

# Computational Companion to “Flexible 3×3 Nets of Equimodular Elliptic Type” — Example 2

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Tested on: Mathematica 14.0

In[1774]:=

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(*=====*)
=====*)
(*=====*)
=====*)
(*=====*)
=====*)
(*Quit*)
(*All angle sets in degrees*)
anglesRad = {
  {ArcCos[1 / Sqrt[5]], ArcCos[7 / (5 Sqrt[2])]},
  {ArcCos[-1 / Sqrt[10]], ArcCos[0]}, (*Vertex 1*)
  {ArcCos[1 / (4 Sqrt[11])], ArcCos[7 Sqrt[7] / (4 Sqrt[22])]},
  {ArcCos[-1 / (2 Sqrt[2])], ArcCos[0]}, (*Vertex 2*)
  {ArcCos[1 / (4 Sqrt[11])], ArcCos[7 Sqrt[7] / (4 Sqrt[22])]},
  {ArcCos[1 / (2 Sqrt[2])], ArcCos[0]}, (*Vertex 3*)
  {ArcCos[1 / Sqrt[5]], ArcCos[7 / (5 Sqrt[2])]},
  {ArcCos[1 / Sqrt[10]], ArcCos[0]} (*Vertex 4*});

anglesDeg = anglesRad * 180 / Pi;

(*-----*)
(*Function to compute sigma from 4 angles*)
computeSigma[{α_, β_, γ_, δ_}] := (α + β + γ + δ) / 2;

(*-----*)
(*Function to compute a,b,c,d from angles*)
computeABCD[{α_, β_, γ_, δ_}] :=
Module[{alpha = α Degree, beta = β Degree, gamma = γ Degree,
  delta = δ Degree, sigma}, sigma = computeSigma[{α, β, γ, δ} Degree];
{Sin[alpha] / Sin[sigma - alpha], Sin[beta] / Sin[sigma - beta],
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Sin[gamma] / Sin[sigma - gamma], Sin[delta] / Sin[sigma - delta]]];

(*-----*)
(*Compute all  $\sigma$  values*)
sigmas = computeSigma /@ anglesDeg;

(*Compute all {a,b,c,d} values*)
results = computeABCD /@ anglesDeg;

(*Optional:extract individual values*)
{a1, b1, c1, d1} = results[[1]];
{a2, b2, c2, d2} = results[[2]];
{a3, b3, c3, d3} = results[[3]];
{a4, b4, c4, d4} = results[[4]];
{ $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$ ,  $\sigma_4$ } = FullSimplify[sigmas];

(*=====
====*)
(*=====
CONDITION (N.0)=====*)
(*=====
====*)
uniqueCombos = {{1, 1, 1, 1}, {1, 1, 1, -1}, {1, 1, -1, -1}, {1, 1, -1, 1},
{1, -1, 1, 1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, -1, -1, -1}};

checkConditionN0Degrees[{ $\alpha$ _,  $\beta$ _,  $\gamma$ _,  $\delta$ _}] :=
Module[{angles = { $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ }, results},
results = Mod[uniqueCombos.angles, 360] // Chop;
! MemberQ[results, 0]];

conditionsN0 = checkConditionN0Degrees /@ anglesDeg;
allVerticesPass = And@@ conditionsN0;

Column[{TextCell[Style["===== CONDITION (N.0) =====",
Darker[Green], Bold, 16], "Text"],
If[allVerticesPass,
Style["✓ All vertices satisfy (N.0).", Darker[Green], Bold],
Style["✗ Some vertices fail (N.0).", Red, Bold]]}]

(*=====
====*)
(*=====
CONDITION (N.3)=====*)
(*=====
====*)

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Ms = FullSimplify[Times@@@results];
allEqualQ = Simplify[Equal@@Ms];

Column[{TextCell[Style["===== CONDITION (N.3) =====",
  Darker[Blue], Bold, 16], "Text"], If[allEqualQ,
  Row[{Style["✓ M1 = M2 = M3 = M4 = ", Bold], Ms[[1]]},
  Style["✗ M_i are not all equal.", Red, Bold]]}]

(*=====
====*)
(*=====CONDITION (N.4)=====*)
(*=====
====*)
aList = results[[All, 1]];
cList = results[[All, 3]];
dList = results[[All, 4]];

rList = FullSimplify /@ (aList * dList); {r1, r2, r3, r4} = rList;
sList = FullSimplify /@ (cList * dList); {s1, s2, s3, s4} = sList;

Column[{TextCell[Style["===== CONDITION (N.4) =====",
  Darker[Purple], Bold, 16], "Text"],
  If[r1 == r2 && r3 == r4 && s1 == s4 && s2 == s3, Column[
    {Row[{Style["✓ r1 = r2 = ", Bold], r1, Style["; ✓ r3 = r4 = ", Bold], r3}],
    Row[{Style["✓ s1 = s4 = ", Bold], s1, Style["; ✓ s2 = s3 = ", Bold],
    s2}]]], Style["✗ Condition (N.4) fails.", Red, Bold]]
}]

(*=====
====*)
(*=====
CONDITION (N.5)=====*)
(*=====
====*)
fList = FullSimplify /@ (aList * cList); {f1, f2, f3, f4} = fList;
M1 = Ms[[1]];

m[M1_] := Piecewise[{{1 - M1, M1 < 1}, {(M1 - 1) / M1, M1 > 1}}] // N;
K = EllipticK[m[M1]] // N;
Kp = EllipticK[1 - m[M1]] // N;

computeTi[i_] :=
Module[{sigma = sigmas[[i]], r = rList[[i]], s = sList[[i]], f = fList[[i]], base},
  base = I * Im[InverseJacobiDN[Piecewise[
    {{Sqrt[f], M1 < 1}, {1 / Sqrt[f], M1 > 1}}], m[M1]]] / Kp;

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Which[sigma < 180, Which[r > 1 && s > 1, base, (r < 1 && s > 1) || (r > 1 && s < 1),
  1 + base, r < 1 && s < 1, 2 + base], sigma > 180, Which[r > 1 && s > 1,
  2 + base, (r < 1 && s > 1) || (r > 1 && s < 1), 3 + base, r < 1 && s < 1, base]]];

tList = computeTi /@ Range[4];
RoundWithTolerance[x_, tol_ : 10^-6] := Module[{nearest}, nearest = Round[x];
  If[Abs[x - nearest] ≤ tol, nearest, x]];

checkValidCombination[M1_, ε_ : 10^-6] :=
Module[{i, combo, dotProd, rePart, imPart, n2, expr, foundQ = False},
  Print[Style["△ Approximate validation using ε-tolerance. For rigorous
    proof, see the referenced paper.", Darker@Orange, Italic]];
  Do[combo = uniqueCombos[[i];
    dotProd = tList.combo;
    rePart = Abs[Re[dotProd]];
    imPart = Abs[Im[dotProd]];
    If[M1 < 1,
      If[Mod[RoundWithTolerance[rePart], 4] < ε,
        If[Mod[RoundWithTolerance[imPart], 2] < ε, expr =
          tList[[1]] + combo[[2]] × tList[[2]] + combo[[3]] × tList[[3]] + combo[[4]] × tList[[4]];
          Print[Style["✔ Valid Combination Found (M < 1):",
            Darker[Green], Bold], "\n", Style["e1 = ", Bold], combo[[2]],
            Style["e2 = ", Bold], combo[[3]], Style["e3 = ", Bold], combo[[4]],
            "\n", Style["t1 = ", Bold], Re[tList[[1]], "K + ", Im[tList[[1]], "iK'",
            "\n", Style["t2 = ", Bold], Re[tList[[2]], "K + ", Im[tList[[2]], "iK'",
            "\n", Style["t3 = ", Bold], Re[tList[[3]], "K + ", Im[tList[[3]], "iK'",
            "\n", Style["t4 = ", Bold], Re[tList[[4]], "K + ", Im[tList[[4]],
            "iK'", "\n", Style["t1 + e1*t2 + e2*t3 + e3*t4 = ", Bold],
            Re[expr], "K + ", Im[expr], "iK'"];
          foundQ = True;
          Break[]]]];
    If[M1 > 1,
      If[Mod[RoundWithTolerance[imPart], 2] < ε,
        n2 = Quotient[RoundWithTolerance[imPart], 2];
        If[Mod[RoundWithTolerance[Abs[rePart - 2 n2]], 4] < ε, expr =
          tList[[1]] + combo[[2]] × tList[[2]] + combo[[3]] × tList[[3]] + combo[[4]] × tList[[4]];
          Print[Style["✔ Valid Combination Found (M > 1):",
            Darker[Green], Bold], "\n", Style["e1 = ", Bold], combo[[2]],
            Style["e2 = ", Bold], combo[[3]], Style["e3 = ", Bold], combo[[4]],
            "\n", Style["t1 = ", Bold], Re[tList[[1]], "K + ", Im[tList[[1]], "iK'",
            "\n", Style["t2 = ", Bold], Re[tList[[2]], "K + ", Im[tList[[2]], "iK'",
            "\n", Style["t3 = ", Bold], Re[tList[[3]], "K + ", Im[tList[[3]], "iK'",
            "\n", Style["t4 = ", Bold], Re[tList[[4]], "K + ", Im[tList[[4]],
            "iK'", "\n", Style["t1 + e1*t2 + e2*t3 + e3*t4 = ", Bold],
            Re[expr], "K + ", Im[expr], "iK'"];

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        foundQ = True;
        Break[]]]], {i, Length[uniqueCombos]}}];
If[! foundQ, Print[Style["✗ No valid combination found.", Red, Bold], "\n",
    Style["t1 = ", Bold], Re[tList[[1]], "K + ", Im[tList[[1]], "iK'", "\n",
    Style["t2 = ", Bold], Re[tList[[2]], "K + ", Im[tList[[2]], "iK'", "\n",
    Style["t3 = ", Bold], Re[tList[[3]], "K + ", Im[tList[[3]], "iK'", "\n",
    Style["t4 = ", Bold], Re[tList[[4]], "K + ", Im[tList[[4]], "iK'", "\n"]];];

Column[{TextCell[Style["===== CONDITION (N.5) =====",
    Darker[Red], Bold, 16], "Text"]}]
res = checkValidCombination[M1];

(*=====
====*)
(*=====
    OTHER PARAMETER=====*)
(*=====
====*)
cosSigmaAlt[Mi_, fi_, ri_, si_] :=
    (Mi (fi + ri + si - 1) + ri fi si - ri fi - ri si - fi si) / (2 Sqrt[ri si fi] (1 - Mi));

Column[
    {TextCell[Style["===== OTHER PARAMETERS =====",
        Darker[Orange], Bold, 16], "Text"],
    Row[{Style["u = ", Bold], 1 - M1}],
    Row[{Style["σ1 = ", Bold], σ1 Degree, Style["", σ2 = ", Bold], σ2 Degree,
        Style["", σ3 = ", Bold], σ3 Degree, Style["", σ4 = ", Bold], σ4 Degree}],
    Row[{Style["σ1 ≈ ", Bold], N[σ1], Style["°", Bold], Style["", σ2 ≈ ", Bold],
        N[σ2], Style["°", Bold], Style["", σ3 ≈ ", Bold], N[σ3],
        Style["°", Bold], Style["", σ4 ≈ ", Bold], N[σ4], Style["°", Bold]}],
    Row[{Style["cosσ1 = ", Bold], Simplify@cosSigmaAlt[M1, f1, r1, s1],
        Style["", cosσ2 = ", Bold], Simplify@cosSigmaAlt[M1, f2, r2, s2],
        Style["", cosσ3 = ", Bold], Simplify@cosSigmaAlt[M1, f3, r3, s3],
        Style["", cosσ4 = ", Bold], Simplify@cosSigmaAlt[M1, f4, r4, s4]}],
    Row[{Style["f1 = ", Bold], f1, Style["", f2 = ", Bold],
        f2, Style["", f3 = ", Bold], f3, Style["", f4 = ", Bold], f4}],
    Row[{Style["z1 = ", Bold], FullSimplify[1 / (f1 - 1)], Style["", z2 = ", Bold],
        FullSimplify[1 / (f2 - 1)], Style["", z3 = ", Bold], FullSimplify[1 / (f3 - 1)],
        Style["", z4 = ", Bold], FullSimplify[1 / (f4 - 1)]}],
    Row[{Style["x1 = ", Bold], FullSimplify[1 / (r1 - 1)], Style["", x2 = ", Bold],
        FullSimplify[1 / (r2 - 1)], Style["", x3 = ", Bold], FullSimplify[1 / (r3 - 1)],
        Style["", x4 = ", Bold], FullSimplify[1 / (r4 - 1)]}],
    Row[{Style["y1 = ", Bold], FullSimplify[1 / (s1 - 1)], Style["", y2 = ", Bold],
        FullSimplify[1 / (s2 - 1)], Style["", y3 = ", Bold], FullSimplify[1 / (s3 - 1)],
        Style["", y4 = ", Bold], FullSimplify[1 / (s4 - 1)]}],

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Row[{Style["p1 = ", Bold], Simplify[Sqrt[r1 - 1]],
  Style["", p2 = ", Bold], Simplify[Sqrt[r2 - 1]], Style["", p3 = ", Bold],
  Simplify[Sqrt[r3 - 1]], Style["", p4 = ", Bold], Simplify[Sqrt[r4 - 1]]}],
Row[{Style["q1 = ", Bold], Simplify[Sqrt[s1 - 1]],
  Style["", q2 = ", Bold], Simplify[Sqrt[s2 - 1]], Style["", q3 = ", Bold],
  Simplify[Sqrt[s3 - 1]], Style["", q4 = ", Bold], Simplify[Sqrt[s4 - 1]]}],
Row[{Style["p1.q1 = ", Bold], Simplify[Sqrt[r1 - 1] Sqrt[s1 - 1]],
  Style["", p2.q2 = ", Bold], Simplify[Sqrt[r2 - 1] Sqrt[s2 - 1]],
  Style["", p3.q3 = ", Bold], Simplify[Sqrt[r3 - 1] Sqrt[s3 - 1]],
  Style["", p4.q4 = ", Bold], Simplify[Sqrt[r4 - 1] Sqrt[s4 - 1]]}]
}]

(*=====
====*)
(*=====
BRICARD'S EQUATIONS=====*)
(*=====
====*)
(*=====
FLEXION 1=====*)
Z[t_] := t;
W1[t_] := 
$$\frac{6t - \sqrt{2(3t^2 - 1)(2 - 3t^2)}}{1 + 3t^2};$$

U[t_] := 
$$\frac{1}{t};$$

W2[t_] := 
$$\frac{5\sqrt{7}t - \sqrt{10(3t^2 - 1)(2 - 3t^2)}}{4 + 9t^2};$$


(*General polynomial constructor*)
BricardsEquation[{α_, β_, γ_, δ_}, σ_, x_, y_] :=
Module[{c22, c20, c02, c11, c00},
  c22 = Sin[σ - δ] Sin[σ - δ - β];
  c20 = Sin[σ - α] Sin[σ - α - β];
  c02 = Sin[σ - γ] Sin[σ - γ - β];
  c11 = -Sin[α] Sin[γ];
  c00 = Sin[σ] Sin[σ - β];
  c22 x^2 y^2 + c20 x^2 + c02 y^2 + 2 c11 x y + c00];

(*Compute and print all P_i for flexion 1*)
TextCell[
  Style["===== FLEXIBILITY (FLEXION 1) =====",
    Darker[Cyan], Bold, 16], "Text"]
funcs = {{Z, W1}, {Z, W2}, {U, W2}, {U, W1}};
Do[angles = anglesDeg[[i]] Degree;
  sigma = sigmas[[i]] Degree;

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{α, β, γ, δ} = angles;
poly = Which[i == 1, BricardsEquation[{α, β, γ, δ}, sigma, Z[t], W1[t]],
  i == 2, BricardsEquation[{α, β, γ, δ}, sigma, Z[t], W2[t]],
  i == 3, BricardsEquation[{α, β, γ, δ}, sigma, U[t], W2[t]],
  i == 4, BricardsEquation[{α, β, γ, δ}, sigma, U[t], W1[t]]];
Print[Row[{Subscript["P", i], "[", funcs[[i, 1],
  ", ", funcs[[i, 2]], "] = ", FullSimplify[poly]]}], {i, 1, 4}];

(*=====
FLEXION 2=====*)
Z2[t_] := t;
W12[t_] := 
$$\frac{6t + \sqrt{2(3t^2 - 1)(2 - 3t^2)}}{1 + 3t^2};$$

U2[t_] := 
$$\frac{1}{t};$$

W22[t_] := 
$$\frac{5\sqrt{7}t + \sqrt{10(3t^2 - 1)(2 - 3t^2)}}{4 + 9t^2};$$


(*General polynomial constructor*)
BricardsEquation[{α_, β_, γ_, δ_}, σ_, x_, y_] :=
Module[{c22, c20, c02, c11, c00},
  c22 = Sin[σ - δ] Sin[σ - δ - β];
  c20 = Sin[σ - α] Sin[σ - α - β];
  c02 = Sin[σ - γ] Sin[σ - γ - β];
  c11 = -Sin[α] Sin[γ];
  c00 = Sin[σ] Sin[σ - β];
  c22 x^2 y^2 + c20 x^2 + c02 y^2 + 2 c11 x y + c00];

(*Compute and print all P_i for flexion 2*)
TextCell[
Style["===== FLEXIBILITY (FLEXION 2) =====",
Cyan, Bold, 16], "Text"]
funcs = {{Z, W1}, {Z, W2}, {U, W2}, {U, W1}};
Do[angles = anglesDeg[[i]] Degree;
sigma = sigmas[[i]] Degree;
{α, β, γ, δ} = angles;
poly = Which[i == 1, BricardsEquation[{α, β, γ, δ}, sigma, Z2[t], W12[t]],
  i == 2, BricardsEquation[{α, β, γ, δ}, sigma, Z2[t], W22[t]],
  i == 3, BricardsEquation[{α, β, γ, δ}, sigma, U2[t], W22[t]],
  i == 4, BricardsEquation[{α, β, γ, δ}, sigma, U2[t], W12[t]]];
Print[Row[{Subscript["P", i], "[", funcs[[i, 1],
  ", ", funcs[[i, 2]], "] = ", FullSimplify[poly]]}], {i, 1, 4}];

(*=====
=====*)

```

```

(*=====
NOT TRIVIAL=====*)
(*=====
=====*)
(*Define domain limits for t*)
tMin = 1 / Sqrt[3];
tMax = Sqrt[2 / 3];

(*=====
FLEXION 1=====*)
(*List of expressions to plot*)
expressions = {Z[t], W2[t], U[t], W1[t]};
labels = {"Z", "W2", "U", "W1"};

Column[{TextCell[
  Style["===== NOT TRIVIAL (FLEXION 1) =====",
    Darker[Brown], Bold, 16], "Text"],

  (*Explanatory text*)
  TextCell[Style["This configuration does not belong to the trivial class –
    even after switching the boundary strips – since none of the
    functions Z, W2, U, or W1 is constant.", GrayLevel[0.3]], "Text"],
  Spacer[12],

  (*Plots in a light panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
    PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
    AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
    Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]

}],

(*=====
FLEXION 2=====*)
(*List of expressions to plot*)
expressions = {Z2[t], W22[t], U2[t], W12[t]};
labels = {"Z", "W2", "U", "W1"};

Column[{TextCell[
  Style["===== NOT TRIVIAL (FLEXION 2) =====",
    Brown, Bold, 16], "Text"],

  (*Explanatory text*)
  TextCell[Style["This configuration does not belong to the trivial class –
    even after switching the boundary strips – since none of the
    functions Z, W2, U, or W1 is constant.", GrayLevel[0.3]], "Text"],
  Spacer[12],

```



```

(*Plots in a light panel*)
Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
  PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
  AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
  Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]

}]

(*=====*)
(*=====*)
NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE=====*)
(*=====*)
(*Define domain limits for t*)
tMin = 1 / Sqrt[3];
tMax = Sqrt[2 / 3];

(*=====*)
FLEXION 1=====*)
(*List of expressions& labels*)
expressions =
  {Z[t] * U[t], Z[t] / U[t], W1[t] * W2[t], W1[t] / W2[t], Z[t] * W2[t], Z[t] / W2[t],
  U[t] * W1[t], U[t] / W1[t], Z[t] * W1[t], Z[t] / W1[t], W2[t] * U[t], W2[t] / U[t]};
labels = {"Z * U", "Z / U", "W1 * W2", "W1 / W2", "Z * W2", "Z / W2",
  "U * W1", "U / W1", "Z * W1", "Z / W1", "W2 * U", "W2 / U"};

Column[
  {TextCell[Style["===== NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE
    (FLEXION 1) =====", Darker[Magenta], Bold, 16], "Text"],

  (*Explanatory text*)TextCell[
    Style["This configuration does not belong to the Linear compound class nor
      to the linear conjugate class – even after switching the boundary
      strips – since none of the functions ZU, Z/U, W1W2, W1/W2, ZW2,
      Z/W2, UW1, and U/W1 is constant. In addition, none of ZW1,
      Z/W1, W2U, W2/U is constant as well.", GrayLevel[0.3]], "Text"],
  Spacer[12],

  (*Plots panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
    PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
    AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
    Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]
  ]
}

```

```

(*=====
FLEXION 2=====*)
(*List of expressions& labels*)
expressions = {Z2[t] * U2[t], Z2[t] / U2[t], W12[t] * W22[t], W12[t] / W22[t],
  Z2[t] * W22[t], Z2[t] / W22[t], U2[t] * W12[t], U2[t] / W12[t],
  Z2[t] * W12[t], Z2[t] / W12[t], W22[t] * U2[t], W22[t] / U2[t]};
labels = {"Z * U", "Z / U", "W1 * W2", "W1 / W2", "Z * W2", "Z / W2",
  "U * W1", "U / W1", "Z * W1", "Z / W1", "W2 * U", "W2 / U"};

Column[
  {TextCell[Style["===== NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE
    (FLEXION 2) =====", Magenta, Bold, 16], "Text"],

  (*Explanatory text*)TextCell[
    Style["This configuration does not belong to the Linear compound class nor
      to the linear conjugate class – even after switching the boundary
      strips – since none of the functions ZU, Z/U, W1W2, W1/W2, ZW2,
      Z/W2, UW1, and U/W1 is constant. In addition, none of ZW1,
      Z/W1, W2U, W2/U is constant as well.", GrayLevel[0.3]], "Text"],
  Spacer[12],

  (*Plots panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
    PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
    AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
    Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]
  ]

(*=====
=====*)
(*=====
SWITCHING BOUNDARY STRIPS=====*)
(*=====
=====*)
SwitchingRightBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={1,2},{1,3},{4,2},{4,3}*)
  modified[[1, 2]] = 180 - anglesDeg[[1, 2]]; (*β1*)
  modified[[1, 3]] = 180 - anglesDeg[[1, 3]]; (*γ1*)
  modified[[4, 2]] = 180 - anglesDeg[[4, 2]]; (*β4*)
  modified[[4, 3]] = 180 - anglesDeg[[4, 3]]; (*γ4*)
  modified]

SwitchingLeftBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={2,2},{2,3},{3,2},{3,3}*)
  modified[[2, 2]] = 180 - anglesDeg[[2, 2]]; (*β2*)

```

```

modified[[2, 3]] = 180 - anglesDeg[[2, 3]]; (*γ2*)
modified[[3, 2]] = 180 - anglesDeg[[3, 2]]; (*β3*)
modified[[3, 3]] = 180 - anglesDeg[[3, 3]]; (*γ3*)
modified]

SwitchingLowerBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={1,1},{1,2},{2,1},{2,2}*)
  modified[[1, 1]] = 180 - anglesDeg[[1, 1]]; (*α1*)
  modified[[1, 2]] = 180 - anglesDeg[[1, 2]]; (*β1*)
  modified[[2, 1]] = 180 - anglesDeg[[2, 1]]; (*α2*)
  modified[[2, 2]] = 180 - anglesDeg[[2, 2]]; (*β2*)
  modified]

SwitchingUpperBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={3,1},{3,2},{4,1},{4,2}*)
  modified[[3, 1]] = 180 - anglesDeg[[3, 1]]; (*α3*)
  modified[[3, 2]] = 180 - anglesDeg[[3, 2]]; (*β3*)
  modified[[4, 1]] = 180 - anglesDeg[[4, 1]]; (*α4*)
  modified[[4, 2]] = 180 - anglesDeg[[4, 2]]; (*β4*)
  modified]

(*=====
====*)
(*=====
NOT CONIC=====*)
(*=====
====*)
Column[{TextCell[Style["===== NOT CONIC =====",
  Pink, Bold, 16], "Text"],
  TextCell[Style["Condition (N.0) is satisfied ⇒
    this configuration is NOT equimodular-conic. Applying
    any boundary-strip switch still preserves (N.0), so
    no conic form emerges.", GrayLevel[0.3]], "Text"]
}]

(*Now the exact same Module for checking all switch combinations...*)
Module[{angles = anglesDeg, switchers, combinations, results},
  (*Define switch functions*)
  switchers = <|"Right" → SwitchingRightBoundaryStrip,
    "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
    "Upper" → SwitchingUpperBoundaryStrip|>;
  (*Generate all combinations of switches (from size 1 to 4)*)
  combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
  (*Evaluate condition after each combination of switches*) results = Table[
    Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];

```

```

passQ = And@@(checkConditionN0Degrees /@ switched);
(*Print the result after switching*)
(*Print["\nSwitch combination: ",name];
Print["Switched anglesDeg:"];
Print[MatrixForm[switched]];*)
{name, passQ}}, {combo, combinations}}];
(*Display results*)
Column[Prepend[Table[Module[{comboName = res[[1]], passQ = res[[2]]},
  Row[{Style[comboName <> ": ", Bold],
    If[passQ,
      Style["Condition (N.0) is still satisfied.", Darker[Green]],
      Style["Condition (N.0) fails.", Red, Bold]
    ]
  }, {res, results}], TextCell[
Style["CONDITION (N.0) AFTER SWITCHING BOUNDARY STRIPS", 14], "Text"]]]]]

(*=====
====*)
(*=====
NOT ORTHODIAGONAL=====*)
(*=====
====*)
Column[
{TextCell[Style["===== NOT ORTHODIAGONAL =====",
  Purple, Bold, 16], "Text"],
TextCell[Style[
"cos( $\alpha_i$ )·cos( $\gamma_i$ )  $\neq$  cos( $\beta_i$ )·cos( $\delta_i$ ) for each i = 1...4  $\Rightarrow$  NOT orthodiagonal.
Switching boundary strips does not
correct this.", GrayLevel[0.3]], "Text"]
}]

Module[{angles = anglesDeg, switchers, combinations, results},
(*Define switch functions*)
switchers = <|"Right" → SwitchingRightBoundaryStrip,
"Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
"Upper" → SwitchingUpperBoundaryStrip|>;
(*Helper function:compute and print difference only*)
formatOrthodiagonalCheck[quad_List] := Module[{vals},
vals = Table[Module[{a, b, c, d, lhs, rhs, diff}, {a, b, c, d} = quad[[i]];
lhs = FullSimplify[Cos[a Degree] Cos[c Degree]];
rhs = FullSimplify[Cos[b Degree] Cos[d Degree]];
diff = Chop[lhs - rhs];
Style[Row[{"cos( $\alpha$ " <> ToString[i] <> ")·cos( $\gamma$ " <> ToString[i] <> ") - ",
"cos( $\beta$ " <> ToString[i] <> ")·cos( $\delta$ " <> ToString[i] <> ") = ", NumberForm[
diff, {5, 3}]}], If[diff == 0, Red, Black]]], {i, Length[quad]}];

```

```

Column[vals]];
(*Orthodiagonal check for anglesDeg before any switching*)
Print[TextCell[Style["\nInitial anglesDeg (no switches):", Bold]]];
Print[MatrixForm[angles]];
Print[TextCell[Style[
  "Orthodiagonal check:  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$ ",
  Italic]]];
Print[formatOrthodiagonalCheck[angles]];
(*Generate all combinations of switches (from size 1 to 4)*)
combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
(*Evaluate condition after each combination of switches*)results = Table[
  Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];
    passQ = And@@ (checkConditionN0Degrees /@switched);
    Print[Style["\nSwitch combination: ", Bold], name];
    Print[Style["Switched anglesDeg:", Italic]];
    Print[MatrixForm[switched]];
    Print[TextCell[Style[
      "Orthodiagonal check:  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$ ",
      Italic]]];
    Print[formatOrthodiagonalCheck[switched]];
    {name, passQ}], {combo, combinations}];]

(*=====
====*)
(*=====
NOT ISOGONAL=====*)
(*=====
====*)
Column[
{TextCell[Style["===== NOT ISOGONAL =====", Orange,
  Bold, 15], "Text"],
TextCell[
Style["Condition (N.0) holds AND for all  $i = 1..4$ :  $\alpha_i \neq \beta_i$ ,  $\alpha_i \neq \gamma_i$ ,  $\alpha_i$ 
 $\neq \delta_i$ ,  $\beta_i \neq \gamma_i$ ,  $\beta_i \neq \delta_i$ ,  $\gamma_i \neq \delta_i$ ,  $\alpha_i + \beta_i \neq \pi \neq \gamma_i + \delta_i$ ,  $\alpha_i + \gamma_i$ 
 $\neq \pi \neq \beta_i + \delta_i$ ,  $\alpha_i + \delta_i \neq \pi \neq \beta_i + \gamma_i \Rightarrow$  NOT isogonal. Switching
boundary strips do not change this.", GrayLevel[0.3]], "Text"]}]]

Module[{angles = anglesDeg, switchers, combinations, results},
(*Define switch functions*)
switchers = <|"Right" → SwitchingRightBoundaryStrip,
  "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
  "Upper" → SwitchingUpperBoundaryStrip|>;
(*Helper function:extended angle relations*)
formatAngleRelations[quad_List] :=
Module[{vals}, vals = Table[Module[{a, b, c, d, exprs}, {a, b, c, d} = quad[[i]]];

```

```

exprs = {Row[{" $\alpha$ " <> ToString[i] <> " -  $\beta$ " <> ToString[i] <> " = ",
  NumberForm[N[a - b], {5, 3}]}], Row[{" $\alpha$ " <> ToString[i] <>
  " -  $\gamma$ " <> ToString[i] <> " = ", NumberForm[N[a - c], {5, 3}]}],
Row[{" $\alpha$ " <> ToString[i] <> " -  $\delta$ " <> ToString[i] <> " = ",
  NumberForm[N[a - d], {5, 3}]}], Row[{" $\beta$ " <> ToString[i] <>
  " -  $\gamma$ " <> ToString[i] <> " = ", NumberForm[N[b - c], {5, 3}]}],
Row[{" $\beta$ " <> ToString[i] <> " -  $\delta$ " <> ToString[i] <> " = ",
  NumberForm[N[b - d], {5, 3}]}], Row[{" $\gamma$ " <> ToString[i] <>
  " -  $\delta$ " <> ToString[i] <> " = ", NumberForm[N[c - d], {5, 3}]}],
Row[{" $\alpha$ " <> ToString[i] <> " +  $\beta$ " <> ToString[i] <> " - 180 = ",
  NumberForm[N[a + b - 180], {5, 3}]}],
Row[{" $\gamma$ " <> ToString[i] <> " +  $\delta$ " <> ToString[i] <> " - 180 = ",
  NumberForm[N[c + d - 180], {5, 3}]}],
Row[{" $\alpha$ " <> ToString[i] <> " +  $\gamma$ " <> ToString[i] <> " - 180 = ",
  NumberForm[N[a + c - 180], {5, 3}]}],
Row[{" $\beta$ " <> ToString[i] <> " +  $\delta$ " <> ToString[i] <> " - 180 = ",
  NumberForm[N[b + d - 180], {5, 3}]}],
Row[{" $\alpha$ " <> ToString[i] <> " +  $\delta$ " <> ToString[i] <> " - 180 = ",
  NumberForm[N[a + d - 180], {5, 3}]}],
Row[{" $\beta$ " <> ToString[i] <> " +  $\gamma$ " <> ToString[i] <> " - 180 = ",
  NumberForm[N[b + c - 180], {5, 3}]}],
Row[{" $\alpha$ " <> ToString[i] <> " +  $\beta$ " <> ToString[i] <> " -  $\gamma$ " <> ToString[i] <>
  " -  $\delta$ " <> ToString[i] <> " = ", NumberForm[N[a + b - c - d], {5, 3}]}],
Row[{" $\alpha$ " <> ToString[i] <> " +  $\gamma$ " <> ToString[i] <> " -  $\beta$ " <> ToString[i] <>
  " -  $\delta$ " <> ToString[i] <> " = ", NumberForm[N[a + c - b - d], {5, 3}]}],
Row[{" $\alpha$ " <> ToString[i] <> " +  $\delta$ " <> ToString[i] <> " -  $\beta$ " <> ToString[i] <>
  " -  $\gamma$ " <> ToString[i] <> " = ", NumberForm[N[a + d - b - c], {5, 3}]}];
Column[Prepend[exprs, Style["Vertex " <> ToString[i], Bold]]],
{i, Length[quad]}}];
Column[vals, Spacings → 1.5]];
(*Angle relation check for anglesDeg before any switching*)
Print[TextCell[Style["\nInitial anglesDeg (no switches):", Bold]]];
Print[MatrixForm[angles]];
Print[TextCell[Style["Angle relation checks for i = 1..4:", Italic]]];
Print[formatAngleRelations[angles]];
(*Generate all combinations of switches (from size 1 to 4)*)
combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
(*Evaluate condition after each combination of switches*)results = Table[
Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
Do[switched = switchers[sw][switched], {sw, combo}];
passQ = And@@ (checkConditionN0Degrees /@ switched);
Print[Style["\nSwitch combination: ", Bold], name];
Print[Style["Switched anglesDeg:", Italic]];
Print[MatrixForm[switched]];
Print[TextCell[Style["Angle relation checks for i = 1..4:", Italic]]];
Print[formatAngleRelations[switched]];
{name, passQ}], {combo, combinations}];]

```

```

(*=====
====*)
(*=====
NOT CONJUGATE-MODULAR=====*)
(*=====
====*)
Column[
{TextCell[Style["===== NOT CONJUGATE-MODULAR =====",
Brown, Bold, 16], "Text"],
TextCell[
Style[" $M_i < 1$  for all  $i = 1...4 \Rightarrow$  NOT conjugate-modular. Boundary-strip
switches preserve this.", GrayLevel[0.3]], "Text"]
}]

Module[{angles = anglesDeg, switchers, combinations, results,
computeConjugateModularInfo}, (*Define switch functions*)
switchers = <|"Right" → SwitchingRightBoundaryStrip,
"Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
"Upper" → SwitchingUpperBoundaryStrip|>;
(*Computes  $M_i$  and  $p_i$  and prints them,
with classification*)computeConjugateModularInfo[quad_List] :=
Module[{abcdList, Ms, summary}, abcdList = computeABCD /@ quad;
Ms = FullSimplify[Times@@@abcdList];
summary = If[AllTrue[Ms, # < 1 &], Style[" $M_i < 1$  for all  $i = 1, \dots, 4$ ",
Bold], Style[" $M_i \geq 1$  for some  $i = 1, \dots, 4$ ", Red, Bold]];
Column[{Style[" $M_i$  values:", Bold], Row[{"M1 = ", Ms[[1]], ", M2 = ",
Ms[[2]], ", M3 = ", Ms[[3]], ", M4 = ", Ms[[4]]}], summary}]];
(*Original anglesDeg check*)
Print[
TextCell[Style["\nInitial configuration (no switches applied):", Bold]]];
Print[MatrixForm[angles]];
Print[computeConjugateModularInfo[angles]];
(*Generate all switch combinations (from size 1 to 4)*)
combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
(*Evaluate each switched configuration*)results = Table[
Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
Do[switched = switchers[sw][switched], {sw, combo}];
passQ = And@@(checkConditionN0Degrees /@switched);
Print[Style["\nSwitch combination: ", Bold], name];
Print[Style["Switched anglesDeg:", Italic]];
Print[MatrixForm[switched]];
Print[computeConjugateModularInfo[switched]];
{name, passQ}], {combo, combinations}];]

```

```
(*)=====
====*)
(*)=====
NOT CHIMERA=====*)
(*)=====
====*)
Column[
{TextCell[Style["===== NOT CHIMERA =====", Blue,
  Bold, 16], "Text"],
TextCell[
Style["Fails conic, orthodiagonal & isogonal tests for all i=1, ...,
4  $\Rightarrow$  NOT chimera. Boundary-strip switches preserve these
failures as demonstrated in the NOT CONIC, NOT ORTHODIAGONAL,
and NOT ISOGONAL sections.", GrayLevel[0.3]], "Text"]
}]
```

Out[1789]=

```
===== CONDITION (N.0) =====
✓ All vertices satisfy (N.0).
```

Out[1792]=

```
===== CONDITION (N.3) =====
✓  $M1 = M2 = M3 = M4 = \frac{1}{2}$ 
```

Out[1798]=

```
===== CONDITION (N.4) =====
✓  $r1 = r2 = \frac{4}{3}$ ; ✓  $r3 = r4 = \frac{5}{2}$ 
✓  $s1 = s4 = 3$ ; ✓  $s2 = s3 = \frac{11}{6}$ 
```

Out[1808]=

```
===== CONDITION (N.5) =====

△ Approximate validation using
 $\epsilon$ -tolerance. For rigorous proof, see the referenced paper.

✓ Valid Combination Found ( $M < 1$ ):
e1 = 1, e2 = 1, e3 = 1
t1 = 0.K + 0.554485iK'
t2 = 0.K + 0.509302iK'
t3 = 0.K + 0.490698iK'
t4 = 0.K + 0.445515iK'
t1 + e1*t2 + e2*t3 + e3*t4 = 0.K + 2.iK'
```



Out[1811]=

===== OTHER PARAMETERS =====

$$\mathbf{u} = \frac{1}{2}$$

$$\sigma_1 = 135^\circ, \sigma_2 = \circ \left( 135 + \frac{90 \operatorname{ArcCos}\left[\frac{15\sqrt{7}}{44}\right]}{\pi} \right)$$

$$, \sigma_3 = \frac{90^\circ \left( \pi + \operatorname{ArcTan}\left[\frac{4\sqrt{7}}{3}\right] \right)}{\pi}, \sigma_4 = \circ \left( 135 - \frac{45 \operatorname{ArcTan}\left[\frac{24}{7}\right]}{\pi} \right)$$

$$\sigma_1 \approx 135.^\circ, \sigma_2 \approx 147.792^\circ, \sigma_3 \approx 127.087^\circ, \sigma_4 \approx 116.565^\circ$$

$$\cos\sigma_1 = -\frac{1}{\sqrt{2}}, \cos\sigma_2 = -\frac{3\sqrt{\frac{7}{22}}}{2}, \cos\sigma_3 = -\frac{2}{\sqrt{11}}, \cos\sigma_4 = -\frac{1}{\sqrt{5}}$$

$$\mathbf{f}_1 = 2, \mathbf{f}_2 = \frac{7}{4}, \mathbf{f}_3 = \frac{5}{3}, \mathbf{f}_4 = \frac{3}{2}$$

$$\mathbf{z}_1 = 1, \mathbf{z}_2 = \frac{4}{3}, \mathbf{z}_3 = \frac{3}{2}, \mathbf{z}_4 = 2$$

$$\mathbf{x}_1 = 3, \mathbf{x}_2 = 3, \mathbf{x}_3 = \frac{2}{3}, \mathbf{x}_4 = \frac{2}{3}$$

$$\mathbf{y}_1 = \frac{1}{2}, \mathbf{y}_2 = \frac{6}{5}, \mathbf{y}_3 = \frac{6}{5}, \mathbf{y}_4 = \frac{1}{2}$$

$$\mathbf{p}_1 = \frac{1}{\sqrt{3}}, \mathbf{p}_2 = \frac{1}{\sqrt{3}}, \mathbf{p}_3 = \sqrt{\frac{3}{2}}, \mathbf{p}_4 = \sqrt{\frac{3}{2}}$$

$$\mathbf{q}_1 = \sqrt{2}, \mathbf{q}_2 = \sqrt{\frac{5}{6}}, \mathbf{q}_3 = \sqrt{\frac{5}{6}}, \mathbf{q}_4 = \sqrt{2}$$

$$\mathbf{p}_1 \cdot \mathbf{q}_1 = \sqrt{\frac{2}{3}}, \mathbf{p}_2 \cdot \mathbf{q}_2 = \frac{\sqrt{\frac{5}{2}}}{3}, \mathbf{p}_3 \cdot \mathbf{q}_3 = \frac{\sqrt{5}}{2}, \mathbf{p}_4 \cdot \mathbf{q}_4 = \sqrt{3}$$

Out[1817]=

===== FLEXIBILITY (FLEXION 1) =====

$$P_1[Z, W1] = 0$$

$$P_2[Z, W2] = 0$$

$$P_3[U, W2] = 0$$

$$P_4[U, W1] = 0$$

Out[1825]=

===== FLEXIBILITY (FLEXION 2) =====

$$P_1[Z, W1] = 0$$

$$P_2[Z, W2] = 0$$

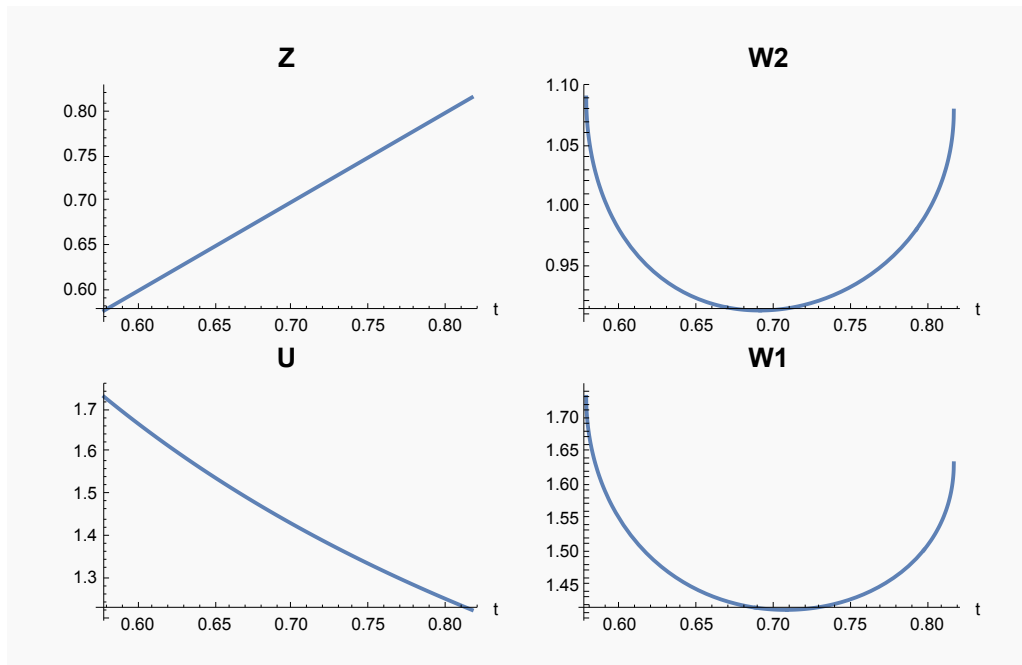
$$P_3[U, W2] = 0$$

$$P_4[U, W1] = 0$$

Out[1832]=

===== NOT TRIVIAL (FLEXION 1) =====

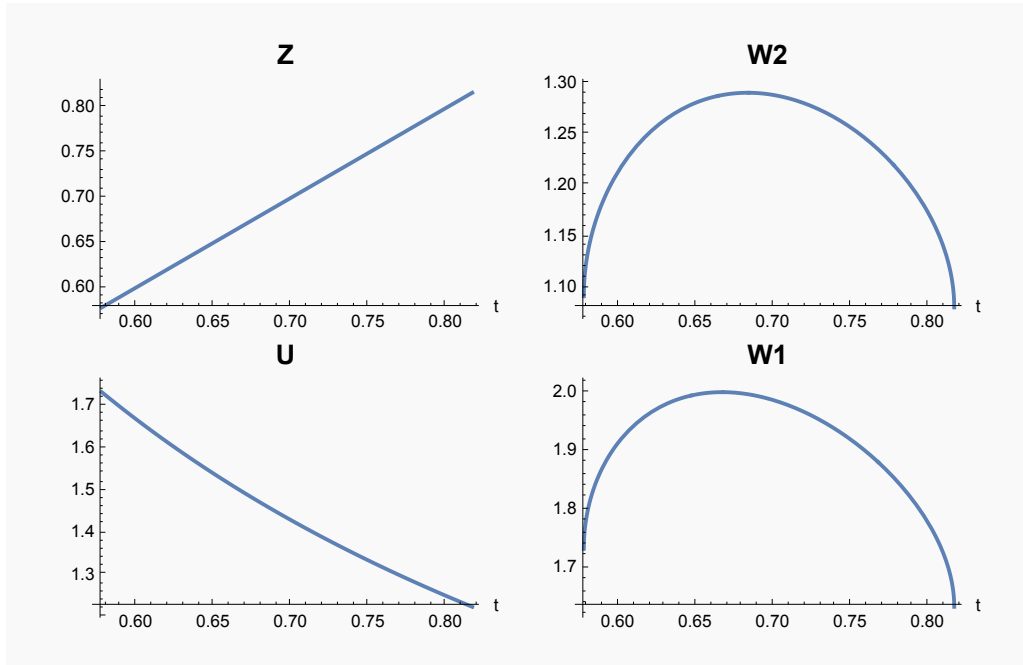
This configuration does not belong to the trivial class – even after switching the boundary strips – since none of the functions  $Z$ ,  $W2$ ,  $U$ , or  $W1$  is constant.



Out[1835]=

===== NOT TRIVIAL (FLEXION 2) =====

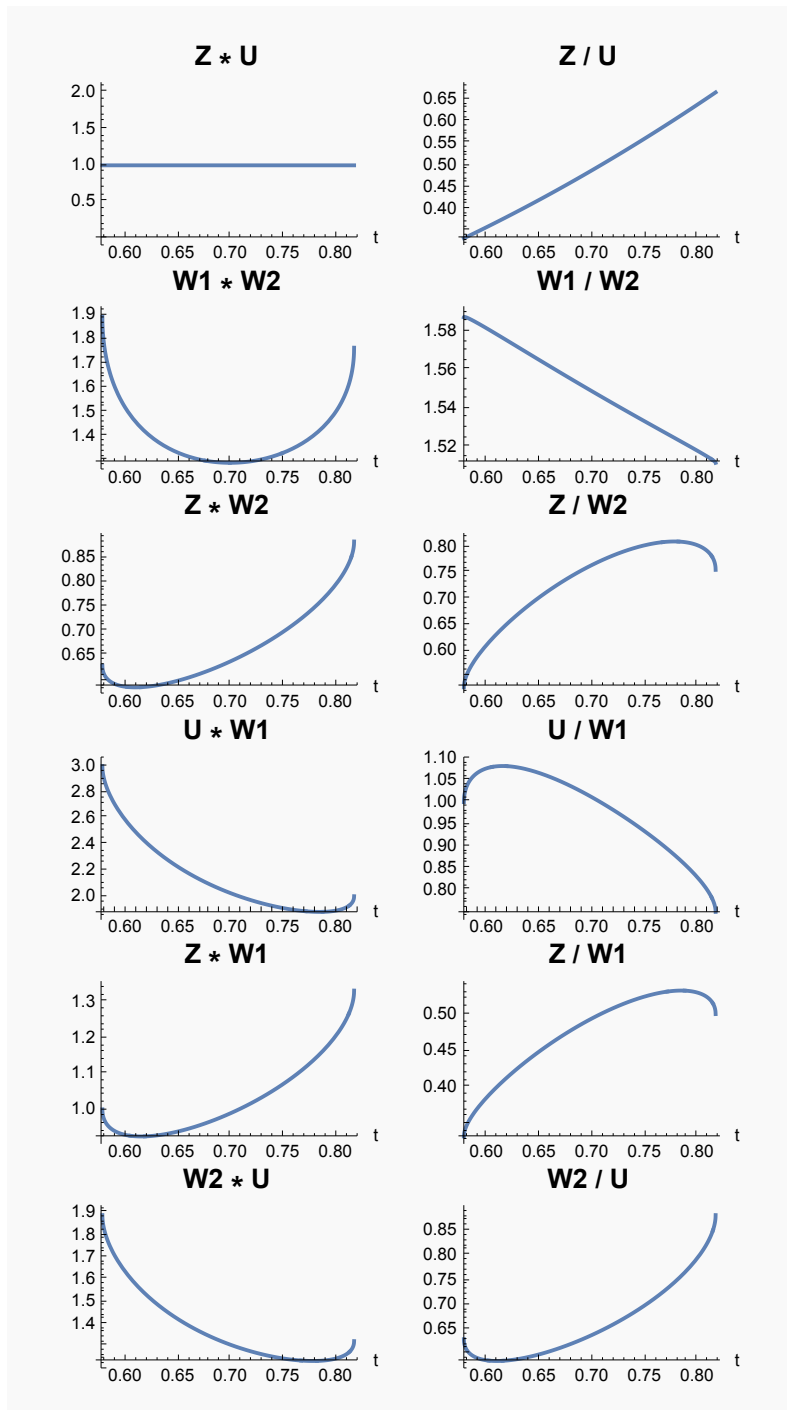
This configuration does not belong to the trivial class – even after switching the boundary strips – since none of the functions  $Z$ ,  $W2$ ,  $U$ , or  $W1$  is constant.



Out[1840]=

===== NOT LINEAR COMPOUND &  
NOT LINEAR CONJUGATE (FLEXION 1) =====

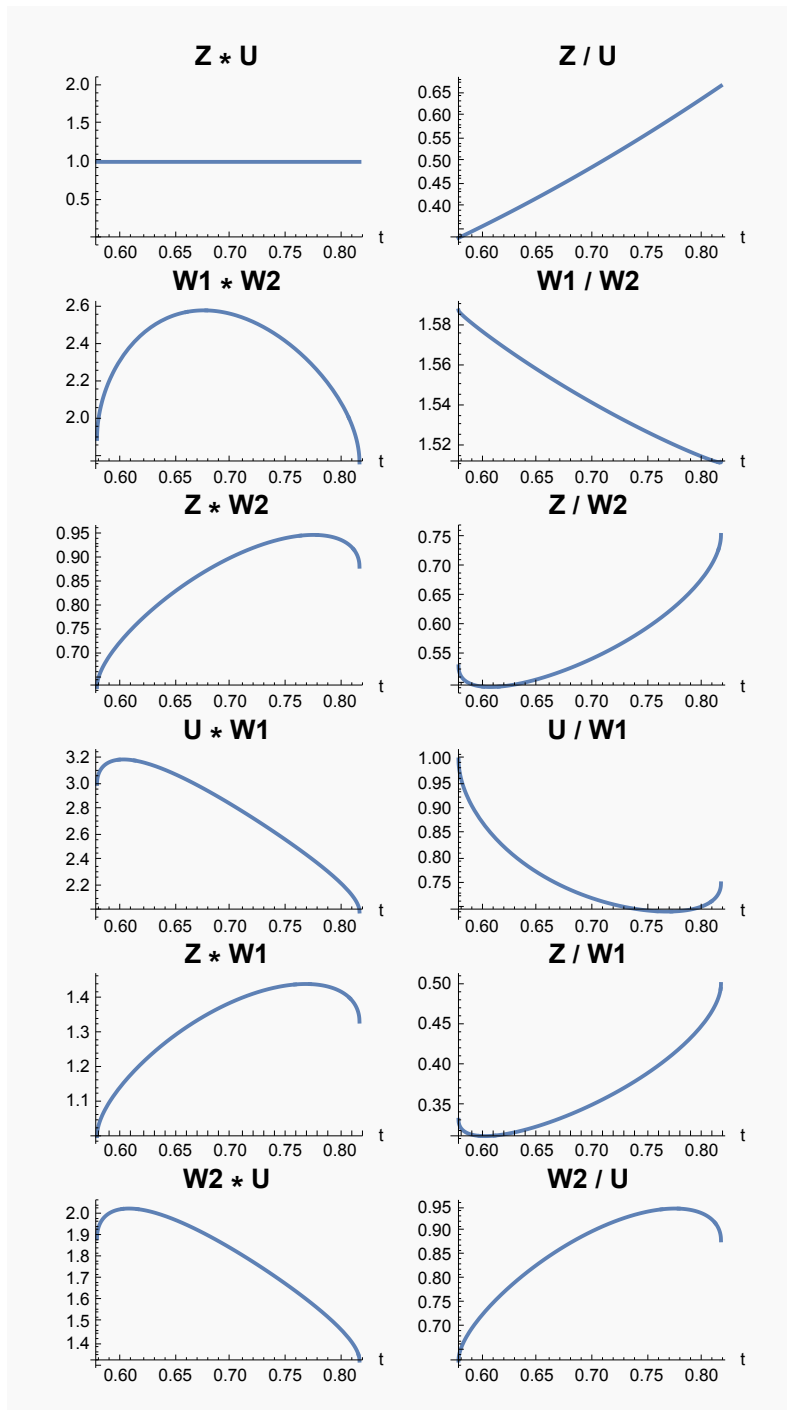
This configuration does not belong to the Linear compound class nor to the linear conjugate class – even after switching the boundary strips – since none of the functions  $ZU$ ,  $Z/U$ ,  $W1W2$ ,  $W1/W2$ ,  $ZW2$ ,  $Z/W2$ ,  $UW1$ , and  $U/W1$  is constant. In addition, none of  $ZW1$ ,  $Z/W1$ ,  $W2U$ ,  $W2/U$  is constant as well.



Out[1843]=

===== NOT LINEAR COMPOUND &  
 NOT LINEAR CONJUGATE (FLEXION 2) =====

This configuration does not belong to the Linear compound class nor to the linear conjugate class – even after switching the boundary strips – since none of the functions  $ZU$ ,  $Z/U$ ,  $W1W2$ ,  $W1/W2$ ,  $ZW2$ ,  $Z/W2$ ,  $UW1$ , and  $U/W1$  is constant. In addition, none of  $ZW1$ ,  $Z/W1$ ,  $W2U$ ,  $W2/U$  is constant as well.



Out[1848]=

===== NOT CONIC =====

Condition (N.0) is satisfied  $\Rightarrow$  this configuration  
is NOT equimodular-conic. Applying any boundary-strip  
switch still preserves (N.0), so no conic form emerges.

Out[1849]=

CONDITION (N.0) AFTER SWITCHING BOUNDARY STRIPS

**Right:** Condition (N.0) is still satisfied.

**Left:** Condition (N.0) is still satisfied.

**Lower:** Condition (N.0) is still satisfied.

**Upper:** Condition (N.0) is still satisfied.

**Right + Left:** Condition (N.0) is still satisfied.

**Right + Lower:** Condition (N.0) is still satisfied.

**Right + Upper:** Condition (N.0) is still satisfied.

**Left + Lower:** Condition (N.0) is still satisfied.

**Left + Upper:** Condition (N.0) is still satisfied.

**Lower + Upper:** Condition (N.0) is still satisfied.

**Right + Left + Lower:** Condition (N.0) is still satisfied.

**Right + Left + Upper:** Condition (N.0) is still satisfied.

**Right + Lower + Upper:** Condition (N.0) is still satisfied.

**Left + Lower + Upper:** Condition (N.0) is still satisfied.

**Right + Left + Lower + Upper:** Condition (N.0) is still satisfied.

Out[1850]=

===== NOT ORTHODIAGONAL =====

$\cos(\alpha_i) \cdot \cos(\gamma_i) \neq \cos(\beta_i) \cdot \cos(\delta_i)$  for each  $i = 1..4 \Rightarrow$  NOT  
orthodiagonal. Switching boundary strips does not correct this.

**Initial anglesDeg (no switches):**

$$\left( \begin{array}{ccc} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{18}}\right]}{\pi} \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{18}}\right]}{\pi} \end{array} \right) \begin{array}{l} 90 \\ 90 \\ 90 \\ 90 \end{array}$$

*Orthodiagonal check:*  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = -\frac{1}{5\sqrt{2}}$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = -\frac{1}{8\sqrt{22}}$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = \frac{1}{8\sqrt{22}}$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = \frac{1}{5\sqrt{2}}$$

**Switch combination:** Right

*Switched anglesDeg:*

$$\begin{pmatrix} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

*Orthodiagonal check:  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$*

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{1}{5\sqrt{2}} \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -\frac{1}{8\sqrt{22}} \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= \frac{1}{8\sqrt{22}} \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -\frac{1}{5\sqrt{2}} \end{aligned}$$

**Switch combination: Left**

*Switched anglesDeg:*

$$\begin{pmatrix} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

*Orthodiagonal check:  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$*

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -\frac{1}{5\sqrt{2}} \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= \frac{1}{8\sqrt{22}} \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -\frac{1}{8\sqrt{22}} \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{1}{5\sqrt{2}} \end{aligned}$$

**Switch combination: Lower**

*Switched anglesDeg:*

$$\begin{pmatrix} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

*Orthodiagonal check:  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$*

$$\begin{aligned}
\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{1}{5\sqrt{2}} \\
\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= \frac{1}{8\sqrt{22}} \\
\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= \frac{1}{8\sqrt{22}} \\
\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{1}{5\sqrt{2}}
\end{aligned}$$

**Switch combination:** Upper

*Switched anglesDeg:*

$$\begin{pmatrix}
\frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90
\end{pmatrix}$$

*Orthodiagonal check:*  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$

$$\begin{aligned}
\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -\frac{1}{5\sqrt{2}} \\
\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -\frac{1}{8\sqrt{22}} \\
\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -\frac{1}{8\sqrt{22}} \\
\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -\frac{1}{5\sqrt{2}}
\end{aligned}$$

**Switch combination:** Right + Left

*Switched anglesDeg:*

$$\begin{pmatrix}
\frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90
\end{pmatrix}$$

*Orthodiagonal check:*  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$

$$\begin{aligned}
\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{1}{5\sqrt{2}} \\
\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= \frac{1}{8\sqrt{22}} \\
\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -\frac{1}{8\sqrt{22}} \\
\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -\frac{1}{5\sqrt{2}}
\end{aligned}$$

**Switch combination:** Right + Lower

*Switched anglesDeg:*



$$\begin{pmatrix} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

*Orthodiagonal check:  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$*

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = -\frac{1}{5\sqrt{2}}$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = \frac{1}{8\sqrt{22}}$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = \frac{1}{8\sqrt{22}}$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = -\frac{1}{5\sqrt{2}}$$

**Switch combination: Right + Upper**

*Switched anglesDeg:*

$$\begin{pmatrix} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

*Orthodiagonal check:  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$*

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = \frac{1}{5\sqrt{2}}$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = -\frac{1}{8\sqrt{22}}$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = -\frac{1}{8\sqrt{22}}$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = \frac{1}{5\sqrt{2}}$$

**Switch combination: Left + Lower**

*Switched anglesDeg:*

$$\begin{pmatrix} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

*Orthodiagonal check:  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$*

$$\begin{aligned}
\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{1}{5\sqrt{2}} \\
\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -\frac{1}{8\sqrt{22}} \\
\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -\frac{1}{8\sqrt{22}} \\
\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{1}{5\sqrt{2}}
\end{aligned}$$

**Switch combination:** Left + Upper

*Switched anglesDeg:*

$$\left( \begin{array}{ccc} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{array} \right)$$

*Orthodiagonal check:*  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$

$$\begin{aligned}
\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -\frac{1}{5\sqrt{2}} \\
\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= \frac{1}{8\sqrt{22}} \\
\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= \frac{1}{8\sqrt{22}} \\
\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -\frac{1}{5\sqrt{2}}
\end{aligned}$$

**Switch combination:** Lower + Upper

*Switched anglesDeg:*

$$\left( \begin{array}{ccc} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{array} \right)$$

*Orthodiagonal check:*  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$

$$\begin{aligned}
\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{1}{5\sqrt{2}} \\
\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= \frac{1}{8\sqrt{22}} \\
\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -\frac{1}{8\sqrt{22}} \\
\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -\frac{1}{5\sqrt{2}}
\end{aligned}$$

**Switch combination:** Right + Left + Lower

*Switched anglesDeg:*

$$\begin{pmatrix} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

*Orthodiagonal check:  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$*

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = -\frac{1}{5\sqrt{2}}$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = -\frac{1}{8\sqrt{22}}$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = -\frac{1}{8\sqrt{22}}$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = -\frac{1}{5\sqrt{2}}$$

**Switch combination:** Right + Left + Upper

*Switched anglesDeg:*

$$\begin{pmatrix} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

*Orthodiagonal check:  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$*

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = \frac{1}{5\sqrt{2}}$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = \frac{1}{8\sqrt{22}}$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = \frac{1}{8\sqrt{22}}$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = \frac{1}{5\sqrt{2}}$$

**Switch combination:** Right + Lower + Upper

*Switched anglesDeg:*

$$\begin{pmatrix} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

*Orthodiagonal check:  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$*

$$\begin{aligned}\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -\frac{1}{5\sqrt{2}} \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= \frac{1}{8\sqrt{22}} \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -\frac{1}{8\sqrt{22}} \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{1}{5\sqrt{2}}\end{aligned}$$

**Switch combination:** Left + Lower + Upper

*Switched anglesDeg:*

$$\begin{pmatrix} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

*Orthodiagonal check:*  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$

$$\begin{aligned}\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= \frac{1}{5\sqrt{2}} \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -\frac{1}{8\sqrt{22}} \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= \frac{1}{8\sqrt{22}} \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -\frac{1}{5\sqrt{2}}\end{aligned}$$

**Switch combination:** Right + Left + Lower + Upper

*Switched anglesDeg:*

$$\begin{pmatrix} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

*Orthodiagonal check:*  $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$  for  $i = 1..4$

$$\begin{aligned}\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -\frac{1}{5\sqrt{2}} \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -\frac{1}{8\sqrt{22}} \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= \frac{1}{8\sqrt{22}} \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= \frac{1}{5\sqrt{2}}\end{aligned}$$

Out[1852]=

===== NOT ISOGONAL =====

Condition (N.0) holds AND for all  $i = 1..4$ :  $\alpha_i \neq \beta_i$ ,

$\alpha_i \neq \gamma_i$ ,  $\alpha_i \neq \delta_i$ ,  $\beta_i \neq \gamma_i$ ,  $\beta_i \neq \delta_i$ ,  $\gamma_i \neq \delta_i$ ,  $\alpha_i + \beta_i \neq$

$\pi \neq \gamma_i + \delta_i$ ,  $\alpha_i + \gamma_i \neq \pi \neq \beta_i + \delta_i$ ,  $\alpha_i + \delta_i \neq \pi \neq \beta_i + \gamma_i \Rightarrow$  NOT

isogonal. Switching boundary strips do not change this.

**Initial anglesDeg (no switches):**

$$\left( \begin{array}{ccc} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{array} \right)$$

*Angle relation checks for  $i = 1..4$ :*

#### Vertex 1

$$\begin{aligned} \alpha 1 - \beta 1 &= 55.305 \\ \alpha 1 - \gamma 1 &= -45.000 \\ \alpha 1 - \delta 1 &= -26.565 \\ \beta 1 - \gamma 1 &= -100.300 \\ \beta 1 - \delta 1 &= -81.870 \\ \gamma 1 - \delta 1 &= 18.435 \\ \alpha 1 + \beta 1 - 180 &= -108.430 \\ \gamma 1 + \delta 1 - 180 &= 18.435 \\ \alpha 1 + \gamma 1 - 180 &= -8.130 \\ \beta 1 + \delta 1 - 180 &= -81.870 \\ \alpha 1 + \delta 1 - 180 &= -26.565 \\ \beta 1 + \gamma 1 - 180 &= -63.435 \\ \alpha 1 + \beta 1 - \gamma 1 - \delta 1 &= -126.870 \\ \alpha 1 + \gamma 1 - \beta 1 - \delta 1 &= 73.740 \\ \alpha 1 + \delta 1 - \beta 1 - \gamma 1 &= 36.870 \end{aligned}$$

#### Vertex 2

$$\begin{aligned} \alpha 2 - \beta 2 &= 76.476 \\ \alpha 2 - \gamma 2 &= -25.028 \\ \alpha 2 - \delta 2 &= -4.323 \\ \beta 2 - \gamma 2 &= -101.500 \\ \beta 2 - \delta 2 &= -80.799 \\ \gamma 2 - \delta 2 &= 20.705 \\ \alpha 2 + \beta 2 - 180 &= -85.122 \\ \gamma 2 + \delta 2 - 180 &= 20.705 \\ \alpha 2 + \gamma 2 - 180 &= 16.382 \\ \beta 2 + \delta 2 - 180 &= -80.799 \\ \alpha 2 + \delta 2 - 180 &= -4.323 \\ \beta 2 + \gamma 2 - 180 &= -60.094 \\ \alpha 2 + \beta 2 - \gamma 2 - \delta 2 &= -105.830 \\ \alpha 2 + \gamma 2 - \beta 2 - \delta 2 &= 97.181 \\ \alpha 2 + \delta 2 - \beta 2 - \gamma 2 &= 55.771 \end{aligned}$$

#### Vertex 3

$$\begin{aligned} \alpha 3 - \beta 3 &= 76.476 \\ \alpha 3 - \gamma 3 &= 16.382 \\ \alpha 3 - \delta 3 &= -4.323 \\ \beta 3 - \gamma 3 &= -60.094 \\ \beta 3 - \delta 3 &= -80.799 \\ \gamma 3 - \delta 3 &= -20.705 \\ \alpha 3 + \beta 3 - 180 &= -85.122 \\ \gamma 3 + \delta 3 - 180 &= -20.705 \\ \alpha 3 + \gamma 3 - 180 &= -25.028 \\ \beta 3 + \delta 3 - 180 &= -80.799 \\ \alpha 3 + \delta 3 - 180 &= -4.323 \\ \beta 3 + \gamma 3 - 180 &= -101.500 \end{aligned}$$

$$\begin{aligned}\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -64.417 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 55.771 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 97.181\end{aligned}$$

**Vertex 4**

$$\begin{aligned}\alpha_4 - \beta_4 &= 55.305 \\ \alpha_4 - \gamma_4 &= -8.130 \\ \alpha_4 - \delta_4 &= -26.565 \\ \beta_4 - \gamma_4 &= -63.435 \\ \beta_4 - \delta_4 &= -81.870 \\ \gamma_4 - \delta_4 &= -18.435 \\ \alpha_4 + \beta_4 - 180 &= -108.430 \\ \gamma_4 + \delta_4 - 180 &= -18.435 \\ \alpha_4 + \gamma_4 - 180 &= -45.000 \\ \beta_4 + \delta_4 - 180 &= -81.870 \\ \alpha_4 + \delta_4 - 180 &= -26.565 \\ \beta_4 + \gamma_4 - 180 &= -100.300 \\ \alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -90.000 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 36.870 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 73.740\end{aligned}$$

**Switch combination: Right***Switched anglesDeg:*

$$\left( \begin{array}{ccc} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{array} \right)$$

*Angle relation checks for i = 1..4:***Vertex 1**

$$\begin{aligned}\alpha_1 - \beta_1 &= -108.430 \\ \alpha_1 - \gamma_1 &= -8.130 \\ \alpha_1 - \delta_1 &= -26.565 \\ \beta_1 - \gamma_1 &= 100.300 \\ \beta_1 - \delta_1 &= 81.870 \\ \gamma_1 - \delta_1 &= -18.435 \\ \alpha_1 + \beta_1 - 180 &= 55.305 \\ \gamma_1 + \delta_1 - 180 &= -18.435 \\ \alpha_1 + \gamma_1 - 180 &= -45.000 \\ \beta_1 + \delta_1 - 180 &= 81.870 \\ \alpha_1 + \delta_1 - 180 &= -26.565 \\ \beta_1 + \gamma_1 - 180 &= 63.435 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 73.740 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -126.870 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -90.000\end{aligned}$$

**Vertex 2**

$$\begin{aligned}\alpha_2 - \beta_2 &= 76.476 \\ \alpha_2 - \gamma_2 &= -25.028 \\ \alpha_2 - \delta_2 &= -4.323 \\ \beta_2 - \gamma_2 &= -101.500 \\ \beta_2 - \delta_2 &= -80.799 \\ \gamma_2 - \delta_2 &= 20.705 \\ \alpha_2 + \beta_2 - 180 &= -85.122 \\ \gamma_2 + \delta_2 - 180 &= 20.705\end{aligned}$$

$$\begin{aligned}
\alpha_2 + \gamma_2 - 180 &= 16.382 \\
\beta_2 + \delta_2 - 180 &= -80.799 \\
\alpha_2 + \delta_2 - 180 &= -4.323 \\
\beta_2 + \gamma_2 - 180 &= -60.094 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -105.830 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 97.181 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 55.771
\end{aligned}$$

**Vertex 3**

$$\begin{aligned}
\alpha_3 - \beta_3 &= 76.476 \\
\alpha_3 - \gamma_3 &= 16.382 \\
\alpha_3 - \delta_3 &= -4.323 \\
\beta_3 - \gamma_3 &= -60.094 \\
\beta_3 - \delta_3 &= -80.799 \\
\gamma_3 - \delta_3 &= -20.705 \\
\alpha_3 + \beta_3 - 180 &= -85.122 \\
\gamma_3 + \delta_3 - 180 &= -20.705 \\
\alpha_3 + \gamma_3 - 180 &= -25.028 \\
\beta_3 + \delta_3 - 180 &= -80.799 \\
\alpha_3 + \delta_3 - 180 &= -4.323 \\
\beta_3 + \gamma_3 - 180 &= -101.500 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -64.417 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 55.771 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 97.181
\end{aligned}$$

**Vertex 4**

$$\begin{aligned}
\alpha_4 - \beta_4 &= -108.430 \\
\alpha_4 - \gamma_4 &= -45.000 \\
\alpha_4 - \delta_4 &= -26.565 \\
\beta_4 - \gamma_4 &= 63.435 \\
\beta_4 - \delta_4 &= 81.870 \\
\gamma_4 - \delta_4 &= 18.435 \\
\alpha_4 + \beta_4 - 180 &= 55.305 \\
\gamma_4 + \delta_4 - 180 &= 18.435 \\
\alpha_4 + \gamma_4 - 180 &= -8.130 \\
\beta_4 + \delta_4 - 180 &= 81.870 \\
\alpha_4 + \delta_4 - 180 &= -26.565 \\
\beta_4 + \gamma_4 - 180 &= 100.300 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 36.870 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -90.000 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -126.870
\end{aligned}$$

**Switch combination:** Left*Switched anglesDeg:*

$$\left( \begin{array}{ccc} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} \end{array} \right) \begin{array}{l} 90 \\ 90 \\ 90 \\ 90 \end{array}$$

*Angle relation checks for i = 1..4:***Vertex 1**

$$\begin{aligned}
\alpha_1 - \beta_1 &= 55.305 \\
\alpha_1 - \gamma_1 &= -45.000 \\
\alpha_1 - \delta_1 &= -26.565 \\
\beta_1 - \gamma_1 &= 63.435 \\
\beta_1 - \delta_1 &= 81.870 \\
\gamma_1 - \delta_1 &= 18.435
\end{aligned}$$

$$\begin{aligned}
\beta_1 - \gamma_1 &= -100.300 \\
\beta_1 - \delta_1 &= -81.870 \\
\gamma_1 - \delta_1 &= 18.435 \\
\alpha_1 + \beta_1 - 180 &= -108.430 \\
\gamma_1 + \delta_1 - 180 &= 18.435 \\
\alpha_1 + \gamma_1 - 180 &= -8.130 \\
\beta_1 + \delta_1 - 180 &= -81.870 \\
\alpha_1 + \delta_1 - 180 &= -26.565 \\
\beta_1 + \gamma_1 - 180 &= -63.435 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -126.870 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 73.740 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 36.870
\end{aligned}$$

**Vertex 2**

$$\begin{aligned}
\alpha_2 - \beta_2 &= -85.122 \\
\alpha_2 - \gamma_2 &= 16.382 \\
\alpha_2 - \delta_2 &= -4.323 \\
\beta_2 - \gamma_2 &= 101.500 \\
\beta_2 - \delta_2 &= 80.799 \\
\gamma_2 - \delta_2 &= -20.705 \\
\alpha_2 + \beta_2 - 180 &= 76.476 \\
\gamma_2 + \delta_2 - 180 &= -20.705 \\
\alpha_2 + \gamma_2 - 180 &= -25.028 \\
\beta_2 + \delta_2 - 180 &= 80.799 \\
\alpha_2 + \delta_2 - 180 &= -4.323 \\
\beta_2 + \gamma_2 - 180 &= 60.094 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 97.181 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -105.830 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -64.417
\end{aligned}$$

**Vertex 3**

$$\begin{aligned}
\alpha_3 - \beta_3 &= -85.122 \\
\alpha_3 - \gamma_3 &= -25.028 \\
\alpha_3 - \delta_3 &= -4.323 \\
\beta_3 - \gamma_3 &= 60.094 \\
\beta_3 - \delta_3 &= 80.799 \\
\gamma_3 - \delta_3 &= 20.705 \\
\alpha_3 + \beta_3 - 180 &= 76.476 \\
\gamma_3 + \delta_3 - 180 &= 20.705 \\
\alpha_3 + \gamma_3 - 180 &= 16.382 \\
\beta_3 + \delta_3 - 180 &= 80.799 \\
\alpha_3 + \delta_3 - 180 &= -4.323 \\
\beta_3 + \gamma_3 - 180 &= 101.500 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 55.771 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -64.417 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -105.830
\end{aligned}$$

**Vertex 4**

$$\begin{aligned}
\alpha_4 - \beta_4 &= 55.305 \\
\alpha_4 - \gamma_4 &= -8.130 \\
\alpha_4 - \delta_4 &= -26.565 \\
\beta_4 - \gamma_4 &= -63.435 \\
\beta_4 - \delta_4 &= -81.870 \\
\gamma_4 - \delta_4 &= -18.435 \\
\alpha_4 + \beta_4 - 180 &= -108.430 \\
\gamma_4 + \delta_4 - 180 &= -18.435 \\
\alpha_4 + \gamma_4 - 180 &= -45.000 \\
\beta_4 + \delta_4 - 180 &= -81.870 \\
\alpha_4 + \delta_4 - 180 &= -26.565 \\
\beta_4 + \gamma_4 - 180 &= -100.300 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -90.000 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 36.870
\end{aligned}$$



$$\alpha_4 + \delta_4 - \beta_4 - \gamma_4 = 73.740$$

**Switch combination:** Lower

*Switched anglesDeg:*

$$\left( \begin{array}{ccc} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{array} \right)$$

*Angle relation checks for  $i = 1..4$ :*

#### Vertex 1

$$\begin{aligned} \alpha_1 - \beta_1 &= -55.305 \\ \alpha_1 - \gamma_1 &= 8.130 \\ \alpha_1 - \delta_1 &= 26.565 \\ \beta_1 - \gamma_1 &= 63.435 \\ \beta_1 - \delta_1 &= 81.870 \\ \gamma_1 - \delta_1 &= 18.435 \\ \alpha_1 + \beta_1 - 180 &= 108.430 \\ \gamma_1 + \delta_1 - 180 &= 18.435 \\ \alpha_1 + \gamma_1 - 180 &= 45.000 \\ \beta_1 + \delta_1 - 180 &= 81.870 \\ \alpha_1 + \delta_1 - 180 &= 26.565 \\ \beta_1 + \gamma_1 - 180 &= 100.300 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 90.000 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -36.870 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -73.740 \end{aligned}$$

#### Vertex 2

$$\begin{aligned} \alpha_2 - \beta_2 &= -76.476 \\ \alpha_2 - \gamma_2 &= -16.382 \\ \alpha_2 - \delta_2 &= 4.323 \\ \beta_2 - \gamma_2 &= 60.094 \\ \beta_2 - \delta_2 &= 80.799 \\ \gamma_2 - \delta_2 &= 20.705 \\ \alpha_2 + \beta_2 - 180 &= 85.122 \\ \gamma_2 + \delta_2 - 180 &= 20.705 \\ \alpha_2 + \gamma_2 - 180 &= 25.028 \\ \beta_2 + \delta_2 - 180 &= 80.799 \\ \alpha_2 + \delta_2 - 180 &= 4.323 \\ \beta_2 + \gamma_2 - 180 &= 101.500 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 64.417 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -55.771 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -97.181 \end{aligned}$$

#### Vertex 3

$$\begin{aligned} \alpha_3 - \beta_3 &= 76.476 \\ \alpha_3 - \gamma_3 &= 16.382 \\ \alpha_3 - \delta_3 &= -4.323 \\ \beta_3 - \gamma_3 &= -60.094 \\ \beta_3 - \delta_3 &= -80.799 \\ \gamma_3 - \delta_3 &= -20.705 \\ \alpha_3 + \beta_3 - 180 &= -85.122 \\ \gamma_3 + \delta_3 - 180 &= -20.705 \\ \alpha_3 + \gamma_3 - 180 &= -25.028 \\ \beta_3 + \delta_3 - 180 &= -80.799 \end{aligned}$$

$$\begin{aligned}
\beta 3 + \delta 3 - 180 &= -80.799 \\
\alpha 3 + \delta 3 - 180 &= -4.323 \\
\beta 3 + \gamma 3 - 180 &= -101.500 \\
\alpha 3 + \beta 3 - \gamma 3 - \delta 3 &= -64.417 \\
\alpha 3 + \gamma 3 - \beta 3 - \delta 3 &= 55.771 \\
\alpha 3 + \delta 3 - \beta 3 - \gamma 3 &= 97.181
\end{aligned}$$

**Vertex 4**

$$\begin{aligned}
\alpha 4 - \beta 4 &= 55.305 \\
\alpha 4 - \gamma 4 &= -8.130 \\
\alpha 4 - \delta 4 &= -26.565 \\
\beta 4 - \gamma 4 &= -63.435 \\
\beta 4 - \delta 4 &= -81.870 \\
\gamma 4 - \delta 4 &= -18.435 \\
\alpha 4 + \beta 4 - 180 &= -108.430 \\
\gamma 4 + \delta 4 - 180 &= -18.435 \\
\alpha 4 + \gamma 4 - 180 &= -45.000 \\
\beta 4 + \delta 4 - 180 &= -81.870 \\
\alpha 4 + \delta 4 - 180 &= -26.565 \\
\beta 4 + \gamma 4 - 180 &= -100.300 \\
\alpha 4 + \beta 4 - \gamma 4 - \delta 4 &= -90.000 \\
\alpha 4 + \gamma 4 - \beta 4 - \delta 4 &= 36.870 \\
\alpha 4 + \delta 4 - \beta 4 - \gamma 4 &= 73.740
\end{aligned}$$

**Switch combination: Upper***Switched anglesDeg:*

$$\left( \begin{array}{ccc}
\frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi}
\end{array} \right) \begin{array}{l} 90 \\ 90 \\ 90 \\ 90 \end{array}$$

*Angle relation checks for i = 1..4:***Vertex 1**

$$\begin{aligned}
\alpha 1 - \beta 1 &= 55.305 \\
\alpha 1 - \gamma 1 &= -45.000 \\
\alpha 1 - \delta 1 &= -26.565 \\
\beta 1 - \gamma 1 &= -100.300 \\
\beta 1 - \delta 1 &= -81.870 \\
\gamma 1 - \delta 1 &= 18.435 \\
\alpha 1 + \beta 1 - 180 &= -108.430 \\
\gamma 1 + \delta 1 - 180 &= 18.435 \\
\alpha 1 + \gamma 1 - 180 &= -8.130 \\
\beta 1 + \delta 1 - 180 &= -81.870 \\
\alpha 1 + \delta 1 - 180 &= -26.565 \\
\beta 1 + \gamma 1 - 180 &= -63.435 \\
\alpha 1 + \beta 1 - \gamma 1 - \delta 1 &= -126.870 \\
\alpha 1 + \gamma 1 - \beta 1 - \delta 1 &= 73.740 \\
\alpha 1 + \delta 1 - \beta 1 - \gamma 1 &= 36.870
\end{aligned}$$

**Vertex 2**

$$\begin{aligned}
\alpha 2 - \beta 2 &= 76.476 \\
\alpha 2 - \gamma 2 &= -25.028 \\
\alpha 2 - \delta 2 &= -4.323 \\
\beta 2 - \gamma 2 &= -101.500 \\
\beta 2 - \delta 2 &= -80.799
\end{aligned}$$

$$\begin{aligned}
\gamma_2 - \delta_2 &= 20.705 \\
\alpha_2 + \beta_2 - 180 &= -85.122 \\
\gamma_2 + \delta_2 - 180 &= 20.705 \\
\alpha_2 + \gamma_2 - 180 &= 16.382 \\
\beta_2 + \delta_2 - 180 &= -80.799 \\
\alpha_2 + \delta_2 - 180 &= -4.323 \\
\beta_2 + \gamma_2 - 180 &= -60.094 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -105.830 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 97.181 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 55.771
\end{aligned}$$

**Vertex 3**

$$\begin{aligned}
\alpha_3 - \beta_3 &= -76.476 \\
\alpha_3 - \gamma_3 &= 25.028 \\
\alpha_3 - \delta_3 &= 4.323 \\
\beta_3 - \gamma_3 &= 101.500 \\
\beta_3 - \delta_3 &= 80.799 \\
\gamma_3 - \delta_3 &= -20.705 \\
\alpha_3 + \beta_3 - 180 &= 85.122 \\
\gamma_3 + \delta_3 - 180 &= -20.705 \\
\alpha_3 + \gamma_3 - 180 &= -16.382 \\
\beta_3 + \delta_3 - 180 &= 80.799 \\
\alpha_3 + \delta_3 - 180 &= 4.323 \\
\beta_3 + \gamma_3 - 180 &= 60.094 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 105.830 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -97.181 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -55.771
\end{aligned}$$

**Vertex 4**

$$\begin{aligned}
\alpha_4 - \beta_4 &= -55.305 \\
\alpha_4 - \gamma_4 &= 45.000 \\
\alpha_4 - \delta_4 &= 26.565 \\
\beta_4 - \gamma_4 &= 100.300 \\
\beta_4 - \delta_4 &= 81.870 \\
\gamma_4 - \delta_4 &= -18.435 \\
\alpha_4 + \beta_4 - 180 &= 108.430 \\
\gamma_4 + \delta_4 - 180 &= -18.435 \\
\alpha_4 + \gamma_4 - 180 &= 8.130 \\
\beta_4 + \delta_4 - 180 &= 81.870 \\
\alpha_4 + \delta_4 - 180 &= 26.565 \\
\beta_4 + \gamma_4 - 180 &= 63.435 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 126.870 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -73.740 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -36.870
\end{aligned}$$

**Switch combination:** Right + Left

*Switched anglesDeg:*

$$\left( \begin{array}{ccc}
\frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90
\end{array} \right)$$

Angle relation checks for  $i = 1..4$ :

**Vertex 1**

$$\alpha_1 - \beta_1 = -108.430$$

$$\begin{aligned}
\alpha_1 - \beta_1 &= -108.430 \\
\alpha_1 - \gamma_1 &= -8.130 \\
\alpha_1 - \delta_1 &= -26.565 \\
\beta_1 - \gamma_1 &= 100.300 \\
\beta_1 - \delta_1 &= 81.870 \\
\gamma_1 - \delta_1 &= -18.435 \\
\alpha_1 + \beta_1 - 180 &= 55.305 \\
\gamma_1 + \delta_1 - 180 &= -18.435 \\
\alpha_1 + \gamma_1 - 180 &= -45.000 \\
\beta_1 + \delta_1 - 180 &= 81.870 \\
\alpha_1 + \delta_1 - 180 &= -26.565 \\
\beta_1 + \gamma_1 - 180 &= 63.435 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 73.740 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -126.870 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -90.000
\end{aligned}$$
**Vertex 2**

$$\begin{aligned}
\alpha_2 - \beta_2 &= -85.122 \\
\alpha_2 - \gamma_2 &= 16.382 \\
\alpha_2 - \delta_2 &= -4.323 \\
\beta_2 - \gamma_2 &= 101.500 \\
\beta_2 - \delta_2 &= 80.799 \\
\gamma_2 - \delta_2 &= -20.705 \\
\alpha_2 + \beta_2 - 180 &= 76.476 \\
\gamma_2 + \delta_2 - 180 &= -20.705 \\
\alpha_2 + \gamma_2 - 180 &= -25.028 \\
\beta_2 + \delta_2 - 180 &= 80.799 \\
\alpha_2 + \delta_2 - 180 &= -4.323 \\
\beta_2 + \gamma_2 - 180 &= 60.094 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 97.181 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -105.830 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -64.417
\end{aligned}$$
**Vertex 3**

$$\begin{aligned}
\alpha_3 - \beta_3 &= -85.122 \\
\alpha_3 - \gamma_3 &= -25.028 \\
\alpha_3 - \delta_3 &= -4.323 \\
\beta_3 - \gamma_3 &= 60.094 \\
\beta_3 - \delta_3 &= 80.799 \\
\gamma_3 - \delta_3 &= 20.705 \\
\alpha_3 + \beta_3 - 180 &= 76.476 \\
\gamma_3 + \delta_3 - 180 &= 20.705 \\
\alpha_3 + \gamma_3 - 180 &= 16.382 \\
\beta_3 + \delta_3 - 180 &= 80.799 \\
\alpha_3 + \delta_3 - 180 &= -4.323 \\
\beta_3 + \gamma_3 - 180 &= 101.500 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 55.771 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -64.417 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -105.830
\end{aligned}$$
**Vertex 4**

$$\begin{aligned}
\alpha_4 - \beta_4 &= -108.430 \\
\alpha_4 - \gamma_4 &= -45.000 \\
\alpha_4 - \delta_4 &= -26.565 \\
\beta_4 - \gamma_4 &= 63.435 \\
\beta_4 - \delta_4 &= 81.870 \\
\gamma_4 - \delta_4 &= 18.435 \\
\alpha_4 + \beta_4 - 180 &= 55.305 \\
\gamma_4 + \delta_4 - 180 &= 18.435 \\
\alpha_4 + \gamma_4 - 180 &= -8.130 \\
\beta_4 + \delta_4 - 180 &= 81.870 \\
\alpha_4 + \delta_4 - 180 &= -26.565
\end{aligned}$$

$$\begin{aligned}
\beta_4 + \gamma_4 - 180 &= 100.300 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 36.870 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -90.000 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -126.870
\end{aligned}$$

**Switch combination:** Right + Lower

*Switched anglesDeg:*

$$\left( \begin{array}{ccc}
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90
\end{array} \right)$$

*Angle relation checks for i = 1..4:*

#### Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= 108.430 \\
\alpha_1 - \gamma_1 &= 45.000 \\
\alpha_1 - \delta_1 &= 26.565 \\
\beta_1 - \gamma_1 &= -63.435 \\
\beta_1 - \delta_1 &= -81.870 \\
\gamma_1 - \delta_1 &= -18.435 \\
\alpha_1 + \beta_1 - 180 &= -55.305 \\
\gamma_1 + \delta_1 - 180 &= -18.435 \\
\alpha_1 + \gamma_1 - 180 &= 8.130 \\
\beta_1 + \delta_1 - 180 &= -81.870 \\
\alpha_1 + \delta_1 - 180 &= 26.565 \\
\beta_1 + \gamma_1 - 180 &= -100.300 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -36.870 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 90.000 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 126.870
\end{aligned}$$

#### Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= -76.476 \\
\alpha_2 - \gamma_2 &= -16.382 \\
\alpha_2 - \delta_2 &= 4.323 \\
\beta_2 - \gamma_2 &= 60.094 \\
\beta_2 - \delta_2 &= 80.799 \\
\gamma_2 - \delta_2 &= 20.705 \\
\alpha_2 + \beta_2 - 180 &= 85.122 \\
\gamma_2 + \delta_2 - 180 &= 20.705 \\
\alpha_2 + \gamma_2 - 180 &= 25.028 \\
\beta_2 + \delta_2 - 180 &= 80.799 \\
\alpha_2 + \delta_2 - 180 &= 4.323 \\
\beta_2 + \gamma_2 - 180 &= 101.500 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 64.417 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -55.771 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -97.181
\end{aligned}$$

#### Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= 76.476 \\
\alpha_3 - \gamma_3 &= 16.382 \\
\alpha_3 - \delta_3 &= -4.323 \\
\beta_3 - \gamma_3 &= -60.094 \\
\beta_3 - \delta_3 &= -80.799 \\
\gamma_3 - \delta_3 &= -20.705 \\
\alpha_3 + \beta_3 - 180 &= -85.122
\end{aligned}$$

$$\begin{aligned}
\gamma_3 + \delta_3 - 180 &= -20.705 \\
\alpha_3 + \gamma_3 - 180 &= -25.028 \\
\beta_3 + \delta_3 - 180 &= -80.799 \\
\alpha_3 + \delta_3 - 180 &= -4.323 \\
\beta_3 + \gamma_3 - 180 &= -101.500 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -64.417 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 55.771 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 97.181
\end{aligned}$$

**Vertex 4**

$$\begin{aligned}
\alpha_4 - \beta_4 &= -108.430 \\
\alpha_4 - \gamma_4 &= -45.000 \\
\alpha_4 - \delta_4 &= -26.565 \\
\beta_4 - \gamma_4 &= 63.435 \\
\beta_4 - \delta_4 &= 81.870 \\
\gamma_4 - \delta_4 &= 18.435 \\
\alpha_4 + \beta_4 - 180 &= 55.305 \\
\gamma_4 + \delta_4 - 180 &= 18.435 \\
\alpha_4 + \gamma_4 - 180 &= -8.130 \\
\beta_4 + \delta_4 - 180 &= 81.870 \\
\alpha_4 + \delta_4 - 180 &= -26.565 \\
\beta_4 + \gamma_4 - 180 &= 100.300 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 36.870 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -90.000 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -126.870
\end{aligned}$$

**Switch combination:** Right + Upper

*Switched anglesDeg:*

$$\left( \begin{array}{ccc}
\frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90
\end{array} \right)$$

*Angle relation checks for i = 1..4:*

**Vertex 1**

$$\begin{aligned}
\alpha_1 - \beta_1 &= -108.430 \\
\alpha_1 - \gamma_1 &= -8.130 \\
\alpha_1 - \delta_1 &= -26.565 \\
\beta_1 - \gamma_1 &= 100.300 \\
\beta_1 - \delta_1 &= 81.870 \\
\gamma_1 - \delta_1 &= -18.435 \\
\alpha_1 + \beta_1 - 180 &= 55.305 \\
\gamma_1 + \delta_1 - 180 &= -18.435 \\
\alpha_1 + \gamma_1 - 180 &= -45.000 \\
\beta_1 + \delta_1 - 180 &= 81.870 \\
\alpha_1 + \delta_1 - 180 &= -26.565 \\
\beta_1 + \gamma_1 - 180 &= 63.435 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 73.740 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -126.870 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -90.000
\end{aligned}$$

**Vertex 2**

$$\begin{aligned}
\alpha_2 - \beta_2 &= 76.476 \\
\alpha_2 - \gamma_2 &= -25.028 \\
\alpha_2 - \delta_2 &= -45.000
\end{aligned}$$

$$\begin{aligned}
\alpha_2 - \delta_2 &= -4.323 \\
\beta_2 - \gamma_2 &= -101.500 \\
\beta_2 - \delta_2 &= -80.799 \\
\gamma_2 - \delta_2 &= 20.705 \\
\alpha_2 + \beta_2 - 180 &= -85.122 \\
\gamma_2 + \delta_2 - 180 &= 20.705 \\
\alpha_2 + \gamma_2 - 180 &= 16.382 \\
\beta_2 + \delta_2 - 180 &= -80.799 \\
\alpha_2 + \delta_2 - 180 &= -4.323 \\
\beta_2 + \gamma_2 - 180 &= -60.094 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -105.830 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 97.181 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 55.771
\end{aligned}$$

### Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= -76.476 \\
\alpha_3 - \gamma_3 &= 25.028 \\
\alpha_3 - \delta_3 &= 4.323 \\
\beta_3 - \gamma_3 &= 101.500 \\
\beta_3 - \delta_3 &= 80.799 \\
\gamma_3 - \delta_3 &= -20.705 \\
\alpha_3 + \beta_3 - 180 &= 85.122 \\
\gamma_3 + \delta_3 - 180 &= -20.705 \\
\alpha_3 + \gamma_3 - 180 &= -16.382 \\
\beta_3 + \delta_3 - 180 &= 80.799 \\
\alpha_3 + \delta_3 - 180 &= 4.323 \\
\beta_3 + \gamma_3 - 180 &= 60.094 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 105.830 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -97.181 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -55.771
\end{aligned}$$

### Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= 108.430 \\
\alpha_4 - \gamma_4 &= 8.130 \\
\alpha_4 - \delta_4 &= 26.565 \\
\beta_4 - \gamma_4 &= -100.300 \\
\beta_4 - \delta_4 &= -81.870 \\
\gamma_4 - \delta_4 &= 18.435 \\
\alpha_4 + \beta_4 - 180 &= -55.305 \\
\gamma_4 + \delta_4 - 180 &= 18.435 \\
\alpha_4 + \gamma_4 - 180 &= 45.000 \\
\beta_4 + \delta_4 - 180 &= -81.870 \\
\alpha_4 + \delta_4 - 180 &= 26.565 \\
\beta_4 + \gamma_4 - 180 &= -63.435 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -73.740 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 126.870 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 90.000
\end{aligned}$$

**Switch combination:** Left + Lower

*Switched anglesDeg:*

$$\left( \begin{array}{ccc}
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
\frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90
\end{array} \right)$$

Angle relation checks for  $i = 1..4$ :

#### Vertex 1

$$\begin{aligned}\alpha_1 - \beta_1 &= -55.305 \\ \alpha_1 - \gamma_1 &= 8.130 \\ \alpha_1 - \delta_1 &= 26.565 \\ \beta_1 - \gamma_1 &= 63.435 \\ \beta_1 - \delta_1 &= 81.870 \\ \gamma_1 - \delta_1 &= 18.435 \\ \alpha_1 + \beta_1 - 180 &= 108.430 \\ \gamma_1 + \delta_1 - 180 &= 18.435 \\ \alpha_1 + \gamma_1 - 180 &= 45.000 \\ \beta_1 + \delta_1 - 180 &= 81.870 \\ \alpha_1 + \delta_1 - 180 &= 26.565 \\ \beta_1 + \gamma_1 - 180 &= 100.300 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 90.000 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -36.870 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -73.740\end{aligned}$$

#### Vertex 2

$$\begin{aligned}\alpha_2 - \beta_2 &= 85.122 \\ \alpha_2 - \gamma_2 &= 25.028 \\ \alpha_2 - \delta_2 &= 4.323 \\ \beta_2 - \gamma_2 &= -60.094 \\ \beta_2 - \delta_2 &= -80.799 \\ \gamma_2 - \delta_2 &= -20.705 \\ \alpha_2 + \beta_2 - 180 &= -76.476 \\ \gamma_2 + \delta_2 - 180 &= -20.705 \\ \alpha_2 + \gamma_2 - 180 &= -16.382 \\ \beta_2 + \delta_2 - 180 &= -80.799 \\ \alpha_2 + \delta_2 - 180 &= 4.323 \\ \beta_2 + \gamma_2 - 180 &= -101.500 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -55.771 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 64.417 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 105.830\end{aligned}$$

#### Vertex 3

$$\begin{aligned}\alpha_3 - \beta_3 &= -85.122 \\ \alpha_3 - \gamma_3 &= -25.028 \\ \alpha_3 - \delta_3 &= -4.323 \\ \beta_3 - \gamma_3 &= 60.094 \\ \beta_3 - \delta_3 &= 80.799 \\ \gamma_3 - \delta_3 &= 20.705 \\ \alpha_3 + \beta_3 - 180 &= 76.476 \\ \gamma_3 + \delta_3 - 180 &= 20.705 \\ \alpha_3 + \gamma_3 - 180 &= 16.382 \\ \beta_3 + \delta_3 - 180 &= 80.799 \\ \alpha_3 + \delta_3 - 180 &= -4.323 \\ \beta_3 + \gamma_3 - 180 &= 101.500 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 55.771 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -64.417 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -105.830\end{aligned}$$

#### Vertex 4

$$\begin{aligned}\alpha_4 - \beta_4 &= 55.305 \\ \alpha_4 - \gamma_4 &= -8.130 \\ \alpha_4 - \delta_4 &= -26.565 \\ \beta_4 - \gamma_4 &= -63.435 \\ \beta_4 - \delta_4 &= -81.870 \\ \gamma_4 - \delta_4 &= -18.435 \\ \alpha_4 + \beta_4 - 180 &= -108.430 \\ \gamma_4 + \delta_4 - 180 &= -18.435\end{aligned}$$



$$\begin{aligned}
\alpha_4 + \gamma_4 - 180 &= -45.000 \\
\beta_4 + \delta_4 - 180 &= -81.870 \\
\alpha_4 + \delta_4 - 180 &= -26.565 \\
\beta_4 + \gamma_4 - 180 &= -100.300 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -90.000 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 36.870 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 73.740
\end{aligned}$$

**Switch combination:** Left + Upper

*Switched anglesDeg:*

$$\left( \begin{array}{ccc}
\frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90
\end{array} \right)$$

*Angle relation checks for i = 1..4:*

#### Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= 55.305 \\
\alpha_1 - \gamma_1 &= -45.000 \\
\alpha_1 - \delta_1 &= -26.565 \\
\beta_1 - \gamma_1 &= -100.300 \\
\beta_1 - \delta_1 &= -81.870 \\
\gamma_1 - \delta_1 &= 18.435 \\
\alpha_1 + \beta_1 - 180 &= -108.430 \\
\gamma_1 + \delta_1 - 180 &= 18.435 \\
\alpha_1 + \gamma_1 - 180 &= -8.130 \\
\beta_1 + \delta_1 - 180 &= -81.870 \\
\alpha_1 + \delta_1 - 180 &= -26.565 \\
\beta_1 + \gamma_1 - 180 &= -63.435 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -126.870 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 73.740 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 36.870
\end{aligned}$$

#### Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= -85.122 \\
\alpha_2 - \gamma_2 &= 16.382 \\
\alpha_2 - \delta_2 &= -4.323 \\
\beta_2 - \gamma_2 &= 101.500 \\
\beta_2 - \delta_2 &= 80.799 \\
\gamma_2 - \delta_2 &= -20.705 \\
\alpha_2 + \beta_2 - 180 &= 76.476 \\
\gamma_2 + \delta_2 - 180 &= -20.705 \\
\alpha_2 + \gamma_2 - 180 &= -25.028 \\
\beta_2 + \delta_2 - 180 &= 80.799 \\
\alpha_2 + \delta_2 - 180 &= -4.323 \\
\beta_2 + \gamma_2 - 180 &= 60.094 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 97.181 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -105.830 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -64.417
\end{aligned}$$

#### Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= 85.122 \\
\alpha_3 - \gamma_3 &= -16.382 \\
\alpha_3 - \delta_3 &= 4.323 \\
\beta_3 - \gamma_3 &= -101.500
\end{aligned}$$

$$\begin{aligned}
\beta_3 - \gamma_3 &= -101.300 \\
\beta_3 - \delta_3 &= -80.799 \\
\gamma_3 - \delta_3 &= 20.705 \\
\alpha_3 + \beta_3 - 180 &= -76.476 \\
\gamma_3 + \delta_3 - 180 &= 20.705 \\
\alpha_3 + \gamma_3 - 180 &= 25.028 \\
\beta_3 + \delta_3 - 180 &= -80.799 \\
\alpha_3 + \delta_3 - 180 &= 4.323 \\
\beta_3 + \gamma_3 - 180 &= -60.094 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -97.181 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 105.830 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 64.417
\end{aligned}$$

**Vertex 4**

$$\begin{aligned}
\alpha_4 - \beta_4 &= -55.305 \\
\alpha_4 - \gamma_4 &= 45.000 \\
\alpha_4 - \delta_4 &= 26.565 \\
\beta_4 - \gamma_4 &= 100.300 \\
\beta_4 - \delta_4 &= 81.870 \\
\gamma_4 - \delta_4 &= -18.435 \\
\alpha_4 + \beta_4 - 180 &= 108.430 \\
\gamma_4 + \delta_4 - 180 &= -18.435 \\
\alpha_4 + \gamma_4 - 180 &= 8.130 \\
\beta_4 + \delta_4 - 180 &= 81.870 \\
\alpha_4 + \delta_4 - 180 &= 26.565 \\
\beta_4 + \gamma_4 - 180 &= 63.435 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 126.870 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -73.740 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -36.870
\end{aligned}$$

**Switch combination:** Lower + Upper*Switched anglesDeg:*

$$\left( \begin{array}{ccc}
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} \quad 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} \quad 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} \quad 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} \quad 90
\end{array} \right)$$

*Angle relation checks for i = 1..4:***Vertex 1**

$$\begin{aligned}
\alpha_1 - \beta_1 &= -55.305 \\
\alpha_1 - \gamma_1 &= 8.130 \\
\alpha_1 - \delta_1 &= 26.565 \\
\beta_1 - \gamma_1 &= 63.435 \\
\beta_1 - \delta_1 &= 81.870 \\
\gamma_1 - \delta_1 &= 18.435 \\
\alpha_1 + \beta_1 - 180 &= 108.430 \\
\gamma_1 + \delta_1 - 180 &= 18.435 \\
\alpha_1 + \gamma_1 - 180 &= 45.000 \\
\beta_1 + \delta_1 - 180 &= 81.870 \\
\alpha_1 + \delta_1 - 180 &= 26.565 \\
\beta_1 + \gamma_1 - 180 &= 100.300 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 90.000 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -36.870 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -73.740
\end{aligned}$$

**Vertex 2**

$$\begin{aligned}
\alpha_2 - \beta_2 &= -76.476 \\
\alpha_2 - \gamma_2 &= -16.382 \\
\alpha_2 - \delta_2 &= 4.323 \\
\beta_2 - \gamma_2 &= 60.094 \\
\beta_2 - \delta_2 &= 80.799 \\
\gamma_2 - \delta_2 &= 20.705 \\
\alpha_2 + \beta_2 - 180 &= 85.122 \\
\gamma_2 + \delta_2 - 180 &= 20.705 \\
\alpha_2 + \gamma_2 - 180 &= 25.028 \\
\beta_2 + \delta_2 - 180 &= 80.799 \\
\alpha_2 + \delta_2 - 180 &= 4.323 \\
\beta_2 + \gamma_2 - 180 &= 101.500 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 64.417 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -55.771 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -97.181
\end{aligned}$$

**Vertex 3**

$$\begin{aligned}
\alpha_3 - \beta_3 &= -76.476 \\
\alpha_3 - \gamma_3 &= 25.028 \\
\alpha_3 - \delta_3 &= 4.323 \\
\beta_3 - \gamma_3 &= 101.500 \\
\beta_3 - \delta_3 &= 80.799 \\
\gamma_3 - \delta_3 &= -20.705 \\
\alpha_3 + \beta_3 - 180 &= 85.122 \\
\gamma_3 + \delta_3 - 180 &= -20.705 \\
\alpha_3 + \gamma_3 - 180 &= -16.382 \\
\beta_3 + \delta_3 - 180 &= 80.799 \\
\alpha_3 + \delta_3 - 180 &= 4.323 \\
\beta_3 + \gamma_3 - 180 &= 60.094 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 105.830 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -97.181 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -55.771
\end{aligned}$$

**Vertex 4**

$$\begin{aligned}
\alpha_4 - \beta_4 &= -55.305 \\
\alpha_4 - \gamma_4 &= 45.000 \\
\alpha_4 - \delta_4 &= 26.565 \\
\beta_4 - \gamma_4 &= 100.300 \\
\beta_4 - \delta_4 &= 81.870 \\
\gamma_4 - \delta_4 &= -18.435 \\
\alpha_4 + \beta_4 - 180 &= 108.430 \\
\gamma_4 + \delta_4 - 180 &= -18.435 \\
\alpha_4 + \gamma_4 - 180 &= 8.130 \\
\beta_4 + \delta_4 - 180 &= 81.870 \\
\alpha_4 + \delta_4 - 180 &= 26.565 \\
\beta_4 + \gamma_4 - 180 &= 63.435 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 126.870 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -73.740 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -36.870
\end{aligned}$$

**Switch combination:** Right + Left + Lower

*Switched anglesDeg:*

$$\begin{pmatrix} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \end{pmatrix}$$

Angle relation checks for  $i = 1..4$ :

#### Vertex 1

$$\begin{aligned} \alpha_1 - \beta_1 &= 108.430 \\ \alpha_1 - \gamma_1 &= 45.000 \\ \alpha_1 - \delta_1 &= 26.565 \\ \beta_1 - \gamma_1 &= -63.435 \\ \beta_1 - \delta_1 &= -81.870 \\ \gamma_1 - \delta_1 &= -18.435 \\ \alpha_1 + \beta_1 - 180 &= -55.305 \\ \gamma_1 + \delta_1 - 180 &= -18.435 \\ \alpha_1 + \gamma_1 - 180 &= 8.130 \\ \beta_1 + \delta_1 - 180 &= -81.870 \\ \alpha_1 + \delta_1 - 180 &= 26.565 \\ \beta_1 + \gamma_1 - 180 &= -100.300 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -36.870 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 90.000 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 126.870 \end{aligned}$$

#### Vertex 2

$$\begin{aligned} \alpha_2 - \beta_2 &= 85.122 \\ \alpha_2 - \gamma_2 &= 25.028 \\ \alpha_2 - \delta_2 &= 4.323 \\ \beta_2 - \gamma_2 &= -60.094 \\ \beta_2 - \delta_2 &= -80.799 \\ \gamma_2 - \delta_2 &= -20.705 \\ \alpha_2 + \beta_2 - 180 &= -76.476 \\ \gamma_2 + \delta_2 - 180 &= -20.705 \\ \alpha_2 + \gamma_2 - 180 &= -16.382 \\ \beta_2 + \delta_2 - 180 &= -80.799 \\ \alpha_2 + \delta_2 - 180 &= 4.323 \\ \beta_2 + \gamma_2 - 180 &= -101.500 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -55.771 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 64.417 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 105.830 \end{aligned}$$

#### Vertex 3

$$\begin{aligned} \alpha_3 - \beta_3 &= -85.122 \\ \alpha_3 - \gamma_3 &= -25.028 \\ \alpha_3 - \delta_3 &= -4.323 \\ \beta_3 - \gamma_3 &= 60.094 \\ \beta_3 - \delta_3 &= 80.799 \\ \gamma_3 - \delta_3 &= 20.705 \\ \alpha_3 + \beta_3 - 180 &= 76.476 \\ \gamma_3 + \delta_3 - 180 &= 20.705 \\ \alpha_3 + \gamma_3 - 180 &= 16.382 \\ \beta_3 + \delta_3 - 180 &= 80.799 \\ \alpha_3 + \delta_3 - 180 &= -4.323 \\ \beta_3 + \gamma_3 - 180 &= 101.500 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 55.771 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -64.417 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -105.830 \end{aligned}$$

**Vertex 4**

$$\begin{aligned}
\alpha_4 - \beta_4 &= -108.430 \\
\alpha_4 - \gamma_4 &= -45.000 \\
\alpha_4 - \delta_4 &= -26.565 \\
\beta_4 - \gamma_4 &= 63.435 \\
\beta_4 - \delta_4 &= 81.870 \\
\gamma_4 - \delta_4 &= 18.435 \\
\alpha_4 + \beta_4 - 180 &= 55.305 \\
\gamma_4 + \delta_4 - 180 &= 18.435 \\
\alpha_4 + \gamma_4 - 180 &= -8.130 \\
\beta_4 + \delta_4 - 180 &= 81.870 \\
\alpha_4 + \delta_4 - 180 &= -26.565 \\
\beta_4 + \gamma_4 - 180 &= 100.300 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 36.870 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -90.000 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -126.870
\end{aligned}$$

**Switch combination:** Right + Left + Upper

*Switched anglesDeg:*

$$\left( \begin{array}{ccc}
\frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{18}}\right]}{\pi} & 90
\end{array} \right)$$

*Angle relation checks for  $i = 1..4$ :*

**Vertex 1**

$$\begin{aligned}
\alpha_1 - \beta_1 &= -108.430 \\
\alpha_1 - \gamma_1 &= -8.130 \\
\alpha_1 - \delta_1 &= -26.565 \\
\beta_1 - \gamma_1 &= 100.300 \\
\beta_1 - \delta_1 &= 81.870 \\
\gamma_1 - \delta_1 &= -18.435 \\
\alpha_1 + \beta_1 - 180 &= 55.305 \\
\gamma_1 + \delta_1 - 180 &= -18.435 \\
\alpha_1 + \gamma_1 - 180 &= -45.000 \\
\beta_1 + \delta_1 - 180 &= 81.870 \\
\alpha_1 + \delta_1 - 180 &= -26.565 \\
\beta_1 + \gamma_1 - 180 &= 63.435 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 73.740 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -126.870 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -90.000
\end{aligned}$$

**Vertex 2**

$$\begin{aligned}
\alpha_2 - \beta_2 &= -85.122 \\
\alpha_2 - \gamma_2 &= 16.382 \\
\alpha_2 - \delta_2 &= -4.323 \\
\beta_2 - \gamma_2 &= 101.500 \\
\beta_2 - \delta_2 &= 80.799 \\
\gamma_2 - \delta_2 &= -20.705 \\
\alpha_2 + \beta_2 - 180 &= 76.476 \\
\gamma_2 + \delta_2 - 180 &= -20.705 \\
\alpha_2 + \gamma_2 - 180 &= -25.028 \\
\beta_2 + \delta_2 - 180 &= 80.799 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -4.323
\end{aligned}$$

$$\begin{aligned}\alpha_2 + \gamma_2 - 180 &= -11.929 \\ \beta_2 + \gamma_2 - 180 &= 60.094 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 97.181 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -105.830 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -64.417\end{aligned}$$

**Vertex 3**

$$\begin{aligned}\alpha_3 - \beta_3 &= 85.122 \\ \alpha_3 - \gamma_3 &= -16.382 \\ \alpha_3 - \delta_3 &= 4.323 \\ \beta_3 - \gamma_3 &= -101.500 \\ \beta_3 - \delta_3 &= -80.799 \\ \gamma_3 - \delta_3 &= 20.705 \\ \alpha_3 + \beta_3 - 180 &= -76.476 \\ \gamma_3 + \delta_3 - 180 &= 20.705 \\ \alpha_3 + \gamma_3 - 180 &= 25.028 \\ \beta_3 + \delta_3 - 180 &= -80.799 \\ \alpha_3 + \delta_3 - 180 &= 4.323 \\ \beta_3 + \gamma_3 - 180 &= -60.094 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -97.181 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 105.830 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 64.417\end{aligned}$$

**Vertex 4**

$$\begin{aligned}\alpha_4 - \beta_4 &= 108.430 \\ \alpha_4 - \gamma_4 &= 8.130 \\ \alpha_4 - \delta_4 &= 26.565 \\ \beta_4 - \gamma_4 &= -100.300 \\ \beta_4 - \delta_4 &= -81.870 \\ \gamma_4 - \delta_4 &= 18.435 \\ \alpha_4 + \beta_4 - 180 &= -55.305 \\ \gamma_4 + \delta_4 - 180 &= 18.435 \\ \alpha_4 + \gamma_4 - 180 &= 45.000 \\ \beta_4 + \delta_4 - 180 &= -81.870 \\ \alpha_4 + \delta_4 - 180 &= 26.565 \\ \beta_4 + \gamma_4 - 180 &= -63.435 \\ \alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -73.740 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 126.870 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 90.000\end{aligned}$$

**Switch combination:** Right + Lower + Upper*Switched anglesDeg:*

$$\left( \begin{array}{ccc} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{array} \right)$$

*Angle relation checks for i = 1..4:***Vertex 1**

$$\begin{aligned}\alpha_1 - \beta_1 &= 108.430 \\ \alpha_1 - \gamma_1 &= 45.000 \\ \alpha_1 - \delta_1 &= 26.565 \\ \beta_1 - \gamma_1 &= -63.435 \\ \beta_1 - \delta_1 &= -81.870 \\ \gamma_1 - \delta_1 &= -18.435\end{aligned}$$

$$\begin{aligned}
\alpha_1 + \beta_1 - 180 &= -55.305 \\
\gamma_1 + \delta_1 - 180 &= -18.435 \\
\alpha_1 + \gamma_1 - 180 &= 8.130 \\
\beta_1 + \delta_1 - 180 &= -81.870 \\
\alpha_1 + \delta_1 - 180 &= 26.565 \\
\beta_1 + \gamma_1 - 180 &= -100.300 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -36.870 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 90.000 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 126.870
\end{aligned}$$

**Vertex 2**

$$\begin{aligned}
\alpha_2 - \beta_2 &= -76.476 \\
\alpha_2 - \gamma_2 &= -16.382 \\
\alpha_2 - \delta_2 &= 4.323 \\
\beta_2 - \gamma_2 &= 60.094 \\
\beta_2 - \delta_2 &= 80.799 \\
\gamma_2 - \delta_2 &= 20.705 \\
\alpha_2 + \beta_2 - 180 &= 85.122 \\
\gamma_2 + \delta_2 - 180 &= 20.705 \\
\alpha_2 + \gamma_2 - 180 &= 25.028 \\
\beta_2 + \delta_2 - 180 &= 80.799 \\
\alpha_2 + \delta_2 - 180 &= 4.323 \\
\beta_2 + \gamma_2 - 180 &= 101.500 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 64.417 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -55.771 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -97.181
\end{aligned}$$

**Vertex 3**

$$\begin{aligned}
\alpha_3 - \beta_3 &= -76.476 \\
\alpha_3 - \gamma_3 &= 25.028 \\
\alpha_3 - \delta_3 &= 4.323 \\
\beta_3 - \gamma_3 &= 101.500 \\
\beta_3 - \delta_3 &= 80.799 \\
\gamma_3 - \delta_3 &= -20.705 \\
\alpha_3 + \beta_3 - 180 &= 85.122 \\
\gamma_3 + \delta_3 - 180 &= -20.705 \\
\alpha_3 + \gamma_3 - 180 &= -16.382 \\
\beta_3 + \delta_3 - 180 &= 80.799 \\
\alpha_3 + \delta_3 - 180 &= 4.323 \\
\beta_3 + \gamma_3 - 180 &= 60.094 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 105.830 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -97.181 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -55.771
\end{aligned}$$

**Vertex 4**

$$\begin{aligned}
\alpha_4 - \beta_4 &= 108.430 \\
\alpha_4 - \gamma_4 &= 8.130 \\
\alpha_4 - \delta_4 &= 26.565 \\
\beta_4 - \gamma_4 &= -100.300 \\
\beta_4 - \delta_4 &= -81.870 \\
\gamma_4 - \delta_4 &= 18.435 \\
\alpha_4 + \beta_4 - 180 &= -55.305 \\
\gamma_4 + \delta_4 - 180 &= 18.435 \\
\alpha_4 + \gamma_4 - 180 &= 45.000 \\
\beta_4 + \delta_4 - 180 &= -81.870 \\
\alpha_4 + \delta_4 - 180 &= 26.565 \\
\beta_4 + \gamma_4 - 180 &= -63.435 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -73.740 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 126.870 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 90.000
\end{aligned}$$

**Switch combination:** Left + Lower + Upper

*Switched anglesDeg:*

$$\left( \begin{array}{ccc} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{array} \right)$$

*Angle relation checks for  $i = 1..4$ :*

#### Vertex 1

$$\begin{aligned} \alpha_1 - \beta_1 &= -55.305 \\ \alpha_1 - \gamma_1 &= 8.130 \\ \alpha_1 - \delta_1 &= 26.565 \\ \beta_1 - \gamma_1 &= 63.435 \\ \beta_1 - \delta_1 &= 81.870 \\ \gamma_1 - \delta_1 &= 18.435 \\ \alpha_1 + \beta_1 - 180 &= 108.430 \\ \gamma_1 + \delta_1 - 180 &= 18.435 \\ \alpha_1 + \gamma_1 - 180 &= 45.000 \\ \beta_1 + \delta_1 - 180 &= 81.870 \\ \alpha_1 + \delta_1 - 180 &= 26.565 \\ \beta_1 + \gamma_1 - 180 &= 100.300 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 90.000 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -36.870 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -73.740 \end{aligned}$$

#### Vertex 2

$$\begin{aligned} \alpha_2 - \beta_2 &= 85.122 \\ \alpha_2 - \gamma_2 &= 25.028 \\ \alpha_2 - \delta_2 &= 4.323 \\ \beta_2 - \gamma_2 &= -60.094 \\ \beta_2 - \delta_2 &= -80.799 \\ \gamma_2 - \delta_2 &= -20.705 \\ \alpha_2 + \beta_2 - 180 &= -76.476 \\ \gamma_2 + \delta_2 - 180 &= -20.705 \\ \alpha_2 + \gamma_2 - 180 &= -16.382 \\ \beta_2 + \delta_2 - 180 &= -80.799 \\ \alpha_2 + \delta_2 - 180 &= 4.323 \\ \beta_2 + \gamma_2 - 180 &= -101.500 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -55.771 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 64.417 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 105.830 \end{aligned}$$

#### Vertex 3

$$\begin{aligned} \alpha_3 - \beta_3 &= 85.122 \\ \alpha_3 - \gamma_3 &= -16.382 \\ \alpha_3 - \delta_3 &= 4.323 \\ \beta_3 - \gamma_3 &= -101.500 \\ \beta_3 - \delta_3 &= -80.799 \\ \gamma_3 - \delta_3 &= 20.705 \\ \alpha_3 + \beta_3 - 180 &= -76.476 \\ \gamma_3 + \delta_3 - 180 &= 20.705 \\ \alpha_3 + \gamma_3 - 180 &= 25.028 \\ \beta_3 + \delta_3 - 180 &= -80.799 \\ \alpha_3 + \delta_3 - 180 &= 4.323 \end{aligned}$$



$$\begin{aligned}
\beta_3 + \gamma_3 - 180 &= -60.094 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -97.181 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 105.830 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 64.417
\end{aligned}$$

**Vertex 4**

$$\begin{aligned}
\alpha_4 - \beta_4 &= -55.305 \\
\alpha_4 - \gamma_4 &= 45.000 \\
\alpha_4 - \delta_4 &= 26.565 \\
\beta_4 - \gamma_4 &= 100.300 \\
\beta_4 - \delta_4 &= 81.870 \\
\gamma_4 - \delta_4 &= -18.435 \\
\alpha_4 + \beta_4 - 180 &= 108.430 \\
\gamma_4 + \delta_4 - 180 &= -18.435 \\
\alpha_4 + \gamma_4 - 180 &= 8.130 \\
\beta_4 + \delta_4 - 180 &= 81.870 \\
\alpha_4 + \delta_4 - 180 &= 26.565 \\
\beta_4 + \gamma_4 - 180 &= 63.435 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 126.870 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -73.740 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -36.870
\end{aligned}$$

**Switch combination:** Right + Left + Lower + Upper

*Switched anglesDeg:*

$$\left( \begin{array}{cccc}
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\
180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90
\end{array} \right)$$

*Angle relation checks for  $i = 1..4$ :*

**Vertex 1**

$$\begin{aligned}
\alpha_1 - \beta_1 &= 108.430 \\
\alpha_1 - \gamma_1 &= 45.000 \\
\alpha_1 - \delta_1 &= 26.565 \\
\beta_1 - \gamma_1 &= -63.435 \\
\beta_1 - \delta_1 &= -81.870 \\
\gamma_1 - \delta_1 &= -18.435 \\
\alpha_1 + \beta_1 - 180 &= -55.305 \\
\gamma_1 + \delta_1 - 180 &= -18.435 \\
\alpha_1 + \gamma_1 - 180 &= 8.130 \\
\beta_1 + \delta_1 - 180 &= -81.870 \\
\alpha_1 + \delta_1 - 180 &= 26.565 \\
\beta_1 + \gamma_1 - 180 &= -100.300 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -36.870 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 90.000 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 126.870
\end{aligned}$$

**Vertex 2**

$$\begin{aligned}
\alpha_2 - \beta_2 &= 85.122 \\
\alpha_2 - \gamma_2 &= 25.028 \\
\alpha_2 - \delta_2 &= 4.323 \\
\beta_2 - \gamma_2 &= -60.094 \\
\beta_2 - \delta_2 &= -80.799 \\
\gamma_2 - \delta_2 &= -20.705
\end{aligned}$$

$$\begin{aligned}
\alpha_2 + \beta_2 - 180 &= -76.476 \\
\gamma_2 + \delta_2 - 180 &= -20.705 \\
\alpha_2 + \gamma_2 - 180 &= -16.382 \\
\beta_2 + \delta_2 - 180 &= -80.799 \\
\alpha_2 + \delta_2 - 180 &= 4.323 \\
\beta_2 + \gamma_2 - 180 &= -101.500 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -55.771 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 64.417 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 105.830
\end{aligned}$$

**Vertex 3**

$$\begin{aligned}
\alpha_3 - \beta_3 &= 85.122 \\
\alpha_3 - \gamma_3 &= -16.382 \\
\alpha_3 - \delta_3 &= 4.323 \\
\beta_3 - \gamma_3 &= -101.500 \\
\beta_3 - \delta_3 &= -80.799 \\
\gamma_3 - \delta_3 &= 20.705 \\
\alpha_3 + \beta_3 - 180 &= -76.476 \\
\gamma_3 + \delta_3 - 180 &= 20.705 \\
\alpha_3 + \gamma_3 - 180 &= 25.028 \\
\beta_3 + \delta_3 - 180 &= -80.799 \\
\alpha_3 + \delta_3 - 180 &= 4.323 \\
\beta_3 + \gamma_3 - 180 &= -60.094 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -97.181 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 105.830 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 64.417
\end{aligned}$$

**Vertex 4**

$$\begin{aligned}
\alpha_4 - \beta_4 &= 108.430 \\
\alpha_4 - \gamma_4 &= 8.130 \\
\alpha_4 - \delta_4 &= 26.565 \\
\beta_4 - \gamma_4 &= -100.300 \\
\beta_4 - \delta_4 &= -81.870 \\
\gamma_4 - \delta_4 &= 18.435 \\
\alpha_4 + \beta_4 - 180 &= -55.305 \\
\gamma_4 + \delta_4 - 180 &= 18.435 \\
\alpha_4 + \gamma_4 - 180 &= 45.000 \\
\beta_4 + \delta_4 - 180 &= -81.870 \\
\alpha_4 + \delta_4 - 180 &= 26.565 \\
\beta_4 + \gamma_4 - 180 &= -63.435 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -73.740 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 126.870 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 90.000
\end{aligned}$$

Out[1854]=

===== NOT CONJUGATE-MODULAR =====

$M_i < 1$  for all  $i = 1..4 \Rightarrow$  NOT

conjugate-modular. Boundary-strip switches preserve this.

**Initial configuration (no switches applied):**

$$\left( \begin{array}{ccc}
\frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{18}}\right]}{\pi} \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} \\
\frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{18}}\right]}{\pi}
\end{array} \right) \begin{array}{l} 90 \\ 90 \\ 90 \\ 90 \end{array}$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Right

*Switched anglesDeg:*

$$\begin{pmatrix} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Left

*Switched anglesDeg:*

$$\begin{pmatrix} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Lower

*Switched anglesDeg:*

$$\begin{pmatrix} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Upper

*Switched anglesDeg:*

$$\left( \begin{array}{ccc} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{array} \right)$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Right + Left

*Switched anglesDeg:*

$$\left( \begin{array}{ccc} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{array} \right)$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Right + Lower

*Switched anglesDeg:*

$$\left( \begin{array}{ccc} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{array} \right)$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Right + Upper

*Switched anglesDeg:*

$$\left( \begin{array}{ccc} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \end{array} \right)$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Left + Lower

*Switched anglesDeg:*

$$\left( \begin{array}{ccc} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \end{array} \right)$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Left + Upper

*Switched anglesDeg:*

$$\left( \begin{array}{ccc} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \end{array} \right)$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Lower + Upper

*Switched anglesDeg:*

$$\begin{pmatrix} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Right + Left + Lower

*Switched anglesDeg:*

$$\begin{pmatrix} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Right + Left + Upper

*Switched anglesDeg:*

$$\begin{pmatrix} \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \\ \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{\frac{7}{22}}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{10}}\right]}{\pi} & 90 \end{pmatrix}$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Right + Lower + Upper

*Switched anglesDeg:*

$$\left( \begin{array}{ccc} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \end{array} \right)$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Left + Lower + Upper

*Switched anglesDeg:*

$$\left( \begin{array}{ccc} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \end{array} \right)$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

**Switch combination:** Right + Left + Lower + Upper

*Switched anglesDeg:*

$$\left( \begin{array}{ccc} 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[-\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{4\sqrt{11}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7\sqrt{22}}{4}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{2\sqrt{2}}\right]}{\pi} & 90 \\ 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{5}}\right]}{\pi} & \frac{180 \operatorname{ArcCos}\left[\frac{7}{5\sqrt{2}}\right]}{\pi} & 180 - \frac{180 \operatorname{ArcCos}\left[\frac{1}{\sqrt{18}}\right]}{\pi} & 90 \end{array} \right)$$

**M<sub>i</sub> values:**

$$M_1 = \frac{1}{2}, M_2 = \frac{1}{2}, M_3 = \frac{1}{2}, M_4 = \frac{1}{2}$$

**M<sub>i</sub> < 1 for all i = 1, ..., 4**

Out[1856]=

===== NOT CHIMERA =====

Fails conic, orthodiagonal & isogonal tests for all  
 $i=1, \dots, 4 \Rightarrow$  NOT chimera. Boundary-strip switches  
 preserve these failures as demonstrated in the NOT  
 CONIC, NOT ORTHODIAGONAL, and NOT ISOGONAL sections.