000$ 
 $\beta_3 + \gamma_3 - 180 = -105.000$ 
 $\alpha_3 + \beta_3 - \gamma_3 - \delta_3 = 60.000$ 
 $\alpha_3 + \gamma_3 - \beta_3 - \delta_3 = -30.000$ 
 $\alpha_3 + \delta_3 - \beta_3 - \gamma_3 = 90.000$ 

```

Vertex 4

```

 $\alpha_4 - \beta_4 = 60.000$ 
 $\alpha_4 - \gamma_4 = 15.000$ 
 $\alpha_4 - \delta_4 = -15.000$ 
 $\beta_4 - \gamma_4 = -45.000$ 
 $\beta_4 - \delta_4 = -75.000$ 
 $\gamma_4 - \delta_4 = -30.000$ 
 $\alpha_4 + \beta_4 - 180 = -90.000$ 
 $\gamma_4 + \delta_4 - 180 = -30.000$ 
 $\alpha_4 + \gamma_4 - 180 = -45.000$ 
 $\beta_4 + \delta_4 - 180 = -75.000$ 
 $\alpha_4 + \delta_4 - 180 = -15.000$ 
 $\beta_4 + \gamma_4 - 180 = -105.000$ 
 $\alpha_4 + \beta_4 - \gamma_4 - \delta_4 = -60.000$ 
 $\alpha_4 + \gamma_4 - \beta_4 - \delta_4 = 30.000$ 
 $\alpha_4 + \delta_4 - \beta_4 - \gamma_4 = 90.000$ 

```

Out[1665]=

===== NOT CONJUGATE-MODULAR =====

$M_i < 1$ and $p_i \in \mathbb{R}$ for all $i = 1..4 \Rightarrow$ NOT

conjugate-modular. Boundary-strip switches preserve this.

Initial configuration (no switches applied):

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$
 M_i values:

$M_1 = -1 + \sqrt{3}$, $M_2 = -1 + \sqrt{3}$, $M_3 = -1 + \sqrt{3}$, $M_4 = -1 + \sqrt{3}$

 p_i values:

$p_1 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$, $p_2 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$, $p_3 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$, $p_4 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$

$M_i < 1$ and $p_i \in \mathbb{R}$ for all $i = 1, \dots, 4$

Switch combination: Right

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

M_i values:

$$M_1 = -1 + \sqrt{3}, M_2 = -1 + \sqrt{3}, M_3 = -1 + \sqrt{3}, M_4 = -1 + \sqrt{3}$$

p_i values:

$$p_1 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_2 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_3 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_4 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Switch combination: Left

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

M_i values:

$$M_1 = -1 + \sqrt{3}, M_2 = -1 + \sqrt{3}, M_3 = -1 + \sqrt{3}, M_4 = -1 + \sqrt{3}$$

p_i values:

$$p_1 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_2 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_3 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_4 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Switch combination: Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

M_i values:

$$M_1 = -1 + \sqrt{3}, M_2 = -1 + \sqrt{3}, M_3 = -1 + \sqrt{3}, M_4 = -1 + \sqrt{3}$$

p_i values:

$$p_1 = 3^{1/4}, p_2 = 3^{1/4}, p_3 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_4 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Switch combination: Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

M_i values:

$$M_1 = -1 + \sqrt{3}, M_2 = -1 + \sqrt{3}, M_3 = -1 + \sqrt{3}, M_4 = -1 + \sqrt{3}$$

p_i values:

$$p_1 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_2 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_3 = 3^{1/4}, p_4 = 3^{1/4}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Switch combination: Right + Left

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

M_i values:

$$M_1 = -1 + \sqrt{3}, M_2 = -1 + \sqrt{3}, M_3 = -1 + \sqrt{3}, M_4 = -1 + \sqrt{3}$$

p_i values:

$$p_1 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_2 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_3 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_4 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Switch combination: Right + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

M_i values:

$$M_1 = -1 + \sqrt{3}, M_2 = -1 + \sqrt{3}, M_3 = -1 + \sqrt{3}, M_4 = -1 + \sqrt{3}$$

p_i values:

$$p_1 = 3^{1/4}, p_2 = 3^{1/4}, p_3 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_4 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Switch combination: Right + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

M_i values:

$$M_1 = -1 + \sqrt{3}, M_2 = -1 + \sqrt{3}, M_3 = -1 + \sqrt{3}, M_4 = -1 + \sqrt{3}$$

p_i values:

$$p_1 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_2 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_3 = 3^{1/4}, p_4 = 3^{1/4}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Switch combination: Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

M_i values:

$$M_1 = -1 + \sqrt{3}, M_2 = -1 + \sqrt{3}, M_3 = -1 + \sqrt{3}, M_4 = -1 + \sqrt{3}$$

p_i values:

$$p_1 = 3^{1/4}, p_2 = 3^{1/4}, p_3 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p_4 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Switch combination: Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

M_i values:

$$M1 = -1 + \sqrt{3}, M2 = -1 + \sqrt{3}, M3 = -1 + \sqrt{3}, M4 = -1 + \sqrt{3}$$

p_i values:

$$p1 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p2 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p3 = 3^{1/4}, p4 = 3^{1/4}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Switch combination: Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

M_i values:

$$M1 = -1 + \sqrt{3}, M2 = -1 + \sqrt{3}, M3 = -1 + \sqrt{3}, M4 = -1 + \sqrt{3}$$

p_i values:

$$p1 = 3^{1/4}, p2 = 3^{1/4}, p3 = 3^{1/4}, p4 = 3^{1/4}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Switch combination: Right + Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

M_i values:

$$M1 = -1 + \sqrt{3}, M2 = -1 + \sqrt{3}, M3 = -1 + \sqrt{3}, M4 = -1 + \sqrt{3}$$

p_i values:

$$p1 = 3^{1/4}, p2 = 3^{1/4}, p3 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p4 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Switch combination: Right + Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

M_i values:

$$M1 = -1 + \sqrt{3}, M2 = -1 + \sqrt{3}, M3 = -1 + \sqrt{3}, M4 = -1 + \sqrt{3}$$

p_i values:

$$p1 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p2 = \sqrt{-1 + \frac{2}{\sqrt{3}}}, p3 = 3^{1/4}, p4 = 3^{1/4}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Switch combination: Right + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

M_i values:

$$M_1 = -1 + \sqrt{3}, M_2 = -1 + \sqrt{3}, M_3 = -1 + \sqrt{3}, M_4 = -1 + \sqrt{3}$$

p_i values:

$$p_1 = 3^{1/4}, p_2 = 3^{1/4}, p_3 = 3^{1/4}, p_4 = 3^{1/4}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Switch combination: Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

M_i values:

$$M_1 = -1 + \sqrt{3}, M_2 = -1 + \sqrt{3}, M_3 = -1 + \sqrt{3}, M_4 = -1 + \sqrt{3}$$

p_i values:

$$p_1 = 3^{1/4}, p_2 = 3^{1/4}, p_3 = 3^{1/4}, p_4 = 3^{1/4}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Switch combination: Right + Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

M_i values:

$$M_1 = -1 + \sqrt{3}, M_2 = -1 + \sqrt{3}, M_3 = -1 + \sqrt{3}, M_4 = -1 + \sqrt{3}$$

p_i values:

$$p_1 = 3^{1/4}, p_2 = 3^{1/4}, p_3 = 3^{1/4}, p_4 = 3^{1/4}$$

M_i < 1 and p_i ∈ ℝ for all i = 1, ..., 4

Out[1667]=

===== NOT CHIMERA =====

Fails conic, orthodiagonal & isogonal tests for all
i=1, ..., 4 ⇒ NOT chimera. Boundary-strip switches
preserve these failures as demonstrated in the NOT
CONIC, NOT ORTHODIAGONAL, and NOT ISOGONAL sections.

Out[1722]=

===== EXTRA INFO (FLEXION 1) =====

This example does NOT have planar parameter
lines (C2A2A1C1, C3A3A4C4, B1A1A4B4, B2A2A3B3)

Coplanarity Check: A2A1-A2C2-A1C1

Determinant of {A2A1, A2C2, A1C1} as a function of t

$$\begin{aligned}
& l1 \left(- \frac{(-1+\sqrt{3}) \sqrt{\frac{1}{2}(2+\sqrt{3})} n1 n2 (-1+3 t^2) (-2 t + \sqrt{1+\sqrt{3}} \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4})}{-2-8 t^2-6 t^4+8 \sqrt{1+\sqrt{3}} t \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4}} + \right. \\
& \left(\sqrt{3} n1 n2 (1+\sqrt{3} + (3+2\sqrt{3}) t^2) \right. \\
& \left((3+2\sqrt{3}) t - \sqrt{1+\sqrt{3}} \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4} \right) \\
& \left. (-1-14 t^2+3 t^4+8 \sqrt{1+\sqrt{3}} t \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4}) \right) \Bigg) / \\
& \left(2 (-1-4 t^2-3 t^4+4 \sqrt{1+\sqrt{3}} t \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4}) \right. \\
& \left. (3+\sqrt{3} + (51+28\sqrt{3}) t^2+3(2+\sqrt{3}) t^4 - \right. \\
& \left. \left. 2 \sqrt{1+\sqrt{3}} (3+2\sqrt{3}) t \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4} \right) \right) \Bigg)
\end{aligned}$$

Coplanarity Check: A3A4-A3C3-A4C4

Determinant of {A3A4, A3C3, A4C4} as a function of t

$$\begin{aligned}
& (-8 \sqrt{3} n3 n4 (1+\sqrt{3} + (3+2\sqrt{3}) t^2) \\
& \left((3+2\sqrt{3}) t - \sqrt{1+\sqrt{3}} \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4} \right) \\
& (-2(2+\sqrt{3}) l3 (1-3 t^2)^2 + \\
& 2 l1 (-1-14 t^2+3 t^4+8 \sqrt{1+\sqrt{3}} t \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4}) + \\
& \sqrt{2} l2 (3+\sqrt{3} - 8(1+2\sqrt{3}) t^2+3(7+5\sqrt{3}) t^4 + \\
& 4(-1+\sqrt{3}) \sqrt{1+\sqrt{3}} t \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4}) \Bigg) + 2(1-\sqrt{3}) \\
& \sqrt{2(2+\sqrt{3})} n3 (1-3 t^2) (2 t - \sqrt{1+\sqrt{3}} \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4}) \\
& (\sqrt{3} ((\sqrt{2} + \sqrt{6}) l2 - 4 l3) n4 ((1+\sqrt{3} + (3+2\sqrt{3}) t^2)^2 - \\
& \left((3+2\sqrt{3}) t - \sqrt{1+\sqrt{3}} \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4} \right)^2) + \\
& (4 l1 + \sqrt{2} (-1+\sqrt{3}) l2) n4 ((1+\sqrt{3} + (3+2\sqrt{3}) t^2)^2 + \\
& \left((3+2\sqrt{3}) t - \sqrt{1+\sqrt{3}} \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4} \right)^2) \Bigg) \Bigg) / \\
& (32 (1+4 t^2+3 t^4 - 4 \sqrt{1+\sqrt{3}} t \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4}) \\
& ((1+\sqrt{3} + (3+2\sqrt{3}) t^2)^2 + \\
& \left((3+2\sqrt{3}) t - \sqrt{1+\sqrt{3}} \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4} \right)^2) \Bigg)
\end{aligned}$$

Coplanarity Check: A2A3-A2B2-A3B3

Determinant of {A2A3, A2B2, A3B3} as a function of t

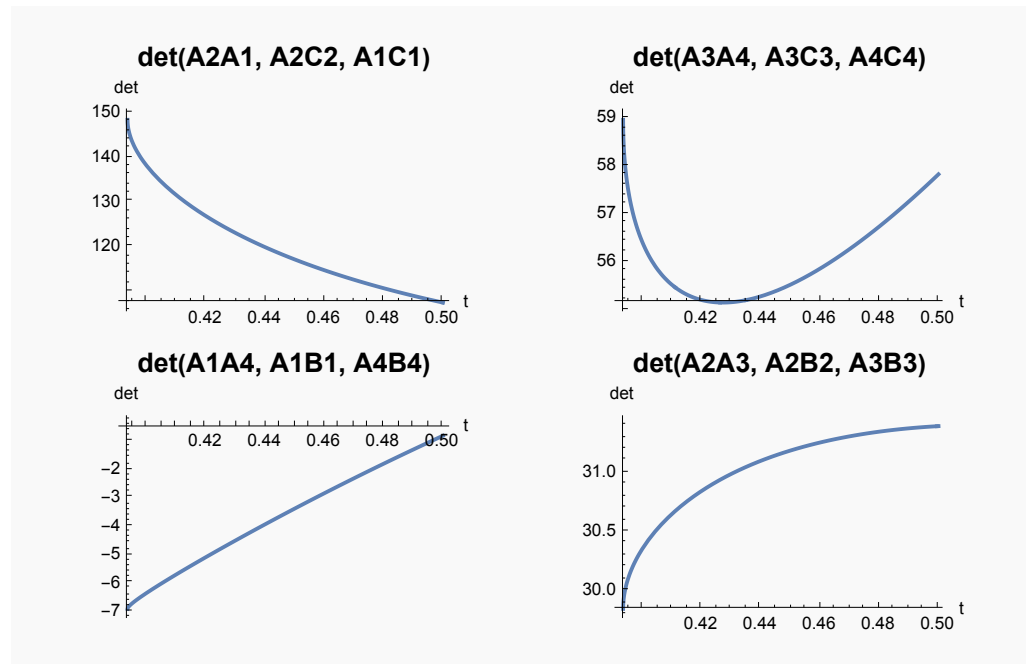
$$\begin{aligned}
& (\sqrt{2} (-1+\sqrt{3}) l2 m2 m3 ((11+8\sqrt{3}) t + 15(55+32\sqrt{3}) t^3 + \\
& 24(109+63\sqrt{3}) t^5 - \sqrt{1+\sqrt{3}} \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4} - \\
& \sqrt{1+\sqrt{3}} (25+12\sqrt{3}) t^2 \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4} + \\
& 12 \sqrt{1+\sqrt{3}} (5+3\sqrt{3}) t^4 \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4}) \Bigg) / \\
& ((1+t^2) (-3-4\sqrt{3} + 7(127+72\sqrt{3}) t^2 + 12(619+357\sqrt{3}) t^4 - \\
& 4 \sqrt{1+\sqrt{3}} (17+12\sqrt{3}) t \sqrt{-1+3(1+\sqrt{3}) t^2 - 3(2+\sqrt{3}) t^4}) \Bigg)
\end{aligned}$$

Coplanarity Check: A1A4-A1B1-A4B4

Determinant of {A1A4, A1B1, A4B4} as a function of t

$$\frac{1}{2} \text{ l3 m1 m4}$$

$$\left(\frac{t}{1+t^2} - \frac{(\sqrt{3}+12(9+5\sqrt{3})t^2) \left(-((36+17\sqrt{3})t) + 2\sqrt{3(1+\sqrt{3})} \sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4} \right)}{3(3+4\sqrt{3}-7(127+72\sqrt{3})t^2-12(619+357\sqrt{3})t^4+4\sqrt{1+\sqrt{3}}(17+12\sqrt{3})t\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4})} \right)$$

Determinant Plots for Our Example

Out[1773]=

===== EXTRA INFO (FLEXION 2) =====

This example does NOT have planar parameter

lines (C2A2A1C1, C3A3A4C4, B1A1A4B4, B2A2A3B3)

Coplanarity Check: A2A1-A2C2-A1C1

Determinant of {A2A1, A2C2, A1C1} as a function of t

$$\frac{1}{2} \left(- \frac{(-1+\sqrt{3}) \sqrt{\frac{1}{2}(2+\sqrt{3})} n_1 n_2 (-1+3t^2) \left(2t + \sqrt{1+\sqrt{3}} \sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4} \right)}{2+8t^2+6t^4+8\sqrt{1+\sqrt{3}}t\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}} + \right. \\ \left. \left(\sqrt{3} n_1 n_2 (1+\sqrt{3} + (3+2\sqrt{3})t^2) \right. \right. \\ \left. \left((3+2\sqrt{3})t + \sqrt{1+\sqrt{3}} \sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4} \right) \right. \\ \left. \left(1+14t^2-3t^4+8\sqrt{1+\sqrt{3}}t\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4} \right) \right) / \\ \left(2(1+4t^2+3t^4+4\sqrt{1+\sqrt{3}}t\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}) \right. \\ \left. (3+\sqrt{3} + (51+28\sqrt{3})t^2+3(2+\sqrt{3})t^4 + \right. \\ \left. \left. 2\sqrt{1+\sqrt{3}}(3+2\sqrt{3})t\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4} \right) \right) \right)$$

Coplanarity Check: A3A4-A3C3-A4C4

Determinant of {A3A4, A3C3, A4C4} as a function of t

$$\left(n3 \left(\sqrt{3} n4 (1 + \sqrt{3} + (3 + 2\sqrt{3}) t^2) ((3 + 2\sqrt{3}) t + \right. \right. \\ \sqrt{1 + \sqrt{3}} \sqrt{-1 + 3(1 + \sqrt{3}) t^2 - 3(2 + \sqrt{3}) t^4}) (2(2 + \sqrt{3}) l3 (1 - 3t^2)^2 + \\ 2 l1 (1 + 14t^2 - 3t^4 + 8\sqrt{1 + \sqrt{3}} t \sqrt{-1 + 3(1 + \sqrt{3}) t^2 - 3(2 + \sqrt{3}) t^4}) + \\ \sqrt{2} l2 (-3 - \sqrt{3} + 8(1 + 2\sqrt{3}) t^2 - 3(7 + 5\sqrt{3}) t^4 + \\ 4(-1 + \sqrt{3}) \sqrt{1 + \sqrt{3}} t \sqrt{-1 + 3(1 + \sqrt{3}) t^2 - 3(2 + \sqrt{3}) t^4})) \Big) \Big) / \\ (3 + \sqrt{3} + (51 + 28\sqrt{3}) t^2 + 3(2 + \sqrt{3}) t^4 + 2\sqrt{1 + \sqrt{3}} (3 + 2\sqrt{3}) t \\ \sqrt{-1 + 3(1 + \sqrt{3}) t^2 - 3(2 + \sqrt{3}) t^4}) + \frac{1}{2} (-1 + \sqrt{3}) \sqrt{\frac{1}{2} (2 + \sqrt{3})} \\ n4 (-1 + 3t^2) (2t + \sqrt{1 + \sqrt{3}} \sqrt{-1 + 3(1 + \sqrt{3}) t^2 - 3(2 + \sqrt{3}) t^4}) \\ \left(4 l1 + \sqrt{2} (-1 + \sqrt{3}) l2 + \right. \\ \left. \frac{4\sqrt{3} \left(-\frac{1}{4} (\sqrt{2} + \sqrt{6}) l2 + l3 \right) \left(-1 + \frac{\left((3 + 2\sqrt{3}) t + \sqrt{1 + \sqrt{3}} \sqrt{-1 + 3(1 + \sqrt{3}) t^2 - 3(2 + \sqrt{3}) t^4} \right)^2}{(1 + \sqrt{3} + (3 + 2\sqrt{3}) t^2)^2} \right)}{1 + \frac{\left((3 + 2\sqrt{3}) t + \sqrt{1 + \sqrt{3}} \sqrt{-1 + 3(1 + \sqrt{3}) t^2 - 3(2 + \sqrt{3}) t^4} \right)^2}{(1 + \sqrt{3} + (3 + 2\sqrt{3}) t^2)^2}} \right) \Big) \Big) / \\ (4 (1 + 4t^2 + 3t^4 + 4\sqrt{1 + \sqrt{3}} t \sqrt{-1 + 3(1 + \sqrt{3}) t^2 - 3(2 + \sqrt{3}) t^4}))$$

Coplanarity Check: A2A3-A2B2-A3B3

Determinant of {A2A3, A2B2, A3B3} as a function of t

$$\left(\sqrt{2} (-1 + \sqrt{3}) l2 m2 m3 \left((11 + 8\sqrt{3}) t + 15(55 + 32\sqrt{3}) t^3 + \right. \right. \\ 24(109 + 63\sqrt{3}) t^5 + \sqrt{1 + \sqrt{3}} \sqrt{-1 + 3(1 + \sqrt{3}) t^2 - 3(2 + \sqrt{3}) t^4} + \\ \sqrt{1 + \sqrt{3}} (25 + 12\sqrt{3}) t^2 \sqrt{-1 + 3(1 + \sqrt{3}) t^2 - 3(2 + \sqrt{3}) t^4} - \\ 12\sqrt{1 + \sqrt{3}} (5 + 3\sqrt{3}) t^4 \sqrt{-1 + 3(1 + \sqrt{3}) t^2 - 3(2 + \sqrt{3}) t^4} \Big) \Big) / \\ \left((1 + t^2) (-3 - 4\sqrt{3} + 7(127 + 72\sqrt{3}) t^2 + 12(619 + 357\sqrt{3}) t^4 + \right. \\ \left. 4\sqrt{1 + \sqrt{3}} (17 + 12\sqrt{3}) t \sqrt{-1 + 3(1 + \sqrt{3}) t^2 - 3(2 + \sqrt{3}) t^4} \right)$$

Coplanarity Check: A1A4-A1B1-A4B4

Determinant of {A1A4, A1B1, A4B4} as a function of t

$$\frac{1}{2} l3 m1 m4 \\ \left(\frac{t}{1 + t^2} - \frac{(\sqrt{3} + 12(9 + 5\sqrt{3}) t^2) ((36 + 17\sqrt{3}) t + 2\sqrt{3(1 + \sqrt{3})} \sqrt{-1 + 3(1 + \sqrt{3}) t^2 - 3(2 + \sqrt{3}) t^4})}{3(-3 - 4\sqrt{3} + 7(127 + 72\sqrt{3}) t^2 + 12(619 + 357\sqrt{3}) t^4 + 4\sqrt{1 + \sqrt{3}} (17 + 12\sqrt{3}) t \sqrt{-1 + 3(1 + \sqrt{3}) t^2 - 3(2 + \sqrt{3}) t^4})} \right)$$

Determinant Plots for Our Example

