Computational Companion to “Flexible 3×3 Nets of Equimodular Elliptic Type” — Example 1

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Tested on: Mathematica 14.0

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(*===========================================================================
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(*Quit*)
(*All angle sets in degrees*)
anglesDeg = {
   {105, 15, 120, 90}, (*Vertex 1*)
   {90, 120, 15, 105}, (*Vertex 2*)
   {90, 60, 165, 75}, (*Vertex 3*)
   {105, 15, 120, 90} (*Vertex 4*)};
(*anglesDeg={
   {26.20863403213998,82.2407675648952,
    21.949109994264898,59.99999999999997},(*Vertex 1*)
   {16.166237389600262,130.87095233025335,
    18.85247535405415,114.99999999999991},(*Vertex 2*)
   {134.65533802039442,34.44439013740831,145.3694664686027,80.0},(*Vertex 3*)
   {117.95117201340666,49.52829397349284,
    149.0275482144225,105.00000000000001}  (*Vertex 4*)};*)

(*--------------------------------------------*)
(*Function to compute sigma from 4 angles*)
computeSigma[{α_, β_, γ_, δ_}] := (α + β + γ + δ) / 2;

(*--------------------------------------------*)
(*Function to compute a,b,c,d from angles*)
computeABCD[{α_, β_, γ_, δ_}] :=
  Module[{alpha = α Degree, beta = β Degree, gamma = γ Degree,
    delta = δ Degree, sigma}, sigma = computeSigma[{α, β, γ, δ} Degree];
   {Sin[alpha] / Sin[sigma - alpha], Sin[beta] / Sin[sigma - beta],
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    Sin[gamma] / Sin[sigma - gamma], Sin[delta] / Sin[sigma - delta]}];

(*-----------------------------------------*)
(*Compute all σ values*)
sigmas = computeSigma /@ anglesDeg;

(*Compute all {a,b,c,d} values*)
results = computeABCD /@ anglesDeg;

(*Optional:extract individual values*)
{a1, b1, c1, d1} = results[[1]];
{a2, b2, c2, d2} = results[[2]];
{a3, b3, c3, d3} = results[[3]];
{a4, b4, c4, d4} = results[[4]];
{σ1, σ2, σ3, σ4} = sigmas;


(*========================================================================
 =====*)
(*================================
   CONDITION (N.0)================================*)
(*========================================================================
 =====*)
uniqueCombos = {{1, 1, 1, 1}, {1, 1, 1, -1}, {1, 1, -1, -1}, {1, 1, -1, 1},
   {1, -1, 1, 1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, -1, -1, -1}};

checkConditionN0Degrees[{α_, β_, γ_, δ_}] :=
  Module[{angles = {α, β, γ, δ}, results},
   results = Mod[uniqueCombos.angles, 360] // Chop;
   ! MemberQ[results, 0]];

conditionsN0 = checkConditionN0Degrees /@ anglesDeg;
allVerticesPass = And @@ conditionsN0;

Column[{TextCell[Style["================= CONDITION (N.0) =================",
     Darker[Green], Bold, 16], "Text"],
  If[allVerticesPass,
   Style["✔ All vertices satisfy (N.0).", Darker[Green], Bold],
   Style["✘ Some vertices fail (N.0).", Red, Bold]]}]


(*========================================================================
 =====*)
(*================================
   CONDITION (N.3)================================*)
(*========================================================================
 =====*)
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Ms = FullSimplify[Times @@@ results];
allEqualQ = Simplify[Equal @@ Ms];

Column[{TextCell[Style["================= CONDITION (N.3) =================",
     Darker[Blue], Bold, 16], "Text"], If[allEqualQ,
   Row[{Style["✓ M1 = M2 = M3 = M4 = ", Bold], Ms⟦1⟧}],
   Style["✗ M_i are not all equal.", Red, Bold]]}]


(*============================================================================
 =====*)
(*==========================CONDITION (N.4)==========================*)
(*============================================================================
 =====*)
aList = results⟦All, 1⟧;
cList = results⟦All, 3⟧;
dList = results⟦All, 4⟧;

rList = FullSimplify /@ (aList * dList); {r1, r2, r3, r4} = rList;
sList = FullSimplify /@ (cList * dList); {s1, s2, s3, s4} = sList;

Column[{TextCell[Style["================= CONDITION (N.4) =================",
     Darker[Purple], Bold, 16], "Text"],
  If[r1 == r2 && r3 == r4 && s1 == s4 && s2 == s3, Column[
    {Row[{Style["✓ r1 = r2 = ", Bold], r1, Style["; ✓ r3 = r4 = ", Bold], r3}],
     Row[{Style["✓ s1 = s4 = ", Bold], s1, Style["; ✓ s2 = s3 = ", Bold],
       s2}]}], Style["✗ Condition (N.4) fails.", Red, Bold]]
 }]


(*============================================================================
 =====*)
(*================================
  CONDITION (N.5)==============================*)
(*============================================================================
 =====*)
fList = FullSimplify /@ (aList * cList); {f1, f2, f3, f4} = fList;
M1 = Ms⟦1⟧;

m[M1_] := Piecewise[{{1 - M1, M1 < 1}, {(M1 - 1) / M1, M1 > 1}}] // N;
K = EllipticK[m[M1]] // N;
Kp = EllipticK[1 - m[M1]] // N;

computeTi[i_] :=
  Module[{sigma = sigmas⟦i⟧, r = rList⟦i⟧, s = sList⟦i⟧, f = fList⟦i⟧, base},
   base = I * Im[InverseJacobiDN[Piecewise[
         {{Sqrt[f], M1 < 1}, {1 / Sqrt[f], M1 > 1}}], m[M1]]] / Kp;
```

```
    Which[sigma < 180, Which[r > 1 && s > 1, base, (r < 1 && s > 1) || (r > 1 && s < 1),
      1 + base, r < 1 && s < 1, 2 + base], sigma > 180, Which[r > 1 && s > 1,
      2 + base, (r < 1 && s > 1) || (r > 1 && s < 1), 3 + base, r < 1 && s < 1, base]]];

tList = computeTi /@ Range[4];
RoundWithTolerance[x_, tol_ : 10^-6] := Module[{nearest}, nearest = Round[x];
    If[Abs[x - nearest] ≤ tol, nearest, x]];

checkValidCombination[M1_, ε_ : 10^-6] :=
  Module[{i, combo, dotProd, rePart, imPart, n2, expr, foundQ = False},
    Print[Style["⚠ Approximate validation using ε-tolerance. For rigorous
        proof, see the referenced paper.", Darker@Orange, Italic]];
    Do[combo = uniqueCombos〚i〛;
     dotProd = tList.combo;
     rePart = Abs[Re[dotProd]];
     imPart = Abs[Im[dotProd]];
     If[M1 < 1,
      If[Mod[RoundWithTolerance[rePart], 4] < ε,
       If[Mod[RoundWithTolerance[imPart], 2] < ε, expr =
         tList〚1〛 + combo〚2〛 × tList〚2〛 + combo〚3〛 × tList〚3〛 + combo〚4〛 × tList〚4〛;
        Print[Style["✅ Valid Combination Found (M < 1):",
          Darker[Green], Bold], "\n", Style["e1 = ", Bold], combo〚2〛,
         Style[", e2 = ", Bold], combo〚3〛, Style[", e3 = ", Bold], combo〚4〛,
         "\n", Style["t1 = ", Bold], Re[tList〚1〛], "K + ", Im[tList〚1〛], "iK'",
         "\n", Style["t2 = ", Bold], Re[tList〚2〛], "K + ", Im[tList〚2〛], "iK'",
         "\n", Style["t3 = ", Bold], Re[tList〚3〛], "K + ", Im[tList〚3〛], "iK'",
         "\n", Style["t4 = ", Bold], Re[tList〚4〛], "K + ", Im[tList〚4〛],
         "iK'", "\n", Style["t1 + e1*t2 + e2*t3 + e3*t4 = ", Bold],
         Re[expr], "K + ", Im[expr], "iK'"];
        foundQ = True;
        Break[]]]];
     If[M1 > 1,
      If[Mod[RoundWithTolerance[imPart], 2] < ε,
       n2 = Quotient[RoundWithTolerance[imPart], 2];
       If[Mod[RoundWithTolerance[Abs[rePart - 2 n2]], 4] < ε, expr =
         tList〚1〛 + combo〚2〛 × tList〚2〛 + combo〚3〛 × tList〚3〛 + combo〚4〛 × tList〚4〛;
        Print[Style["✅ Valid Combination Found (M > 1):",
          Darker[Green], Bold], "\n", Style["e1 = ", Bold], combo〚2〛,
         Style[", e2 = ", Bold], combo〚3〛, Style[", e3 = ", Bold], combo〚4〛,
         "\n", Style["t1 = ", Bold], Re[tList〚1〛], "K + ", Im[tList〚1〛], "iK'",
         "\n", Style["t2 = ", Bold], Re[tList〚2〛], "K + ", Im[tList〚2〛], "iK'",
         "\n", Style["t3 = ", Bold], Re[tList〚3〛], "K + ", Im[tList〚3〛], "iK'",
         "\n", Style["t4 = ", Bold], Re[tList〚4〛], "K + ", Im[tList〚4〛],
         "iK'", "\n", Style["t1 + e1*t2 + e2*t3 + e3*t4 = ", Bold],
         Re[expr], "K + ", Im[expr], "iK'"];
```

```
        foundQ = True;
        Break[]]]], {i, Length[uniqueCombos]}];
    If[! foundQ, Print[Style["❌ No valid combination found.", Red, Bold], "\n",
      Style["t1 = ", Bold], Re[tList[[1]]], "K + ", Im[tList[[1]]], "iK'", "\n",
      Style["t2 = ", Bold], Re[tList[[2]]], "K + ", Im[tList[[2]]], "iK'", "\n",
      Style["t3 = ", Bold], Re[tList[[3]]], "K + ", Im[tList[[3]]], "iK'", "\n",
      Style["t4 = ", Bold], Re[tList[[4]]], "K + ", Im[tList[[4]]], "iK'", "\n"]];];


Column[{TextCell[Style["================= CONDITION (N.5) =================",
     Darker[Red], Bold, 16], "Text"]}]
res = checkValidCombination[M1];


(*=====================================================================
 =====*)
(*================================
   OTHER PARAMETER================================*)
(*=====================================================================
 =====*)
Column[
 {TextCell[Style["================= OTHER PARAMETERS =================",
     Darker[Orange], Bold, 16], "Text"],
  Row[{Style["u = ", Bold], 1 - M1}],
  Row[{Style["σ1 = ", Bold], σ1 Degree, Style[", σ2 = ", Bold], σ2 Degree,
     Style[", σ3 = ", Bold], σ3 Degree, Style[", σ4 = ", Bold], σ4 Degree}],
  Row[{Style["f1 = ", Bold], f1, Style[", f2 = ", Bold],
     f2, Style[", f3 = ", Bold], f3, Style[", f4 = ", Bold], f4}],
  Row[{Style["z1 = ", Bold], FullSimplify[1 / (f1 - 1)], Style[", z2 = ", Bold],
     FullSimplify[1 / (f2 - 1)], Style[", z3 = ", Bold], FullSimplify[1 / (f3 - 1)],
     Style[", z4 = ", Bold], FullSimplify[1 / (f4 - 1)]}],
  Row[{Style["x1 = ", Bold], FullSimplify[1 / (r1 - 1)], Style[", x2 = ", Bold],
     FullSimplify[1 / (r2 - 1)], Style[", x3 = ", Bold], FullSimplify[1 / (r3 - 1)],
     Style[", x4 = ", Bold], FullSimplify[1 / (r4 - 1)]}],
  Row[{Style["y1 = ", Bold], FullSimplify[1 / (s1 - 1)], Style[", y2 = ", Bold],
     FullSimplify[1 / (s2 - 1)], Style[", y3 = ", Bold], FullSimplify[1 / (s3 - 1)],
     Style[", y4 = ", Bold], FullSimplify[1 / (s4 - 1)]}],
  Row[{Style["p1 = ", Bold], Simplify[Sqrt[r1 - 1]],
     Style[", p2 = ", Bold], Simplify[Sqrt[r2 - 1]], Style[", p3 = ", Bold],
     Simplify[Sqrt[r3 - 1]], Style[", p4 = ", Bold], Simplify[Sqrt[r4 - 1]]}],
  Row[{Style["q1 = ", Bold], Simplify[Sqrt[s1 - 1]],
     Style[", q2 = ", Bold], Simplify[Sqrt[s2 - 1]], Style[", q3 = ", Bold],
     Simplify[Sqrt[s3 - 1]], Style[", q4 = ", Bold], Simplify[Sqrt[s4 - 1]]}],
  Row[{Style["p1·q1 = ", Bold], Simplify[Sqrt[r1 - 1] Sqrt[s1 - 1]],
     Style[", p2·q2 = ", Bold], Simplify[Sqrt[r2 - 1] Sqrt[s2 - 1]],
     Style[", p3·q3 = ", Bold], Simplify[Sqrt[r3 - 1] Sqrt[s3 - 1]],
     Style[", p4·q4 = ", Bold], Simplify[Sqrt[r4 - 1] Sqrt[s4 - 1]]}],
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```
  Row[{Style["\!\(\*OverscriptBox[\(α1\), \(_\)]\) = ", Bold],
    σ1 - anglesDeg⟦1, 1⟧, "°", Style[
     ", \!\(\*OverscriptBox[\(β1\), \(_\)]\) = ", Bold], σ1 - anglesDeg⟦1, 2⟧,
    "°", Style[", \!\(\*OverscriptBox[\(γ1\), \(_\)]\) = ", Bold],
    σ1 - anglesDeg⟦1, 3⟧, "°",
    Style[", \!\(\*OverscriptBox[\(δ1\), \(_\)]\) = ", Bold],
    σ1 - anglesDeg⟦1, 4⟧, "°"}],
  Row[Module[{αb1 = σ1 - anglesDeg⟦1, 1⟧, βb1 = σ1 - anglesDeg⟦1, 2⟧,
     γb1 = σ1 - anglesDeg⟦1, 3⟧, δb1 = σ1 - anglesDeg⟦1, 4⟧, ineq1, ineq2, ineq3},
    ineq1 = Sin[anglesDeg⟦1, 1⟧ Degree] > Sin[αb1 Degree] Sin[δb1 Degree];
    ineq2 = Sin[anglesDeg⟦1, 3⟧ Degree] > Sin[γb1 Degree] Sin[δb1 Degree];
    ineq3 = Sin[anglesDeg⟦1, 1⟧ Degree] Sin[anglesDeg⟦1, 2⟧ Degree] <
      Sin[αb1 Degree] Sin[βb1 Degree];
    {Style["Inequalities (vertex 1): ", Bold], TraditionalForm@
      (Sin[Subscript[α, 1]] > Sin[Overscript[Subscript[α, 1], _]] *
         Sin[Overscript[Subscript[δ, 1], _]]),
     " : ", Simplify[ineq1], "   ,   ", TraditionalForm@
      (Sin[Subscript[γ, 1]] > Sin[Overscript[Subscript[γ, 1], _]] *
         Sin[Overscript[Subscript[δ, 1], _]]), " : ", Simplify[ineq2],
     "   ,   ", TraditionalForm@(Sin[Subscript[α, 1]] Sin[Subscript[β, 1]] <
        Sin[Overscript[Subscript[α, 1], _]] *
         Sin[Overscript[Subscript[β, 1], _]]), " : ", Simplify[ineq3]}]]

 }]


(*============================================================================
 =====*)
(*==============================
  BRICARD's EQUATIONS==============================*)
(*============================================================================
 =====*)
(*==================================
 FLEXION 1==================================*)
Z[t_] := t;
```

$$W1[t\_] := \frac{\left(3+2\sqrt{3}\right) t - \sqrt{1+\sqrt{3}}\ \sqrt{\left(\sqrt{3}\,t^2-1\right)\left(1-\left(2\sqrt{3}+3\right)t^2\right)}}{1+\sqrt{3}+\left(3+2\sqrt{3}\right)t^2};$$

$$U[t\_] := \frac{\left(36+17\sqrt{3}\right) t - 2\sqrt{3\left(1+\sqrt{3}\right)}\ \sqrt{\left(\sqrt{3}\,t^2-1\right)\left(1-\left(2\sqrt{3}+3\right)t^2\right)}}{\sqrt{3}+12\left(9+5\sqrt{3}\right)t^2};$$

$$W2[t\_] := \frac{\sqrt{2}\left(-2t+\sqrt{1+\sqrt{3}}\ \sqrt{\left(\sqrt{3}\,t^2-1\right)\left(1-\left(2\sqrt{3}+3\right)t^2\right)}\right)}{\left(1+\sqrt{3}\right)\left(-1+3t^2\right)};$$

```
(*General polynomial constructor*)
BricardsEquation[{α_, β_, γ_, δ_}, σ_, x_, y_] :=
  Module[{c22, c20, c02, c11, c00},
   c22 = Sin[σ - δ] Sin[σ - δ - β];
   c20 = Sin[σ - α] Sin[σ - α - β];
   c02 = Sin[σ - γ] Sin[σ - γ - β];
   c11 = -Sin[α] Sin[γ];
   c00 = Sin[σ] Sin[σ - β];
   c22 x^2 y^2 + c20 x^2 + c02 y^2 + 2 c11 x y + c00];

(*Compute and print all P_i for flexion 1*)
TextCell[
 Style["================= FLEXIBILITY (FLEXION 1) =================",
  Darker[Cyan], Bold, 16], "Text"]
funcs = {{Z, W1}, {Z, W2}, {U, W2}, {U, W1}};
Do[angles = anglesDeg[[i]] Degree;
  sigma = sigmas[[i]] Degree;
  {α, β, γ, δ} = angles;
  poly = Which[i == 1, BricardsEquation[{α, β, γ, δ}, sigma, Z[t], W1[t]],
    i == 2, BricardsEquation[{α, β, γ, δ}, sigma, Z[t], W2[t]],
    i == 3, BricardsEquation[{α, β, γ, δ}, sigma, U[t], W2[t]],
    i == 4, BricardsEquation[{α, β, γ, δ}, sigma, U[t], W1[t]]];
  Print[Row[{Subscript["P", i], "[", funcs[[i, 1]],
     ", ", funcs[[i, 2]], "] = ", FullSimplify[poly]}]], {i, 1, 4}];

(*==================================
 FLEXION 2==================================*)
Z2[t_] := t;
```

$$\texttt{W12[t\_]} := \frac{\left(3+2\sqrt{3}\right)t+\sqrt{1+\sqrt{3}}\ \sqrt{\left(\sqrt{3}\,t^2-1\right)\left(1-\left(2\sqrt{3}+3\right)t^2\right)}}{1+\sqrt{3}+\left(3+2\sqrt{3}\right)t^2};$$

$$\texttt{U2[t\_]} := \frac{\left(36+17\sqrt{3}\right)t+2\sqrt{3\left(1+\sqrt{3}\right)}\ \sqrt{\left(\sqrt{3}\,t^2-1\right)\left(1-\left(2\sqrt{3}+3\right)t^2\right)}}{\sqrt{3}+12\left(9+5\sqrt{3}\right)t^2};$$

$$\texttt{W22[t\_]} := \frac{\sqrt{2}\left(-2t-\sqrt{1+\sqrt{3}}\ \sqrt{\left(\sqrt{3}\,t^2-1\right)\left(1-\left(2\sqrt{3}+3\right)t^2\right)}\right)}{\left(1+\sqrt{3}\right)\left(-1+3t^2\right)};$$

```
(*General polynomial constructor*)
BricardsEquation[{α_, β_, γ_, δ_}, σ_, x_, y_] :=
  Module[{c22, c20, c02, c11, c00},
   c22 = Sin[σ - δ] Sin[σ - δ - β];
   c20 = Sin[σ - α] Sin[σ - α - β];
   c02 = Sin[σ - γ] Sin[σ - γ - β];
   c11 = -Sin[α] Sin[γ];
```

```
    c00 = Sin[σ] Sin[σ - β];
    c22 x^2 y^2 + c20 x^2 + c02 y^2 + 2 c11 x y + c00];

(*Compute and print all P_i for flexion 2*)
TextCell[
 Style["================= FLEXIBILITY (FLEXION 2) =================",
  Cyan, Bold, 16], "Text"]
funcs = {{Z, W1}, {Z, W2}, {U, W2}, {U, W1}};
Do[angles = anglesDeg⟦i⟧ Degree;
  sigma = sigmas⟦i⟧ Degree;
  {α, β, γ, δ} = angles;
  poly = Which[i == 1, BricardsEquation[{α, β, γ, δ}, sigma, Z2[t], W12[t]],
    i == 2, BricardsEquation[{α, β, γ, δ}, sigma, Z2[t], W22[t]],
    i == 3, BricardsEquation[{α, β, γ, δ}, sigma, U2[t], W22[t]],
    i == 4, BricardsEquation[{α, β, γ, δ}, sigma, U2[t], W12[t]]];
  Print[Row[{Subscript["P", i], "[", funcs⟦i, 1⟧,
     ", ", funcs⟦i, 2⟧, "] = ", FullSimplify[poly]}]], {i, 1, 4}];


(*===========================================================================
 =====*)
(*==================================
 NOT TRIVIAL==================================*)
(*===========================================================================
 =====*)
(*Define domain limits for t*)
tMin = 1 / Sqrt[2 Sqrt[3] + 3];
tMax = 1 / 2;

(*==================================
 FLEXION 1==================================*)
(*List of expressions to plot*)
expressions = {Z[t], W2[t], U[t], W1[t]};
labels = {"Z", "W2", "U", "W1"};

Column[{TextCell[
    Style["================= NOT TRIVIAL (FLEXION 1) =================",
     Darker[Brown], Bold, 16], "Text"],

   (*Explanatory text*)
  TextCell[Style["This configuration does not belong to the trivial class –
      even after switching the boundary strips – since none of the
      functions Z, W2, U, or W1 is constant.", GrayLevel[0.3]], "Text"],
  Spacer[12],

   (*Plots in a light panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions⟦i⟧, {t, tMin, tMax},
```

```
        PlotLabel → Style[labels⟦i⟧, Bold, 14], PlotRange → All,
        AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
     Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]

 }]

(*==================================
 FLEXION 2==================================*)
(*List of expressions to plot*)
expressions = {Z2[t], W22[t], U2[t], W12[t]};
labels = {"Z", "W2", "U", "W1"};

Column[{TextCell[
    Style["================= NOT TRIVIAL (FLEXION 2) =================",
     Brown, Bold, 16], "Text"],

   (*Explanatory text*)
   TextCell[Style["This configuration does not belong to the trivial class –
       even after switching the boundary strips – since none of the
       functions Z, W2, U, or W1 is constant.", GrayLevel[0.3]], "Text"],
   Spacer[12],

   (*Plots in a light panel*)
   Panel[GraphicsGrid[Partition[Table[Plot[expressions⟦i⟧, {t, tMin, tMax},
        PlotLabel → Style[labels⟦i⟧, Bold, 14], PlotRange → All,
        AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
     Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]

 }]


(*=========================================================================
 =====*)
(*===================
 NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE==================*)
(*=========================================================================
 =====*)
(*Define domain limits for t*)
tMin = 1 / Sqrt[2 Sqrt[3] + 3];
tMax = 1 / 2;

(*==================================
 FLEXION 1==================================*)
(*List of expressions& labels*)
expressions =
  {Z[t] * U[t], Z[t] / U[t], W1[t] * W2[t], W1[t] / W2[t], Z[t] * W2[t], Z[t] / W2[t],
   U[t] * W1[t], U[t] / W1[t], Z[t] * W1[t], Z[t] / W1[t], W2[t] * U[t], W2[t] / U[t]};
```

```
labels = {"Z * U", "Z / U", "W1 * W2", "W1 / W2", "Z * W2", "Z / W2",
   "U * W1", "U / W1", "Z * W1", "Z / W1", "W2 * U", "W2 / U"};

Column[
 {TextCell[Style["============= NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE
      (FLEXION 1) =============", Darker[Magenta], Bold, 16], "Text"],

  (*Explanatory text*)TextCell[
   Style["This configuration does not belong to the Linear compound class nor
      to the linear conjugate class – even after switching the boundary
      strips – since none of the functions ZU, Z/U, W1W2, W1/W2, ZW2,
      Z/W2, UW1, and U/W1 is constant. In addition, none of ZW1,
      Z/W1, W2U, W2/U is constant as well.", GrayLevel[0.3]], "Text"],
  Spacer[12],

  (*Plots panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions⟦i⟧, {t, tMin, tMax},
       PlotLabel → Style[labels⟦i⟧, Bold, 14], PlotRange → All,
       AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
    Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]
 }]

(*=================================
 FLEXION 2=================================*)
(*List of expressions& labels*)
expressions = {Z2[t] * U2[t], Z2[t] / U2[t], W12[t] * W22[t], W12[t] / W22[t],
   Z2[t] * W22[t], Z2[t] / W22[t], U2[t] * W12[t], U2[t] / W12[t],
   Z2[t] * W12[t], Z2[t] / W12[t], W22[t] * U2[t], W22[t] / U2[t]};
labels = {"Z * U", "Z / U", "W1 * W2", "W1 / W2", "Z * W2", "Z / W2",
   "U * W1", "U / W1", "Z * W1", "Z / W1", "W2 * U", "W2 / U"};

Column[
 {TextCell[Style["============= NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE
      (FLEXION 2) =============", Magenta, Bold, 16], "Text"],

  (*Explanatory text*)TextCell[
   Style["This configuration does not belong to the Linear compound class nor
      to the linear conjugate class – even after switching the boundary
      strips – since none of the functions ZU, Z/U, W1W2, W1/W2, ZW2,
      Z/W2, UW1, and U/W1 is constant. In addition, none of ZW1,
      Z/W1, W2U, W2/U is constant as well.", GrayLevel[0.3]], "Text"],
  Spacer[12],

  (*Plots panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions⟦i⟧, {t, tMin, tMax},
       PlotLabel → Style[labels⟦i⟧, Bold, 14], PlotRange → All,
       AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
```

```
    Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]
 }]


(*============================================================================
 =====*)
(*===========================
 SWITCHING BOUNDARY STRIPS===========================*)
(*============================================================================
 =====*)
SwitchingRightBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={1,2},{1,3},{4,2},{4,3}*)
  modified⟦1, 2⟧ = 180 - anglesDeg⟦1, 2⟧; (*β1*)
  modified⟦1, 3⟧ = 180 - anglesDeg⟦1, 3⟧; (*γ1*)
  modified⟦4, 2⟧ = 180 - anglesDeg⟦4, 2⟧; (*β4*)
  modified⟦4, 3⟧ = 180 - anglesDeg⟦4, 3⟧; (*γ4*)
  modified]

SwitchingLeftBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={2,2},{2,3},{3,2},{3,3}*)
  modified⟦2, 2⟧ = 180 - anglesDeg⟦2, 2⟧; (*β2*)
  modified⟦2, 3⟧ = 180 - anglesDeg⟦2, 3⟧; (*γ2*)
  modified⟦3, 2⟧ = 180 - anglesDeg⟦3, 2⟧; (*β3*)
  modified⟦3, 3⟧ = 180 - anglesDeg⟦3, 3⟧; (*γ3*)
  modified]

SwitchingLowerBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={1,1},{1,2},{2,1},{2,2}*)
  modified⟦1, 1⟧ = 180 - anglesDeg⟦1, 1⟧; (*α1*)
  modified⟦1, 2⟧ = 180 - anglesDeg⟦1, 2⟧; (*β1*)
  modified⟦2, 1⟧ = 180 - anglesDeg⟦2, 1⟧; (*α2*)
  modified⟦2, 2⟧ = 180 - anglesDeg⟦2, 2⟧; (*β2*)
  modified]

SwitchingUpperBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={3,1},{3,2},{4,1},{4,2}*)
  modified⟦3, 1⟧ = 180 - anglesDeg⟦3, 1⟧; (*α3*)
  modified⟦3, 2⟧ = 180 - anglesDeg⟦3, 2⟧; (*β3*)
  modified⟦4, 1⟧ = 180 - anglesDeg⟦4, 1⟧; (*α4*)
  modified⟦4, 2⟧ = 180 - anglesDeg⟦4, 2⟧; (*β4*)
  modified]


(*============================================================================
 =====*)
(*==================================
   NOT CONIC==================================*)
```

```
(*==========================================================================
 =====*)
Column[{TextCell[Style["================= NOT CONIC =================",
    Pink, Bold, 16], "Text"],
  TextCell[Style["Condition (N.0) is satisfied ⇒
      this configuration is NOT equimodular-conic. Applying
      any boundary-strip switch still preserves (N.0), so
      no conic form emerges.", GrayLevel[0.3]], "Text"]
 }]

(*Now the exact same Module for checking all switch combinations…*)
Module[{angles = anglesDeg, switchers, combinations, results},
 (*Define switch functions*)
 switchers = <|"Right" → SwitchingRightBoundaryStrip,
   "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
   "Upper" → SwitchingUpperBoundaryStrip|>;
 (*Generate all combinations of switches (from size 1 to 4)*)
 combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
 (*Evaluate condition after each combination of switches*)results = Table[
   Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];
    passQ = And @@ (checkConditionN0Degrees /@ switched);
    (*Print the result after switching*)
    (*Print["\nSwitch combination: ",name];
    Print["Switched anglesDeg:"];
    Print[MatrixForm[switched]];*)
    {name, passQ}], {combo, combinations}];
 (*Display results*)
 Column[Prepend[Table[Module[{comboName = res[[1]], passQ = res[[2]]},
     Row[{Style[comboName <> ": ", Bold],
       If[passQ,
        Style["Condition (N.0) is still satisfied.", Darker[Green]],
        Style["Condition (N.0) fails.", Red, Bold]
       ]
      }
     ]], {res, results}], TextCell[
    Style["CONDITION (N.0) AFTER SWITCHING BOUNDARY STRIPS", 14], "Text"]]]]


(*==========================================================================
 =====*)
(*================================
 NOT ORTHODIAGONAL================================*)
(*==========================================================================
 =====*)
Column[
 {TextCell[Style["================= NOT ORTHODIAGONAL =================",
```

```
    Purple, Bold, 16], "Text"],
   TextCell[Style[
    "cos(αi)·cos(γi) ≠ cos(βi)·cos(δi) for each i = 1…4 ⇒ NOT orthodiagonal.
      Switching boundary strips does not
      correct this.", GrayLevel[0.3]], "Text"]
 }]

Module[{angles = anglesDeg, switchers, combinations, results},
 (*Define switch functions*)
 switchers = <|"Right" → SwitchingRightBoundaryStrip,
   "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
   "Upper" → SwitchingUpperBoundaryStrip|>;
 (*Helper function:compute and print difference only*)
 formatOrthodiagonalCheck[quad_List] := Module[{vals},
   vals = Table[Module[{a, b, c, d, lhs, rhs, diff}, {a, b, c, d} = quad[[i]];
      lhs = FullSimplify[Cos[a Degree] Cos[c Degree]];
      rhs = FullSimplify[Cos[b Degree] Cos[d Degree]];
      diff = Chop[lhs - rhs];
      Style[Row[{"cos(α" <> ToString[i] <> ")·cos(γ" <> ToString[i] <> ") - ",
         "cos(β" <> ToString[i] <> ")·cos(δ" <> ToString[i] <> ") = ", NumberForm[
          diff, {5, 3}]}], If[diff == 0, Red, Black]]], {i, Length[quad]}];
   Column[vals]];
 (*Orthodiagonal check for anglesDeg before any switching*)
 Print[TextCell[Style["\nInitial anglesDeg (no switches):", Bold]]];
 Print[MatrixForm[angles]];
 Print[TextCell[Style[
    "Orthodiagonal check: cos(αi)·cos(γi) - cos(βi)·cos(δi) for i = 1..4",
    Italic]]];
 Print[formatOrthodiagonalCheck[angles]];
 (*Generate all combinations of switches (from size 1 to 4)*)
 combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
 (*Evaluate condition after each combination of switches*)results = Table[
   Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];
    passQ = And @@ (checkConditionN0Degrees /@ switched);
    Print[Style["\nSwitch combination: ", Bold], name];
    Print[Style["Switched anglesDeg:", Italic]];
    Print[MatrixForm[switched]];
    Print[TextCell[Style[
       "Orthodiagonal check: cos(αi)·cos(γi) - cos(βi)·cos(δi) for i = 1..4",
       Italic]]];
    Print[formatOrthodiagonalCheck[switched]];
    {name, passQ}], {combo, combinations}];]


(*=========================================================================
 =====*)
```

```
(*=================================
 NOT ISOGONAL=================================*)
(*===================================================================
 =====*)
Column[
 {TextCell[Style["================= NOT ISOGONAL =================", Orange,
    Bold, 15], "Text"],
  TextCell[
   Style["Condition (N.0) holds AND for all i = 1…4: αi ≠ βi, αi ≠ γi, αi
      ≠ δi, βi ≠ γi, βi ≠ δi, γi ≠ δi, αi+βi ≠ π ≠ γi+δi, αi+γi
      ≠ π ≠ βi+δi, αi+δi ≠ π ≠ βi+γi ⇒ NOT isogonal. Switching
      boundary strips do not change this.", GrayLevel[0.3]], "Text"]}]

Module[{angles = anglesDeg, switchers, combinations, results},
 (*Define switch functions*)
 switchers = <|"Right" → SwitchingRightBoundaryStrip,
   "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
   "Upper" → SwitchingUpperBoundaryStrip|>;
 (*Helper function:extended angle relations*)
 formatAngleRelations[quad_List] :=
  Module[{vals}, vals = Table[Module[{a, b, c, d, exprs}, {a, b, c, d} = quad[[i]];
      exprs = {Row[{"α" <> ToString[i] <> " - β" <> ToString[i] <> " = ",
           NumberForm[N[a - b], {5, 3}]}], Row[{"α" <> ToString[i] <>
            " - γ" <> ToString[i] <> " = ", NumberForm[N[a - c], {5, 3}]}],
        Row[{"α" <> ToString[i] <> " - δ" <> ToString[i] <> " = ",
          NumberForm[N[a - d], {5, 3}]}], Row[{"β" <> ToString[i] <>
            " - γ" <> ToString[i] <> " = ", NumberForm[N[b - c], {5, 3}]}],
        Row[{"β" <> ToString[i] <> " - δ" <> ToString[i] <> " = ",
          NumberForm[N[b - d], {5, 3}]}], Row[{"γ" <> ToString[i] <>
            " - δ" <> ToString[i] <> " = ", NumberForm[N[c - d], {5, 3}]}],
        Row[{"α" <> ToString[i] <> " + β" <> ToString[i] <> " - 180 = ",
          NumberForm[N[a + b - 180], {5, 3}]}],
        Row[{"γ" <> ToString[i] <> " + δ" <> ToString[i] <> " - 180 = ",
          NumberForm[N[c + d - 180], {5, 3}]}],
        Row[{"α" <> ToString[i] <> " + γ" <> ToString[i] <> " - 180 = ",
          NumberForm[N[a + c - 180], {5, 3}]}],
        Row[{"β" <> ToString[i] <> " + δ" <> ToString[i] <> " - 180 = ",
          NumberForm[N[b + d - 180], {5, 3}]}],
        Row[{"α" <> ToString[i] <> " + δ" <> ToString[i] <> " - 180 = ",
          NumberForm[N[a + d - 180], {5, 3}]}],
        Row[{"β" <> ToString[i] <> " + γ" <> ToString[i] <> " - 180 = ",
          NumberForm[N[b + c - 180], {5, 3}]}],
        Row[{"α" <> ToString[i] <> " + β" <> ToString[i] <> " - γ" <> ToString[i] <>
            " - δ" <> ToString[i] <> " = ", NumberForm[N[a + b - c - d], {5, 3}]}],
        Row[{"α" <> ToString[i] <> " + γ" <> ToString[i] <> " - β" <> ToString[i] <>
            " - δ" <> ToString[i] <> " = ", NumberForm[N[a + c - b - d], {5, 3}]}],
        Row[{"α" <> ToString[i] <> " + δ" <> ToString[i] <> " - β" <> ToString[i] <>
```

```
        " - γ" <> ToString[i] <> " = ", NumberForm[N[a + d - b - c], {5, 3}]}]};
      Column[Prepend[exprs, Style["Vertex " <> ToString[i], Bold]]]],
     {i, Length[quad]}];
   Column[vals, Spacings → 1.5]];
 (*Angle relation check for anglesDeg before any switching*)
 Print[TextCell[Style["\nInitial anglesDeg (no switches):", Bold]]];
 Print[MatrixForm[angles]];
 Print[TextCell[Style["Angle relation checks for i = 1..4:", Italic]]];
 Print[formatAngleRelations[angles]];
 (*Generate all combinations of switches (from size 1 to 4)*)
 combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
 (*Evaluate condition after each combination of switches*)results = Table[
   Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];
    passQ = And @@ (checkConditionN0Degrees /@ switched);
    Print[Style["\nSwitch combination: ", Bold], name];
    Print[Style["Switched anglesDeg:", Italic]];
    Print[MatrixForm[switched]];
    Print[TextCell[Style["Angle relation checks for i = 1..4:", Italic]]];
    Print[formatAngleRelations[switched]];
    {name, passQ}], {combo, combinations}];]


(*===========================================================================
 =====*)
(*=============================
   NOT CONJUGATE-MODULAR=============================*)
(*===========================================================================
 =====*)
Column[
 {TextCell[Style["================= NOT CONJUGATE-MODULAR =================",
    Brown, Bold, 16], "Text"],
  TextCell[Style["Mi < 1 and pi ∈ ℝ for all i = 1…4 ⇒ NOT conjugate-modular.
      Boundary-strip switches preserve this.", GrayLevel[0.3]], "Text"]
 }]


Module[{angles = anglesDeg, switchers, combinations, results,
  computeConjugateModularInfo}, (*Define switch functions*)
 switchers = <|"Right" → SwitchingRightBoundaryStrip,
   "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
   "Upper" → SwitchingUpperBoundaryStrip|>;
 (*Computes Mi and pi and prints them,
 with classification*)computeConjugateModularInfo[quad_List] :=
  Module[{abcdList, Ms, aList, dList, rList, pList, summary},
   abcdList = computeABCD /@ quad;
   Ms = FullSimplify[Times @@@ abcdList];
```

```
    aList = abcdList⟦All, 1⟧;
    dList = abcdList⟦All, 4⟧;
    rList = FullSimplify /@ (aList * dList);
    pList = FullSimplify[Sqrt[# - 1]] & /@ rList;
    summary = If[AllTrue[Ms, # < 1 &] && AllTrue[pList, Element[#, Reals] &],
      Style["Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4", Bold],
      Style["Either Mi ≥ 1 or pi ∉ ℝ for some i = 1, ..., 4", Red, Bold]];
    Column[{Style["Mi values:", Bold],
      Row[{"M1 = ", Ms⟦1⟧, ", M2 = ", Ms⟦2⟧, ", M3 = ", Ms⟦3⟧, ", M4 = ", Ms⟦4⟧}],
      Style["pi values:", Bold], Row[{"p1 = ", pList⟦1⟧, ", p2 = ",
        pList⟦2⟧, ", p3 = ", pList⟦3⟧, ", p4 = ", pList⟦4⟧}], summary}]];
  (*Original anglesDeg check*)
  Print[
   TextCell[Style["\nInitial configuration (no switches applied):", Bold]]];
  Print[MatrixForm[angles]];
  Print[computeConjugateModularInfo[angles]];
  (*Generate all switch combinations (from size 1 to 4)*)
  combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
  (*Evaluate each switched configuration*) results = Table[
    Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
     Do[switched = switchers[sw][switched], {sw, combo}];
     passQ = And @@ (checkConditionN0Degrees /@ switched);
     Print[Style["\nSwitch combination: ", Bold], name];
     Print[Style["Switched anglesDeg:", Italic]];
     Print[MatrixForm[switched]];
     Print[computeConjugateModularInfo[switched]];
     {name, passQ}], {combo, combinations}];]


(*=============================================================================
 =====*)
(*==================================
 NOT CHIMERA==================================*)
(*=============================================================================
 =====*)
Column[
 {TextCell[Style["================= NOT CHIMERA =================", Blue,
    Bold, 16], "Text"],
  TextCell[
   Style["Fails conic, orthodiagonal & isogonal tests for all i=1, ...,
      4 ⇒ NOT chimera. Boundary-strip switches preserve these
      failures as demonstrated in the NOT CONIC, NOT ORTHODIAGONAL,
      and NOT ISOGONAL sections.", GrayLevel[0.3]], "Text"]
}]


(*=============================================================================
```

```
 =====*)
(*==================================
   EXTRA INFO===================================*)
(*==========================================================================
 =====*)
{α1, β1, γ1, δ1} = anglesDeg⟦1⟧ * Degree;
{α2, β2, γ2, δ2} = anglesDeg⟦2⟧ * Degree;
{α3, β3, γ3, δ3} = anglesDeg⟦3⟧ * Degree;
{α4, β4, γ4, δ4} = anglesDeg⟦4⟧ * Degree;

(*=================================
 FLEXION 1=================================*)
(*---Link lengths (parameters)---*)
(*A-links:l1=A1A2,l2=A2A3,l3=A1A4,l4=A3A4*)
Clear[l1, l2, l3, l4];
(*B-links:m1=A1B1,m2=A2B2,m3=A3B3,m4=A4B4*)
Clear[m1, m2, m3, m4];
(*C-links:n1=A1C1,n2=A2C2,n3=A3C3,n4=A4C4*)
Clear[n1, n2, n3, n4];

(*---Joint half-angle formulas:cosθi,sinθi---*)
cosθ1[t_] := FullSimplify[((Z[t])^2 - 1) / ((Z[t])^2 + 1)];
sinθ1[t_] := FullSimplify[2 Z[t] / ((Z[t])^2 + 1)];
cosθ2[t_] := FullSimplify[((W2[t])^2 - 1) / ((W2[t])^2 + 1)];
sinθ2[t_] := FullSimplify[2 W2[t] / ((W2[t])^2 + 1)];
cosθ3[t_] := FullSimplify[((U[t])^2 - 1) / ((U[t])^2 + 1)];
sinθ3[t_] := FullSimplify[2 U[t] / ((U[t])^2 + 1)];
cosθ4[t_] := FullSimplify[((W1[t])^2 - 1) / ((W1[t])^2 + 1)];
sinθ4[t_] := FullSimplify[2 W1[t] / ((W1[t])^2 + 1)];

(*==========================A-POINTS==========================*)
xA2 = 0;
xA1 = l1;
xA3 = l2 Cos[δ2];
xA4 = l1 - l3 Cos[δ1];
yA2 = 0;
yA1 = 0;
yA3 = l2 Sin[δ2];
yA4 = l3 Sin[δ1];
zA2 = 0; zA1 = 0; zA3 = 0; zA4 = 0;

(*==========================B-POINTS==========================*)
xB1[t_] := xA1 - m1 Cos[α1];
yB1[t_] := yA1 + m1 Sin[α1] cosθ1[t];
zB1[t_] := zA1 + m1 Sin[α1] sinθ1[t];
xB2[t_] := xA2 + m2 Cos[α2];
yB2[t_] := yA2 + m2 Sin[α2] cosθ1[t];
```

```
zB2[t_] := zA2 + m2 Sin[α2] sinθ1[t];
xB3[t_] := xA3 - m3 (Cos[α3] Cos[δ2 + δ3] + Sin[α3] cosθ3[t] Sin[δ2 + δ3]);
yB3[t_] := yA3 - m3 (Cos[α3] Sin[δ2 + δ3] - Sin[α3] cosθ3[t] Cos[δ2 + δ3]);
zB3[t_] := zA3 + m3 Sin[α3] sinθ3[t];
xB4[t_] := xA4 + m4 (Cos[α4] Cos[δ2 + δ3] - Sin[α4] cosθ3[t] Sin[δ2 + δ3]);
yB4[t_] := yA4 + m4 (Cos[α4] Sin[δ2 + δ3] + Sin[α4] cosθ3[t] Cos[δ2 + δ3]);
zB4[t_] := zA4 + m4 Sin[α4] sinθ3[t];

(*==========================C-POINTS==========================*)
xC1[t_] := xA1 - n1 (Cos[γ1] Cos[δ1] + Sin[γ1] cosθ4[t] Sin[δ1]);
yC1[t_] := yA1 + n1 (Cos[γ1] Sin[δ1] - Sin[γ1] cosθ4[t] Cos[δ1]);
zC1[t_] := zA1 + n1 Sin[γ1] sinθ4[t];
xC2[t_] := xA2 + n2 (Cos[γ2] Cos[δ2] + Sin[γ2] cosθ2[t] Sin[δ2]);
yC2[t_] := yA2 + n2 (Cos[γ2] Sin[δ2] - Sin[γ2] cosθ2[t] Cos[δ2]);
zC2[t_] := zA2 + n2 Sin[γ2] sinθ2[t];
xC3[t_] := xA3 - n3 (Cos[γ3] Cos[δ2] - Sin[γ3] cosθ2[t] Sin[δ2]);
yC3[t_] := yA3 - n3 (Cos[γ3] Sin[δ2] + Sin[γ3] cosθ2[t] Cos[δ2]);
zC3[t_] := zA3 + n3 Sin[γ3] sinθ2[t];
xC4[t_] := xA4 + n4 (Cos[γ4] Cos[δ1] - Sin[γ4] cosθ4[t] Sin[δ1]);
yC4[t_] := yA4 - n4 (Cos[γ4] Sin[δ1] + Sin[γ4] cosθ4[t] Cos[δ1]);
zC4[t_] := zA4 + n4 Sin[γ4] sinθ4[t];

(*COPLANARITY (DET-VALUE) FUNCTIONS*)detA2A1A2C2A1C1[t_] :=
  Module[{vA2A1, vA2C2, vA1C1}, vA2A1 = {xA1, yA1, zA1} - {xA2, yA2, zA2};
   vA2C2 = {xC2[t], yC2[t], zC2[t]} - {xA2, yA2, zA2};
   vA1C1 = {xC1[t], yC1[t], zC1[t]} - {xA1, yA1, zA1};
   Simplify[Det[{vA2A1, vA2C2, vA1C1}]]];

detA3A4A3C3A4C4[t_] :=
  Module[{vA3A4, vA3C3, vA4C4}, vA3A4 = {xA4, yA4, zA4} - {xA3, yA3, zA3};
   vA3C3 = {xC3[t], yC3[t], zC3[t]} - {xA3, yA3, zA3};
   vA4C4 = {xC4[t], yC4[t], zC4[t]} - {xA4, yA4, zA4};
   Simplify[Det[{vA3A4, vA3C3, vA4C4}]]];

detA1A4A1B1A4B4[t_] :=
  Module[{vA1A4, vA1B1, vA4B4}, vA1A4 = {xA4, yA4, zA4} - {xA1, yA1, zA1};
   vA1B1 = {xB1[t], yB1[t], zB1[t]} - {xA1, yA1, zA1};
   vA4B4 = {xB4[t], yB4[t], zB4[t]} - {xA4, yA4, zA4};
   Simplify[Det[{vA1A4, vA1B1, vA4B4}]]];

detA2A3A2B2A3B3[t_] :=
  Module[{vA2A3, vA2B2, vA3B3}, vA2A3 = {xA3, yA3, zA3} - {xA2, yA2, zA2};
   vA2B2 = {xB2[t], yB2[t], zB2[t]} - {xA2, yA2, zA2};
   vA3B3 = {xB3[t], yB3[t], zB3[t]} - {xA3, yA3, zA3};
   Simplify[Det[{vA2A3, vA2B2, vA3B3}]]];

(*---t-range---*)
```

```
tMin = 1 / Sqrt[2 * Sqrt[3] + 3];
tMax = 1 / 2;

Column[{TextCell[
   Style["================= EXTRA INFO (FLEXION 1) =================",
    Darker[Red], Bold, 16], "Text"], (*Explanatory text*)TextCell[
   Style["This example does NOT have planar parameter lines (C2A2A1C1,
      C3A3A4C4, B1A1A4B4, B2A2A3B3)", GrayLevel[0.3]], "Text"],
  Spacer[12],

  TextCell[Style["Coplanarity Check: A2A1-A2C2-A1C1",
    Darker[Green], Bold, 12], "Text"],
  TextCell[Style["Determinant of {A2A1, A2C2, A1C1} as a function of t",
    GrayLevel[0.3]], "Text"],
  Spacer[5],
  detA2A1A2C2A1C1[t],
  Spacer[5],

  TextCell[Style["Coplanarity Check: A3A4-A3C3-A4C4",
    Darker[Blue], Bold, 12], "Text"],
  TextCell[Style["Determinant of {A3A4, A3C3, A4C4} as a function of t",
    GrayLevel[0.3]], "Text"],
  Spacer[5],
  detA3A4A3C3A4C4[t],
  Spacer[5],

  TextCell[Style["Coplanarity Check: A2A3-A2B2-A3B3",
    Darker[Purple], Bold, 12], "Text"],
  TextCell[Style["Determinant of {A2A3, A2B2, A3B3} as a function of t",
    GrayLevel[0.3]], "Text"],
  Spacer[5],
  detA2A3A2B2A3B3[t],
  Spacer[5],

  TextCell[Style["Coplanarity Check: A1A4-A1B1-A4B4",
    Darker[Brown], Bold, 12], "Text"],
  TextCell[Style["Determinant of {A1A4, A1B1, A4B4} as a function of t",
    GrayLevel[0.3]], "Text"],
  Spacer[5],
  detA1A4A1B1A4B4[t],
  Spacer[5],

  (*---Numeric parameter values ---*)
  l1 = 4; l2 = 4; l3 = 3.863703305156273; l4 = 5.035276180410083;
m1 = 4.528845917006416; m2 = 5.05545590735599995;
  m3 = 6.363904177855777; m4 = 5.700997492662467;
  n1 = 6.181925288250036; n2 = 6.954665949281291;
```

```
n3 = 3.090962644125018; n4 = 6.181925288250037;

(*Plots in a light panel arranged in a 2×2 grid*)
TextCell[
 Style["Determinant Plots for Our Example", Darker[Cyan], Bold, 14], "Text"],
Panel[GraphicsGrid[{{(*1) A2A1-A2C2-A1C1*)Plot[detA2A1A2C2A1C1[t],
     {t, tMin, tMax}, PlotLabel → Style["det(A2A1, A2C2, A1C1)", Bold, 14],
     AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250],
    (*2) A3A4-A3C3-A4C4*)Plot[detA3A4A3C3A4C4[t], {t, tMin, tMax},
     PlotLabel → Style["det(A3A4, A3C3, A4C4)", Bold, 14],
     AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250]},
   {(*3) A1A4-A1B1-A4B4*)Plot[detA1A4A1B1A4B4[t], {t, tMin, tMax},
     PlotLabel → Style["det(A1A4, A1B1, A4B4)", Bold, 14],
     AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250],
    (*4) A2A3-A2B2-A3B3*)Plot[detA2A3A2B2A3B3[t], {t, tMin, tMax},
     PlotLabel → Style["det(A2A3, A2B2, A3B3)", Bold, 14],
     AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250]}}],
 Background → Lighter[Gray, 0.95], FrameMargins → 15]}]


(*==================================
 FLEXION 2=================================*)
(*---Link lengths (parameters)---*)
(*A-links:l1=A1A2,l2=A2A3,l3=A1A4,l4=A3A4*)
Clear[l1, l2, l3, l4];
(*B-links:m1=A1B1,m2=A2B2,m3=A3B3,m4=A4B4*)
Clear[m1, m2, m3, m4];
(*C-links:n1=A1C1,n2=A2C2,n3=A3C3,n4=A4C4*)
Clear[n1, n2, n3, n4];

(*---Joint half-angle formulas:cosθi,sinθi---*)
cosθ1[t_] := FullSimplify[((Z2[t])^2 - 1) / ((Z2[t])^2 + 1)];
sinθ1[t_] := FullSimplify[2 Z2[t] / ((Z2[t])^2 + 1)];
cosθ2[t_] := FullSimplify[((W22[t])^2 - 1) / ((W22[t])^2 + 1)];
sinθ2[t_] := FullSimplify[2 W22[t] / ((W22[t])^2 + 1)];
cosθ3[t_] := FullSimplify[((U2[t])^2 - 1) / ((U2[t])^2 + 1)];
sinθ3[t_] := FullSimplify[2 U2[t] / ((U2[t])^2 + 1)];
cosθ4[t_] := FullSimplify[((W12[t])^2 - 1) / ((W12[t])^2 + 1)];
sinθ4[t_] := FullSimplify[2 W12[t] / ((W12[t])^2 + 1)];

(*==========================A-POINTS==========================*)
xA2 = 0;
xA1 = l1;
xA3 = l2 Cos[δ2];
xA4 = l1 - l3 Cos[δ1];
yA2 = 0;
yA1 = 0;
```

```
yA3 = l2 Sin[δ2];
yA4 = l3 Sin[δ1];
zA2 = 0; zA1 = 0; zA3 = 0; zA4 = 0;

(*=========================B-POINTS==========================*)
xB1[t_] := xA1 - m1 Cos[α1];
yB1[t_] := yA1 + m1 Sin[α1] cosθ1[t];
zB1[t_] := zA1 + m1 Sin[α1] sinθ1[t];
xB2[t_] := xA2 + m2 Cos[α2];
yB2[t_] := yA2 + m2 Sin[α2] cosθ1[t];
zB2[t_] := zA2 + m2 Sin[α2] sinθ1[t];
xB3[t_] := xA3 - m3 (Cos[α3] Cos[δ2 + δ3] + Sin[α3] cosθ3[t] Sin[δ2 + δ3]);
yB3[t_] := yA3 - m3 (Cos[α3] Sin[δ2 + δ3] - Sin[α3] cosθ3[t] Cos[δ2 + δ3]);
zB3[t_] := zA3 + m3 Sin[α3] sinθ3[t];
xB4[t_] := xA4 + m4 (Cos[α4] Cos[δ2 + δ3] - Sin[α4] cosθ3[t] Sin[δ2 + δ3]);
yB4[t_] := yA4 + m4 (Cos[α4] Sin[δ2 + δ3] + Sin[α4] cosθ3[t] Cos[δ2 + δ3]);
zB4[t_] := zA4 + m4 Sin[α4] sinθ3[t];

(*=========================C-POINTS==========================*)
xC1[t_] := xA1 - n1 (Cos[γ1] Cos[δ1] + Sin[γ1] cosθ4[t] Sin[δ1]);
yC1[t_] := yA1 + n1 (Cos[γ1] Sin[δ1] - Sin[γ1] cosθ4[t] Cos[δ1]);
zC1[t_] := zA1 + n1 Sin[γ1] sinθ4[t];
xC2[t_] := xA2 + n2 (Cos[γ2] Cos[δ2] + Sin[γ2] cosθ2[t] Sin[δ2]);
yC2[t_] := yA2 + n2 (Cos[γ2] Sin[δ2] - Sin[γ2] cosθ2[t] Cos[δ2]);
zC2[t_] := zA2 + n2 Sin[γ2] sinθ2[t];
xC3[t_] := xA3 - n3 (Cos[γ3] Cos[δ2] - Sin[γ3] cosθ2[t] Sin[δ2]);
yC3[t_] := yA3 - n3 (Cos[γ3] Sin[δ2] + Sin[γ3] cosθ2[t] Cos[δ2]);
zC3[t_] := zA3 + n3 Sin[γ3] sinθ2[t];
xC4[t_] := xA4 + n4 (Cos[γ4] Cos[δ1] - Sin[γ4] cosθ4[t] Sin[δ1]);
yC4[t_] := yA4 - n4 (Cos[γ4] Sin[δ1] + Sin[γ4] cosθ4[t] Cos[δ1]);
zC4[t_] := zA4 + n4 Sin[γ4] sinθ4[t];

(*COPLANARITY (DET-VALUE) FUNCTIONS*)detA2A1A2C2A1C1[t_] :=
  Module[{vA2A1, vA2C2, vA1C1}, vA2A1 = {xA1, yA1, zA1} - {xA2, yA2, zA2};
   vA2C2 = {xC2[t], yC2[t], zC2[t]} - {xA2, yA2, zA2};
   vA1C1 = {xC1[t], yC1[t], zC1[t]} - {xA1, yA1, zA1};
   Simplify[Det[{vA2A1, vA2C2, vA1C1}]]];

detA3A4A3C3A4C4[t_] :=
  Module[{vA3A4, vA3C3, vA4C4}, vA3A4 = {xA4, yA4, zA4} - {xA3, yA3, zA3};
   vA3C3 = {xC3[t], yC3[t], zC3[t]} - {xA3, yA3, zA3};
   vA4C4 = {xC4[t], yC4[t], zC4[t]} - {xA4, yA4, zA4};
   Simplify[Det[{vA3A4, vA3C3, vA4C4}]]];

detA1A4A1B1A4B4[t_] :=
  Module[{vA1A4, vA1B1, vA4B4}, vA1A4 = {xA4, yA4, zA4} - {xA1, yA1, zA1};
   vA1B1 = {xB1[t], yB1[t], zB1[t]} - {xA1, yA1, zA1};
```

```
   vA4B4 = {xB4[t], yB4[t], zB4[t]} - {xA4, yA4, zA4};
   Simplify[Det[{vA1A4, vA1B1, vA4B4}]]];

detA2A3A2B2A3B3[t_] :=
  Module[{vA2A3, vA2B2, vA3B3}, vA2A3 = {xA3, yA3, zA3} - {xA2, yA2, zA2};
   vA2B2 = {xB2[t], yB2[t], zB2[t]} - {xA2, yA2, zA2};
   vA3B3 = {xB3[t], yB3[t], zB3[t]} - {xA3, yA3, zA3};
   Simplify[Det[{vA2A3, vA2B2, vA3B3}]]];

(*---t-range---*)
tMin = 1 / Sqrt[2 * Sqrt[3] + 3];
tMax = 1 / 2;

Column[{TextCell[
   Style["================= EXTRA INFO (FLEXION 2) =================",
    Red, Bold, 16], "Text"], (*Explanatory text*)TextCell[
   Style["This example does NOT have planar parameter lines (C2A2A1C1,
      C3A3A4C4, B1A1A4B4, B2A2A3B3)", GrayLevel[0.3]], "Text"],
  Spacer[12],

  TextCell[
   Style["Coplanarity Check: A2A1-A2C2-A1C1", Green, Bold, 12], "Text"],
  TextCell[Style["Determinant of {A2A1, A2C2, A1C1} as a function of t",
    GrayLevel[0.3]], "Text"],
  Spacer[5],
  detA2A1A2C2A1C1[t],
  Spacer[5],

  TextCell[
   Style["Coplanarity Check: A3A4-A3C3-A4C4", Blue, Bold, 12], "Text"],
  TextCell[Style["Determinant of {A3A4, A3C3, A4C4} as a function of t",
    GrayLevel[0.3]], "Text"],
  Spacer[5],
  detA3A4A3C3A4C4[t],
  Spacer[5],

  TextCell[
   Style["Coplanarity Check: A2A3-A2B2-A3B3", Purple, Bold, 12], "Text"],
  TextCell[Style["Determinant of {A2A3, A2B2, A3B3} as a function of t",
    GrayLevel[0.3]], "Text"],
  Spacer[5],
  detA2A3A2B2A3B3[t],
  Spacer[5],

  TextCell[
   Style["Coplanarity Check: A1A4-A1B1-A4B4", Brown, Bold, 12], "Text"],
  TextCell[Style["Determinant of {A1A4, A1B1, A4B4} as a function of t",
```

```
     GraylLevel[0.3]], "Text"],
   Spacer[5],
   detA1A4A1B1A4B4[t],
   Spacer[5],

   (*---Numeric parameter values ---*)
   l1 = 4; l2 = 4; l3 = 3.863703305156273; l4 = 5.035276180410083;
m1 = 4.528845917006416; m2 = 5.05545590735599995;
   m3 = 6.363904177855777; m4 = 5.700997492662467;
   n1 = 6.181925288250036; n2 = 6.954665949281291;
   n3 = 3.090962644125018; n4 = 6.181925288250037;

   (*Plots in a light panel arranged in a 2x2 grid*)
   TextCell[Style["Determinant Plots for Our Example", Cyan, Bold, 14], "Text"],
   Panel[GraphicsGrid[{{(*1) A2A1-A2C2-A1C1*)Plot[detA2A1A2C2A1C1[t],
        {t, tMin, tMax}, PlotLabel → Style["det(A2A1, A2C2, A1C1)", Bold, 14],
        AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250],
       (*2) A3A4-A3C3-A4C4*)Plot[detA3A4A3C3A4C4[t], {t, tMin, tMax},
        PlotLabel → Style["det(A3A4, A3C3, A4C4)", Bold, 14],
        AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250]},
      {(*3) A1A4-A1B1-A4B4*)Plot[detA1A4A1B1A4B4[t], {t, tMin, tMax},
        PlotLabel → Style["det(A1A4, A1B1, A4B4)", Bold, 14],
        AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250],
       (*4) A2A3-A2B2-A3B3*)Plot[detA2A3A2B2A3B3[t], {t, tMin, tMax},
        PlotLabel → Style["det(A2A3, A2B2, A3B3)", Bold, 14],
        AxesLabel → {"t", "det"}, PlotRange → All, ImageSize → 250]}}],
    Background → Lighter[Gray, 0.95], FrameMargins → 15]}]
```

Out[ ]=

================= CONDITION (N.0) =================

✔ All vertices satisfy (N.0).

Out[ ]=

================= CONDITION (N.3) =================

✔ M1 = M2 = M3 = M4 = $-1+\sqrt{3}$

Out[ ]=

================= CONDITION (N.4) =================

✔ r1 = r2 = $\frac{2}{\sqrt{3}}$; ✔ r3 = r4 = $\frac{2}{\sqrt{3}}$

✔ s1 = s4 = $3-\sqrt{3}$; ✔ s2 = s3 = $\frac{1}{\sqrt{3}}$

Out[ ]=

================= CONDITION (N.5) =================

⚠ *Approximate validation using*
ε-tolerance. For rigorous proof, see the referenced paper.

✅ **Valid Combination Found** (M < 1):

e1 = −1, **e2** = −1, **e3** = 1

t1 = 0.K + 0.446521iK'

t2 = 1.K + 0.446521iK'

t3 = 3.K + 0.446521iK'

t4 = 0.K + 0.446521iK'

t1 + **e1*t2** + **e2*t3** + **e3*t4** = −4.K + 0.iK'

Out[∘]=

================== OTHER PARAMETERS ==================

u = $2-\sqrt{3}$

σ1 = 165 °, **σ2** = 165 °, **σ3** = 195 °, **σ4** = 165 °

f1 = $\frac{1}{2}\left(1+\sqrt{3}\right)$, **f2** = $4-2\sqrt{3}$, **f3** = $4-2\sqrt{3}$, **f4** = $\frac{1}{2}\left(1+\sqrt{3}\right)$

z1 = $1+\sqrt{3}$, **z2** = $-1-\frac{2}{\sqrt{3}}$, **z3** = $-1-\frac{2}{\sqrt{3}}$, **z4** = $1+\sqrt{3}$

x1 = $3+2\sqrt{3}$, **x2** = $3+2\sqrt{3}$, **x3** = $3+2\sqrt{3}$, **x4** = $3+2\sqrt{3}$

y1 = $2+\sqrt{3}$, **y2** = $\frac{1}{-1+\frac{1}{\sqrt{3}}}$, **y3** = $\frac{1}{-1+\frac{1}{\sqrt{3}}}$, **y4** = $2+\sqrt{3}$

p1 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, **p2** = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, **p3** = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, **p4** = $\sqrt{-1+\frac{2}{\sqrt{3}}}$

q1 = $\sqrt{2-\sqrt{3}}$, **q2** = $i\sqrt{1-\frac{1}{\sqrt{3}}}$, **q3** = $i\sqrt{1-\frac{1}{\sqrt{3}}}$, **q4** = $\sqrt{2-\sqrt{3}}$

$\overline{\alpha 1}$ = 60°, $\overline{\beta 1}$ = 150°, $\overline{\gamma 1}$ = 45°, $\overline{\delta 1}$ = 75°

Inequalities (vertex 1): $\sin(105\,°_1) > \sin(\overline{90\,°_1})\sin(\overline{105\,°_1})$
: True , $\sin(120\,°_1) > \sin(\overline{90\,°_1})\sin(\overline{120\,°_1})$: True
, $\sin(15\,°_1)\sin(105\,°_1) < \sin(\overline{15\,°_1})\sin(\overline{105\,°_1})$: True

Out[∘]=

================== FLEXIBILITY (FLEXION 1) ==================

$P_1[Z, W1] = 0$

$P_2[Z, W2] = 0$

$P_3[U, W2] = 0$

$P_4[U, W1] = 0$

Out[∘]=

================== FLEXIBILITY (FLEXION 2) ==================

$P_1[Z, W1] = 0$

$P_2[Z, W2] = 0$

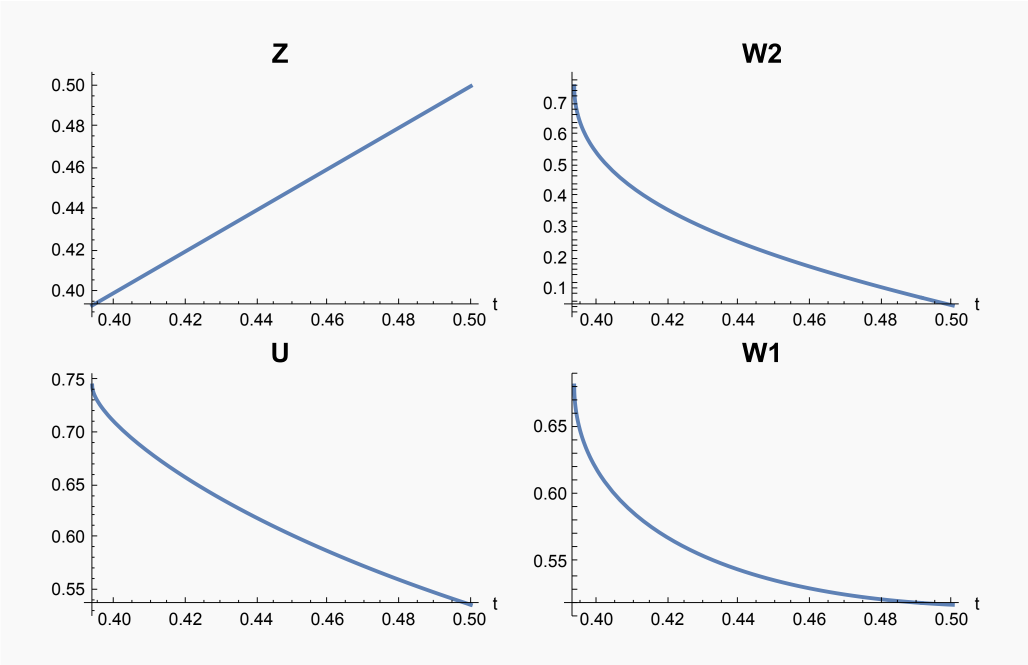
$P_3[U, W2] = 0$

$P_4[U, W1] = 0$

Out[ ]=

================= NOT TRIVIAL (FLEXION 1) =================

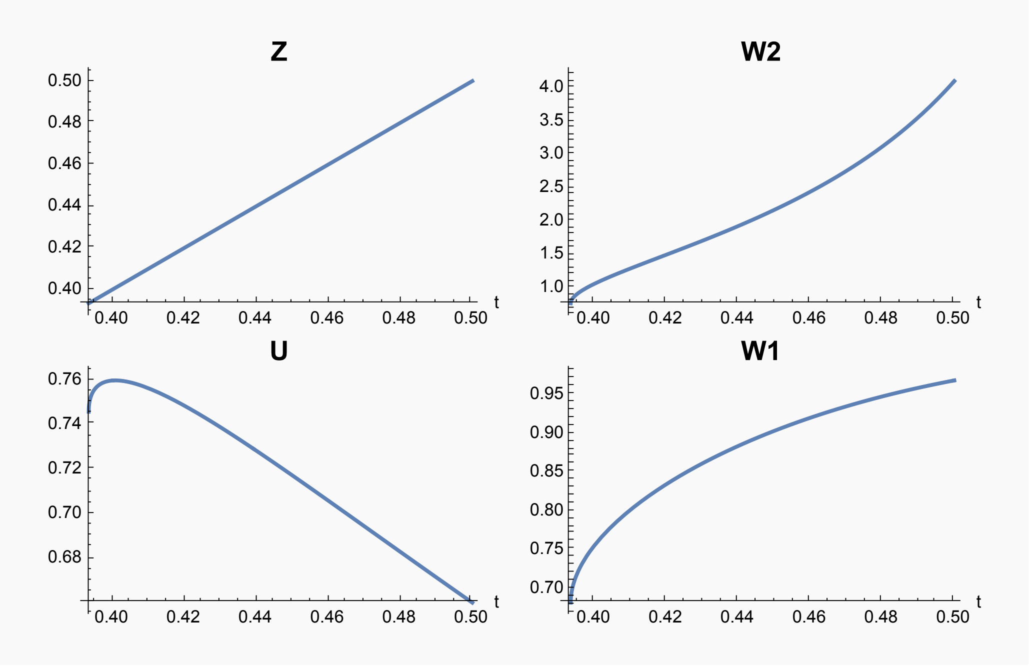
This configuration does not belong to the trivial
class – even after switching the boundary strips –
since none of the functions Z, W2, U, or W1 is constant.



Out[ ]=

================= NOT TRIVIAL (FLEXION 2) =================

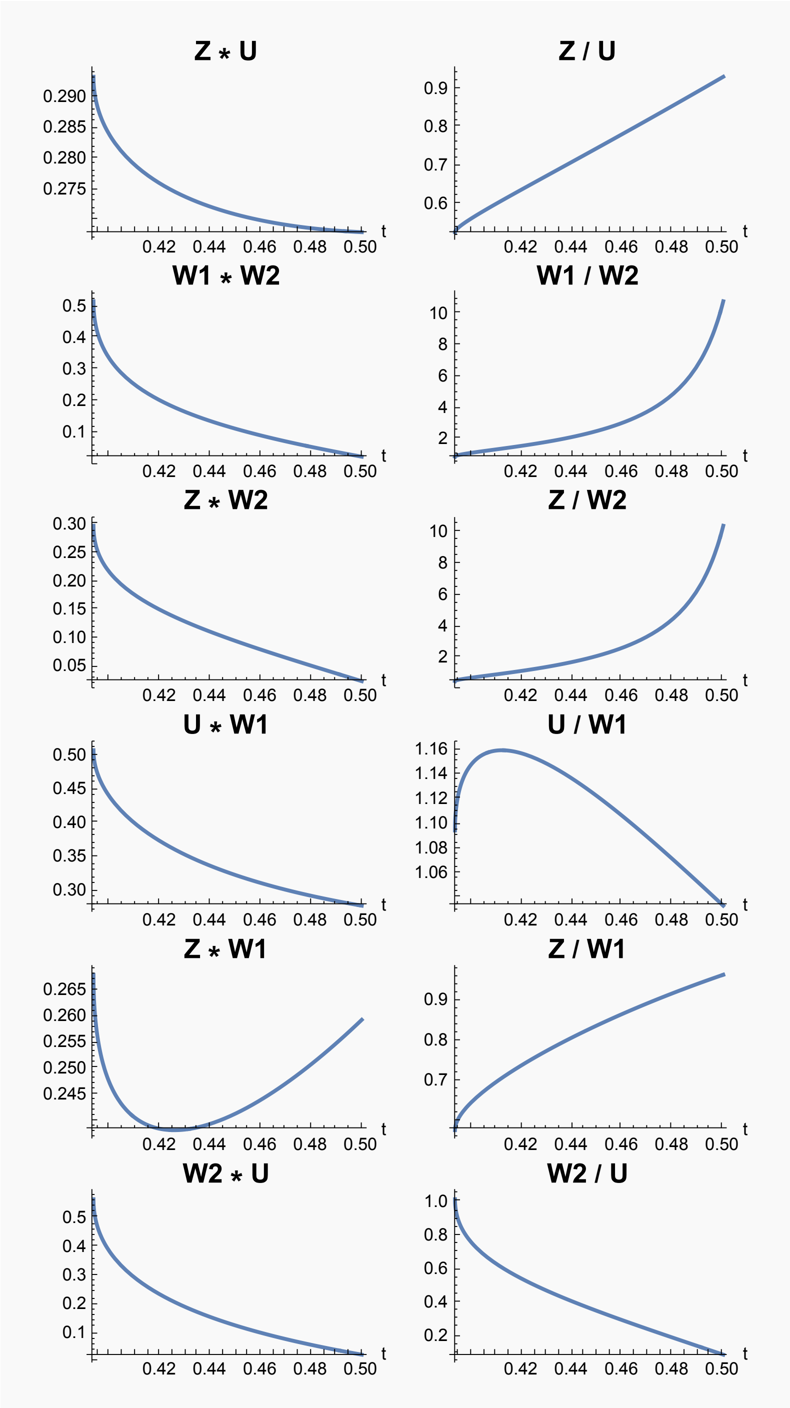
This configuration does not belong to the trivial class — even after switching the boundary strips — since none of the functions Z, W2, U, or W1 is constant.



Out[ ]=

============= NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE (FLEXION 1) =============

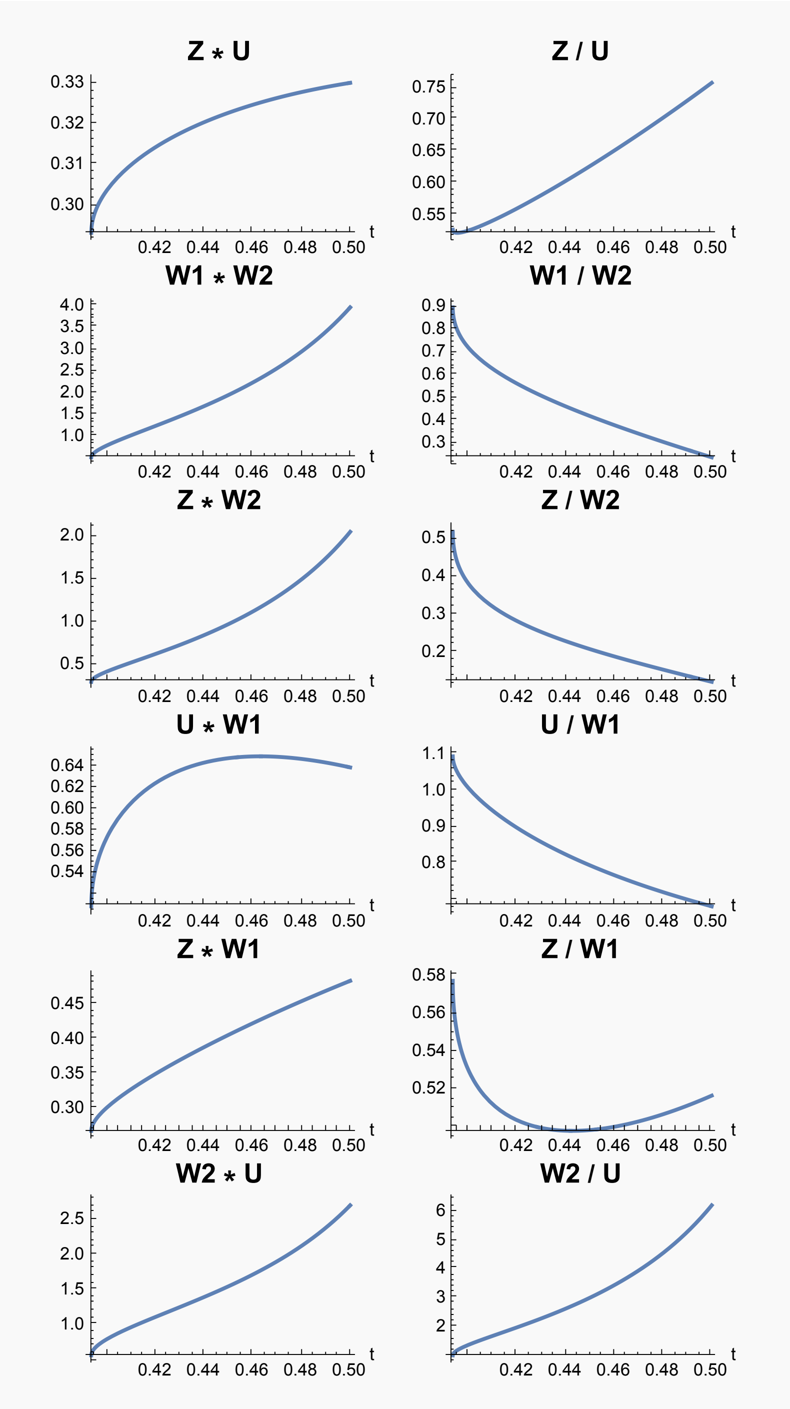
This configuration does not belong to the Linear compound class nor to the linear conjugate class — even after switching the boundary strips — since none of the functions ZU, Z/U, W1W2, W1/W2, ZW2, Z/W2, UW1, and U/W1 is constant. In addition, none of ZW1, Z/W1, W2U, W2/U is constant as well.



Out[ ]=

============= NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE (FLEXION 2) =============

This configuration does not belong to the Linear compound class nor to the linear conjugate class — even after switching the boundary strips — since none of the functions ZU, Z/U, W1W2, W1/W2, ZW2, Z/W2, UW1, and U/W1 is constant. In addition, none of ZW1, Z/W1, W2U, W2/U is constant as well.



Out[•]=

================== **NOT CONIC** ==================

Condition (N.0) is satisfied ⇒ this configuration is NOT equimodular-conic. Applying any boundary-strip switch still preserves (N.0), so no conic form emerges.

Out[•]=

CONDITION (N.0) AFTER SWITCHING BOUNDARY STRIPS

Right: Condition (N.0) is still satisfied.
Left: Condition (N.0) is still satisfied.
Lower: Condition (N.0) is still satisfied.
Upper: Condition (N.0) is still satisfied.
Right + Left: Condition (N.0) is still satisfied.
Right + Lower: Condition (N.0) is still satisfied.
Right + Upper: Condition (N.0) is still satisfied.
Left + Lower: Condition (N.0) is still satisfied.
Left + Upper: Condition (N.0) is still satisfied.
Lower + Upper: Condition (N.0) is still satisfied.
Right + Left + Lower: Condition (N.0) is still satisfied.
Right + Left + Upper: Condition (N.0) is still satisfied.
Right + Lower + Upper: Condition (N.0) is still satisfied.
Left + Lower + Upper: Condition (N.0) is still satisfied.
Right + Left + Lower + Upper: Condition (N.0) is still satisfied.

Out[•]=

================== **NOT ORTHODIAGONAL** ==================

$\cos(\alpha_i)\cdot\cos(\gamma_i) \neq \cos(\beta_i)\cdot\cos(\delta_i)$ for each $i = 1\ldots4$ ⇒ NOT orthodiagonal. Switching boundary strips does not correct this.

Initial anglesDeg (no switches):

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$$\cos(\alpha1)\cdot\cos(\gamma1) - \cos(\beta1)\cdot\cos(\delta1) = \frac{\sqrt{2-\sqrt{3}}}{4}$$

$$\cos(\alpha2)\cdot\cos(\gamma2) - \cos(\beta2)\cdot\cos(\delta2) = -\frac{1}{4}\sqrt{2-\sqrt{3}}$$

$$\cos(\alpha3)\cdot\cos(\gamma3) - \cos(\beta3)\cdot\cos(\delta3) = -\frac{1}{4}\sqrt{2-\sqrt{3}}$$

$$\cos(\alpha4)\cdot\cos(\gamma4) - \cos(\beta4)\cdot\cos(\delta4) = \frac{\sqrt{2-\sqrt{3}}}{4}$$

Switch combination: Right

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$\cos(\alpha 1)\cdot\cos(\gamma 1) - \cos(\beta 1)\cdot\cos(\delta 1) = \frac{1}{8}\left(\sqrt{2}-\sqrt{6}\right)$

$\cos(\alpha 2)\cdot\cos(\gamma 2) - \cos(\beta 2)\cdot\cos(\delta 2) = -\frac{1}{4}\sqrt{2-\sqrt{3}}$

$\cos(\alpha 3)\cdot\cos(\gamma 3) - \cos(\beta 3)\cdot\cos(\delta 3) = -\frac{1}{4}\sqrt{2-\sqrt{3}}$

$\cos(\alpha 4)\cdot\cos(\gamma 4) - \cos(\beta 4)\cdot\cos(\delta 4) = \frac{1}{8}\left(\sqrt{2}-\sqrt{6}\right)$

Switch combination: Left

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$\cos(\alpha 1)\cdot\cos(\gamma 1) - \cos(\beta 1)\cdot\cos(\delta 1) = \frac{\sqrt{2-\sqrt{3}}}{4}$

$\cos(\alpha 2)\cdot\cos(\gamma 2) - \cos(\beta 2)\cdot\cos(\delta 2) = \frac{1}{8}\left(-\sqrt{2}+\sqrt{6}\right)$

$\cos(\alpha 3)\cdot\cos(\gamma 3) - \cos(\beta 3)\cdot\cos(\delta 3) = \frac{1}{8}\left(-\sqrt{2}+\sqrt{6}\right)$

$\cos(\alpha 4)\cdot\cos(\gamma 4) - \cos(\beta 4)\cdot\cos(\delta 4) = \frac{\sqrt{2-\sqrt{3}}}{4}$

Switch combination: Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$\cos(\alpha 1)\cdot\cos(\gamma 1) - \cos(\beta 1)\cdot\cos(\delta 1) = \frac{1}{8}\left(\sqrt{2}-\sqrt{6}\right)$

$\cos(\alpha 2)\cdot\cos(\gamma 2) - \cos(\beta 2)\cdot\cos(\delta 2) = \frac{1}{8}\left(-\sqrt{2}+\sqrt{6}\right)$

$\cos(\alpha 3)\cdot\cos(\gamma 3) - \cos(\beta 3)\cdot\cos(\delta 3) = -\frac{1}{4}\sqrt{2-\sqrt{3}}$

$\cos(\alpha 4)\cdot\cos(\gamma 4) - \cos(\beta 4)\cdot\cos(\delta 4) = \frac{\sqrt{2-\sqrt{3}}}{4}$

Switch combination: Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$\cos(\alpha 1)\cdot\cos(\gamma 1) - \cos(\beta 1)\cdot\cos(\delta 1) = \frac{\sqrt{2-\sqrt{3}}}{4}$

$\cos(\alpha 2)\cdot\cos(\gamma 2) - \cos(\beta 2)\cdot\cos(\delta 2) = -\frac{1}{4}\sqrt{2-\sqrt{3}}$

$\cos(\alpha 3)\cdot\cos(\gamma 3) - \cos(\beta 3)\cdot\cos(\delta 3) = \frac{1}{8}\left(-\sqrt{2}+\sqrt{6}\right)$

$\cos(\alpha 4)\cdot\cos(\gamma 4) - \cos(\beta 4)\cdot\cos(\delta 4) = \frac{1}{8}\left(\sqrt{2}-\sqrt{6}\right)$

Switch combination: Right + Left

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$\cos(\alpha 1)\cdot\cos(\gamma 1) - \cos(\beta 1)\cdot\cos(\delta 1) = \frac{1}{8}\left(\sqrt{2} - \sqrt{6}\right)$

$\cos(\alpha 2)\cdot\cos(\gamma 2) - \cos(\beta 2)\cdot\cos(\delta 2) = \frac{1}{8}\left(-\sqrt{2} + \sqrt{6}\right)$

$\cos(\alpha 3)\cdot\cos(\gamma 3) - \cos(\beta 3)\cdot\cos(\delta 3) = \frac{1}{8}\left(-\sqrt{2} + \sqrt{6}\right)$

$\cos(\alpha 4)\cdot\cos(\gamma 4) - \cos(\beta 4)\cdot\cos(\delta 4) = \frac{1}{8}\left(\sqrt{2} - \sqrt{6}\right)$

Switch combination: Right + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$\cos(\alpha 1)\cdot\cos(\gamma 1) - \cos(\beta 1)\cdot\cos(\delta 1) = \frac{\sqrt{2-\sqrt{3}}}{4}$

$\cos(\alpha 2)\cdot\cos(\gamma 2) - \cos(\beta 2)\cdot\cos(\delta 2) = \frac{1}{8}\left(-\sqrt{2} + \sqrt{6}\right)$

$\cos(\alpha 3)\cdot\cos(\gamma 3) - \cos(\beta 3)\cdot\cos(\delta 3) = -\frac{1}{4}\sqrt{2 - \sqrt{3}}$

$\cos(\alpha 4)\cdot\cos(\gamma 4) - \cos(\beta 4)\cdot\cos(\delta 4) = \frac{1}{8}\left(\sqrt{2} - \sqrt{6}\right)$

Switch combination: Right + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$\cos(\alpha 1)\cdot\cos(\gamma 1) - \cos(\beta 1)\cdot\cos(\delta 1) = \frac{1}{8}\left(\sqrt{2} - \sqrt{6}\right)$

$\cos(\alpha 2)\cdot\cos(\gamma 2) - \cos(\beta 2)\cdot\cos(\delta 2) = -\frac{1}{4}\sqrt{2 - \sqrt{3}}$

$\cos(\alpha 3)\cdot\cos(\gamma 3) - \cos(\beta 3)\cdot\cos(\delta 3) = \frac{1}{8}\left(-\sqrt{2} + \sqrt{6}\right)$

$\cos(\alpha 4)\cdot\cos(\gamma 4) - \cos(\beta 4)\cdot\cos(\delta 4) = \frac{\sqrt{2-\sqrt{3}}}{4}$

Switch combination: Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$\cos(\alpha1)\cdot\cos(\gamma1) - \cos(\beta1)\cdot\cos(\delta1) = \frac{1}{8}\left(\sqrt{2}-\sqrt{6}\right)$

$\cos(\alpha2)\cdot\cos(\gamma2) - \cos(\beta2)\cdot\cos(\delta2) = -\frac{1}{4}\sqrt{2-\sqrt{3}}$

$\cos(\alpha3)\cdot\cos(\gamma3) - \cos(\beta3)\cdot\cos(\delta3) = \frac{1}{8}\left(-\sqrt{2}+\sqrt{6}\right)$

$\cos(\alpha4)\cdot\cos(\gamma4) - \cos(\beta4)\cdot\cos(\delta4) = \frac{\sqrt{2-\sqrt{3}}}{4}$

Switch combination: Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$\cos(\alpha1)\cdot\cos(\gamma1) - \cos(\beta1)\cdot\cos(\delta1) = \frac{\sqrt{2-\sqrt{3}}}{4}$

$\cos(\alpha2)\cdot\cos(\gamma2) - \cos(\beta2)\cdot\cos(\delta2) = \frac{1}{8}\left(-\sqrt{2}+\sqrt{6}\right)$

$\cos(\alpha3)\cdot\cos(\gamma3) - \cos(\beta3)\cdot\cos(\delta3) = -\frac{1}{4}\sqrt{2-\sqrt{3}}$

$\cos(\alpha4)\cdot\cos(\gamma4) - \cos(\beta4)\cdot\cos(\delta4) = \frac{1}{8}\left(\sqrt{2}-\sqrt{6}\right)$

Switch combination: Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$\cos(\alpha1)\cdot\cos(\gamma1) - \cos(\beta1)\cdot\cos(\delta1) = \frac{1}{8}\left(\sqrt{2}-\sqrt{6}\right)$

$\cos(\alpha2)\cdot\cos(\gamma2) - \cos(\beta2)\cdot\cos(\delta2) = \frac{1}{8}\left(-\sqrt{2}+\sqrt{6}\right)$

$\cos(\alpha3)\cdot\cos(\gamma3) - \cos(\beta3)\cdot\cos(\delta3) = \frac{1}{8}\left(-\sqrt{2}+\sqrt{6}\right)$

$\cos(\alpha4)\cdot\cos(\gamma4) - \cos(\beta4)\cdot\cos(\delta4) = \frac{1}{8}\left(\sqrt{2}-\sqrt{6}\right)$

Switch combination: Right + Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$\cos(\alpha1)\cdot\cos(\gamma1) - \cos(\beta1)\cdot\cos(\delta1) = \frac{\sqrt{2-\sqrt{3}}}{4}$

$\cos(\alpha2)\cdot\cos(\gamma2) - \cos(\beta2)\cdot\cos(\delta2) = -\frac{1}{4}\sqrt{2-\sqrt{3}}$

$\cos(\alpha3)\cdot\cos(\gamma3) - \cos(\beta3)\cdot\cos(\delta3) = \frac{1}{8}\left(-\sqrt{2}+\sqrt{6}\right)$

$\cos(\alpha4)\cdot\cos(\gamma4) - \cos(\beta4)\cdot\cos(\delta4) = \frac{1}{8}\left(\sqrt{2}-\sqrt{6}\right)$

Switch combination: Right + Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$\cos(\alpha 1)\cdot\cos(\gamma 1) - \cos(\beta 1)\cdot\cos(\delta 1) = \frac{1}{8}\left(\sqrt{2}-\sqrt{6}\right)$

$\cos(\alpha 2)\cdot\cos(\gamma 2) - \cos(\beta 2)\cdot\cos(\delta 2) = \frac{1}{8}\left(-\sqrt{2}+\sqrt{6}\right)$

$\cos(\alpha 3)\cdot\cos(\gamma 3) - \cos(\beta 3)\cdot\cos(\delta 3) = -\frac{1}{4}\sqrt{2-\sqrt{3}}$

$\cos(\alpha 4)\cdot\cos(\gamma 4) - \cos(\beta 4)\cdot\cos(\delta 4) = \frac{\sqrt{2-\sqrt{3}}}{4}$

Switch combination: Right + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$\cos(\alpha 1)\cdot\cos(\gamma 1) - \cos(\beta 1)\cdot\cos(\delta 1) = \frac{\sqrt{2-\sqrt{3}}}{4}$

$\cos(\alpha 2)\cdot\cos(\gamma 2) - \cos(\beta 2)\cdot\cos(\delta 2) = \frac{1}{8}\left(-\sqrt{2}+\sqrt{6}\right)$

$\cos(\alpha 3)\cdot\cos(\gamma 3) - \cos(\beta 3)\cdot\cos(\delta 3) = \frac{1}{8}\left(-\sqrt{2}+\sqrt{6}\right)$

$\cos(\alpha 4)\cdot\cos(\gamma 4) - \cos(\beta 4)\cdot\cos(\delta 4) = \frac{\sqrt{2-\sqrt{3}}}{4}$

Switch combination: Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i)\cdot\cos(\gamma_i) - \cos(\beta_i)\cdot\cos(\delta_i)$ *for* $i = 1..4$

$\cos(\alpha 1)\cdot\cos(\gamma 1) - \cos(\beta 1)\cdot\cos(\delta 1) = \frac{1}{8}\left(\sqrt{2}-\sqrt{6}\right)$

$\cos(\alpha 2)\cdot\cos(\gamma 2) - \cos(\beta 2)\cdot\cos(\delta 2) = -\frac{1}{4}\sqrt{2-\sqrt{3}}$

$\cos(\alpha 3)\cdot\cos(\gamma 3) - \cos(\beta 3)\cdot\cos(\delta 3) = -\frac{1}{4}\sqrt{2-\sqrt{3}}$

$\cos(\alpha 4)\cdot\cos(\gamma 4) - \cos(\beta 4)\cdot\cos(\delta 4) = \frac{1}{8}\left(\sqrt{2}-\sqrt{6}\right)$

Switch combination: Right + Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Orthodiagonal check: cos(αi)·cos(γi) − cos(βi)·cos(δi) for i = 1..4

$\cos(\alpha 1)\cdot\cos(\gamma 1) - \cos(\beta 1)\cdot\cos(\delta 1) = \frac{\sqrt{2-\sqrt{3}}}{4}$

$\cos(\alpha 2)\cdot\cos(\gamma 2) - \cos(\beta 2)\cdot\cos(\delta 2) = -\frac{1}{4}\sqrt{2-\sqrt{3}}$

$\cos(\alpha 3)\cdot\cos(\gamma 3) - \cos(\beta 3)\cdot\cos(\delta 3) = -\frac{1}{4}\sqrt{2-\sqrt{3}}$

$\cos(\alpha 4)\cdot\cos(\gamma 4) - \cos(\beta 4)\cdot\cos(\delta 4) = \frac{\sqrt{2-\sqrt{3}}}{4}$

Out[◦]=

================= NOT ISOGONAL =================

Condition (N.0) holds AND for all i = 1…4: $\alpha_i \neq \beta_i$, $\alpha_i \neq \gamma_i$, $\alpha_i \neq \delta_i$, $\beta_i \neq \gamma_i$, $\beta_i \neq \delta_i$, $\gamma_i \neq \delta_i$, $\alpha_i+\beta_i \neq \pi \neq \gamma_i+\delta_i$, $\alpha_i+\gamma_i \neq \pi \neq \beta_i+\delta_i$, $\alpha_i+\delta_i \neq \pi \neq \beta_i+\gamma_i \Rightarrow$ NOT isogonal. Switching boundary strips do not change this.

Initial anglesDeg (no switches):

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1

$\alpha 1 - \beta 1 = 90.000$
$\alpha 1 - \gamma 1 = -15.000$
$\alpha 1 - \delta 1 = 15.000$
$\beta 1 - \gamma 1 = -105.000$
$\beta 1 - \delta 1 = -75.000$
$\gamma 1 - \delta 1 = 30.000$
$\alpha 1 + \beta 1 - 180 = -60.000$
$\gamma 1 + \delta 1 - 180 = 30.000$
$\alpha 1 + \gamma 1 - 180 = 45.000$
$\beta 1 + \delta 1 - 180 = -75.000$
$\alpha 1 + \delta 1 - 180 = 15.000$
$\beta 1 + \gamma 1 - 180 = -45.000$
$\alpha 1 + \beta 1 - \gamma 1 - \delta 1 = -90.000$
$\alpha 1 + \gamma 1 - \beta 1 - \delta 1 = 120.000$
$\alpha 1 + \delta 1 - \beta 1 - \gamma 1 = 60.000$

Vertex 2

$\alpha 2 - \beta 2 = -30.000$
$\alpha 2 - \gamma 2 = 75.000$
$\alpha 2 - \delta 2 = -15.000$
$\beta 2 - \gamma 2 = 105.000$
$\beta 2 - \delta 2 = 15.000$
$\gamma 2 - \delta 2 = -90.000$
$\alpha 2 + \beta 2 - 180 = 30.000$
$\gamma 2 + \delta 2 - 180 = -60.000$
$\alpha 2 + \gamma 2 - 180 = -75.000$
$\beta 2 + \delta 2 - 180 = 45.000$
$\alpha 2 + \delta 2 - 180 = 15.000$
$\beta 2 + \gamma 2 - 180 = -45.000$
$\alpha 2 + \beta 2 - \gamma 2 - \delta 2 = 90.000$
$\alpha 2 + \gamma 2 - \beta 2 - \delta 2 = -120.000$
$\alpha 2 + \delta 2 - \beta 2 - \gamma 2 = 60.000$

Vertex 3

$\alpha 3 - \beta 3 = 30.000$
$\alpha 3 - \gamma 3 = -75.000$
$\alpha 3 - \delta 3 = 15.000$
$\beta 3 - \gamma 3 = -105.000$
$\beta 3 - \delta 3 = -15.000$
$\gamma 3 - \delta 3 = 90.000$
$\alpha 3 + \beta 3 - 180 = -30.000$
$\gamma 3 + \delta 3 - 180 = 60.000$
$\alpha 3 + \gamma 3 - 180 = 75.000$
$\beta 3 + \delta 3 - 180 = -45.000$
$\alpha 3 + \delta 3 - 180 = -15.000$
$\beta 3 + \gamma 3 - 180 = 45.000$
$\alpha 3 + \beta 3 - \gamma 3 - \delta 3 = -90.000$
$\alpha 3 + \gamma 3 - \beta 3 - \delta 3 = 120.000$
$\alpha 3 + \delta 3 - \beta 3 - \gamma 3 = -60.000$

Vertex 4
$\alpha 4 - \beta 4 = 90.000$
$\alpha 4 - \gamma 4 = -15.000$
$\alpha 4 - \delta 4 = 15.000$
$\beta 4 - \gamma 4 = -105.000$
$\beta 4 - \delta 4 = -75.000$
$\gamma 4 - \delta 4 = 30.000$
$\alpha 4 + \beta 4 - 180 = -60.000$
$\gamma 4 + \delta 4 - 180 = 30.000$
$\alpha 4 + \gamma 4 - 180 = 45.000$
$\beta 4 + \delta 4 - 180 = -75.000$
$\alpha 4 + \delta 4 - 180 = 15.000$
$\beta 4 + \gamma 4 - 180 = -45.000$
$\alpha 4 + \beta 4 - \gamma 4 - \delta 4 = -90.000$
$\alpha 4 + \gamma 4 - \beta 4 - \delta 4 = 120.000$
$\alpha 4 + \delta 4 - \beta 4 - \gamma 4 = 60.000$

Switch combination: Right

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1
$\alpha 1 - \beta 1 = -60.000$
$\alpha 1 - \gamma 1 = 45.000$
$\alpha 1 - \delta 1 = 15.000$
$\beta 1 - \gamma 1 = 105.000$
$\beta 1 - \delta 1 = 75.000$
$\gamma 1 - \delta 1 = -30.000$
$\alpha 1 + \beta 1 - 180 = 90.000$
$\gamma 1 + \delta 1 - 180 = -30.000$
$\alpha 1 + \gamma 1 - 180 = -15.000$
$\beta 1 + \delta 1 - 180 = 75.000$
$\alpha 1 + \delta 1 - 180 = 15.000$
$\beta 1 + \gamma 1 - 180 = 45.000$
$\alpha 1 + \beta 1 - \gamma 1 - \delta 1 = 120.000$
$\alpha 1 + \gamma 1 - \beta 1 - \delta 1 = -90.000$
$\alpha 1 + \delta 1 - \beta 1 - \gamma 1 = -30.000$

Vertex 2
$\alpha 2 - \beta 2 = -30.000$

$\alpha 2 - \gamma 2 = 75.000$
$\alpha 2 - \delta 2 = -15.000$
$\beta 2 - \gamma 2 = 105.000$
$\beta 2 - \delta 2 = 15.000$
$\gamma 2 - \delta 2 = -90.000$
$\alpha 2 + \beta 2 - 180 = 30.000$
$\gamma 2 + \delta 2 - 180 = -60.000$
$\alpha 2 + \gamma 2 - 180 = -75.000$
$\beta 2 + \delta 2 - 180 = 45.000$
$\alpha 2 + \delta 2 - 180 = 15.000$
$\beta 2 + \gamma 2 - 180 = -45.000$
$\alpha 2 + \beta 2 - \gamma 2 - \delta 2 = 90.000$
$\alpha 2 + \gamma 2 - \beta 2 - \delta 2 = -120.000$
$\alpha 2 + \delta 2 - \beta 2 - \gamma 2 = 60.000$

Vertex 3
$\alpha 3 - \beta 3 = 30.000$
$\alpha 3 - \gamma 3 = -75.000$
$\alpha 3 - \delta 3 = 15.000$
$\beta 3 - \gamma 3 = -105.000$
$\beta 3 - \delta 3 = -15.000$
$\gamma 3 - \delta 3 = 90.000$
$\alpha 3 + \beta 3 - 180 = -30.000$
$\gamma 3 + \delta 3 - 180 = 60.000$
$\alpha 3 + \gamma 3 - 180 = 75.000$
$\beta 3 + \delta 3 - 180 = -45.000$
$\alpha 3 + \delta 3 - 180 = -15.000$
$\beta 3 + \gamma 3 - 180 = 45.000$
$\alpha 3 + \beta 3 - \gamma 3 - \delta 3 = -90.000$
$\alpha 3 + \gamma 3 - \beta 3 - \delta 3 = 120.000$
$\alpha 3 + \delta 3 - \beta 3 - \gamma 3 = -60.000$

Vertex 4
$\alpha 4 - \beta 4 = -60.000$
$\alpha 4 - \gamma 4 = 45.000$
$\alpha 4 - \delta 4 = 15.000$
$\beta 4 - \gamma 4 = 105.000$
$\beta 4 - \delta 4 = 75.000$
$\gamma 4 - \delta 4 = -30.000$
$\alpha 4 + \beta 4 - 180 = 90.000$
$\gamma 4 + \delta 4 - 180 = -30.000$
$\alpha 4 + \gamma 4 - 180 = -15.000$
$\beta 4 + \delta 4 - 180 = 75.000$
$\alpha 4 + \delta 4 - 180 = 15.000$
$\beta 4 + \gamma 4 - 180 = 45.000$
$\alpha 4 + \beta 4 - \gamma 4 - \delta 4 = 120.000$
$\alpha 4 + \gamma 4 - \beta 4 - \delta 4 = -90.000$
$\alpha 4 + \delta 4 - \beta 4 - \gamma 4 = -30.000$

Switch combination: Left

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1
$\alpha 1 - \beta 1 = 90.000$
$\alpha 1 - \gamma 1 = -15.000$

$\alpha1 - \gamma1 = -15.000$
$\alpha1 - \delta1 = 15.000$
$\beta1 - \gamma1 = -105.000$
$\beta1 - \delta1 = -75.000$
$\gamma1 - \delta1 = 30.000$
$\alpha1 + \beta1 - 180 = -60.000$
$\gamma1 + \delta1 - 180 = 30.000$
$\alpha1 + \gamma1 - 180 = 45.000$
$\beta1 + \delta1 - 180 = -75.000$
$\alpha1 + \delta1 - 180 = 15.000$
$\beta1 + \gamma1 - 180 = -45.000$
$\alpha1 + \beta1 - \gamma1 - \delta1 = -90.000$
$\alpha1 + \gamma1 - \beta1 - \delta1 = 120.000$
$\alpha1 + \delta1 - \beta1 - \gamma1 = 60.000$

Vertex 2
$\alpha2 - \beta2 = 30.000$
$\alpha2 - \gamma2 = -75.000$
$\alpha2 - \delta2 = -15.000$
$\beta2 - \gamma2 = -105.000$
$\beta2 - \delta2 = -45.000$
$\gamma2 - \delta2 = 60.000$
$\alpha2 + \beta2 - 180 = -30.000$
$\gamma2 + \delta2 - 180 = 90.000$
$\alpha2 + \gamma2 - 180 = 75.000$
$\beta2 + \delta2 - 180 = -15.000$
$\alpha2 + \delta2 - 180 = 15.000$
$\beta2 + \gamma2 - 180 = 45.000$
$\alpha2 + \beta2 - \gamma2 - \delta2 = -120.000$
$\alpha2 + \gamma2 - \beta2 - \delta2 = 90.000$
$\alpha2 + \delta2 - \beta2 - \gamma2 = -30.000$

Vertex 3
$\alpha3 - \beta3 = -30.000$
$\alpha3 - \gamma3 = 75.000$
$\alpha3 - \delta3 = 15.000$
$\beta3 - \gamma3 = 105.000$
$\beta3 - \delta3 = 45.000$
$\gamma3 - \delta3 = -60.000$
$\alpha3 + \beta3 - 180 = 30.000$
$\gamma3 + \delta3 - 180 = -90.000$
$\alpha3 + \gamma3 - 180 = -75.000$
$\beta3 + \delta3 - 180 = 15.000$
$\alpha3 + \delta3 - 180 = -15.000$
$\beta3 + \gamma3 - 180 = -45.000$
$\alpha3 + \beta3 - \gamma3 - \delta3 = 120.000$
$\alpha3 + \gamma3 - \beta3 - \delta3 = -90.000$
$\alpha3 + \delta3 - \beta3 - \gamma3 = 30.000$

Vertex 4
$\alpha4 - \beta4 = 90.000$
$\alpha4 - \gamma4 = -15.000$
$\alpha4 - \delta4 = 15.000$
$\beta4 - \gamma4 = -105.000$
$\beta4 - \delta4 = -75.000$
$\gamma4 - \delta4 = 30.000$
$\alpha4 + \beta4 - 180 = -60.000$
$\gamma4 + \delta4 - 180 = 30.000$
$\alpha4 + \gamma4 - 180 = 45.000$
$\beta4 + \delta4 - 180 = -75.000$
$\alpha4 + \delta4 - 180 = 15.000$
$\beta4 + \gamma4 - 180 = -45.000$

$\alpha4 + \beta4 - \gamma4 - \delta4 = -90.000$
$\alpha4 + \gamma4 - \beta4 - \delta4 = 120.000$
$\alpha4 + \delta4 - \beta4 - \gamma4 = 60.000$

Switch combination: Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1
$\alpha1 - \beta1 = -90.000$
$\alpha1 - \gamma1 = -45.000$
$\alpha1 - \delta1 = -15.000$
$\beta1 - \gamma1 = 45.000$
$\beta1 - \delta1 = 75.000$
$\gamma1 - \delta1 = 30.000$
$\alpha1 + \beta1 - 180 = 60.000$
$\gamma1 + \delta1 - 180 = 30.000$
$\alpha1 + \gamma1 - 180 = 15.000$
$\beta1 + \delta1 - 180 = 75.000$
$\alpha1 + \delta1 - 180 = -15.000$
$\beta1 + \gamma1 - 180 = 105.000$
$\alpha1 + \beta1 - \gamma1 - \delta1 = 30.000$
$\alpha1 + \gamma1 - \beta1 - \delta1 = -60.000$
$\alpha1 + \delta1 - \beta1 - \gamma1 = -120.000$

Vertex 2
$\alpha2 - \beta2 = 30.000$
$\alpha2 - \gamma2 = 75.000$
$\alpha2 - \delta2 = -15.000$
$\beta2 - \gamma2 = 45.000$
$\beta2 - \delta2 = -45.000$
$\gamma2 - \delta2 = -90.000$
$\alpha2 + \beta2 - 180 = -30.000$
$\gamma2 + \delta2 - 180 = -60.000$
$\alpha2 + \gamma2 - 180 = -75.000$
$\beta2 + \delta2 - 180 = -15.000$
$\alpha2 + \delta2 - 180 = 15.000$
$\beta2 + \gamma2 - 180 = -105.000$
$\alpha2 + \beta2 - \gamma2 - \delta2 = 30.000$
$\alpha2 + \gamma2 - \beta2 - \delta2 = -60.000$
$\alpha2 + \delta2 - \beta2 - \gamma2 = 120.000$

Vertex 3
$\alpha3 - \beta3 = 30.000$
$\alpha3 - \gamma3 = -75.000$
$\alpha3 - \delta3 = 15.000$
$\beta3 - \gamma3 = -105.000$
$\beta3 - \delta3 = -15.000$
$\gamma3 - \delta3 = 90.000$
$\alpha3 + \beta3 - 180 = -30.000$
$\gamma3 + \delta3 - 180 = 60.000$
$\alpha3 + \gamma3 - 180 = 75.000$
$\beta3 + \delta3 - 180 = -45.000$
$\alpha3 + \delta3 - 180 = -15.000$
$\beta3 + \gamma3 - 180 = 45.000$
$\alpha3 + \beta3 - \gamma3 - \delta3 = -90.000$

$\alpha 3 + \gamma 3 - \beta 3 - \delta 3 = 120.000$
$\alpha 3 + \delta 3 - \beta 3 - \gamma 3 = -60.000$

Vertex 4
$\alpha 4 - \beta 4 = 90.000$
$\alpha 4 - \gamma 4 = -15.000$
$\alpha 4 - \delta 4 = 15.000$
$\beta 4 - \gamma 4 = -105.000$
$\beta 4 - \delta 4 = -75.000$
$\gamma 4 - \delta 4 = 30.000$
$\alpha 4 + \beta 4 - 180 = -60.000$
$\gamma 4 + \delta 4 - 180 = 30.000$
$\alpha 4 + \gamma 4 - 180 = 45.000$
$\beta 4 + \delta 4 - 180 = -75.000$
$\alpha 4 + \delta 4 - 180 = 15.000$
$\beta 4 + \gamma 4 - 180 = -45.000$
$\alpha 4 + \beta 4 - \gamma 4 - \delta 4 = -90.000$
$\alpha 4 + \gamma 4 - \beta 4 - \delta 4 = 120.000$
$\alpha 4 + \delta 4 - \beta 4 - \gamma 4 = 60.000$

Switch combination: Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1
$\alpha 1 - \beta 1 = 90.000$
$\alpha 1 - \gamma 1 = -15.000$
$\alpha 1 - \delta 1 = 15.000$
$\beta 1 - \gamma 1 = -105.000$
$\beta 1 - \delta 1 = -75.000$
$\gamma 1 - \delta 1 = 30.000$
$\alpha 1 + \beta 1 - 180 = -60.000$
$\gamma 1 + \delta 1 - 180 = 30.000$
$\alpha 1 + \gamma 1 - 180 = 45.000$
$\beta 1 + \delta 1 - 180 = -75.000$
$\alpha 1 + \delta 1 - 180 = 15.000$
$\beta 1 + \gamma 1 - 180 = -45.000$
$\alpha 1 + \beta 1 - \gamma 1 - \delta 1 = -90.000$
$\alpha 1 + \gamma 1 - \beta 1 - \delta 1 = 120.000$
$\alpha 1 + \delta 1 - \beta 1 - \gamma 1 = 60.000$

Vertex 2
$\alpha 2 - \beta 2 = -30.000$
$\alpha 2 - \gamma 2 = 75.000$
$\alpha 2 - \delta 2 = -15.000$
$\beta 2 - \gamma 2 = 105.000$
$\beta 2 - \delta 2 = 15.000$
$\gamma 2 - \delta 2 = -90.000$
$\alpha 2 + \beta 2 - 180 = 30.000$
$\gamma 2 + \delta 2 - 180 = -60.000$
$\alpha 2 + \gamma 2 - 180 = -75.000$
$\beta 2 + \delta 2 - 180 = 45.000$
$\alpha 2 + \delta 2 - 180 = 15.000$
$\beta 2 + \gamma 2 - 180 = -45.000$
$\alpha 2 + \beta 2 - \gamma 2 - \delta 2 = 90.000$
$\alpha 2 + \gamma 2 - \beta 2 - \delta 2 = -120.000$

$\alpha 2 + \delta 2 - \beta 2 - \gamma 2 = 60.000$

Vertex 3

$\alpha 3 - \beta 3 = -30.000$
$\alpha 3 - \gamma 3 = -75.000$
$\alpha 3 - \delta 3 = 15.000$
$\beta 3 - \gamma 3 = -45.000$
$\beta 3 - \delta 3 = 45.000$
$\gamma 3 - \delta 3 = 90.000$
$\alpha 3 + \beta 3 - 180 = 30.000$
$\gamma 3 + \delta 3 - 180 = 60.000$
$\alpha 3 + \gamma 3 - 180 = 75.000$
$\beta 3 + \delta 3 - 180 = 15.000$
$\alpha 3 + \delta 3 - 180 = -15.000$
$\beta 3 + \gamma 3 - 180 = 105.000$
$\alpha 3 + \beta 3 - \gamma 3 - \delta 3 = -30.000$
$\alpha 3 + \gamma 3 - \beta 3 - \delta 3 = 60.000$
$\alpha 3 + \delta 3 - \beta 3 - \gamma 3 = -120.000$

Vertex 4

$\alpha 4 - \beta 4 = -90.000$
$\alpha 4 - \gamma 4 = -45.000$
$\alpha 4 - \delta 4 = -15.000$
$\beta 4 - \gamma 4 = 45.000$
$\beta 4 - \delta 4 = 75.000$
$\gamma 4 - \delta 4 = 30.000$
$\alpha 4 + \beta 4 - 180 = 60.000$
$\gamma 4 + \delta 4 - 180 = 30.000$
$\alpha 4 + \gamma 4 - 180 = 15.000$
$\beta 4 + \delta 4 - 180 = 75.000$
$\alpha 4 + \delta 4 - 180 = -15.000$
$\beta 4 + \gamma 4 - 180 = 105.000$
$\alpha 4 + \beta 4 - \gamma 4 - \delta 4 = 30.000$
$\alpha 4 + \gamma 4 - \beta 4 - \delta 4 = -60.000$
$\alpha 4 + \delta 4 - \beta 4 - \gamma 4 = -120.000$

Switch combination: Right + Left

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1

$\alpha 1 - \beta 1 = -60.000$
$\alpha 1 - \gamma 1 = 45.000$
$\alpha 1 - \delta 1 = 15.000$
$\beta 1 - \gamma 1 = 105.000$
$\beta 1 - \delta 1 = 75.000$
$\gamma 1 - \delta 1 = -30.000$
$\alpha 1 + \beta 1 - 180 = 90.000$
$\gamma 1 + \delta 1 - 180 = -30.000$
$\alpha 1 + \gamma 1 - 180 = -15.000$
$\beta 1 + \delta 1 - 180 = 75.000$
$\alpha 1 + \delta 1 - 180 = 15.000$
$\beta 1 + \gamma 1 - 180 = 45.000$
$\alpha 1 + \beta 1 - \gamma 1 - \delta 1 = 120.000$
$\alpha 1 + \gamma 1 - \beta 1 - \delta 1 = -90.000$
$\alpha 1 + \delta 1 - \beta 1 - \gamma 1 = -30.000$

Vertex 2
$\alpha2 - \beta2 = 30.000$
$\alpha2 - \gamma2 = -75.000$
$\alpha2 - \delta2 = -15.000$
$\beta2 - \gamma2 = -105.000$
$\beta2 - \delta2 = -45.000$
$\gamma2 - \delta2 = 60.000$
$\alpha2 + \beta2 - 180 = -30.000$
$\gamma2 + \delta2 - 180 = 90.000$
$\alpha2 + \gamma2 - 180 = 75.000$
$\beta2 + \delta2 - 180 = -15.000$
$\alpha2 + \delta2 - 180 = 15.000$
$\beta2 + \gamma2 - 180 = 45.000$
$\alpha2 + \beta2 - \gamma2 - \delta2 = -120.000$
$\alpha2 + \gamma2 - \beta2 - \delta2 = 90.000$
$\alpha2 + \delta2 - \beta2 - \gamma2 = -30.000$

Vertex 3
$\alpha3 - \beta3 = -30.000$
$\alpha3 - \gamma3 = 75.000$
$\alpha3 - \delta3 = 15.000$
$\beta3 - \gamma3 = 105.000$
$\beta3 - \delta3 = 45.000$
$\gamma3 - \delta3 = -60.000$
$\alpha3 + \beta3 - 180 = 30.000$
$\gamma3 + \delta3 - 180 = -90.000$
$\alpha3 + \gamma3 - 180 = -75.000$
$\beta3 + \delta3 - 180 = 15.000$
$\alpha3 + \delta3 - 180 = -15.000$
$\beta3 + \gamma3 - 180 = -45.000$
$\alpha3 + \beta3 - \gamma3 - \delta3 = 120.000$
$\alpha3 + \gamma3 - \beta3 - \delta3 = -90.000$
$\alpha3 + \delta3 - \beta3 - \gamma3 = 30.000$

Vertex 4
$\alpha4 - \beta4 = -60.000$
$\alpha4 - \gamma4 = 45.000$
$\alpha4 - \delta4 = 15.000$
$\beta4 - \gamma4 = 105.000$
$\beta4 - \delta4 = 75.000$
$\gamma4 - \delta4 = -30.000$
$\alpha4 + \beta4 - 180 = 90.000$
$\gamma4 + \delta4 - 180 = -30.000$
$\alpha4 + \gamma4 - 180 = -15.000$
$\beta4 + \delta4 - 180 = 75.000$
$\alpha4 + \delta4 - 180 = 15.000$
$\beta4 + \gamma4 - 180 = 45.000$
$\alpha4 + \beta4 - \gamma4 - \delta4 = 120.000$
$\alpha4 + \gamma4 - \beta4 - \delta4 = -90.000$
$\alpha4 + \delta4 - \beta4 - \gamma4 = -30.000$

Switch combination: Right + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1
$\alpha 1 - \beta 1 = 60.000$
$\alpha 1 - \gamma 1 = 15.000$
$\alpha 1 - \delta 1 = -15.000$
$\beta 1 - \gamma 1 = -45.000$
$\beta 1 - \delta 1 = -75.000$
$\gamma 1 - \delta 1 = -30.000$
$\alpha 1 + \beta 1 - 180 = -90.000$
$\gamma 1 + \delta 1 - 180 = -30.000$
$\alpha 1 + \gamma 1 - 180 = -45.000$
$\beta 1 + \delta 1 - 180 = -75.000$
$\alpha 1 + \delta 1 - 180 = -15.000$
$\beta 1 + \gamma 1 - 180 = -105.000$
$\alpha 1 + \beta 1 - \gamma 1 - \delta 1 = -60.000$
$\alpha 1 + \gamma 1 - \beta 1 - \delta 1 = 30.000$
$\alpha 1 + \delta 1 - \beta 1 - \gamma 1 = 90.000$

Vertex 2
$\alpha 2 - \beta 2 = 30.000$
$\alpha 2 - \gamma 2 = 75.000$
$\alpha 2 - \delta 2 = -15.000$
$\beta 2 - \gamma 2 = 45.000$
$\beta 2 - \delta 2 = -45.000$
$\gamma 2 - \delta 2 = -90.000$
$\alpha 2 + \beta 2 - 180 = -30.000$
$\gamma 2 + \delta 2 - 180 = -60.000$
$\alpha 2 + \gamma 2 - 180 = -75.000$
$\beta 2 + \delta 2 - 180 = -15.000$
$\alpha 2 + \delta 2 - 180 = 15.000$
$\beta 2 + \gamma 2 - 180 = -105.000$
$\alpha 2 + \beta 2 - \gamma 2 - \delta 2 = 30.000$
$\alpha 2 + \gamma 2 - \beta 2 - \delta 2 = -60.000$
$\alpha 2 + \delta 2 - \beta 2 - \gamma 2 = 120.000$

Vertex 3
$\alpha 3 - \beta 3 = 30.000$
$\alpha 3 - \gamma 3 = -75.000$
$\alpha 3 - \delta 3 = 15.000$
$\beta 3 - \gamma 3 = -105.000$
$\beta 3 - \delta 3 = -15.000$
$\gamma 3 - \delta 3 = 90.000$
$\alpha 3 + \beta 3 - 180 = -30.000$
$\gamma 3 + \delta 3 - 180 = 60.000$
$\alpha 3 + \gamma 3 - 180 = 75.000$
$\beta 3 + \delta 3 - 180 = -45.000$
$\alpha 3 + \delta 3 - 180 = -15.000$
$\beta 3 + \gamma 3 - 180 = 45.000$
$\alpha 3 + \beta 3 - \gamma 3 - \delta 3 = -90.000$
$\alpha 3 + \gamma 3 - \beta 3 - \delta 3 = 120.000$
$\alpha 3 + \delta 3 - \beta 3 - \gamma 3 = -60.000$

Vertex 4
$\alpha 4 - \beta 4 = -60.000$
$\alpha 4 - \gamma 4 = 45.000$
$\alpha 4 - \delta 4 = 15.000$
$\beta 4 - \gamma 4 = 105.000$
$\beta 4 - \delta 4 = 75.000$
$\gamma 4 - \delta 4 = -30.000$
$\alpha 4 + \beta 4 - 180 = 90.000$
$\gamma 4 + \delta 4 - 180 = -30.000$
$\alpha 4 + \gamma 4 - 180 = -15.000$
$\beta 4 + \delta 4 - 180 = 75.000$

$\beta 4 + \delta 4 - 180 = 75.000$
$\alpha 4 + \delta 4 - 180 = 15.000$
$\beta 4 + \gamma 4 - 180 = 45.000$
$\alpha 4 + \beta 4 - \gamma 4 - \delta 4 = 120.000$
$\alpha 4 + \gamma 4 - \beta 4 - \delta 4 = -90.000$
$\alpha 4 + \delta 4 - \beta 4 - \gamma 4 = -30.000$

Switch combination: Right + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1
$\alpha 1 - \beta 1 = -60.000$
$\alpha 1 - \gamma 1 = 45.000$
$\alpha 1 - \delta 1 = 15.000$
$\beta 1 - \gamma 1 = 105.000$
$\beta 1 - \delta 1 = 75.000$
$\gamma 1 - \delta 1 = -30.000$
$\alpha 1 + \beta 1 - 180 = 90.000$
$\gamma 1 + \delta 1 - 180 = -30.000$
$\alpha 1 + \gamma 1 - 180 = -15.000$
$\beta 1 + \delta 1 - 180 = 75.000$
$\alpha 1 + \delta 1 - 180 = 15.000$
$\beta 1 + \gamma 1 - 180 = 45.000$
$\alpha 1 + \beta 1 - \gamma 1 - \delta 1 = 120.000$
$\alpha 1 + \gamma 1 - \beta 1 - \delta 1 = -90.000$
$\alpha 1 + \delta 1 - \beta 1 - \gamma 1 = -30.000$

Vertex 2
$\alpha 2 - \beta 2 = -30.000$
$\alpha 2 - \gamma 2 = 75.000$
$\alpha 2 - \delta 2 = -15.000$
$\beta 2 - \gamma 2 = 105.000$
$\beta 2 - \delta 2 = 15.000$
$\gamma 2 - \delta 2 = -90.000$
$\alpha 2 + \beta 2 - 180 = 30.000$
$\gamma 2 + \delta 2 - 180 = -60.000$
$\alpha 2 + \gamma 2 - 180 = -75.000$
$\beta 2 + \delta 2 - 180 = 45.000$
$\alpha 2 + \delta 2 - 180 = 15.000$
$\beta 2 + \gamma 2 - 180 = -45.000$
$\alpha 2 + \beta 2 - \gamma 2 - \delta 2 = 90.000$
$\alpha 2 + \gamma 2 - \beta 2 - \delta 2 = -120.000$
$\alpha 2 + \delta 2 - \beta 2 - \gamma 2 = 60.000$

Vertex 3
$\alpha 3 - \beta 3 = -30.000$
$\alpha 3 - \gamma 3 = -75.000$
$\alpha 3 - \delta 3 = 15.000$
$\beta 3 - \gamma 3 = -45.000$
$\beta 3 - \delta 3 = 45.000$
$\gamma 3 - \delta 3 = 90.000$
$\alpha 3 + \beta 3 - 180 = 30.000$
$\gamma 3 + \delta 3 - 180 = 60.000$
$\alpha 3 + \gamma 3 - 180 = 75.000$
$\beta 3 + \delta 3 - 180 = 15.000$
$\alpha 3 + \delta 3 - 180 = -15.000$

$\alpha 3 + \delta 3 - 180 = -15.000$
$\beta 3 + \gamma 3 - 180 = 105.000$
$\alpha 3 + \beta 3 - \gamma 3 - \delta 3 = -30.000$
$\alpha 3 + \gamma 3 - \beta 3 - \delta 3 = 60.000$
$\alpha 3 + \delta 3 - \beta 3 - \gamma 3 = -120.000$

Vertex 4
$\alpha 4 - \beta 4 = 60.000$
$\alpha 4 - \gamma 4 = 15.000$
$\alpha 4 - \delta 4 = -15.000$
$\beta 4 - \gamma 4 = -45.000$
$\beta 4 - \delta 4 = -75.000$
$\gamma 4 - \delta 4 = -30.000$
$\alpha 4 + \beta 4 - 180 = -90.000$
$\gamma 4 + \delta 4 - 180 = -30.000$
$\alpha 4 + \gamma 4 - 180 = -45.000$
$\beta 4 + \delta 4 - 180 = -75.000$
$\alpha 4 + \delta 4 - 180 = -15.000$
$\beta 4 + \gamma 4 - 180 = -105.000$
$\alpha 4 + \beta 4 - \gamma 4 - \delta 4 = -60.000$
$\alpha 4 + \gamma 4 - \beta 4 - \delta 4 = 30.000$
$\alpha 4 + \delta 4 - \beta 4 - \gamma 4 = 90.000$

Switch combination: Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1
$\alpha 1 - \beta 1 = -90.000$
$\alpha 1 - \gamma 1 = -45.000$
$\alpha 1 - \delta 1 = -15.000$
$\beta 1 - \gamma 1 = 45.000$
$\beta 1 - \delta 1 = 75.000$
$\gamma 1 - \delta 1 = 30.000$
$\alpha 1 + \beta 1 - 180 = 60.000$
$\gamma 1 + \delta 1 - 180 = 30.000$
$\alpha 1 + \gamma 1 - 180 = 15.000$
$\beta 1 + \delta 1 - 180 = 75.000$
$\alpha 1 + \delta 1 - 180 = -15.000$
$\beta 1 + \gamma 1 - 180 = 105.000$
$\alpha 1 + \beta 1 - \gamma 1 - \delta 1 = 30.000$
$\alpha 1 + \gamma 1 - \beta 1 - \delta 1 = -60.000$
$\alpha 1 + \delta 1 - \beta 1 - \gamma 1 = -120.000$

Vertex 2
$\alpha 2 - \beta 2 = -30.000$
$\alpha 2 - \gamma 2 = -75.000$
$\alpha 2 - \delta 2 = -15.000$
$\beta 2 - \gamma 2 = -45.000$
$\beta 2 - \delta 2 = 15.000$
$\gamma 2 - \delta 2 = 60.000$
$\alpha 2 + \beta 2 - 180 = 30.000$
$\gamma 2 + \delta 2 - 180 = 90.000$
$\alpha 2 + \gamma 2 - 180 = 75.000$
$\beta 2 + \delta 2 - 180 = 45.000$
$\alpha 2 + \delta 2 - 180 = 15.000$
$\beta 2 + \gamma 2 - 180 = 105.000$

$\beta2 + \gamma2 - 180 = 105.000$
$\alpha2 + \beta2 - \gamma2 - \delta2 = -60.000$
$\alpha2 + \gamma2 - \beta2 - \delta2 = 30.000$
$\alpha2 + \delta2 - \beta2 - \gamma2 = -90.000$

Vertex 3
$\alpha3 - \beta3 = -30.000$
$\alpha3 - \gamma3 = 75.000$
$\alpha3 - \delta3 = 15.000$
$\beta3 - \gamma3 = 105.000$
$\beta3 - \delta3 = 45.000$
$\gamma3 - \delta3 = -60.000$
$\alpha3 + \beta3 - 180 = 30.000$
$\gamma3 + \delta3 - 180 = -90.000$
$\alpha3 + \gamma3 - 180 = -75.000$
$\beta3 + \delta3 - 180 = 15.000$
$\alpha3 + \delta3 - 180 = -15.000$
$\beta3 + \gamma3 - 180 = -45.000$
$\alpha3 + \beta3 - \gamma3 - \delta3 = 120.000$
$\alpha3 + \gamma3 - \beta3 - \delta3 = -90.000$
$\alpha3 + \delta3 - \beta3 - \gamma3 = 30.000$

Vertex 4
$\alpha4 - \beta4 = 90.000$
$\alpha4 - \gamma4 = -15.000$
$\alpha4 - \delta4 = 15.000$
$\beta4 - \gamma4 = -105.000$
$\beta4 - \delta4 = -75.000$
$\gamma4 - \delta4 = 30.000$
$\alpha4 + \beta4 - 180 = -60.000$
$\gamma4 + \delta4 - 180 = 30.000$
$\alpha4 + \gamma4 - 180 = 45.000$
$\beta4 + \delta4 - 180 = -75.000$
$\alpha4 + \delta4 - 180 = 15.000$
$\beta4 + \gamma4 - 180 = -45.000$
$\alpha4 + \beta4 - \gamma4 - \delta4 = -90.000$
$\alpha4 + \gamma4 - \beta4 - \delta4 = 120.000$
$\alpha4 + \delta4 - \beta4 - \gamma4 = 60.000$

Switch combination: Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1
$\alpha1 - \beta1 = 90.000$
$\alpha1 - \gamma1 = -15.000$
$\alpha1 - \delta1 = 15.000$
$\beta1 - \gamma1 = -105.000$
$\beta1 - \delta1 = -75.000$
$\gamma1 - \delta1 = 30.000$
$\alpha1 + \beta1 - 180 = -60.000$
$\gamma1 + \delta1 - 180 = 30.000$
$\alpha1 + \gamma1 - 180 = 45.000$
$\beta1 + \delta1 - 180 = -75.000$
$\alpha1 + \delta1 - 180 = 15.000$
$\beta1 + \gamma1 - 180 = -45.000$

$\alpha1 + \beta1 - \gamma1 - \delta1 = -90.000$
$\alpha1 + \gamma1 - \beta1 - \delta1 = 120.000$
$\alpha1 + \delta1 - \beta1 - \gamma1 = 60.000$

Vertex 2
$\alpha2 - \beta2 = 30.000$
$\alpha2 - \gamma2 = -75.000$
$\alpha2 - \delta2 = -15.000$
$\beta2 - \gamma2 = -105.000$
$\beta2 - \delta2 = -45.000$
$\gamma2 - \delta2 = 60.000$
$\alpha2 + \beta2 - 180 = -30.000$
$\gamma2 + \delta2 - 180 = 90.000$
$\alpha2 + \gamma2 - 180 = 75.000$
$\beta2 + \delta2 - 180 = -15.000$
$\alpha2 + \delta2 - 180 = 15.000$
$\beta2 + \gamma2 - 180 = 45.000$
$\alpha2 + \beta2 - \gamma2 - \delta2 = -120.000$
$\alpha2 + \gamma2 - \beta2 - \delta2 = 90.000$
$\alpha2 + \delta2 - \beta2 - \gamma2 = -30.000$

Vertex 3
$\alpha3 - \beta3 = 30.000$
$\alpha3 - \gamma3 = 75.000$
$\alpha3 - \delta3 = 15.000$
$\beta3 - \gamma3 = 45.000$
$\beta3 - \delta3 = -15.000$
$\gamma3 - \delta3 = -60.000$
$\alpha3 + \beta3 - 180 = -30.000$
$\gamma3 + \delta3 - 180 = -90.000$
$\alpha3 + \gamma3 - 180 = -75.000$
$\beta3 + \delta3 - 180 = -45.000$
$\alpha3 + \delta3 - 180 = -15.000$
$\beta3 + \gamma3 - 180 = -105.000$
$\alpha3 + \beta3 - \gamma3 - \delta3 = 60.000$
$\alpha3 + \gamma3 - \beta3 - \delta3 = -30.000$
$\alpha3 + \delta3 - \beta3 - \gamma3 = 90.000$

Vertex 4
$\alpha4 - \beta4 = -90.000$
$\alpha4 - \gamma4 = -45.000$
$\alpha4 - \delta4 = -15.000$
$\beta4 - \gamma4 = 45.000$
$\beta4 - \delta4 = 75.000$
$\gamma4 - \delta4 = 30.000$
$\alpha4 + \beta4 - 180 = 60.000$
$\gamma4 + \delta4 - 180 = 30.000$
$\alpha4 + \gamma4 - 180 = 15.000$
$\beta4 + \delta4 - 180 = 75.000$
$\alpha4 + \delta4 - 180 = -15.000$
$\beta4 + \gamma4 - 180 = 105.000$
$\alpha4 + \beta4 - \gamma4 - \delta4 = 30.000$
$\alpha4 + \gamma4 - \beta4 - \delta4 = -60.000$
$\alpha4 + \delta4 - \beta4 - \gamma4 = -120.000$

Switch combination: Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1

$\alpha 1 - \beta 1 = -90.000$
$\alpha 1 - \gamma 1 = -45.000$
$\alpha 1 - \delta 1 = -15.000$
$\beta 1 - \gamma 1 = 45.000$
$\beta 1 - \delta 1 = 75.000$
$\gamma 1 - \delta 1 = 30.000$
$\alpha 1 + \beta 1 - 180 = 60.000$
$\gamma 1 + \delta 1 - 180 = 30.000$
$\alpha 1 + \gamma 1 - 180 = 15.000$
$\beta 1 + \delta 1 - 180 = 75.000$
$\alpha 1 + \delta 1 - 180 = -15.000$
$\beta 1 + \gamma 1 - 180 = 105.000$
$\alpha 1 + \beta 1 - \gamma 1 - \delta 1 = 30.000$
$\alpha 1 + \gamma 1 - \beta 1 - \delta 1 = -60.000$
$\alpha 1 + \delta 1 - \beta 1 - \gamma 1 = -120.000$

Vertex 2

$\alpha 2 - \beta 2 = 30.000$
$\alpha 2 - \gamma 2 = 75.000$
$\alpha 2 - \delta 2 = -15.000$
$\beta 2 - \gamma 2 = 45.000$
$\beta 2 - \delta 2 = -45.000$
$\gamma 2 - \delta 2 = -90.000$
$\alpha 2 + \beta 2 - 180 = -30.000$
$\gamma 2 + \delta 2 - 180 = -60.000$
$\alpha 2 + \gamma 2 - 180 = -75.000$
$\beta 2 + \delta 2 - 180 = -15.000$
$\alpha 2 + \delta 2 - 180 = 15.000$
$\beta 2 + \gamma 2 - 180 = -105.000$
$\alpha 2 + \beta 2 - \gamma 2 - \delta 2 = 30.000$
$\alpha 2 + \gamma 2 - \beta 2 - \delta 2 = -60.000$
$\alpha 2 + \delta 2 - \beta 2 - \gamma 2 = 120.000$

Vertex 3

$\alpha 3 - \beta 3 = -30.000$
$\alpha 3 - \gamma 3 = -75.000$
$\alpha 3 - \delta 3 = 15.000$
$\beta 3 - \gamma 3 = -45.000$
$\beta 3 - \delta 3 = 45.000$
$\gamma 3 - \delta 3 = 90.000$
$\alpha 3 + \beta 3 - 180 = 30.000$
$\gamma 3 + \delta 3 - 180 = 60.000$
$\alpha 3 + \gamma 3 - 180 = 75.000$
$\beta 3 + \delta 3 - 180 = 15.000$
$\alpha 3 + \delta 3 - 180 = -15.000$
$\beta 3 + \gamma 3 - 180 = 105.000$
$\alpha 3 + \beta 3 - \gamma 3 - \delta 3 = -30.000$
$\alpha 3 + \gamma 3 - \beta 3 - \delta 3 = 60.000$
$\alpha 3 + \delta 3 - \beta 3 - \gamma 3 = -120.000$

Vertex 4

$\alpha 4 - \beta 4 = -90.000$
$\alpha 4 - \gamma 4 = -45.000$
$\alpha 4 - \delta 4 = -15.000$
$\beta 4 - \gamma 4 = 45.000$

$\beta4 - \gamma4 = 45.000$
$\beta4 - \delta4 = 75.000$
$\gamma4 - \delta4 = 30.000$
$\alpha4 + \beta4 - 180 = 60.000$
$\gamma4 + \delta4 - 180 = 30.000$
$\alpha4 + \gamma4 - 180 = 15.000$
$\beta4 + \delta4 - 180 = 75.000$
$\alpha4 + \delta4 - 180 = -15.000$
$\beta4 + \gamma4 - 180 = 105.000$
$\alpha4 + \beta4 - \gamma4 - \delta4 = 30.000$
$\alpha4 + \gamma4 - \beta4 - \delta4 = -60.000$
$\alpha4 + \delta4 - \beta4 - \gamma4 = -120.000$

Switch combination: Right + Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1
$\alpha1 - \beta1 = 60.000$
$\alpha1 - \gamma1 = 15.000$
$\alpha1 - \delta1 = -15.000$
$\beta1 - \gamma1 = -45.000$
$\beta1 - \delta1 = -75.000$
$\gamma1 - \delta1 = -30.000$
$\alpha1 + \beta1 - 180 = -90.000$
$\gamma1 + \delta1 - 180 = -30.000$
$\alpha1 + \gamma1 - 180 = -45.000$
$\beta1 + \delta1 - 180 = -75.000$
$\alpha1 + \delta1 - 180 = -15.000$
$\beta1 + \gamma1 - 180 = -105.000$
$\alpha1 + \beta1 - \gamma1 - \delta1 = -60.000$
$\alpha1 + \gamma1 - \beta1 - \delta1 = 30.000$
$\alpha1 + \delta1 - \beta1 - \gamma1 = 90.000$

Vertex 2
$\alpha2 - \beta2 = -30.000$
$\alpha2 - \gamma2 = -75.000$
$\alpha2 - \delta2 = -15.000$
$\beta2 - \gamma2 = -45.000$
$\beta2 - \delta2 = 15.000$
$\gamma2 - \delta2 = 60.000$
$\alpha2 + \beta2 - 180 = 30.000$
$\gamma2 + \delta2 - 180 = 90.000$
$\alpha2 + \gamma2 - 180 = 75.000$
$\beta2 + \delta2 - 180 = 45.000$
$\alpha2 + \delta2 - 180 = 15.000$
$\beta2 + \gamma2 - 180 = 105.000$
$\alpha2 + \beta2 - \gamma2 - \delta2 = -60.000$
$\alpha2 + \gamma2 - \beta2 - \delta2 = 30.000$
$\alpha2 + \delta2 - \beta2 - \gamma2 = -90.000$

Vertex 3
$\alpha3 - \beta3 = -30.000$
$\alpha3 - \gamma3 = 75.000$
$\alpha3 - \delta3 = 15.000$
$\beta3 - \gamma3 = 105.000$
$\beta3 - \delta3 = 45.000$

$\beta 3 - \delta 3 = 45.000$
$\gamma 3 - \delta 3 = -60.000$
$\alpha 3 + \beta 3 - 180 = 30.000$
$\gamma 3 + \delta 3 - 180 = -90.000$
$\alpha 3 + \gamma 3 - 180 = -75.000$
$\beta 3 + \delta 3 - 180 = 15.000$
$\alpha 3 + \delta 3 - 180 = -15.000$
$\beta 3 + \gamma 3 - 180 = -45.000$
$\alpha 3 + \beta 3 - \gamma 3 - \delta 3 = 120.000$
$\alpha 3 + \gamma 3 - \beta 3 - \delta 3 = -90.000$
$\alpha 3 + \delta 3 - \beta 3 - \gamma 3 = 30.000$

Vertex 4
$\alpha 4 - \beta 4 = -60.000$
$\alpha 4 - \gamma 4 = 45.000$
$\alpha 4 - \delta 4 = 15.000$
$\beta 4 - \gamma 4 = 105.000$
$\beta 4 - \delta 4 = 75.000$
$\gamma 4 - \delta 4 = -30.000$
$\alpha 4 + \beta 4 - 180 = 90.000$
$\gamma 4 + \delta 4 - 180 = -30.000$
$\alpha 4 + \gamma 4 - 180 = -15.000$
$\beta 4 + \delta 4 - 180 = 75.000$
$\alpha 4 + \delta 4 - 180 = 15.000$
$\beta 4 + \gamma 4 - 180 = 45.000$
$\alpha 4 + \beta 4 - \gamma 4 - \delta 4 = 120.000$
$\alpha 4 + \gamma 4 - \beta 4 - \delta 4 = -90.000$
$\alpha 4 + \delta 4 - \beta 4 - \gamma 4 = -30.000$

Switch combination: Right + Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1
$\alpha 1 - \beta 1 = -60.000$
$\alpha 1 - \gamma 1 = 45.000$
$\alpha 1 - \delta 1 = 15.000$
$\beta 1 - \gamma 1 = 105.000$
$\beta 1 - \delta 1 = 75.000$
$\gamma 1 - \delta 1 = -30.000$
$\alpha 1 + \beta 1 - 180 = 90.000$
$\gamma 1 + \delta 1 - 180 = -30.000$
$\alpha 1 + \gamma 1 - 180 = -15.000$
$\beta 1 + \delta 1 - 180 = 75.000$
$\alpha 1 + \delta 1 - 180 = 15.000$
$\beta 1 + \gamma 1 - 180 = 45.000$
$\alpha 1 + \beta 1 - \gamma 1 - \delta 1 = 120.000$
$\alpha 1 + \gamma 1 - \beta 1 - \delta 1 = -90.000$
$\alpha 1 + \delta 1 - \beta 1 - \gamma 1 = -30.000$

Vertex 2
$\alpha 2 - \beta 2 = 30.000$
$\alpha 2 - \gamma 2 = -75.000$
$\alpha 2 - \delta 2 = -15.000$
$\beta 2 - \gamma 2 = -105.000$
$\beta 2 - \delta 2 = -45.000$
$\gamma 2 - \delta 2 = 60.000$

$\gamma 2 - \delta 2 = 60.000$
$\alpha 2 + \beta 2 - 180 = -30.000$
$\gamma 2 + \delta 2 - 180 = 90.000$
$\alpha 2 + \gamma 2 - 180 = 75.000$
$\beta 2 + \delta 2 - 180 = -15.000$
$\alpha 2 + \delta 2 - 180 = 15.000$
$\beta 2 + \gamma 2 - 180 = 45.000$
$\alpha 2 + \beta 2 - \gamma 2 - \delta 2 = -120.000$
$\alpha 2 + \gamma 2 - \beta 2 - \delta 2 = 90.000$
$\alpha 2 + \delta 2 - \beta 2 - \gamma 2 = -30.000$

Vertex 3
$\alpha 3 - \beta 3 = 30.000$
$\alpha 3 - \gamma 3 = 75.000$
$\alpha 3 - \delta 3 = 15.000$
$\beta 3 - \gamma 3 = 45.000$
$\beta 3 - \delta 3 = -15.000$
$\gamma 3 - \delta 3 = -60.000$
$\alpha 3 + \beta 3 - 180 = -30.000$
$\gamma 3 + \delta 3 - 180 = -90.000$
$\alpha 3 + \gamma 3 - 180 = -75.000$
$\beta 3 + \delta 3 - 180 = -45.000$
$\alpha 3 + \delta 3 - 180 = -15.000$
$\beta 3 + \gamma 3 - 180 = -105.000$
$\alpha 3 + \beta 3 - \gamma 3 - \delta 3 = 60.000$
$\alpha 3 + \gamma 3 - \beta 3 - \delta 3 = -30.000$
$\alpha 3 + \delta 3 - \beta 3 - \gamma 3 = 90.000$

Vertex 4
$\alpha 4 - \beta 4 = 60.000$
$\alpha 4 - \gamma 4 = 15.000$
$\alpha 4 - \delta 4 = -15.000$
$\beta 4 - \gamma 4 = -45.000$
$\beta 4 - \delta 4 = -75.000$
$\gamma 4 - \delta 4 = -30.000$
$\alpha 4 + \beta 4 - 180 = -90.000$
$\gamma 4 + \delta 4 - 180 = -30.000$
$\alpha 4 + \gamma 4 - 180 = -45.000$
$\beta 4 + \delta 4 - 180 = -75.000$
$\alpha 4 + \delta 4 - 180 = -15.000$
$\beta 4 + \gamma 4 - 180 = -105.000$
$\alpha 4 + \beta 4 - \gamma 4 - \delta 4 = -60.000$
$\alpha 4 + \gamma 4 - \beta 4 - \delta 4 = 30.000$
$\alpha 4 + \delta 4 - \beta 4 - \gamma 4 = 90.000$

Switch combination: Right + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1
$\alpha 1 - \beta 1 = 60.000$
$\alpha 1 - \gamma 1 = 15.000$
$\alpha 1 - \delta 1 = -15.000$
$\beta 1 - \gamma 1 = -45.000$
$\beta 1 - \delta 1 = -75.000$
$\gamma 1 - \delta 1 = -30.000$

$\alpha1 + \beta1 - 180 = -90.000$
$\gamma1 + \delta1 - 180 = -30.000$
$\alpha1 + \gamma1 - 180 = -45.000$
$\beta1 + \delta1 - 180 = -75.000$
$\alpha1 + \delta1 - 180 = -15.000$
$\beta1 + \gamma1 - 180 = -105.000$
$\alpha1 + \beta1 - \gamma1 - \delta1 = -60.000$
$\alpha1 + \gamma1 - \beta1 - \delta1 = 30.000$
$\alpha1 + \delta1 - \beta1 - \gamma1 = 90.000$

Vertex 2
$\alpha2 - \beta2 = 30.000$
$\alpha2 - \gamma2 = 75.000$
$\alpha2 - \delta2 = -15.000$
$\beta2 - \gamma2 = 45.000$
$\beta2 - \delta2 = -45.000$
$\gamma2 - \delta2 = -90.000$
$\alpha2 + \beta2 - 180 = -30.000$
$\gamma2 + \delta2 - 180 = -60.000$
$\alpha2 + \gamma2 - 180 = -75.000$
$\beta2 + \delta2 - 180 = -15.000$
$\alpha2 + \delta2 - 180 = 15.000$
$\beta2 + \gamma2 - 180 = -105.000$
$\alpha2 + \beta2 - \gamma2 - \delta2 = 30.000$
$\alpha2 + \gamma2 - \beta2 - \delta2 = -60.000$
$\alpha2 + \delta2 - \beta2 - \gamma2 = 120.000$

Vertex 3
$\alpha3 - \beta3 = -30.000$
$\alpha3 - \gamma3 = -75.000$
$\alpha3 - \delta3 = 15.000$
$\beta3 - \gamma3 = -45.000$
$\beta3 - \delta3 = 45.000$
$\gamma3 - \delta3 = 90.000$
$\alpha3 + \beta3 - 180 = 30.000$
$\gamma3 + \delta3 - 180 = 60.000$
$\alpha3 + \gamma3 - 180 = 75.000$
$\beta3 + \delta3 - 180 = 15.000$
$\alpha3 + \delta3 - 180 = -15.000$
$\beta3 + \gamma3 - 180 = 105.000$
$\alpha3 + \beta3 - \gamma3 - \delta3 = -30.000$
$\alpha3 + \gamma3 - \beta3 - \delta3 = 60.000$
$\alpha3 + \delta3 - \beta3 - \gamma3 = -120.000$

Vertex 4
$\alpha4 - \beta4 = 60.000$
$\alpha4 - \gamma4 = 15.000$
$\alpha4 - \delta4 = -15.000$
$\beta4 - \gamma4 = -45.000$
$\beta4 - \delta4 = -75.000$
$\gamma4 - \delta4 = -30.000$
$\alpha4 + \beta4 - 180 = -90.000$
$\gamma4 + \delta4 - 180 = -30.000$
$\alpha4 + \gamma4 - 180 = -45.000$
$\beta4 + \delta4 - 180 = -75.000$
$\alpha4 + \delta4 - 180 = -15.000$
$\beta4 + \gamma4 - 180 = -105.000$
$\alpha4 + \beta4 - \gamma4 - \delta4 = -60.000$
$\alpha4 + \gamma4 - \beta4 - \delta4 = 30.000$
$\alpha4 + \delta4 - \beta4 - \gamma4 = 90.000$

Switch combination: Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1

$\alpha1 - \beta1 = -90.000$
$\alpha1 - \gamma1 = -45.000$
$\alpha1 - \delta1 = -15.000$
$\beta1 - \gamma1 = 45.000$
$\beta1 - \delta1 = 75.000$
$\gamma1 - \delta1 = 30.000$
$\alpha1 + \beta1 - 180 = 60.000$
$\gamma1 + \delta1 - 180 = 30.000$
$\alpha1 + \gamma1 - 180 = 15.000$
$\beta1 + \delta1 - 180 = 75.000$
$\alpha1 + \delta1 - 180 = -15.000$
$\beta1 + \gamma1 - 180 = 105.000$
$\alpha1 + \beta1 - \gamma1 - \delta1 = 30.000$
$\alpha1 + \gamma1 - \beta1 - \delta1 = -60.000$
$\alpha1 + \delta1 - \beta1 - \gamma1 = -120.000$

Vertex 2

$\alpha2 - \beta2 = -30.000$
$\alpha2 - \gamma2 = -75.000$
$\alpha2 - \delta2 = -15.000$
$\beta2 - \gamma2 = -45.000$
$\beta2 - \delta2 = 15.000$
$\gamma2 - \delta2 = 60.000$
$\alpha2 + \beta2 - 180 = 30.000$
$\gamma2 + \delta2 - 180 = 90.000$
$\alpha2 + \gamma2 - 180 = 75.000$
$\beta2 + \delta2 - 180 = 45.000$
$\alpha2 + \delta2 - 180 = 15.000$
$\beta2 + \gamma2 - 180 = 105.000$
$\alpha2 + \beta2 - \gamma2 - \delta2 = -60.000$
$\alpha2 + \gamma2 - \beta2 - \delta2 = 30.000$
$\alpha2 + \delta2 - \beta2 - \gamma2 = -90.000$

Vertex 3

$\alpha3 - \beta3 = 30.000$
$\alpha3 - \gamma3 = 75.000$
$\alpha3 - \delta3 = 15.000$
$\beta3 - \gamma3 = 45.000$
$\beta3 - \delta3 = -15.000$
$\gamma3 - \delta3 = -60.000$
$\alpha3 + \beta3 - 180 = -30.000$
$\gamma3 + \delta3 - 180 = -90.000$
$\alpha3 + \gamma3 - 180 = -75.000$
$\beta3 + \delta3 - 180 = -45.000$
$\alpha3 + \delta3 - 180 = -15.000$
$\beta3 + \gamma3 - 180 = -105.000$
$\alpha3 + \beta3 - \gamma3 - \delta3 = 60.000$
$\alpha3 + \gamma3 - \beta3 - \delta3 = -30.000$
$\alpha3 + \delta3 - \beta3 - \gamma3 = 90.000$

Vertex 4
$\alpha 4 - \beta 4 = -90.000$
$\alpha 4 - \gamma 4 = -45.000$
$\alpha 4 - \delta 4 = -15.000$
$\beta 4 - \gamma 4 = 45.000$
$\beta 4 - \delta 4 = 75.000$
$\gamma 4 - \delta 4 = 30.000$
$\alpha 4 + \beta 4 - 180 = 60.000$
$\gamma 4 + \delta 4 - 180 = 30.000$
$\alpha 4 + \gamma 4 - 180 = 15.000$
$\beta 4 + \delta 4 - 180 = 75.000$
$\alpha 4 + \delta 4 - 180 = -15.000$
$\beta 4 + \gamma 4 - 180 = 105.000$
$\alpha 4 + \beta 4 - \gamma 4 - \delta 4 = 30.000$
$\alpha 4 + \gamma 4 - \beta 4 - \delta 4 = -60.000$
$\alpha 4 + \delta 4 - \beta 4 - \gamma 4 = -120.000$

Switch combination: Right + Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1
$\alpha 1 - \beta 1 = 60.000$
$\alpha 1 - \gamma 1 = 15.000$
$\alpha 1 - \delta 1 = -15.000$
$\beta 1 - \gamma 1 = -45.000$
$\beta 1 - \delta 1 = -75.000$
$\gamma 1 - \delta 1 = -30.000$
$\alpha 1 + \beta 1 - 180 = -90.000$
$\gamma 1 + \delta 1 - 180 = -30.000$
$\alpha 1 + \gamma 1 - 180 = -45.000$
$\beta 1 + \delta 1 - 180 = -75.000$
$\alpha 1 + \delta 1 - 180 = -15.000$
$\beta 1 + \gamma 1 - 180 = -105.000$
$\alpha 1 + \beta 1 - \gamma 1 - \delta 1 = -60.000$
$\alpha 1 + \gamma 1 - \beta 1 - \delta 1 = 30.000$
$\alpha 1 + \delta 1 - \beta 1 - \gamma 1 = 90.000$

Vertex 2
$\alpha 2 - \beta 2 = -30.000$
$\alpha 2 - \gamma 2 = -75.000$
$\alpha 2 - \delta 2 = -15.000$
$\beta 2 - \gamma 2 = -45.000$
$\beta 2 - \delta 2 = 15.000$
$\gamma 2 - \delta 2 = 60.000$
$\alpha 2 + \beta 2 - 180 = 30.000$
$\gamma 2 + \delta 2 - 180 = 90.000$
$\alpha 2 + \gamma 2 - 180 = 75.000$
$\beta 2 + \delta 2 - 180 = 45.000$
$\alpha 2 + \delta 2 - 180 = 15.000$
$\beta 2 + \gamma 2 - 180 = 105.000$
$\alpha 2 + \beta 2 - \gamma 2 - \delta 2 = -60.000$
$\alpha 2 + \gamma 2 - \beta 2 - \delta 2 = 30.000$
$\alpha 2 + \delta 2 - \beta 2 - \gamma 2 = -90.000$

Vertex 3

$\alpha3 - \beta3 = 30.000$
$\alpha3 - \gamma3 = 75.000$
$\alpha3 - \delta3 = 15.000$
$\beta3 - \gamma3 = 45.000$
$\beta3 - \delta3 = -15.000$
$\gamma3 - \delta3 = -60.000$
$\alpha3 + \beta3 - 180 = -30.000$
$\gamma3 + \delta3 - 180 = -90.000$
$\alpha3 + \gamma3 - 180 = -75.000$
$\beta3 + \delta3 - 180 = -45.000$
$\alpha3 + \delta3 - 180 = -15.000$
$\beta3 + \gamma3 - 180 = -105.000$
$\alpha3 + \beta3 - \gamma3 - \delta3 = 60.000$
$\alpha3 + \gamma3 - \beta3 - \delta3 = -30.000$
$\alpha3 + \delta3 - \beta3 - \gamma3 = 90.000$

Vertex 4
$\alpha4 - \beta4 = 60.000$
$\alpha4 - \gamma4 = 15.000$
$\alpha4 - \delta4 = -15.000$
$\beta4 - \gamma4 = -45.000$
$\beta4 - \delta4 = -75.000$
$\gamma4 - \delta4 = -30.000$
$\alpha4 + \beta4 - 180 = -90.000$
$\gamma4 + \delta4 - 180 = -30.000$
$\alpha4 + \gamma4 - 180 = -45.000$
$\beta4 + \delta4 - 180 = -75.000$
$\alpha4 + \delta4 - 180 = -15.000$
$\beta4 + \gamma4 - 180 = -105.000$
$\alpha4 + \beta4 - \gamma4 - \delta4 = -60.000$
$\alpha4 + \gamma4 - \beta4 - \delta4 = 30.000$
$\alpha4 + \delta4 - \beta4 - \gamma4 = 90.000$

Out[ ]=

================= NOT CONJUGATE-MODULAR =================

Mi < 1 and pi ∈ ℝ for all i = 1…4 ⇒ NOT conjugate-modular. Boundary-strip switches preserve this.

Initial configuration (no switches applied):

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

M_i values:

$M1 = -1 + \sqrt{3}$, $M2 = -1 + \sqrt{3}$, $M3 = -1 + \sqrt{3}$, $M4 = -1 + \sqrt{3}$

p_i values:

$p1 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$, $p2 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$, $p3 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$, $p4 = \sqrt{-1 + \frac{2}{\sqrt{3}}}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Right

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

M_i values:

M1 = $-1+\sqrt{3}$, M2 = $-1+\sqrt{3}$, M3 = $-1+\sqrt{3}$, M4 = $-1+\sqrt{3}$

p_i values:

p1 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p2 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p3 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p4 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Left

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

M_i values:

M1 = $-1+\sqrt{3}$, M2 = $-1+\sqrt{3}$, M3 = $-1+\sqrt{3}$, M4 = $-1+\sqrt{3}$

p_i values:

p1 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p2 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p3 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p4 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

M_i values:

M1 = $-1+\sqrt{3}$, M2 = $-1+\sqrt{3}$, M3 = $-1+\sqrt{3}$, M4 = $-1+\sqrt{3}$

p_i values:

p1 = $3^{1/4}$, p2 = $3^{1/4}$, p3 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p4 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

M_i values:

M1 = $-1+\sqrt{3}$, M2 = $-1+\sqrt{3}$, M3 = $-1+\sqrt{3}$, M4 = $-1+\sqrt{3}$

p_i values:

p1 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p2 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p3 = $3^{1/4}$, p4 = $3^{1/4}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Right + Left

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

M_i values:

M1 = $-1+\sqrt{3}$, M2 = $-1+\sqrt{3}$, M3 = $-1+\sqrt{3}$, M4 = $-1+\sqrt{3}$

p_i values:

p1 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p2 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p3 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p4 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Right + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 60 & 165 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

M_i values:

M1 = $-1+\sqrt{3}$, M2 = $-1+\sqrt{3}$, M3 = $-1+\sqrt{3}$, M4 = $-1+\sqrt{3}$

p_i values:

p1 = $3^{1/4}$, p2 = $3^{1/4}$, p3 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p4 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Right + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 120 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

M_i values:

M1 = $-1+\sqrt{3}$, M2 = $-1+\sqrt{3}$, M3 = $-1+\sqrt{3}$, M4 = $-1+\sqrt{3}$

p_i values:

p1 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p2 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p3 = $3^{1/4}$, p4 = $3^{1/4}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 15 & 120 & 90 \end{pmatrix}$$

M_i values:

M1 = $-1+\sqrt{3}$, M2 = $-1+\sqrt{3}$, M3 = $-1+\sqrt{3}$, M4 = $-1+\sqrt{3}$

p_i values:

p1 = $3^{1/4}$, p2 = $3^{1/4}$, p3 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p4 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 15 & 120 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

M_i values:

M1 = $-1+\sqrt{3}$, M2 = $-1+\sqrt{3}$, M3 = $-1+\sqrt{3}$, M4 = $-1+\sqrt{3}$

p_i values:

p1 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p2 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p3 = $3^{1/4}$, p4 = $3^{1/4}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

M_i values:

M1 = $-1+\sqrt{3}$, M2 = $-1+\sqrt{3}$, M3 = $-1+\sqrt{3}$, M4 = $-1+\sqrt{3}$

p_i values:

p1 = $3^{1/4}$, p2 = $3^{1/4}$, p3 = $3^{1/4}$, p4 = $3^{1/4}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Right + Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 120 & 15 & 75 \\ 105 & 165 & 60 & 90 \end{pmatrix}$$

M_i values:

M1 = $-1+\sqrt{3}$, M2 = $-1+\sqrt{3}$, M3 = $-1+\sqrt{3}$, M4 = $-1+\sqrt{3}$

p_i values:

p1 = $3^{1/4}$, p2 = $3^{1/4}$, p3 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p4 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Right + Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 105 & 165 & 60 & 90 \\ 90 & 60 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

M_i values:

M1 = $-1+\sqrt{3}$, M2 = $-1+\sqrt{3}$, M3 = $-1+\sqrt{3}$, M4 = $-1+\sqrt{3}$

p_i values:

p1 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p2 = $\sqrt{-1+\frac{2}{\sqrt{3}}}$, p3 = $3^{1/4}$, p4 = $3^{1/4}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Right + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 60 & 15 & 105 \\ 90 & 120 & 165 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

M_i values:

$M1 = -1+\sqrt{3}$, $M2 = -1+\sqrt{3}$, $M3 = -1+\sqrt{3}$, $M4 = -1+\sqrt{3}$

p_i values:

$p1 = 3^{1/4}$, $p2 = 3^{1/4}$, $p3 = 3^{1/4}$, $p4 = 3^{1/4}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 165 & 120 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 165 & 120 & 90 \end{pmatrix}$$

M_i values:

$M1 = -1+\sqrt{3}$, $M2 = -1+\sqrt{3}$, $M3 = -1+\sqrt{3}$, $M4 = -1+\sqrt{3}$

p_i values:

$p1 = 3^{1/4}$, $p2 = 3^{1/4}$, $p3 = 3^{1/4}$, $p4 = 3^{1/4}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Switch combination: Right + Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 75 & 15 & 60 & 90 \\ 90 & 120 & 165 & 105 \\ 90 & 60 & 15 & 75 \\ 75 & 15 & 60 & 90 \end{pmatrix}$$

M_i values:

$M1 = -1+\sqrt{3}$, $M2 = -1+\sqrt{3}$, $M3 = -1+\sqrt{3}$, $M4 = -1+\sqrt{3}$

p_i values:

$p1 = 3^{1/4}$, $p2 = 3^{1/4}$, $p3 = 3^{1/4}$, $p4 = 3^{1/4}$

Mi < 1 and pi ∈ ℝ for all i = 1, ..., 4

Out[∘]=

================== NOT CHIMERA ==================

Fails conic, orthodiagonal & isogonal tests for all i=1, ..., 4 ⇒ NOT chimera. Boundary-strip switches preserve these failures as demonstrated in the NOT CONIC, NOT ORTHODIAGONAL, and NOT ISOGONAL sections.

Out[∘]=

================== EXTRA INFO (FLEXION 1) ==================

This example does NOT have planar parameter lines (C2A2A1C1, C3A3A4C4, B1A1A4B4, B2A2A3B3)

Coplanarity Check: A2A1-A2C2-A1C1

Determinant of {A2A1, A2C2, A1C1} as a function of t

$$\mathrm{l1}\left(-\frac{(-1+\sqrt{3})\sqrt{\frac{1}{2}(2+\sqrt{3})}\,\mathrm{n1}\,\mathrm{n2}\,(-1+3t^2)\left(-2t+\sqrt{1+\sqrt{3}}\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)}{-2-8t^2-6t^4+8\sqrt{1+\sqrt{3}}\,t\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}}+\right.$$
$$\left(\sqrt{3}\,\mathrm{n1}\,\mathrm{n2}\left(1+\sqrt{3}+(3+2\sqrt{3})t^2\right)\right.$$
$$\left((3+2\sqrt{3})t-\sqrt{1+\sqrt{3}}\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)$$
$$\left.\left(-1-14t^2+3t^4+8\sqrt{1+\sqrt{3}}\,t\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)\right)\Big/$$
$$\left(2\left(-1-4t^2-3t^4+4\sqrt{1+\sqrt{3}}\,t\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)\right.$$
$$\left(3+\sqrt{3}+(51+28\sqrt{3})t^2+3(2+\sqrt{3})t^4-\right.$$
$$\left.\left.\left.2\sqrt{1+\sqrt{3}}\,(3+2\sqrt{3})\,t\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)\right)\right)$$

Coplanarity Check: A3A4-A3C3-A4C4

Determinant of {A3A4, A3C3, A4C4} as a function of t

$$\left(-8\sqrt{3}\,\mathrm{n3}\,\mathrm{n4}\left(1+\sqrt{3}+(3+2\sqrt{3})t^2\right)\right.$$
$$\left((3+2\sqrt{3})t-\sqrt{1+\sqrt{3}}\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)$$
$$\left(-2(2+\sqrt{3})\,\mathrm{l3}\,(1-3t^2)^2+\right.$$
$$2\,\mathrm{l1}\left(-1-14t^2+3t^4+8\sqrt{1+\sqrt{3}}\,t\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)+$$
$$\sqrt{2}\,\mathrm{l2}\left(3+\sqrt{3}-8(1+2\sqrt{3})t^2+3(7+5\sqrt{3})t^4+\right.$$
$$\left.\left.4(-1+\sqrt{3})\sqrt{1+\sqrt{3}}\,t\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)\right)+2(1-\sqrt{3})$$
$$\sqrt{2(2+\sqrt{3})}\,\mathrm{n3}\,(1-3t^2)\left(2t-\sqrt{1+\sqrt{3}}\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)$$
$$\left(\sqrt{3}\left((\sqrt{2}+\sqrt{6})\,\mathrm{l2}-4\,\mathrm{l3}\right)\mathrm{n4}\left(\left(1+\sqrt{3}+(3+2\sqrt{3})t^2\right)^2-\right.\right.$$
$$\left.\left((3+2\sqrt{3})t-\sqrt{1+\sqrt{3}}\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)^2\right)+$$
$$\left(4\,\mathrm{l1}+\sqrt{2}(-1+\sqrt{3})\,\mathrm{l2}\right)\mathrm{n4}\left(\left(1+\sqrt{3}+(3+2\sqrt{3})t^2\right)^2+\right.$$
$$\left.\left.\left.\left((3+2\sqrt{3})t-\sqrt{1+\sqrt{3}}\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)^2\right)\right)\right)\Big/$$
$$\left(32\left(1+4t^2+3t^4-4\sqrt{1+\sqrt{3}}\,t\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)\right.$$
$$\left(\left(1+\sqrt{3}+(3+2\sqrt{3})t^2\right)^2+\right.$$
$$\left.\left.\left((3+2\sqrt{3})t-\sqrt{1+\sqrt{3}}\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)^2\right)\right)$$

Coplanarity Check: A2A3-A2B2-A3B3

Determinant of {A2A3, A2B2, A3B3} as a function of t

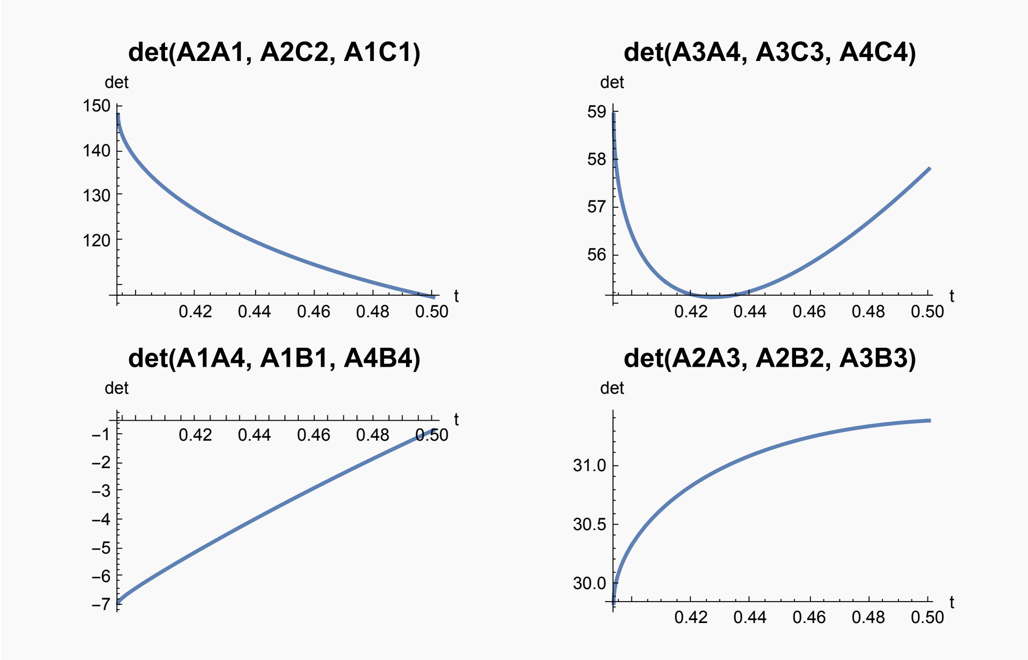
$$\left(\sqrt{2}(-1+\sqrt{3})\,\mathrm{l2}\,\mathrm{m2}\,\mathrm{m3}\left((11+8\sqrt{3})t+15(55+32\sqrt{3})t^3+\right.\right.$$
$$24(109+63\sqrt{3})t^5-\sqrt{1+\sqrt{3}}\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}-$$
$$\sqrt{1+\sqrt{3}}\,(25+12\sqrt{3})t^2\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}+$$
$$\left.\left.12\sqrt{1+\sqrt{3}}\,(5+3\sqrt{3})t^4\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)\right)\Big/$$
$$\left((1+t^2)\left(-3-4\sqrt{3}+7(127+72\sqrt{3})t^2+12(619+357\sqrt{3})t^4-\right.\right.$$
$$\left.\left.4\sqrt{1+\sqrt{3}}\,(17+12\sqrt{3})\,t\sqrt{-1+3(1+\sqrt{3})t^2-3(2+\sqrt{3})t^4}\right)\right)$$

Coplanarity Check: A1A4-A1B1-A4B4

Determinant of {A1A4, A1B1, A4B4} as a function of t

$$\frac{1}{2}\, l3\, m1\, m4 \left(\frac{t}{1+t^2} - \frac{\left(\sqrt{3}+12\left(9+5\sqrt{3}\right)t^2\right)\left(-\left(\left(36+17\sqrt{3}\right)t\right)+2\sqrt{3\left(1+\sqrt{3}\right)}\sqrt{-1+3\left(1+\sqrt{3}\right)t^2-3\left(2+\sqrt{3}\right)t^4}\right)}{3\left(3+4\sqrt{3}-7\left(127+72\sqrt{3}\right)t^2-12\left(619+357\sqrt{3}\right)t^4+4\sqrt{1+\sqrt{3}}\left(17+12\sqrt{3}\right)t\sqrt{-1+3\left(1+\sqrt{3}\right)t^2-3\left(2+\sqrt{3}\right)t^4}\right)}\right)$$

Determinant Plots for Our Example



Out[∘]=

================= EXTRA INFO (FLEXION 2) =================

This example does NOT have planar parameter lines (C2A2A1C1, C3A3A4C4, B1A1A4B4, B2A2A3B3)

Coplanarity Check: A2A1-A2C2-A1C1

Determinant of {A2A1, A2C2, A1C1} as a function of t

$$\begin{aligned} l1 \Bigg(& -\frac{\left(-1+\sqrt{3}\right)\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\, n1\, n2\left(-1+3t^2\right)\left(2t+\sqrt{1+\sqrt{3}}\sqrt{-1+3\left(1+\sqrt{3}\right)t^2-3\left(2+\sqrt{3}\right)t^4}\right)}{2+8t^2+6t^4+8\sqrt{1+\sqrt{3}}\,t\sqrt{-1+3\left(1+\sqrt{3}\right)t^2-3\left(2+\sqrt{3}\right)t^4}} + \\ & \Bigg(\sqrt{3}\, n1\, n2\left(1+\sqrt{3}+\left(3+2\sqrt{3}\right)t^2\right) \\ & \quad \left(\left(3+2\sqrt{3}\right)t+\sqrt{1+\sqrt{3}}\sqrt{-1+3\left(1+\sqrt{3}\right)t^2-3\left(2+\sqrt{3}\right)t^4}\right) \\ & \quad \left(1+14t^2-3t^4+8\sqrt{1+\sqrt{3}}\,t\sqrt{-1+3\left(1+\sqrt{3}\right)t^2-3\left(2+\sqrt{3}\right)t^4}\right)\Bigg) \Big/ \\ & \Bigg(2\left(1+4t^2+3t^4+4\sqrt{1+\sqrt{3}}\,t\sqrt{-1+3\left(1+\sqrt{3}\right)t^2-3\left(2+\sqrt{3}\right)t^4}\right) \\ & \quad \Bigg(3+\sqrt{3}+\left(51+28\sqrt{3}\right)t^2+3\left(2+\sqrt{3}\right)t^4+ \\ & \qquad 2\sqrt{1+\sqrt{3}}\left(3+2\sqrt{3}\right)t\sqrt{-1+3\left(1+\sqrt{3}\right)t^2-3\left(2+\sqrt{3}\right)t^4}\Bigg)\Bigg)\Bigg) \end{aligned}$$

Coplanarity Check: A3A4-A3C3-A4C4

Determinant of {A3A4, A3C3, A4C4} as a function of t

$$
\begin{aligned}
&\mathrm{n3}\,\Big(\Big(\sqrt{3}\,\mathrm{n4}\,\big(1+\sqrt{3}+(3+2\sqrt{3})\,t^2\big)\,\Big((3+2\sqrt{3})\,t+{}\\
&\quad\sqrt{1+\sqrt{3}}\,\sqrt{-1+3\,(1+\sqrt{3})\,t^2-3\,(2+\sqrt{3})\,t^4}\Big)\,\Big(2\,(2+\sqrt{3})\,\mathrm{l3}\,(1-3\,t^2)^2+{}\\
&\quad 2\,\mathrm{l1}\,\Big(1+14\,t^2-3\,t^4+8\,\sqrt{1+\sqrt{3}}\,t\,\sqrt{-1+3\,(1+\sqrt{3})\,t^2-3\,(2+\sqrt{3})\,t^4}\Big)+{}\\
&\quad \sqrt{2}\,\mathrm{l2}\,\Big(-3-\sqrt{3}+8\,(1+2\sqrt{3})\,t^2-3\,(7+5\sqrt{3})\,t^4+{}\\
&\qquad 4\,(-1+\sqrt{3})\,\sqrt{1+\sqrt{3}}\,t\,\sqrt{-1+3\,(1+\sqrt{3})\,t^2-3\,(2+\sqrt{3})\,t^4}\Big)\Big)\Big)\Big/\\
&\quad\Big(3+\sqrt{3}+(51+28\sqrt{3})\,t^2+3\,(2+\sqrt{3})\,t^4+2\,\sqrt{1+\sqrt{3}}\,(3+2\sqrt{3})\,t\\
&\qquad\sqrt{-1+3\,(1+\sqrt{3})\,t^2-3\,(2+\sqrt{3})\,t^4}\Big)+\frac{1}{2}\,(-1+\sqrt{3})\,\sqrt{\frac{1}{2}\,(2+\sqrt{3})}\\
&\quad\mathrm{n4}\,(-1+3\,t^2)\,\Big(2\,t+\sqrt{1+\sqrt{3}}\,\sqrt{-1+3\,(1+\sqrt{3})\,t^2-3\,(2+\sqrt{3})\,t^4}\Big)\\
&\quad\Bigg(4\,\mathrm{l1}+\sqrt{2}\,(-1+\sqrt{3})\,\mathrm{l2}+{}\\
&\qquad\frac{4\sqrt{3}\,\Big(-\frac{1}{4}\,(\sqrt{2}+\sqrt{6})\,\mathrm{l2}+\mathrm{l3}\Big)\left(-1+\frac{\Big((3+2\sqrt{3})\,t+\sqrt{1+\sqrt{3}}\,\sqrt{-1+3\,(1+\sqrt{3})\,t^2-3\,(2+\sqrt{3})\,t^4}\Big)^2}{\big(1+\sqrt{3}+(3+2\sqrt{3})\,t^2\big)^2}\right)}{1+\frac{\Big((3+2\sqrt{3})\,t+\sqrt{1+\sqrt{3}}\,\sqrt{-1+3\,(1+\sqrt{3})\,t^2-3\,(2+\sqrt{3})\,t^4}\Big)^2}{\big(1+\sqrt{3}+(3+2\sqrt{3})\,t^2\big)^2}}\Bigg)\Bigg)\Bigg)\Big/\\
&\Big(4\,\Big(1+4\,t^2+3\,t^4+4\,\sqrt{1+\sqrt{3}}\,t\,\sqrt{-1+3\,(1+\sqrt{3})\,t^2-3\,(2+\sqrt{3})\,t^4}\Big)\Big)
\end{aligned}
$$

Coplanarity Check: A2A3-A2B2-A3B3

Determinant of {A2A3, A2B2, A3B3} as a function of t

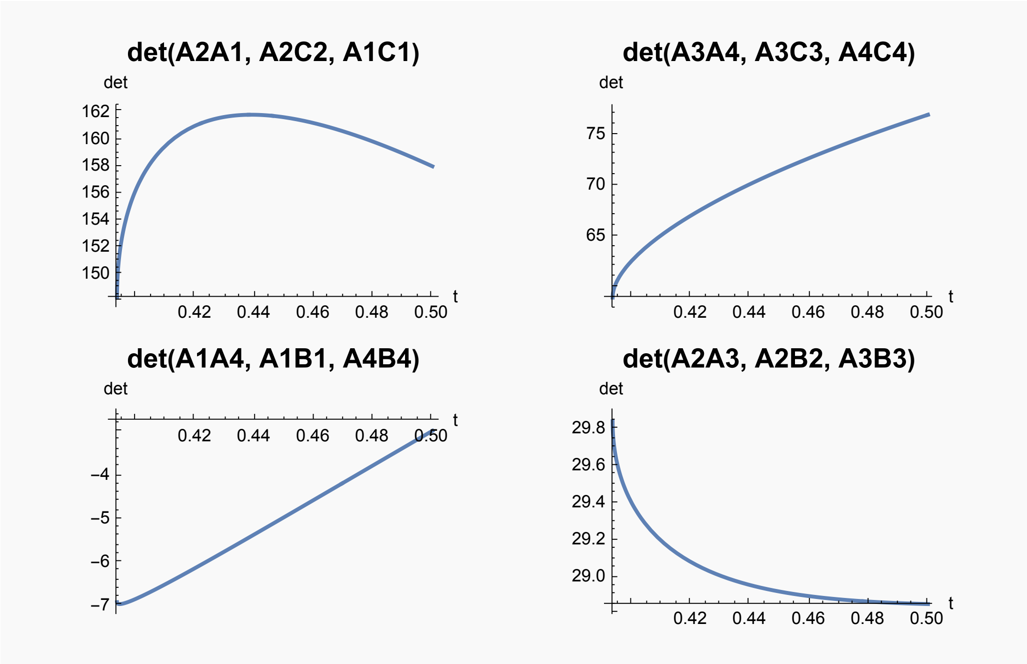
$$
\begin{aligned}
&\Big(\sqrt{2}\,(-1+\sqrt{3})\,\mathrm{l2}\,\mathrm{m2}\,\mathrm{m3}\,\Big((11+8\sqrt{3})\,t+15\,(55+32\sqrt{3})\,t^3+{}\\
&\quad 24\,(109+63\sqrt{3})\,t^5+\sqrt{1+\sqrt{3}}\,\sqrt{-1+3\,(1+\sqrt{3})\,t^2-3\,(2+\sqrt{3})\,t^4}+{}\\
&\quad \sqrt{1+\sqrt{3}}\,(25+12\sqrt{3})\,t^2\,\sqrt{-1+3\,(1+\sqrt{3})\,t^2-3\,(2+\sqrt{3})\,t^4}-{}\\
&\quad 12\,\sqrt{1+\sqrt{3}}\,(5+3\sqrt{3})\,t^4\,\sqrt{-1+3\,(1+\sqrt{3})\,t^2-3\,(2+\sqrt{3})\,t^4}\Big)\Big)\Big/\\
&\Big((1+t^2)\,\Big(-3-4\sqrt{3}+7\,(127+72\sqrt{3})\,t^2+12\,(619+357\sqrt{3})\,t^4+{}\\
&\quad 4\,\sqrt{1+\sqrt{3}}\,(17+12\sqrt{3})\,t\,\sqrt{-1+3\,(1+\sqrt{3})\,t^2-3\,(2+\sqrt{3})\,t^4}\Big)\Big)
\end{aligned}
$$

Coplanarity Check: A1A4-A1B1-A4B4

Determinant of {A1A4, A1B1, A4B4} as a function of t

$$
\begin{aligned}
&\frac{1}{2}\,\mathrm{l3}\,\mathrm{m1}\,\mathrm{m4}\\
&\left(\frac{t}{1+t^2}-\frac{\big(\sqrt{3}+12\,(9+5\sqrt{3})\,t^2\big)\,\Big((36+17\sqrt{3})\,t+2\,\sqrt{3\,(1+\sqrt{3})}\,\sqrt{-1+3\,(1+\sqrt{3})\,t^2-3\,(2+\sqrt{3})\,t^4}\Big)}{3\,\Big(-3-4\sqrt{3}+7\,(127+72\sqrt{3})\,t^2+12\,(619+357\sqrt{3})\,t^4+4\,\sqrt{1+\sqrt{3}}\,(17+12\sqrt{3})\,t\,\sqrt{-1+3\,(1+\sqrt{3})\,t^2-3\,(2+\sqrt{3})\,t^4}\Big)}\right)
\end{aligned}
$$

Determinant Plots for Our Example


det(A2A1, A2C2, A1C1)
det
162
160
158
156
154
152
150
0.42
0.44
0.46
0.48
0.50
t
det(A3A4, A3C3, A4C4)
det
75
70
65
0.42
0.44
0.46
0.48
0.50
t
det(A1A4, A1B1, A4B4)
det
0.42
0.44
0.46
0.48
0.50
t
−4
−5
−6
−7
det(A2A3, A2B2, A3B3)
det
29.8
29.6
29.4
29.2
29.0
0.42
0.44
0.46
0.48
0.50
t