

Computational Companion to “Flexible 3×3 Nets of Equimodular Elliptic Type” — Example 4

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Tested on: Mathematica 14.0

In[1480]:=

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(*=====*)
=====*)
(*=====*)
=====*)
(*=====*)
=====*)
(*Quit*)
(*All angle sets in degrees*)
anglesDeg = {
  {26.20863403213998, 82.2407675648952, 21.949109994264898, 60}, (*Vertex 1*)
  {16.166237389600262,
   130.87095233025335, 18.85247535405415, 115}, (*Vertex 2*)
  {134.65533802039442,
   34.44439013740831, 145.3694664686027, 80}, (*Vertex 3*)
  {117.95117201340666,
   49.52829397349284, 149.0275482144225, 105} (*Vertex 4*)};

(*-----*)
(*Function to compute sigma from 4 angles*)
computeSigma[{ $\alpha$ _,  $\beta$ _,  $\gamma$ _,  $\delta$ _}] := ( $\alpha$  +  $\beta$  +  $\gamma$  +  $\delta$ ) / 2;

(*-----*)
(*Function to compute a,b,c,d from angles*)
computeABCD[{ $\alpha$ _,  $\beta$ _,  $\gamma$ _,  $\delta$ _}] :=
Module[{alpha =  $\alpha$  Degree, beta =  $\beta$  Degree, gamma =  $\gamma$  Degree,
  delta =  $\delta$  Degree, sigma}, sigma = computeSigma[{ $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ ] Degree];
{Sin[alpha] / Sin[sigma - alpha], Sin[beta] / Sin[sigma - beta],
 Sin[gamma] / Sin[sigma - gamma], Sin[delta] / Sin[sigma - delta]};

(*-----*)
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(*Compute all  $\sigma$  values*)
sigmas = computeSigma /@ anglesDeg;

(*Compute all {a,b,c,d} values*)
results = computeABCD /@ anglesDeg;

(*Optional:extract individual values*)
{a1, b1, c1, d1} = results[[1]];
{a2, b2, c2, d2} = results[[2]];
{a3, b3, c3, d3} = results[[3]];
{a4, b4, c4, d4} = results[[4]];
{ $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$ ,  $\sigma_4$ } = FullSimplify[sigmas];

(*=====
====*)
(*=====
CONDITION (N.0) =====*)
(*=====
====*)
(*uniqueCombos={{1,1,1,1},{1,1,1,-1},{1,1,-1,-1},
{1,1,-1,1},{1,-1,1,1},{1,-1,-1,1},{1,-1,1,-1},{1,-1,-1,-1}};

checkConditionN0Degrees[{ $\alpha$ _, $\beta$ _, $\gamma$ _, $\delta$ _}]:=Module[
{angles={ $\alpha$ , $\beta$ , $\gamma$ , $\delta$ },results},results=Mod[uniqueCombos.angles,360]//Chop;
!MemberQ[results,0]];

conditionsN0=checkConditionN0Degrees/@anglesDeg;
allVerticesPass=And@@conditionsN0;

Column[{TextCell[Style["===== CONDITION (N.0) =====",
Darker[Green],Bold, 16],"Text"],
If[allVerticesPass,
Style["✓ All vertices satisfy (N.0).",Darker[Green],Bold],
Style["✗ Some vertices fail (N.0).",Red,Bold]]]*),
uniqueCombos = {{1, 1, 1, 1}, {1, 1, 1, -1}, {1, 1, -1, -1}, {1, 1, -1, 1},
{1, -1, 1, 1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, -1, -1, -1}};

checkConditionN0Degrees[{ $\alpha$ _,  $\beta$ _,  $\gamma$ _,  $\delta$ _}] :=
Module[{angles = { $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ }, results},
results = Mod[uniqueCombos.angles, 360] // Chop;
results];

(*apply to all vertices*)
resultsPerVertex = checkConditionN0Degrees /@ anglesDeg;

(*check pass/fail*)

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conditionsN0 = Map[! MemberQ[#, 0] &, resultsPerVertex];
allVerticesPass = And@@conditionsN0;

Column[{TextCell[Style["===== CONDITION (N.0) =====",
  Darker[Green], Bold, 16], "Text"], (*summary line*)If[allVerticesPass,
  Style["✓ All vertices satisfy (N.0).", Darker[Green], Bold],
  Style["✗ Some vertices fail (N.0).", Red, Bold]],
  (*detailed results for each vertex*)
  Grid[Prepend[Table[{"Vertex " <> ToString[i], resultsPerVertex[[i],
    If[conditionsN0[[i], "✓ Pass", "✗ Fail"]}, {i, Length[anglesDeg]}],
    {"Vertex", "Combinations (mod 360)", "Status"}], Frame → All]]]

(*=====*)
(*=====
  CONDITION (N.3)=====*)
(*=====*)
Ms = FullSimplify[Times@@@results];
allEqualQ = Simplify[Equal@@Ms];

Column[{TextCell[Style["===== CONDITION (N.3) =====",
  Darker[Blue], Bold, 16], "Text"], If[allEqualQ,
  Row[{Style["✓ M1 = M2 = M3 = M4 = ", Bold], Ms[[1]]},
  Style["✗ M_i are not all equal.", Red, Bold]]]}]

(*=====*)
(*=====CONDITION (N.4)=====*)
(*=====*)
aList = results[[All, 1]];
cList = results[[All, 3]];
dList = results[[All, 4]];

rList = FullSimplify /@ (aList * dList); {r1, r2, r3, r4} = rList;
sList = FullSimplify /@ (cList * dList); {s1, s2, s3, s4} = sList;

Column[{TextCell[Style["===== CONDITION (N.4) =====",
  Darker[Purple], Bold, 16], "Text"],
  If[r1 == r2 && r3 == r4 && s1 == s4 && s2 == s3, Column[
    {Row[{Style["✓ r1 = r2 = ", Bold], r1, Style["; ✓ r3 = r4 = ", Bold], r3}],
    Row[{Style["✓ s1 = s4 = ", Bold], s1, Style["; ✓ s2 = s3 = ", Bold],
      s2}]]], Style["✗ Condition (N.4) fails.", Red, Bold]]
}]

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(*=====
====*)
(*=====
CONDITION (N.5)=====*)
(*=====
====*)
fList = FullSimplify /@ (aList * cList); {f1, f2, f3, f4} = fList;
M1 = Ms[[1]];

m[M1_] := Piecewise[{{1 - M1, M1 < 1}, {(M1 - 1) / M1, M1 > 1}}] // N;
K = EllipticK[m[M1]] // N;
Kp = EllipticK[1 - m[M1]] // N;

computeTi[i_] :=
Module[{sigma = sigmas[[i]], r = rList[[i]], s = sList[[i]], f = fList[[i]], base},
  base = I * Im[InverseJacobiDN[Piecewise[
    {{Sqrt[f], M1 < 1}, {1 / Sqrt[f], M1 > 1}}], m[M1]]] / Kp;
  Which[sigma < 180, Which[r > 1 && s > 1, base, (r < 1 && s > 1) || (r > 1 && s < 1),
    1 + base, r < 1 && s < 1, 2 + base], sigma > 180, Which[r > 1 && s > 1,
    2 + base, (r < 1 && s > 1) || (r > 1 && s < 1), 3 + base, r < 1 && s < 1, base]]];

tList = computeTi /@ Range[4];
RoundWithTolerance[x_, tol_ : 10^(-14)] :=
Module[{nearest}, nearest = Round[x];
  If[Abs[x - nearest] ≤ tol, nearest, x]];

checkValidCombination[M1_, ε_ : 10^(-14)] :=
Module[{i, combo, dotProd, rePart, imPart, n2, expr, foundQ = False},
  Print[Style["△ Approximate validation using ε-tolerance. For rigorous
    proof, see the referenced paper.", Darker@Orange, Italic]];
  Do[combo = uniqueCombos[[i]];
    dotProd = tList.combo;
    rePart = Abs[Re[dotProd]];
    imPart = Abs[Im[dotProd]];
    If[M1 < 1,
      If[Mod[RoundWithTolerance[rePart], 4] < ε,
        If[Mod[RoundWithTolerance[imPart], 2] < ε, expr =
          tList[[1]] + combo[[2]] × tList[[2]] + combo[[3]] × tList[[3]] + combo[[4]] × tList[[4]];
          Print[Style["✔ Valid Combination Found (M < 1):",
            Darker[Green], Bold], "\n", Style["e1 = ", Bold], combo[[2]],
            Style["", e2 = ", Bold], combo[[3]], Style["", e3 = ", Bold], combo[[4]],
            "\n", Style["t1 = ", Bold], Re[tList[[1]], "K + ", Im[tList[[1]], "iK'",
            "\n", Style["t2 = ", Bold], Re[tList[[2]], "K + ", Im[tList[[2]], "iK'",
            "\n", Style["t3 = ", Bold], Re[tList[[3]], "K + ", Im[tList[[3]], "iK'"]];

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"\n", Style["t4 = ", Bold], Re[tList[[4]], "K + ", Im[tList[[4]],
"iK'", "\n", Style["t1 + e1*t2 + e2*t3 + e3*t4 = ", Bold],
Re[expr], "K + ", Im[expr], "iK'"];
foundQ = True;
Break[]]]];
If[M1 > 1,
If[Mod[RoundWithTolerance[imPart], 2] < ε,
n2 = Quotient[RoundWithTolerance[imPart], 2];
If[Mod[RoundWithTolerance[Abs[rePart - 2 n2]], 4] < ε, expr =
tList[[1]] + combo[[2]] × tList[[2]] + combo[[3]] × tList[[3]] + combo[[4]] × tList[[4]];
Print[Style["✔ Valid Combination Found (M > 1):",
Darker[Green], Bold], "\n", Style["e1 = ", Bold], combo[[2]],
Style["e2 = ", Bold], combo[[3]], Style["e3 = ", Bold], combo[[4]],
"\n", Style["t1 = ", Bold], Re[tList[[1]], "K + ", Im[tList[[1]], "iK'",
"\n", Style["t2 = ", Bold], Re[tList[[2]], "K + ", Im[tList[[2]], "iK'",
"\n", Style["t3 = ", Bold], Re[tList[[3]], "K + ", Im[tList[[3]], "iK'",
"\n", Style["t4 = ", Bold], Re[tList[[4]], "K + ", Im[tList[[4]],
"iK'", "\n", Style["t1 + e1*t2 + e2*t3 + e3*t4 = ", Bold],
Re[expr], "K + ", Im[expr], "iK'"];
foundQ = True;
Break[]]]], {i, Length[uniqueCombos]}}];
If[! foundQ, Print[Style["✗ No valid combination found.", Red, Bold], "\n",
Style["t1 = ", Bold], Re[tList[[1]], "K + ", Im[tList[[1]], "iK'", "\n",
Style["t2 = ", Bold], Re[tList[[2]], "K + ", Im[tList[[2]], "iK'", "\n",
Style["t3 = ", Bold], Re[tList[[3]], "K + ", Im[tList[[3]], "iK'", "\n",
Style["t4 = ", Bold], Re[tList[[4]], "K + ", Im[tList[[4]], "iK'", "\n"]];];];

Column[{TextCell[Style["===== CONDITION (N.5) =====",
Darker[Red], Bold, 16], "Text"]}]
res = checkValidCombination[M1];

(*=====
====*)
(*=====
OTHER PARAMETER=====*)
(*=====
====*)
cosSigmaAlt[Mi_, fi_, ri_, si_] :=
(Mi (fi + ri + si - 1) + ri fi si - ri fi - ri si - fi si) / (2 Sqrt[ri si fi] (1 - Mi));

Column[
{TextCell[Style["===== OTHER PARAMETERS =====",
Darker[Orange], Bold, 16], "Text"],
Row[{Style["u = ", Bold], 1 - M1}],

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Row[{Style["σ1 = ", Bold], σ1 Degree, Style["", σ2 = ", Bold], σ2 Degree,
  Style["", σ3 = ", Bold], σ3 Degree, Style["", σ4 = ", Bold], σ4 Degree}],
Row[{Style["σ1 ≈ ", Bold], N[σ1], Style["°", Bold], Style["", σ2 ≈ ", Bold],
  N[σ2], Style["°", Bold], Style["", σ3 ≈ ", Bold], N[σ3],
  Style["°", Bold], Style["", σ4 ≈ ", Bold], N[σ4], Style["°", Bold]}],
Row[{Style["cosσ1 = ", Bold], Simplify@cosSigmaAlt[M1, f1, r1, s1],
  Style["", cosσ2 = ", Bold], Simplify@cosSigmaAlt[M1, f2, r2, s2],
  Style["", cosσ3 = ", Bold], Simplify@cosSigmaAlt[M1, f3, r3, s3],
  Style["", cosσ4 = ", Bold], Simplify@cosSigmaAlt[M1, f4, r4, s4]}],
Row[{Style["f1 = ", Bold], f1, Style["", f2 = ", Bold],
  f2, Style["", f3 = ", Bold], f3, Style["", f4 = ", Bold], f4}],
Row[{Style["z1 = ", Bold], FullSimplify[1 / (f1 - 1)], Style["", z2 = ", Bold],
  FullSimplify[1 / (f2 - 1)], Style["", z3 = ", Bold], FullSimplify[1 / (f3 - 1)],
  Style["", z4 = ", Bold], FullSimplify[1 / (f4 - 1)]}],
Row[{Style["x1 = ", Bold], FullSimplify[1 / (r1 - 1)], Style["", x2 = ", Bold],
  FullSimplify[1 / (r2 - 1)], Style["", x3 = ", Bold], FullSimplify[1 / (r3 - 1)],
  Style["", x4 = ", Bold], FullSimplify[1 / (r4 - 1)]}],
Row[{Style["y1 = ", Bold], FullSimplify[1 / (s1 - 1)], Style["", y2 = ", Bold],
  FullSimplify[1 / (s2 - 1)], Style["", y3 = ", Bold], FullSimplify[1 / (s3 - 1)],
  Style["", y4 = ", Bold], FullSimplify[1 / (s4 - 1)]}],
Row[{Style["p1 = ", Bold], Simplify[Sqrt[r1 - 1]],
  Style["", p2 = ", Bold], Simplify[Sqrt[r2 - 1]], Style["", p3 = ", Bold],
  Simplify[Sqrt[r3 - 1]], Style["", p4 = ", Bold], Simplify[Sqrt[r4 - 1]]}],
Row[{Style["q1 = ", Bold], Simplify[Sqrt[s1 - 1]],
  Style["", q2 = ", Bold], Simplify[Sqrt[s2 - 1]], Style["", q3 = ", Bold],
  Simplify[Sqrt[s3 - 1]], Style["", q4 = ", Bold], Simplify[Sqrt[s4 - 1]]}],
Row[{Style["p1.q1 = ", Bold], Simplify[Sqrt[r1 - 1] Sqrt[s1 - 1]],
  Style["", p2.q2 = ", Bold], Simplify[Sqrt[r2 - 1] Sqrt[s2 - 1]],
  Style["", p3.q3 = ", Bold], Simplify[Sqrt[r3 - 1] Sqrt[s3 - 1]],
  Style["", p4.q4 = ", Bold], Simplify[Sqrt[r4 - 1] Sqrt[s4 - 1]]}]
}]

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(*=====
====*)
(*=====
BRICARD's EQUATIONS=====*)
(*=====
====*)
(*=====
FLEXION 1=====*)

Z[t_] := t;
W1[t_] := (1.8303883744906646` (0.739190870110122` t - 0.8185802872931142`
  Sqrt[(1 + 3.4575776313801847` t^2) (1 - 0.7811714739558353` t^2)])) /
  (-1.2264950862699229` - 3.4575776313801847` t^2);

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U[t_] := (0.18029302872898165` (11.610011024543208` t + 6.663793331850769`
  Sqrt[(1 + 3.4575776313801847` t^2) (1 - 0.7811714739558353` t^2)])) /
  (2.6091661366212175` + 3.845171738795376` t^2);
W2[t_] := (0.8842187622039149` (0.8494336559689466` t - 1.1387226496890441`
  Sqrt[(1 + 3.4575776313801847` t^2) (1 - 0.7811714739558353` t^2)])) /
  (-1.1465569838598522` - 3.4575776313801847` t^2);

(*General polynomial constructor*)
BricardsEquation[{α_, β_, γ_, δ_}, σ_, x_, y_] :=
Module[{c22, c20, c02, c11, c00},
  c22 = Sin[σ - δ] Sin[σ - δ - β];
  c20 = Sin[σ - α] Sin[σ - α - β];
  c02 = Sin[σ - γ] Sin[σ - γ - β];
  c11 = -Sin[α] Sin[γ];
  c00 = Sin[σ] Sin[σ - β];
  c22 x^2 y^2 + c20 x^2 + c02 y^2 + 2 c11 x y + c00];

(*Compute and print all P_i for flexion 1*)
TextCell[
  Style["===== FLEXIBILITY (FLEXION 1) =====",
    Darker[Cyan], Bold, 16], "Text"]
funcs = {{Z, W1}, {Z, W2}, {U, W2}, {U, W1}};
Do[angles = anglesDeg[[i]] Degree;
  sigma = sigmas[[i]] Degree;
  {α, β, γ, δ} = angles;
  poly = Which[i == 1, BricardsEquation[{α, β, γ, δ}, sigma, Z[t], W1[t]],
    i == 2, BricardsEquation[{α, β, γ, δ}, sigma, Z[t], W2[t]],
    i == 3, BricardsEquation[{α, β, γ, δ}, sigma, U[t], W2[t]],
    i == 4, BricardsEquation[{α, β, γ, δ}, sigma, U[t], W1[t]]];
  Print[Row[{Subscript["P", i], "[", funcs[[i, 1],
    ", ", funcs[[i, 2], "] = ", FullSimplify[poly]]}], {i, 1, 4}];

(*t-range*)
tMin = 0;
tMax = 1;

(*Build P_i(t) consistently with your printing logic*)
Clear[PiExpr, labels, funcs];
funcs = {{Z, W1}, {Z, W2}, {U, W2}, {U, W1}};

PiExpr = Table[Module[{angles = anglesDeg[[i]] Degree,
  sigma = sigmas[[i]] Degree, α, β, γ, δ, poly}, {α, β, γ, δ} = angles;
  poly = BricardsEquation[{α, β, γ, δ}, sigma, funcs[[i, 1][t], funcs[[i, 2][t]];
  FullSimplify[poly]], {i, 1, 4}];

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labels = Table[Row[{Subscript["P", i], "[",
ToString@funcs[[i, 1]], ", ", ToString@funcs[[i, 2]], "]" }], {i, 1, 4}];

(*Pretty panel with plots like your style*)
Column[{TextCell[
Style["===== FLEXIBILITY (FLEXION 1) =====",
Darker[Cyan], Bold, 16], "Text"], TextCell[
Style["Polynomials P_i(t) built from Bricard's equations for flexion 1.",
GrayLevel[0.3]], "Text"], Spacer[12],
Panel[GraphicsGrid[Partition[Table[Plot[Evaluate@PiExpr[[i],
{t, tMin, tMax}], PlotLabel -> Style[labels[[i], Bold, 14],
PlotRange -> {-10^(-13), 10^(-13)}, AxesLabel -> {"t", None},
ImageSize -> 250], {i, Length[PiExpr]}], 2], Spacings -> {2, 2}],
Background -> Lighter[Gray, 0.95], FrameMargins -> 15]]]

(*=====
FLEXION 2=====*)
Z2[t_] := t;
W12[t_] := (1.8303883744906646` (0.739190870110122` t + 0.8185802872931142`

$$\frac{\sqrt{(1 + 3.4575776313801847` t^2) (1 - 0.7811714739558353` t^2)}}{(-1.2264950862699229` - 3.4575776313801847` t^2)};
U2[t_] := (0.18029302872898165` (11.610011024543208` t - 6.663793331850769`

$$\frac{\sqrt{(1 + 3.4575776313801847` t^2) (1 - 0.7811714739558353` t^2)}}{(2.6091661366212175` + 3.845171738795376` t^2)};
W22[t_] := (0.8842187622039149` (0.8494336559689466` t + 1.1387226496890441`

$$\frac{\sqrt{(1 + 3.4575776313801847` t^2) (1 - 0.7811714739558353` t^2)}}{(-1.1465569838598522` - 3.4575776313801847` t^2)};

(*General polynomial constructor*)
BricardsEquation[{α_, β_, γ_, δ_}, σ_, x_, y_] :=
Module[{c22, c20, c02, c11, c00},
c22 = Sin[σ - δ] Sin[σ - δ - β];
c20 = Sin[σ - α] Sin[σ - α - β];
c02 = Sin[σ - γ] Sin[σ - γ - β];
c11 = -Sin[α] Sin[γ];
c00 = Sin[σ] Sin[σ - β];
c22 x^2 y^2 + c20 x^2 + c02 y^2 + 2 c11 x y + c00];

(*Compute and print all P_i for flexion 2*)
TextCell[
Style["===== FLEXIBILITY (FLEXION 2) =====",
Cyan, Bold, 16], "Text"]
funcs = {{Z, W1}, {Z, W2}, {U, W2}, {U, W1}};$$$$$$

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```

Do[angles = anglesDeg[[i]] Degree;
  sigma = sigmas[[i]] Degree;
  {α, β, γ, δ} = angles;
  poly = Which[i == 1, BricardsEquation[{α, β, γ, δ}, sigma, Z2[t], W12[t]],
    i == 2, BricardsEquation[{α, β, γ, δ}, sigma, Z2[t], W22[t]],
    i == 3, BricardsEquation[{α, β, γ, δ}, sigma, U2[t], W22[t]],
    i == 4, BricardsEquation[{α, β, γ, δ}, sigma, U2[t], W12[t]]];
  Print[Row[{Subscript["P", i], "[", funcs[[i, 1],
    ", ", funcs[[i, 2]], "] = ", FullSimplify[poly]]}], {i, 1, 4}];

(*t-range*)
tMin = 0;
tMax = 1;

(*Build P_i(t) consistently with your printing logic*)
Clear[PiExpr, labels, funcs];
funcs = {{Z2, W12}, {Z2, W22}, {U2, W22}, {U2, W12}};

PiExpr = Table[Module[{angles = anglesDeg[[i]] Degree,
  sigma = sigmas[[i]] Degree, α, β, γ, δ, poly}, {α, β, γ, δ} = angles;
  poly = BricardsEquation[{α, β, γ, δ}, sigma, funcs[[i, 1]][t], funcs[[i, 2]][t]];
  FullSimplify[poly]], {i, 1, 4}];

labels = Table[Row[{Subscript["P", i], "[",
  ToString@funcs[[i, 1]], ", ", ToString@funcs[[i, 2]], "]" }], {i, 1, 4}];

(*Pretty panel with plots like your style*)
Column[{TextCell[
  Style["===== FLEXIBILITY (FLEXION 2) =====",
    Cyan, Bold, 16], "Text"], TextCell[
  Style["Polynomials P_i(t) built from Bricard's equations for flexion 2.",
    GrayLevel[0.3]], "Text"], Spacer[12],
  Panel[GraphicsGrid[Partition[Table[Plot[Evaluate@PiExpr[[i],
    {t, tMin, tMax}], PlotLabel → Style[labels[[i]], Bold, 14],
    PlotRange → {-10^(-13), 10^(-13)}, AxesLabel → {"t", None},
    ImageSize → 250], {i, Length[PiExpr]}], 2], Spacings → {2, 2}],
  Background → Lighter[Gray, 0.95], FrameMargins → 15]]]

(*=====
====*)
(*=====
NOT TRIVIAL=====*)
(*=====
====*)
(*Define domain limits for t*)
tMin = 0;

```

```

tMax = 1;

(*=====
FLEXION 1=====*)
(*List of expressions to plot*)
expressions = {Z[t], W2[t], U[t], W1[t]};
labels = {"Z", "W2", "U", "W1"};

Column[{TextCell[
  Style["===== NOT TRIVIAL (FLEXION 1) =====",
    Darker[Brown], Bold, 16], "Text"],

  (*Explanatory text*)
  TextCell[Style["This configuration does not belong to the trivial class –
    even after switching the boundary strips – since none of the
    functions Z, W2, U, or W1 is constant.", GrayLevel[0.3]], "Text"],
  Spacer[12],

  (*Plots in a light panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
    PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
    AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
    Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]

  ]}]

(*=====
FLEXION 2=====*)
(*Define domain limits for t*)
tMin = 0;
tMax = 1;
(*List of expressions to plot*)
expressions = {Z2[t], W22[t], U2[t], W12[t]};
labels = {"Z", "W2", "U", "W1"};

Column[{TextCell[
  Style["===== NOT TRIVIAL (FLEXION 2) =====",
    Brown, Bold, 16], "Text"],

  (*Explanatory text*)
  TextCell[Style["This configuration does not belong to the trivial class –
    even after switching the boundary strips – since none of the
    functions Z, W2, U, or W1 is constant.", GrayLevel[0.3]], "Text"],
  Spacer[12],

  (*Plots in a light panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},

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        PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
        AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
        Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]

    ]]

(*=====
====*)
(*=====
NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE=====*)
(*=====
====*)
(*Define domain limits for t*)
tMin = 0;
tMax = 1;

(*=====
FLEXION 1=====*)
(*List of expressions& labels*)
expressions =
    {Z[t] * U[t], Z[t] / U[t], W1[t] * W2[t], W1[t] / W2[t], Z[t] * W2[t], Z[t] / W2[t],
    U[t] * W1[t], U[t] / W1[t], Z[t] * W1[t], Z[t] / W1[t], W2[t] * U[t], W2[t] / U[t]};
labels = {"Z * U", "Z / U", "W1 * W2", "W1 / W2", "Z * W2", "Z / W2",
    "U * W1", "U / W1", "Z * W1", "Z / W1", "W2 * U", "W2 / U"};

Column[
    {TextCell[Style["===== NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE
(FLEXION 1) =====", Darker[Magenta], Bold, 16], "Text"],

    (*Explanatory text*)TextCell[
        Style["This configuration does not belong to the Linear compound class nor
to the linear conjugate class – even after switching the boundary
strips – since none of the functions ZU, Z/U, W1W2, W1/W2, ZW2,
Z/W2, UW1, and U/W1 is constant. In addition, none of ZW1,
Z/W1, W2U, W2/U is constant as well.", GrayLevel[0.3]], "Text"],
    Spacer[12],

    (*Plots panel*)
    Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
        PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
        AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
        Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]
    ]]

(*=====
FLEXION 2=====*)

```

```

(*Define domain limits for t*)
tMin = 0;
tMax = 0.65;
(*List of expressions& labels*)
expressions = {Z2[t] * U2[t], Z2[t] / U2[t], W12[t] * W22[t], W12[t] / W22[t],
  Z2[t] * W22[t], Z2[t] / W22[t], U2[t] * W12[t], U2[t] / W12[t],
  Z2[t] * W12[t], Z2[t] / W12[t], W22[t] * U2[t], W22[t] / U2[t]};
labels = {"Z * U", "Z / U", "W1 * W2", "W1 / W2", "Z * W2", "Z / W2",
  "U * W1", "U / W1", "Z * W1", "Z / W1", "W2 * U", "W2 / U"};

Column[
  {TextCell[Style["===== NOT LINEAR COMPOUND & NOT LINEAR CONJUGATE
    (FLEXION 2) =====", Magenta, Bold, 16], "Text"],

  (*Explanatory text*)TextCell[
    Style["This configuration does not belong to the Linear compound class nor
      to the linear conjugate class – even after switching the boundary
      strips – since none of the functions ZU, Z/U, W1W2, W1/W2, ZW2,
      Z/W2, UW1, and U/W1 is constant. In addition, none of ZW1,
      Z/W1, W2U, W2/U is constant as well.", GrayLevel[0.3]], "Text"],
  Spacer[12],

  (*Plots panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions[[i]], {t, tMin, tMax},
    PlotLabel → Style[labels[[i]], Bold, 14], PlotRange → All,
    AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
    Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]
  ]

(*=====
====*)
(*=====
SWITCHING BOUNDARY STRIPS=====*)
(*=====
====*)
SwitchingRightBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={1,2},{1,3},{4,2},{4,3}*)
  modified[[1, 2]] = 180 - anglesDeg[[1, 2]]; (*β1*)
  modified[[1, 3]] = 180 - anglesDeg[[1, 3]]; (*γ1*)
  modified[[4, 2]] = 180 - anglesDeg[[4, 2]]; (*β4*)
  modified[[4, 3]] = 180 - anglesDeg[[4, 3]]; (*γ4*)
  modified]

SwitchingLeftBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={2,2},{2,3},{3,2},{3,3}*)
  modified[[2, 2]] = 180 - anglesDeg[[2, 2]]; (*β2*)

```

```

modified[[2, 3]] = 180 - anglesDeg[[2, 3]]; (*γ2*)
modified[[3, 2]] = 180 - anglesDeg[[3, 2]]; (*β3*)
modified[[3, 3]] = 180 - anglesDeg[[3, 3]]; (*γ3*)
modified]

SwitchingLowerBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={1,1},{1,2},{2,1},{2,2}*)
  modified[[1, 1]] = 180 - anglesDeg[[1, 1]]; (*α1*)
  modified[[1, 2]] = 180 - anglesDeg[[1, 2]]; (*β1*)
  modified[[2, 1]] = 180 - anglesDeg[[2, 1]]; (*α2*)
  modified[[2, 2]] = 180 - anglesDeg[[2, 2]]; (*β2*)
  modified]

SwitchingUpperBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={3,1},{3,2},{4,1},{4,2}*)
  modified[[3, 1]] = 180 - anglesDeg[[3, 1]]; (*α3*)
  modified[[3, 2]] = 180 - anglesDeg[[3, 2]]; (*β3*)
  modified[[4, 1]] = 180 - anglesDeg[[4, 1]]; (*α4*)
  modified[[4, 2]] = 180 - anglesDeg[[4, 2]]; (*β4*)
  modified]

(*=====
====*)
(*=====
NOT CONIC=====*)
(*=====
====*)
uniqueCombos = {{1, 1, 1, 1}, {1, 1, 1, -1}, {1, 1, -1, -1}, {1, 1, -1, 1},
  {1, -1, 1, 1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, -1, -1, -1}};

checkConditionN0Degrees[{α_, β_, γ_, δ_}] :=
  Module[{angles = {α, β, γ, δ}, results},
    results = Mod[uniqueCombos.angles, 360] // Chop;
    ! MemberQ[results, 0]];

conditionsN0 = checkConditionN0Degrees /@ anglesDeg;
allVerticesPass = And@@ conditionsN0;

Column[{TextCell[Style["===== NOT CONIC =====",
  Pink, Bold, 16], "Text"],
  TextCell[Style["Condition (N.0) is satisfied ⇒
    this configuration is NOT equimodular-conic. Applying
    any boundary-strip switch still preserves (N.0), so
    no conic form emerges.", GrayLevel[0.3]], "Text"]
}]

```

```

(*Now the exact same Module for checking all switch combinations...*)
Module[{angles = anglesDeg, switchers, combinations, results},
  (*Define switch functions*)
  switchers = <|"Right" → SwitchingRightBoundaryStrip,
    "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
    "Upper" → SwitchingUpperBoundaryStrip|>;
  (*Generate all combinations of switches (from size 1 to 4)*)
  combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
  (*Evaluate condition after each combination of switches*) results = Table[
    Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];
    passQ = And@@ (checkConditionN0Degrees /@ switched);
    (*Print the result after switching*)
    (*Print["\nSwitch combination: ", name];
    Print["Switched anglesDeg:"];
    Print[MatrixForm[switched]]];*)
    {name, passQ}], {combo, combinations}];
  (*Display results*)
  Column[Prepend[Table[Module[{comboName = res[[1]], passQ = res[[2]]},
    Row[{Style[comboName <> ": ", Bold],
      If[passQ,
        Style["Condition (N.0) is still satisfied.", Darker[Green]],
        Style["Condition (N.0) fails.", Red, Bold]
      ]
    }, {res, results}], TextCell[
    Style["CONDITION (N.0) AFTER SWITCHING BOUNDARY STRIPS", 14], "Text"]]]]

  (*=====
  =====*)
  (*=====
  NOT ORTHODIAGONAL=====*)
  (*=====
  =====*)
  Column[
    {TextCell[Style["===== NOT ORTHODIAGONAL =====",
      Purple, Bold, 16], "Text"],
    TextCell[Style[
      "cos(αi) · cos(γi) ≠ cos(βi) · cos(δi) for each i = 1...4 ⇒ NOT orthodiagonal.
      Switching boundary strips does not
      correct this.", GrayLevel[0.3]], "Text"]
  ]
}
Module[{angles = anglesDeg, switchers, combinations, results},
  (*Define switch functions*)
  switchers = <|"Right" → SwitchingRightBoundaryStrip,

```

```

"Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
"Upper" → SwitchingUpperBoundaryStrip|>;
(*Helper function:compute and print difference only*)
formatOrthodiagonalCheck[quad_List] := Module[{vals},
  vals = Table[Module[{a, b, c, d, lhs, rhs, diff}, {a, b, c, d} = quad[[i]];
    lhs = FullSimplify[Cos[a Degree] Cos[c Degree]];
    rhs = FullSimplify[Cos[b Degree] Cos[d Degree]];
    diff = Chop[lhs - rhs];
    Style[Row["cos(α" <> ToString[i] <> ") · cos(γ" <> ToString[i] <> ") - ",
      "cos(β" <> ToString[i] <> ") · cos(δ" <> ToString[i] <> ") = ", NumberForm[
        diff, {5, 3}]], If[diff == 0, Red, Black]], {i, Length[quad]}];
  Column[vals]];
(*Orthodiagonal check for anglesDeg before any switching*)
Print[TextCell[Style["\nInitial anglesDeg (no switches):", Bold]]];
Print[MatrixForm[angles]];
Print[TextCell[Style[
  "Orthodiagonal check: cos(αi) · cos(γi) - cos(βi) · cos(δi) for i = 1..4",
  Italic]]];
Print[formatOrthodiagonalCheck[angles]];
(*Generate all combinations of switches (from size 1 to 4)*)
combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
(*Evaluate condition after each combination of switches*)results = Table[
  Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];
    passQ = And@@ (checkConditionN0Degrees /@switched);
    Print[Style["\nSwitch combination: ", Bold], name];
    Print[Style["Switched anglesDeg:", Italic]];
    Print[MatrixForm[switched]];
    Print[TextCell[Style[
      "Orthodiagonal check: cos(αi) · cos(γi) - cos(βi) · cos(δi) for i = 1..4",
      Italic]]];
    Print[formatOrthodiagonalCheck[switched]];
    {name, passQ}], {combo, combinations}];

(*=====
====*)
(*=====
NOT ISOGONAL=====*)
(*=====
====*)
Column[
{TextCell[Style["===== NOT ISOGONAL =====", Orange,
  Bold, 15], "Text"],
TextCell[
Style["Condition (N.0) holds AND for all i = 1..4: αi ≠ βi, αi ≠ γi, αi
  ≠ δi, βi ≠ γi, βi ≠ δi, γi ≠ δi, αi+βi ≠ π ≠ γi+δi, αi+γi

```

```

 $\neq \pi \neq \beta_i + \delta_i, \alpha_i + \delta_i \neq \pi \neq \beta_i + \gamma_i \Rightarrow$  NOT isogonal. Switching
boundary strips do not change this.", GrayLevel[0.3]], "Text"]]]

Module[{angles = anglesDeg, switchers, combinations, results},
  (*Define switch functions*)
  switchers = <|"Right" → SwitchingRightBoundaryStrip,
    "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
    "Upper" → SwitchingUpperBoundaryStrip|>;
  (*Helper function:extended angle relations*)
  formatAngleRelations[quad_List] :=
    Module[{vals}, vals = Table[Module[{a, b, c, d, exprs}, {a, b, c, d} = quad[[i];
      exprs = {Row[{" $\alpha$ " <> ToString[i] <> " -  $\beta$ " <> ToString[i] <> " = ",
        NumberForm[N[a - b], {5, 3}]}], Row[{" $\alpha$ " <> ToString[i] <>
        " -  $\gamma$ " <> ToString[i] <> " = ", NumberForm[N[a - c], {5, 3}]}],
      Row[{" $\alpha$ " <> ToString[i] <> " -  $\delta$ " <> ToString[i] <> " = ",
        NumberForm[N[a - d], {5, 3}]}], Row[{" $\beta$ " <> ToString[i] <>
        " -  $\gamma$ " <> ToString[i] <> " = ", NumberForm[N[b - c], {5, 3}]}],
      Row[{" $\beta$ " <> ToString[i] <> " -  $\delta$ " <> ToString[i] <> " = ",
        NumberForm[N[b - d], {5, 3}]}], Row[{" $\gamma$ " <> ToString[i] <>
        " -  $\delta$ " <> ToString[i] <> " = ", NumberForm[N[c - d], {5, 3}]}],
      Row[{" $\alpha$ " <> ToString[i] <> " +  $\beta$ " <> ToString[i] <> " - 180 = ",
        NumberForm[N[a + b - 180], {5, 3}]}],
      Row[{" $\gamma$ " <> ToString[i] <> " +  $\delta$ " <> ToString[i] <> " - 180 = ",
        NumberForm[N[c + d - 180], {5, 3}]}],
      Row[{" $\alpha$ " <> ToString[i] <> " +  $\gamma$ " <> ToString[i] <> " - 180 = ",
        NumberForm[N[a + c - 180], {5, 3}]}],
      Row[{" $\beta$ " <> ToString[i] <> " +  $\delta$ " <> ToString[i] <> " - 180 = ",
        NumberForm[N[b + d - 180], {5, 3}]}],
      Row[{" $\alpha$ " <> ToString[i] <> " +  $\delta$ " <> ToString[i] <> " - 180 = ",
        NumberForm[N[a + d - 180], {5, 3}]}],
      Row[{" $\beta$ " <> ToString[i] <> " +  $\gamma$ " <> ToString[i] <> " - 180 = ",
        NumberForm[N[b + c - 180], {5, 3}]}],
      Row[{" $\alpha$ " <> ToString[i] <> " +  $\beta$ " <> ToString[i] <> " -  $\gamma$ " <> ToString[i] <>
        " -  $\delta$ " <> ToString[i] <> " = ", NumberForm[N[a + b - c - d], {5, 3}]}],
      Row[{" $\alpha$ " <> ToString[i] <> " +  $\gamma$ " <> ToString[i] <> " -  $\beta$ " <> ToString[i] <>
        " -  $\delta$ " <> ToString[i] <> " = ", NumberForm[N[a + c - b - d], {5, 3}]}],
      Row[{" $\alpha$ " <> ToString[i] <> " +  $\delta$ " <> ToString[i] <> " -  $\beta$ " <> ToString[i] <>
        " -  $\gamma$ " <> ToString[i] <> " = ", NumberForm[N[a + d - b - c], {5, 3}]}];
      Column[Prepend[exprs, Style["Vertex " <> ToString[i], Bold]]],
      {i, Length[quad]}];
    Column[vals, Spacings → 1.5]];
  (*Angle relation check for anglesDeg before any switching*)
  Print[TextCell[Style["\nInitial anglesDeg (no switches):", Bold]]];
  Print[MatrixForm[angles]];
  Print[TextCell[Style["Angle relation checks for i = 1..4:", Italic]]];
  Print[formatAngleRelations[angles]];
  (*Generate all combinations of switches (from size 1 to 4)*)

```



```

combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
(*Evaluate condition after each combination of switches*)results = Table[
  Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];
    passQ = And@@ (checkConditionN0Degrees /@switched);
    Print[Style["\nSwitch combination: ", Bold], name];
    Print[Style["Switched anglesDeg:", Italic]];
    Print[MatrixForm[switched]];
    Print[TextCell[Style["Angle relation checks for i = 1..4:", Italic]]];
    Print[formatAngleRelations[switched]];
    {name, passQ}], {combo, combinations}];]

(*****
====*)
(*=====
      NOT CONJUGATE-MODULAR=====*)
(*****
====*)
Column[
  {TextCell[Style["===== NOT CONJUGATE-MODULAR =====",
    Brown, Bold, 16], "Text"],
    TextCell[Style[
      "M1 = M2 = M3 = M4 = M and M ≠ 2 ⇒ NOT conjugate-modular. Boundary-strip
        switches preserve this.", GrayLevel[0.3]], "Text"]
  ]}
Ms = FullSimplify[Times@@@results];
allEqualQ = Simplify[Equal@@Ms];

Module[{angles = anglesDeg, switchers, combinations, results,
  computeConjugateModularInfo}, (*Define switch functions*)
switchers = <|"Right" → SwitchingRightBoundaryStrip,
  "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
  "Upper" → SwitchingUpperBoundaryStrip|>;
(*Computes Mi and pi and prints them,
with classification*)computeConjugateModularInfo[quad_List] :=
Module[{abcdList, Ms, summary}, abcdList = computeABCD /@quad;
  Ms = FullSimplify[Times@@@abcdList];
  summary = If[Simplify[Equal@@Ms] && Ms[[1]] != 2,
    Style["M1 = M2 = M3 = M4 = M and M ≠ 2", Bold],
    Style["M1 = M2 = M3 = M4 = M and M = 2", Red, Bold]];
  Column[{Style["Mi values:", Bold], Row[{"M1 = ", Ms[[1]], ", M2 = ",
    Ms[[2]], ", M3 = ", Ms[[3]], ", M4 = ", Ms[[4]]}], summary}]]];
(*Original anglesDeg check*)
Print[
  TextCell[Style["\nInitial configuration (no switches applied):", Bold]]];
Print[MatrixForm[angles]];

```

```

Print[computeConjugateModularInfo[angles]];
(*Generate all switch combinations (from size 1 to 4)*)
combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
(*Evaluate each switched configuration*)results = Table[
  Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];
    passQ = And@@ (checkConditionN0Degrees /@ switched);
    Print[Style["\nSwitch combination: ", Bold], name];
    Print[Style["Switched anglesDeg:", Italic]];
    Print[MatrixForm[switched]];
    Print[computeConjugateModularInfo[switched]];
    {name, passQ}], {combo, combinations}];]

```

```

(*=====
====*)
(*=====
NOT CHIMERA=====*)
(*=====
====*)
Column[
{TextCell[Style["===== NOT CHIMERA =====", Blue,
  Bold, 16], "Text"],
TextCell[
Style["Fails conic, orthodiagonal & isogonal tests for all i=1, ...,
4  $\Rightarrow$  NOT chimera. Boundary-strip switches preserve these
failures as demonstrated in the NOT CONIC, NOT ORTHODIAGONAL,
and NOT ISOGONAL sections.", GrayLevel[0.3]], "Text"]
}]

```

Out[1495]=

```

===== CONDITION (N.0) =====
✓ All vertices satisfy (N.0).

```

Vertex	Combinations (mod 360)	Status
Vertex 1	{190.399, 70.3985, 26.5003, 146.5, 25.917, 342.019, 265.917, 222.019}	✓ Pass
Vertex 2	{280.89, 50.8897, 13.1847, 243.185, 19.1478, 341.443, 149.148, 111.443}	✓ Pass
Vertex 3	{34.4692, 234.469, 303.73, 103.73, 325.58, 34.8415, 165.58, 234.841}	✓ Pass
Vertex 4	{61.507, 211.507, 273.452, 123.452, 322.45, 24.3953, 112.45, 174.395}	✓ Pass

Out[1498]=

```

===== CONDITION (N.3) =====
✓ M1 = M2 = M3 = M4 = 1.22593

```

Out[1504]=

```
===== CONDITION (N.4) =====
✓ r1 = r2 = 0.71078; ✓ r3 = r4 = 0.887609
✓ s1 = s4 = 0.58646; ✓ s2 = s3 = 0.800228
```

Out[1514]=

```
===== CONDITION (N.5) =====
```

△ Approximate validation using ε -tolerance. For rigorous proof, see the referenced paper.

✓ Valid Combination Found (M > 1):

```
e1 = -1, e2 = -1, e3 = 1
t1 = 2.K + 0.801037iK'
t2 = 2.K + 0.837691iK'
t3 = 0.K + 0.579573iK'
t4 = 0.K + 0.616227iK'
t1 + e1*t2 + e2*t3 + e3*t4 = 0.K + 2.33147 × 10-15iK'
```

Out[1517]=

```
===== OTHER PARAMETERS =====
```

```
u = -0.22593
σ1 = 1.66154, σ2 = 2.45122, σ3 = 3.44239, σ4 = 3.67834
σ1 ≈ 95.1993°, σ2 ≈ 140.445°, σ3 ≈ 197.235°, σ4 ≈ 210.754°
cosσ1 = -0.0906196, cosσ2 = -0.771012, cosσ3 = -0.9551, cosσ4 = -0.859375
f1 = 0.184669, f2 = 0.127824, f3 = 0.578993, f4 = 0.516796
z1 = -1.2265, z2 = -1.14656, z3 = -2.37526, z4 = -2.06952
x1 = -3.45758, x2 = -3.45758, x3 = -8.89754, x4 = -8.89754
y1 = -2.41814, y2 = -5.00571, y3 = -5.00571, y4 = -2.41814
p1 = 0. + 0.537792 i, p2 = 0. + 0.537792 i
, p3 = 0. + 0.335247 i, p4 = 0. + 0.335247 i
q1 = 0. + 0.643071 i, q2 = 0. + 0.446958 i
, q3 = 0. + 0.446958 i, q4 = 0. + 0.643071 i
p1·q1 = -0.345838 + 0. i, p2·q2 = -0.24037 + 0. i
, p3·q3 = -0.149841 + 0. i, p4·q4 = -0.215588 + 0. i
```

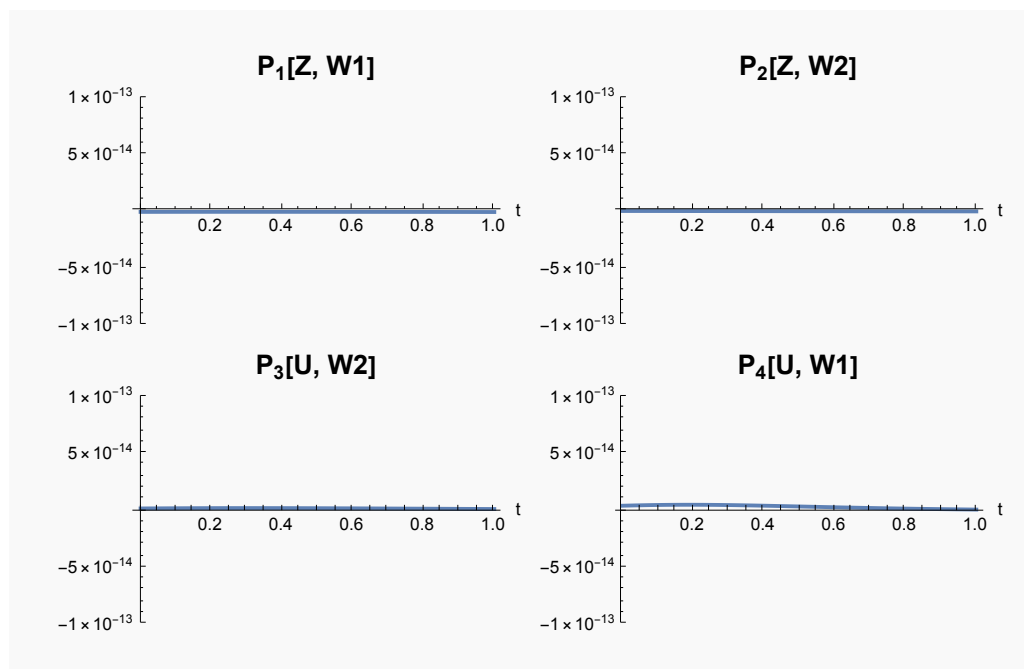
Out[1523]=

```
===== FLEXIBILITY (FLEXION 1) =====
```

$$\begin{aligned}
P_1[Z, W1] &= \frac{1}{(0.354727 + 1. t^2)^2} \left(-1.12836 \times 10^{-16} + \right. \\
&\quad \left. t \left(7.63278 \times 10^{-17} \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} + t \left(-8.55179 \times 10^{-16} - 1.52032 \times 10^{-15} t^2 + \right. \right. \right. \\
&\quad \left. \left. 1.9004 \times 10^{-16} t^4 + 3.88578 \times 10^{-16} t \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} \right) \right) \Big) \\
P_2[Z, W2] &= \frac{1}{(0.331607 + 1. t^2)^2} \\
&\quad \left(-1.0532 \times 10^{-17} + t \left(-2.08167 \times 10^{-17} \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} + t \left(-1.89577 \times 10^{-16} - \right. \right. \right. \\
&\quad \left. \left. 5.6873 \times 10^{-16} t^2 - 1.0532 \times 10^{-16} t^4 + 7.63278 \times 10^{-17} t \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} \right) \right) \Big) \\
P_3[U, W2] &= \frac{1}{(0.225014 + 1.01016 t^2 + 1. t^4)^2} \\
&\quad \left(8.02692 \times 10^{-17} + t \left(9.7296 \times 10^{-17} \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} + t \left(6.42255 \times 10^{-16} + \right. \right. \right. \\
&\quad \left. \left. t \left(4.21616 \times 10^{-16} \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} + t \left(1.82917 \times 10^{-15} + 1.66701 \times 10^{-15} \right. \right. \right. \\
&\quad \left. \left. t^2 + 2.20538 \times 10^{-16} t^4 + 4.54048 \times 10^{-16} t \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} \right) \right) \right) \Big) \\
P_4[U, W1] &= \frac{1}{(0.240702 + 1.03328 t^2 + 1. t^4)^2} \\
&\quad \left(2.31221 \times 10^{-16} + t \left(4.95008 \times 10^{-16} \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} + t \left(1.00956 \times 10^{-15} + \right. \right. \right. \\
&\quad \left. \left. t \left(1.30265 \times 10^{-15} \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} + t \left(9.90016 \times 10^{-16} - 1.0812 \times 10^{-15} \right. \right. \right. \\
&\quad \left. \left. t^2 - 2.60531 \times 10^{-16} t^4 + 4.81982 \times 10^{-16} t \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} \right) \right) \right) \Big)
\end{aligned}$$

Out[1532]=

===== FLEXIBILITY (FLEXION 1) =====
 Polynomials $P_i(t)$ built from Bricard's equations for flexion 1.



Out[1538]=

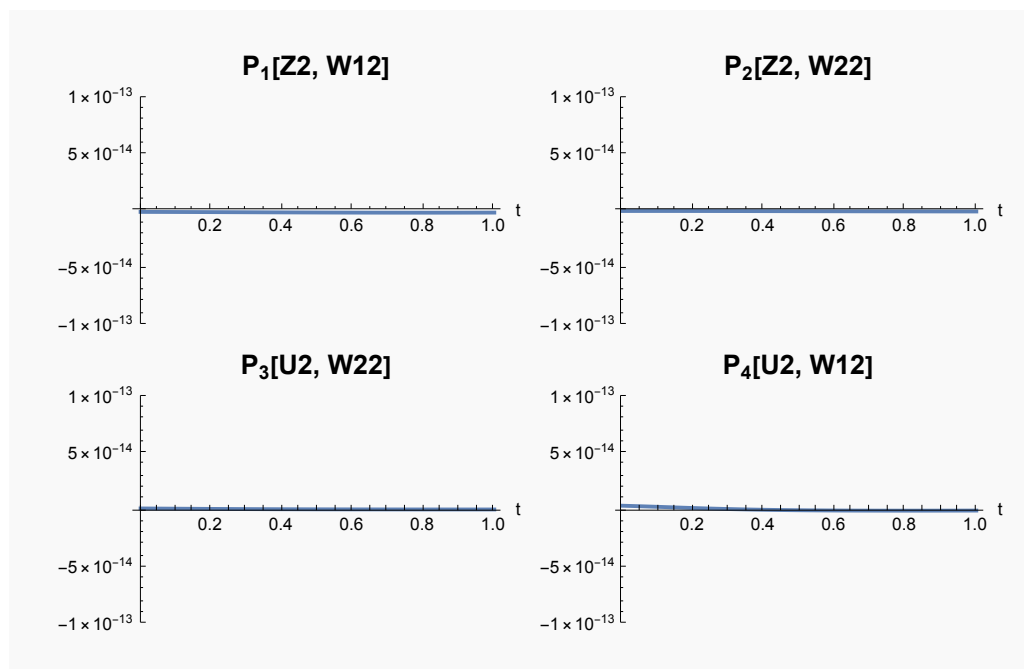
===== FLEXIBILITY (FLEXION 2) =====

$$\begin{aligned}
P_1[Z, W1] &= \frac{1}{(0.354727 + 1. t^2)^2} \left(-1.12836 \times 10^{-16} + \right. \\
&\quad \left. t \left(-7.63278 \times 10^{-17} \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} + t \left(-8.55179 \times 10^{-16} - 1.52032 \times 10^{-15} t^2 + \right. \right. \right. \\
&\quad \left. \left. 1.9004 \times 10^{-16} t^4 - 3.88578 \times 10^{-16} t \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} \right) \right) \\
P_2[Z, W2] &= \frac{1}{(0.331607 + 1. t^2)^2} \\
&\quad \left(-1.0532 \times 10^{-17} + t \left(2.08167 \times 10^{-17} \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} + t \left(-1.89577 \times 10^{-16} - \right. \right. \right. \\
&\quad \left. \left. 5.6873 \times 10^{-16} t^2 - 1.0532 \times 10^{-16} t^4 - 7.63278 \times 10^{-17} t \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} \right) \right) \\
P_3[U, W2] &= \frac{1}{(0.225014 + 1.01016 t^2 + 1. t^4)^2} \\
&\quad \left(8.02692 \times 10^{-17} + t \left(-9.7296 \times 10^{-17} \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} + t \left(6.42255 \times 10^{-16} + \right. \right. \right. \\
&\quad \left. \left. t \left(-4.21616 \times 10^{-16} \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} + t \left(1.82917 \times 10^{-15} + 1.66701 \times 10^{-15} \right. \right. \right. \\
&\quad \left. \left. \left. t^2 + 2.20538 \times 10^{-16} t^4 - 4.54048 \times 10^{-16} t \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} \right) \right) \right) \\
P_4[U, W1] &= \frac{1}{(0.240702 + 1.03328 t^2 + 1. t^4)^2} \\
&\quad \left(2.31221 \times 10^{-16} + t \left(-4.95008 \times 10^{-16} \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} + t \left(1.00956 \times 10^{-15} + \right. \right. \right. \\
&\quad \left. \left. t \left(-1.30265 \times 10^{-15} \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} + t \left(9.90016 \times 10^{-16} - 1.0812 \times 10^{-15} \right. \right. \right. \\
&\quad \left. \left. \left. t^2 - 2.60531 \times 10^{-16} t^4 - 4.81982 \times 10^{-16} t \sqrt{1 + 2.67641 t^2 - 2.70096 t^4} \right) \right) \right)
\end{aligned}$$

Out[1547]=

===== FLEXIBILITY (FLEXION 2) =====

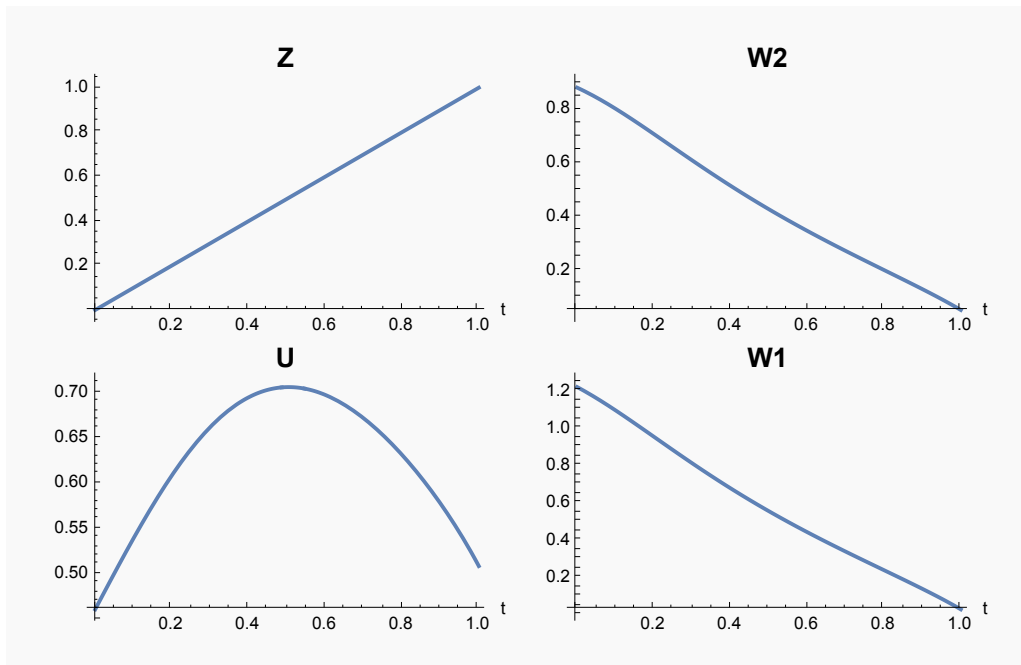
Polynomials $P_i(t)$ built from Bricard's equations for flexion 2.



Out[1552]=

===== NOT TRIVIAL (FLEXION 1) =====

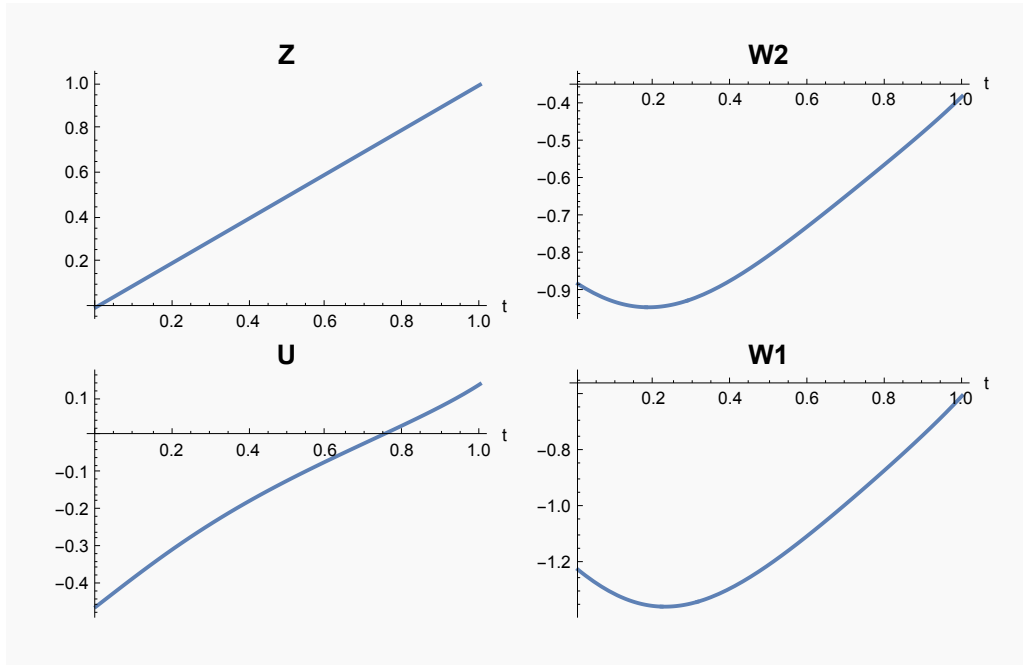
This configuration does not belong to the trivial class – even after switching the boundary strips – since none of the functions Z , $W2$, U , or $W1$ is constant.



Out[1557]=

===== NOT TRIVIAL (FLEXION 2) =====

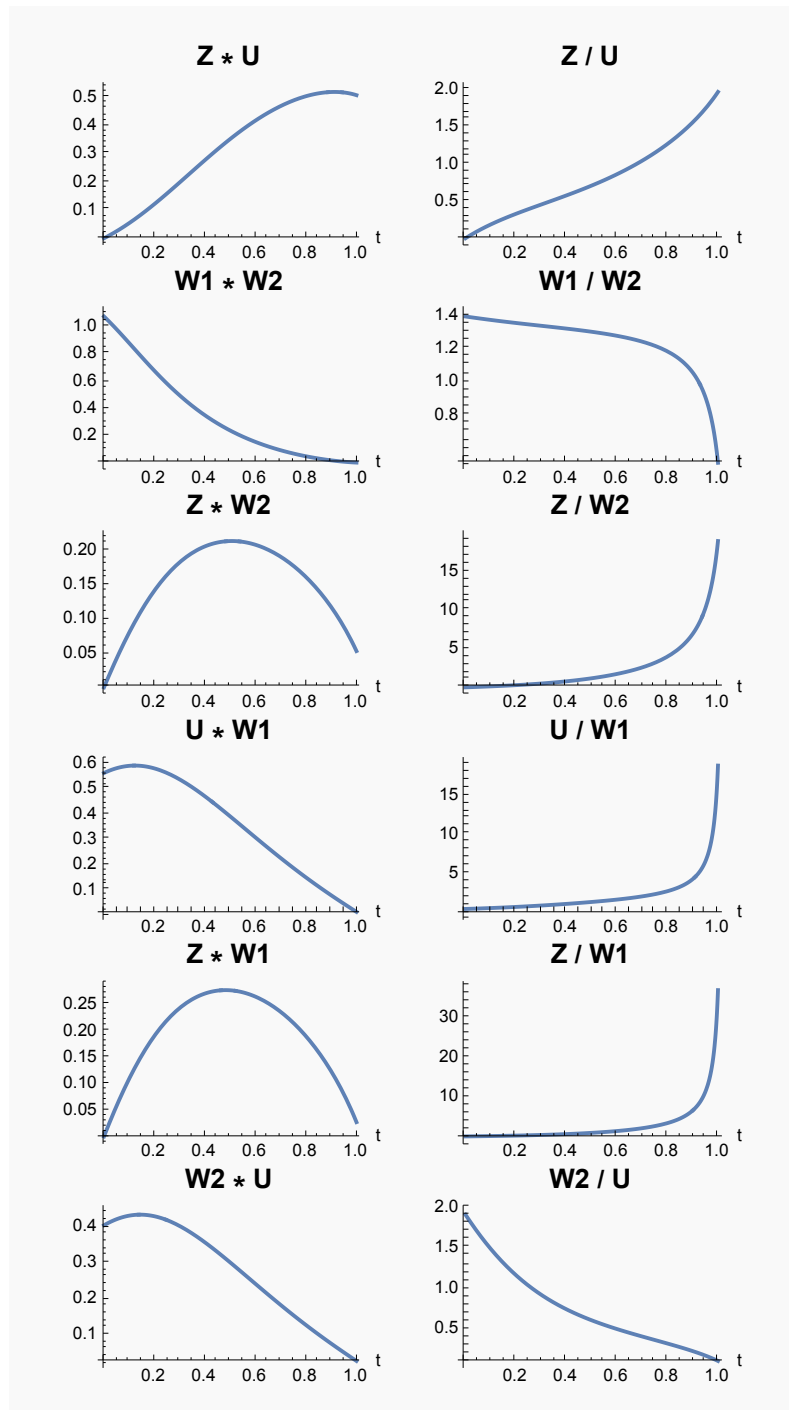
This configuration does not belong to the trivial class – even after switching the boundary strips – since none of the functions Z , $W2$, U , or $W1$ is constant.



Out[1562]=

===== NOT LINEAR COMPOUND &
NOT LINEAR CONJUGATE (FLEXION 1) =====

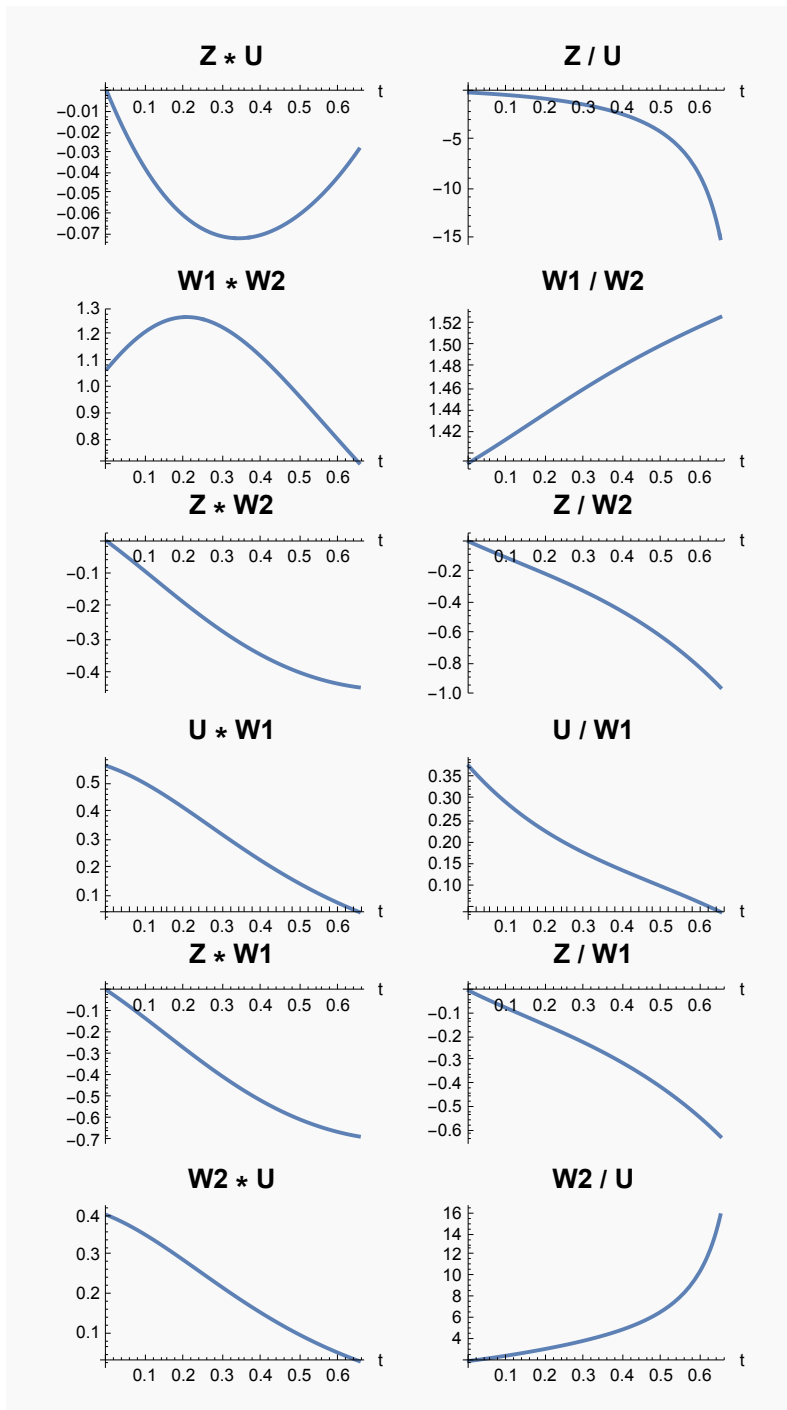
This configuration does not belong to the Linear compound class nor to the linear conjugate class – even after switching the boundary strips – since none of the functions ZU , Z/U , $W1W2$, $W1/W2$, $ZW2$, $Z/W2$, $UW1$, and $U/W1$ is constant. In addition, none of $ZW1$, $Z/W1$, $W2U$, $W2/U$ is constant as well.



Out[1567]=

===== NOT LINEAR COMPOUND &
 NOT LINEAR CONJUGATE (FLEXION 2) =====

This configuration does not belong to the Linear compound class nor to the linear conjugate class – even after switching the boundary strips – since none of the functions ZU , Z/U , $W1W2$, $W1/W2$, $ZW2$, $Z/W2$, $UW1$, and $U/W1$ is constant. In addition, none of $ZW1$, $Z/W1$, $W2U$, $W2/U$ is constant as well.



Out[1576]=

===== **NOT CONIC** =====

Condition (N.0) is satisfied \Rightarrow this configuration
is NOT equimodular-conic. Applying any boundary-strip
switch still preserves (N.0), so no conic form emerges.

Out[1577]=

CONDITION (N.0) AFTER SWITCHING BOUNDARY STRIPS

Right: Condition (N.0) is still satisfied.
Left: Condition (N.0) is still satisfied.
Lower: Condition (N.0) is still satisfied.
Upper: Condition (N.0) is still satisfied.
Right + Left: Condition (N.0) is still satisfied.
Right + Lower: Condition (N.0) is still satisfied.
Right + Upper: Condition (N.0) is still satisfied.
Left + Lower: Condition (N.0) is still satisfied.
Left + Upper: Condition (N.0) is still satisfied.
Lower + Upper: Condition (N.0) is still satisfied.
Right + Left + Lower: Condition (N.0) is still satisfied.
Right + Left + Upper: Condition (N.0) is still satisfied.
Right + Lower + Upper: Condition (N.0) is still satisfied.
Left + Lower + Upper: Condition (N.0) is still satisfied.
Right + Left + Lower + Upper: Condition (N.0) is still satisfied.

Out[1578]=

===== **NOT ORTHODIAGONAL** =====

$\cos(\alpha_i) \cdot \cos(\gamma_i) \neq \cos(\beta_i) \cdot \cos(\delta_i)$ for each $i = 1..4 \Rightarrow$ NOT
orthodiagonal. Switching boundary strips does not correct this.

Initial anglesDeg (no switches):

$$\begin{pmatrix} 26.2086 & 82.2408 & 21.9491 & 60 \\ 16.1662 & 130.871 & 18.8525 & 115 \\ 134.655 & 34.4444 & 145.369 & 80 \\ 117.951 & 49.5283 & 149.028 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = 0.765$
 $\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = 0.632$
 $\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = 0.435$
 $\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = 0.570$

Switch combination: Right

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 97.7592 & 158.051 & 60 \\ 16.1662 & 130.871 & 18.8525 & 115 \\ 134.655 & 34.4444 & 145.369 & 80 \\ 117.951 & 130.472 & 30.9725 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = -0.765$
 $\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = 0.632$
 $\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = 0.435$
 $\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = -0.570$

Switch combination: Left*Switched anglesDeg:*

$$\begin{pmatrix} 26.2086 & 82.2408 & 21.9491 & 60 \\ 16.1662 & 49.129 & 161.148 & 115 \\ 134.655 & 145.556 & 34.6305 & 80 \\ 117.951 & 49.5283 & 149.028 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= 0.765 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -0.632 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -0.435 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= 0.570 \end{aligned}$$

Switch combination: Lower*Switched anglesDeg:*

$$\begin{pmatrix} 153.791 & 97.7592 & 21.9491 & 60 \\ 163.834 & 49.129 & 18.8525 & 115 \\ 134.655 & 34.4444 & 145.369 & 80 \\ 117.951 & 49.5283 & 149.028 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -0.765 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -0.632 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= 0.435 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= 0.570 \end{aligned}$$

Switch combination: Upper*Switched anglesDeg:*

$$\begin{pmatrix} 26.2086 & 82.2408 & 21.9491 & 60 \\ 16.1662 & 130.871 & 18.8525 & 115 \\ 45.3447 & 145.556 & 145.369 & 80 \\ 62.0488 & 130.472 & 149.028 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= 0.765 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= 0.632 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -0.435 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -0.570 \end{aligned}$$

Switch combination: Right + Left*Switched anglesDeg:*

$$\begin{pmatrix} 26.2086 & 97.7592 & 158.051 & 60 \\ 16.1662 & 49.129 & 161.148 & 115 \\ 134.655 & 145.556 & 34.6305 & 80 \\ 117.951 & 130.472 & 30.9725 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -0.765 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -0.632 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -0.435 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -0.570 \end{aligned}$$

Switch combination: Right + Lower

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 82.2408 & 158.051 & 60 \\ 163.834 & 49.129 & 18.8525 & 115 \\ 134.655 & 34.4444 & 145.369 & 80 \\ 117.951 & 130.472 & 30.9725 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= 0.765 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -0.632 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= 0.435 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -0.570 \end{aligned}$$

Switch combination: Right + Upper

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 97.7592 & 158.051 & 60 \\ 16.1662 & 130.871 & 18.8525 & 115 \\ 45.3447 & 145.556 & 145.369 & 80 \\ 62.0488 & 49.5283 & 30.9725 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -0.765 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= 0.632 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -0.435 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= 0.570 \end{aligned}$$

Switch combination: Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 97.7592 & 21.9491 & 60 \\ 163.834 & 130.871 & 161.148 & 115 \\ 134.655 & 145.556 & 34.6305 & 80 \\ 117.951 & 49.5283 & 149.028 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= -0.765 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= 0.632 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= -0.435 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= 0.570 \end{aligned}$$

Switch combination: Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 82.2408 & 21.9491 & 60 \\ 16.1662 & 49.129 & 161.148 & 115 \\ 45.3447 & 34.4444 & 34.6305 & 80 \\ 62.0488 & 130.472 & 149.028 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\begin{aligned} \cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) &= 0.765 \\ \cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) &= -0.632 \\ \cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) &= 0.435 \\ \cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) &= -0.570 \end{aligned}$$

Switch combination: Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 97.7592 & 21.9491 & 60 \\ 163.834 & 49.129 & 18.8525 & 115 \\ 45.3447 & 145.556 & 145.369 & 80 \\ 62.0488 & 130.472 & 149.028 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = -0.765$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = -0.632$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = -0.435$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = -0.570$$

Switch combination: Right + Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 82.2408 & 158.051 & 60 \\ 163.834 & 130.871 & 161.148 & 115 \\ 134.655 & 145.556 & 34.6305 & 80 \\ 117.951 & 130.472 & 30.9725 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = 0.765$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = 0.632$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = -0.435$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = -0.570$$

Switch combination: Right + Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 97.7592 & 158.051 & 60 \\ 16.1662 & 49.129 & 161.148 & 115 \\ 45.3447 & 34.4444 & 34.6305 & 80 \\ 62.0488 & 49.5283 & 30.9725 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = -0.765$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = -0.632$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = 0.435$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = 0.570$$

Switch combination: Right + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 82.2408 & 158.051 & 60 \\ 163.834 & 49.129 & 18.8525 & 115 \\ 45.3447 & 145.556 & 145.369 & 80 \\ 62.0488 & 49.5283 & 30.9725 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = 0.765$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = -0.632$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = -0.435$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = 0.570$$

Switch combination: Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 97.7592 & 21.9491 & 60 \\ 163.834 & 130.871 & 161.148 & 115 \\ 45.3447 & 34.4444 & 34.6305 & 80 \\ 62.0488 & 130.472 & 149.028 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = -0.765$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = 0.632$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = 0.435$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = -0.570$$

Switch combination: Right + Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 82.2408 & 158.051 & 60 \\ 163.834 & 130.871 & 161.148 & 115 \\ 45.3447 & 34.4444 & 34.6305 & 80 \\ 62.0488 & 49.5283 & 30.9725 & 105 \end{pmatrix}$$

Orthodiagonal check: $\cos(\alpha_i) \cdot \cos(\gamma_i) - \cos(\beta_i) \cdot \cos(\delta_i)$ for $i = 1..4$

$$\cos(\alpha_1) \cdot \cos(\gamma_1) - \cos(\beta_1) \cdot \cos(\delta_1) = 0.765$$

$$\cos(\alpha_2) \cdot \cos(\gamma_2) - \cos(\beta_2) \cdot \cos(\delta_2) = 0.632$$

$$\cos(\alpha_3) \cdot \cos(\gamma_3) - \cos(\beta_3) \cdot \cos(\delta_3) = 0.435$$

$$\cos(\alpha_4) \cdot \cos(\gamma_4) - \cos(\beta_4) \cdot \cos(\delta_4) = 0.570$$

Out[1580]=

===== NOT ISOGONAL =====

Condition (N.0) holds AND for all $i = 1..4$: $\alpha_i \neq \beta_i$,

$$\alpha_i \neq \gamma_i, \alpha_i \neq \delta_i, \beta_i \neq \gamma_i, \beta_i \neq \delta_i, \gamma_i \neq \delta_i, \alpha_i + \beta_i \neq$$

$$\pi \neq \gamma_i + \delta_i, \alpha_i + \gamma_i \neq \pi \neq \beta_i + \delta_i, \alpha_i + \delta_i \neq \pi \neq \beta_i + \gamma_i \Rightarrow \text{NOT}$$

isogonal. Switching boundary strips do not change this.

Initial anglesDeg (no switches):

$$\begin{pmatrix} 26.2086 & 82.2408 & 21.9491 & 60 \\ 16.1662 & 130.871 & 18.8525 & 115 \\ 134.655 & 34.4444 & 145.369 & 80 \\ 117.951 & 49.5283 & 149.028 & 105 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\alpha_1 - \beta_1 = -56.032$$

$$\alpha_1 - \gamma_1 = 4.260$$

$$\alpha_1 - \delta_1 = -33.791$$

$$\beta_1 - \gamma_1 = 60.292$$

$$\beta_1 - \delta_1 = 22.241$$

$$\gamma_1 - \delta_1 = -38.051$$

$$\alpha_1 + \beta_1 - 180 = -71.551$$

$$\gamma_1 + \delta_1 - 180 = -98.051$$

$$\alpha_1 + \gamma_1 - 180 = -131.840$$

$$\beta_1 + \delta_1 - 180 = -37.759$$

$$\alpha_1 + \delta_1 - 180 = -93.791$$

$$\beta_1 + \gamma_1 - 180 = -75.810$$

$$\alpha_1 + \beta_1 - \gamma_1 - \delta_1 = 26.500$$

$$\alpha_1 + \gamma_1 - \beta_1 - \delta_1 = -94.083$$

$$\alpha_1 + \delta_1 - \beta_1 - \gamma_1 = -17.981$$

Vertex 2

$$\alpha_2 - \beta_2 = -114.700$$

$$\begin{aligned}
\alpha_2 - \beta_2 &= -22.180 \\
\alpha_2 - \gamma_2 &= -2.686 \\
\alpha_2 - \delta_2 &= -98.834 \\
\beta_2 - \gamma_2 &= 112.020 \\
\beta_2 - \delta_2 &= 15.871 \\
\gamma_2 - \delta_2 &= -96.148 \\
\alpha_2 + \beta_2 - 180 &= -32.963 \\
\gamma_2 + \delta_2 - 180 &= -46.148 \\
\alpha_2 + \gamma_2 - 180 &= -144.980 \\
\beta_2 + \delta_2 - 180 &= 65.871 \\
\alpha_2 + \delta_2 - 180 &= -48.834 \\
\beta_2 + \gamma_2 - 180 &= -30.277 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 13.185 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -210.850 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -18.557
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= 100.210 \\
\alpha_3 - \gamma_3 &= -10.714 \\
\alpha_3 - \delta_3 &= 54.655 \\
\beta_3 - \gamma_3 &= -110.930 \\
\beta_3 - \delta_3 &= -45.556 \\
\gamma_3 - \delta_3 &= 65.369 \\
\alpha_3 + \beta_3 - 180 &= -10.900 \\
\gamma_3 + \delta_3 - 180 &= 45.369 \\
\alpha_3 + \gamma_3 - 180 &= 100.020 \\
\beta_3 + \delta_3 - 180 &= -65.556 \\
\alpha_3 + \delta_3 - 180 &= 34.655 \\
\beta_3 + \gamma_3 - 180 &= -0.186 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -56.270 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 165.580 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 34.841
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= 68.423 \\
\alpha_4 - \gamma_4 &= -31.076 \\
\alpha_4 - \delta_4 &= 12.951 \\
\beta_4 - \gamma_4 &= -99.499 \\
\beta_4 - \delta_4 &= -55.472 \\
\gamma_4 - \delta_4 &= 44.028 \\
\alpha_4 + \beta_4 - 180 &= -12.521 \\
\gamma_4 + \delta_4 - 180 &= 74.028 \\
\alpha_4 + \gamma_4 - 180 &= 86.979 \\
\beta_4 + \delta_4 - 180 &= -25.472 \\
\alpha_4 + \delta_4 - 180 &= 42.951 \\
\beta_4 + \gamma_4 - 180 &= 18.556 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -86.548 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 112.450 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 24.395
\end{aligned}$$

Switch combination: Right*Switched anglesDeg:*

$$\begin{pmatrix} 26.2086 & 97.7592 & 158.051 & 60 \\ 16.1662 & 130.871 & 18.8525 & 115 \\ 134.655 & 34.4444 & 145.369 & 80 \\ 117.951 & 130.472 & 30.9725 & 105 \end{pmatrix}$$

*Angle relation checks for i = 1..4:***Vertex 1**

$$\begin{aligned}
\alpha_1 - \beta_1 &= -71.551 \\
\alpha_1 - \gamma_1 &= 121.840
\end{aligned}$$

```

 $\alpha_1 - \gamma_1 = -131.840$ 
 $\alpha_1 - \delta_1 = -33.791$ 
 $\beta_1 - \gamma_1 = -60.292$ 
 $\beta_1 - \delta_1 = 37.759$ 
 $\gamma_1 - \delta_1 = 98.051$ 
 $\alpha_1 + \beta_1 - 180 = -56.032$ 
 $\gamma_1 + \delta_1 - 180 = 38.051$ 
 $\alpha_1 + \gamma_1 - 180 = 4.260$ 
 $\beta_1 + \delta_1 - 180 = -22.241$ 
 $\alpha_1 + \delta_1 - 180 = -93.791$ 
 $\beta_1 + \gamma_1 - 180 = 75.810$ 
 $\alpha_1 + \beta_1 - \gamma_1 - \delta_1 = -94.083$ 
 $\alpha_1 + \gamma_1 - \beta_1 - \delta_1 = 26.500$ 
 $\alpha_1 + \delta_1 - \beta_1 - \gamma_1 = -169.600$ 

```

Vertex 2

```

 $\alpha_2 - \beta_2 = -114.700$ 
 $\alpha_2 - \gamma_2 = -2.686$ 
 $\alpha_2 - \delta_2 = -98.834$ 
 $\beta_2 - \gamma_2 = 112.020$ 
 $\beta_2 - \delta_2 = 15.871$ 
 $\gamma_2 - \delta_2 = -96.148$ 
 $\alpha_2 + \beta_2 - 180 = -32.963$ 
 $\gamma_2 + \delta_2 - 180 = -46.148$ 
 $\alpha_2 + \gamma_2 - 180 = -144.980$ 
 $\beta_2 + \delta_2 - 180 = 65.871$ 
 $\alpha_2 + \delta_2 - 180 = -48.834$ 
 $\beta_2 + \gamma_2 - 180 = -30.277$ 
 $\alpha_2 + \beta_2 - \gamma_2 - \delta_2 = 13.185$ 
 $\alpha_2 + \gamma_2 - \beta_2 - \delta_2 = -210.850$ 
 $\alpha_2 + \delta_2 - \beta_2 - \gamma_2 = -18.557$ 

```

Vertex 3

```

 $\alpha_3 - \beta_3 = 100.210$ 
 $\alpha_3 - \gamma_3 = -10.714$ 
 $\alpha_3 - \delta_3 = 54.655$ 
 $\beta_3 - \gamma_3 = -110.930$ 
 $\beta_3 - \delta_3 = -45.556$ 
 $\gamma_3 - \delta_3 = 65.369$ 
 $\alpha_3 + \beta_3 - 180 = -10.900$ 
 $\gamma_3 + \delta_3 - 180 = 45.369$ 
 $\alpha_3 + \gamma_3 - 180 = 100.020$ 
 $\beta_3 + \delta_3 - 180 = -65.556$ 
 $\alpha_3 + \delta_3 - 180 = 34.655$ 
 $\beta_3 + \gamma_3 - 180 = -0.186$ 
 $\alpha_3 + \beta_3 - \gamma_3 - \delta_3 = -56.270$ 
 $\alpha_3 + \gamma_3 - \beta_3 - \delta_3 = 165.580$ 
 $\alpha_3 + \delta_3 - \beta_3 - \gamma_3 = 34.841$ 

```

Vertex 4

```

 $\alpha_4 - \beta_4 = -12.521$ 
 $\alpha_4 - \gamma_4 = 86.979$ 
 $\alpha_4 - \delta_4 = 12.951$ 
 $\beta_4 - \gamma_4 = 99.499$ 
 $\beta_4 - \delta_4 = 25.472$ 
 $\gamma_4 - \delta_4 = -74.028$ 
 $\alpha_4 + \beta_4 - 180 = 68.423$ 
 $\gamma_4 + \delta_4 - 180 = -44.028$ 
 $\alpha_4 + \gamma_4 - 180 = -31.076$ 
 $\beta_4 + \delta_4 - 180 = 55.472$ 
 $\alpha_4 + \delta_4 - 180 = 42.951$ 
 $\beta_4 + \gamma_4 - 180 = -18.556$ 

```


$$\begin{aligned}\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 112.450 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -86.548 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 61.507\end{aligned}$$

Switch combination: Left

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 82.2408 & 21.9491 & 60 \\ 16.1662 & 49.129 & 161.148 & 115 \\ 134.655 & 145.556 & 34.6305 & 80 \\ 117.951 & 49.5283 & 149.028 & 105 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}\alpha_1 - \beta_1 &= -56.032 \\ \alpha_1 - \gamma_1 &= 4.260 \\ \alpha_1 - \delta_1 &= -33.791 \\ \beta_1 - \gamma_1 &= 60.292 \\ \beta_1 - \delta_1 &= 22.241 \\ \gamma_1 - \delta_1 &= -38.051 \\ \alpha_1 + \beta_1 - 180 &= -71.551 \\ \gamma_1 + \delta_1 - 180 &= -98.051 \\ \alpha_1 + \gamma_1 - 180 &= -131.840 \\ \beta_1 + \delta_1 - 180 &= -37.759 \\ \alpha_1 + \delta_1 - 180 &= -93.791 \\ \beta_1 + \gamma_1 - 180 &= -75.810 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 26.500 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -94.083 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -17.981\end{aligned}$$

Vertex 2

$$\begin{aligned}\alpha_2 - \beta_2 &= -32.963 \\ \alpha_2 - \gamma_2 &= -144.980 \\ \alpha_2 - \delta_2 &= -98.834 \\ \beta_2 - \gamma_2 &= -112.020 \\ \beta_2 - \delta_2 &= -65.871 \\ \gamma_2 - \delta_2 &= 46.148 \\ \alpha_2 + \beta_2 - 180 &= -114.700 \\ \gamma_2 + \delta_2 - 180 &= 96.148 \\ \alpha_2 + \gamma_2 - 180 &= -2.686 \\ \beta_2 + \delta_2 - 180 &= -15.871 \\ \alpha_2 + \delta_2 - 180 &= -48.834 \\ \beta_2 + \gamma_2 - 180 &= 30.277 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -210.850 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 13.185 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -79.110\end{aligned}$$

Vertex 3

$$\begin{aligned}\alpha_3 - \beta_3 &= -10.900 \\ \alpha_3 - \gamma_3 &= 100.020 \\ \alpha_3 - \delta_3 &= 54.655 \\ \beta_3 - \gamma_3 &= 110.930 \\ \beta_3 - \delta_3 &= 65.556 \\ \gamma_3 - \delta_3 &= -45.369 \\ \alpha_3 + \beta_3 - 180 &= 100.210 \\ \gamma_3 + \delta_3 - 180 &= -65.369 \\ \alpha_3 + \gamma_3 - 180 &= -10.714 \\ \beta_3 + \delta_3 - 180 &= 45.556 \\ \alpha_3 + \delta_3 - 180 &= 34.655 \\ \beta_3 + \gamma_3 - 180 &= 0.186 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 165.580\end{aligned}$$

$$\alpha_3 + \gamma_3 - \beta_3 - \delta_3 = -56.270$$

$$\alpha_3 + \delta_3 - \beta_3 - \gamma_3 = 34.469$$

Vertex 4

$$\alpha_4 - \beta_4 = 68.423$$

$$\alpha_4 - \gamma_4 = -31.076$$

$$\alpha_4 - \delta_4 = 12.951$$

$$\beta_4 - \gamma_4 = -99.499$$

$$\beta_4 - \delta_4 = -55.472$$

$$\gamma_4 - \delta_4 = 44.028$$

$$\alpha_4 + \beta_4 - 180 = -12.521$$

$$\gamma_4 + \delta_4 - 180 = 74.028$$

$$\alpha_4 + \gamma_4 - 180 = 86.979$$

$$\beta_4 + \delta_4 - 180 = -25.472$$

$$\alpha_4 + \delta_4 - 180 = 42.951$$

$$\beta_4 + \gamma_4 - 180 = 18.556$$

$$\alpha_4 + \beta_4 - \gamma_4 - \delta_4 = -86.548$$

$$\alpha_4 + \gamma_4 - \beta_4 - \delta_4 = 112.450$$

$$\alpha_4 + \delta_4 - \beta_4 - \gamma_4 = 24.395$$

Switch combination: Lower

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 97.7592 & 21.9491 & 60 \\ 163.834 & 49.129 & 18.8525 & 115 \\ 134.655 & 34.4444 & 145.369 & 80 \\ 117.951 & 49.5283 & 149.028 & 105 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\alpha_1 - \beta_1 = 56.032$$

$$\alpha_1 - \gamma_1 = 131.840$$

$$\alpha_1 - \delta_1 = 93.791$$

$$\beta_1 - \gamma_1 = 75.810$$

$$\beta_1 - \delta_1 = 37.759$$

$$\gamma_1 - \delta_1 = -38.051$$

$$\alpha_1 + \beta_1 - 180 = 71.551$$

$$\gamma_1 + \delta_1 - 180 = -98.051$$

$$\alpha_1 + \gamma_1 - 180 = -4.260$$

$$\beta_1 + \delta_1 - 180 = -22.241$$

$$\alpha_1 + \delta_1 - 180 = 33.791$$

$$\beta_1 + \gamma_1 - 180 = -60.292$$

$$\alpha_1 + \beta_1 - \gamma_1 - \delta_1 = 169.600$$

$$\alpha_1 + \gamma_1 - \beta_1 - \delta_1 = 17.981$$

$$\alpha_1 + \delta_1 - \beta_1 - \gamma_1 = 94.083$$

Vertex 2

$$\alpha_2 - \beta_2 = 114.700$$

$$\alpha_2 - \gamma_2 = 144.980$$

$$\alpha_2 - \delta_2 = 48.834$$

$$\beta_2 - \gamma_2 = 30.277$$

$$\beta_2 - \delta_2 = -65.871$$

$$\gamma_2 - \delta_2 = -96.148$$

$$\alpha_2 + \beta_2 - 180 = 32.963$$

$$\gamma_2 + \delta_2 - 180 = -46.148$$

$$\alpha_2 + \gamma_2 - 180 = 2.686$$

$$\beta_2 + \delta_2 - 180 = -15.871$$

$$\alpha_2 + \delta_2 - 180 = 98.834$$

$$\beta_2 + \gamma_2 - 180 = -112.020$$

$$\alpha_2 + \beta_2 - \gamma_2 - \delta_2 = 79.110$$

$$\alpha_2 + \gamma_2 - \beta_2 - \delta_2 = 18.557$$

$$\alpha_2 + \delta_2 - \beta_2 - \gamma_2 = 210.850$$

Vertex 3

$$\begin{aligned}\alpha_3 - \beta_3 &= 100.210 \\ \alpha_3 - \gamma_3 &= -10.714 \\ \alpha_3 - \delta_3 &= 54.655 \\ \beta_3 - \gamma_3 &= -110.930 \\ \beta_3 - \delta_3 &= -45.556 \\ \gamma_3 - \delta_3 &= 65.369 \\ \alpha_3 + \beta_3 - 180 &= -10.900 \\ \gamma_3 + \delta_3 - 180 &= 45.369 \\ \alpha_3 + \gamma_3 - 180 &= 100.020 \\ \beta_3 + \delta_3 - 180 &= -65.556 \\ \alpha_3 + \delta_3 - 180 &= 34.655 \\ \beta_3 + \gamma_3 - 180 &= -0.186 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -56.270 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 165.580 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 34.841\end{aligned}$$

Vertex 4

$$\begin{aligned}\alpha_4 - \beta_4 &= 68.423 \\ \alpha_4 - \gamma_4 &= -31.076 \\ \alpha_4 - \delta_4 &= 12.951 \\ \beta_4 - \gamma_4 &= -99.499 \\ \beta_4 - \delta_4 &= -55.472 \\ \gamma_4 - \delta_4 &= 44.028 \\ \alpha_4 + \beta_4 - 180 &= -12.521 \\ \gamma_4 + \delta_4 - 180 &= 74.028 \\ \alpha_4 + \gamma_4 - 180 &= 86.979 \\ \beta_4 + \delta_4 - 180 &= -25.472 \\ \alpha_4 + \delta_4 - 180 &= 42.951 \\ \beta_4 + \gamma_4 - 180 &= 18.556 \\ \alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -86.548 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 112.450 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 24.395\end{aligned}$$

Switch combination: Upper

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 82.2408 & 21.9491 & 60 \\ 16.1662 & 130.871 & 18.8525 & 115 \\ 45.3447 & 145.556 & 145.369 & 80 \\ 62.0488 & 130.472 & 149.028 & 105 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}\alpha_1 - \beta_1 &= -56.032 \\ \alpha_1 - \gamma_1 &= 4.260 \\ \alpha_1 - \delta_1 &= -33.791 \\ \beta_1 - \gamma_1 &= 60.292 \\ \beta_1 - \delta_1 &= 22.241 \\ \gamma_1 - \delta_1 &= -38.051 \\ \alpha_1 + \beta_1 - 180 &= -71.551 \\ \gamma_1 + \delta_1 - 180 &= -98.051 \\ \alpha_1 + \gamma_1 - 180 &= -131.840 \\ \beta_1 + \delta_1 - 180 &= -37.759 \\ \alpha_1 + \delta_1 - 180 &= -93.791 \\ \beta_1 + \gamma_1 - 180 &= -75.810 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 26.500 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -94.083 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -17.981\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= -114.700 \\
\alpha_2 - \gamma_2 &= -2.686 \\
\alpha_2 - \delta_2 &= -98.834 \\
\beta_2 - \gamma_2 &= 112.020 \\
\beta_2 - \delta_2 &= 15.871 \\
\gamma_2 - \delta_2 &= -96.148 \\
\alpha_2 + \beta_2 - 180 &= -32.963 \\
\gamma_2 + \delta_2 - 180 &= -46.148 \\
\alpha_2 + \gamma_2 - 180 &= -144.980 \\
\beta_2 + \delta_2 - 180 &= 65.871 \\
\alpha_2 + \delta_2 - 180 &= -48.834 \\
\beta_2 + \gamma_2 - 180 &= -30.277 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 13.185 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -210.850 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -18.557
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= -100.210 \\
\alpha_3 - \gamma_3 &= -100.020 \\
\alpha_3 - \delta_3 &= -34.655 \\
\beta_3 - \gamma_3 &= 0.186 \\
\beta_3 - \delta_3 &= 65.556 \\
\gamma_3 - \delta_3 &= 65.369 \\
\alpha_3 + \beta_3 - 180 &= 10.900 \\
\gamma_3 + \delta_3 - 180 &= 45.369 \\
\alpha_3 + \gamma_3 - 180 &= 10.714 \\
\beta_3 + \delta_3 - 180 &= 45.556 \\
\alpha_3 + \delta_3 - 180 &= -54.655 \\
\beta_3 + \gamma_3 - 180 &= 110.930 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -34.469 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -34.841 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -165.580
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= -68.423 \\
\alpha_4 - \gamma_4 &= -86.979 \\
\alpha_4 - \delta_4 &= -42.951 \\
\beta_4 - \gamma_4 &= -18.556 \\
\beta_4 - \delta_4 &= 25.472 \\
\gamma_4 - \delta_4 &= 44.028 \\
\alpha_4 + \beta_4 - 180 &= 12.521 \\
\gamma_4 + \delta_4 - 180 &= 74.028 \\
\alpha_4 + \gamma_4 - 180 &= 31.076 \\
\beta_4 + \delta_4 - 180 &= 55.472 \\
\alpha_4 + \delta_4 - 180 &= -12.951 \\
\beta_4 + \gamma_4 - 180 &= 99.499 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -61.507 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -24.395 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -112.450
\end{aligned}$$

Switch combination: Right + Left*Switched anglesDeg:*

$$\begin{pmatrix} 26.2086 & 97.7592 & 158.051 & 60 \\ 16.1662 & 49.129 & 161.148 & 115 \\ 134.655 & 145.556 & 34.6305 & 80 \\ 117.951 & 130.472 & 30.9725 & 105 \end{pmatrix}$$

Angle relation checks for i = 1..4:

Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= -71.551 \\
\alpha_1 - \gamma_1 &= -131.840 \\
\alpha_1 - \delta_1 &= -33.791 \\
\beta_1 - \gamma_1 &= -60.292 \\
\beta_1 - \delta_1 &= 37.759 \\
\gamma_1 - \delta_1 &= 98.051 \\
\alpha_1 + \beta_1 - 180 &= -56.032 \\
\gamma_1 + \delta_1 - 180 &= 38.051 \\
\alpha_1 + \gamma_1 - 180 &= 4.260 \\
\beta_1 + \delta_1 - 180 &= -22.241 \\
\alpha_1 + \delta_1 - 180 &= -93.791 \\
\beta_1 + \gamma_1 - 180 &= 75.810 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -94.083 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 26.500 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -169.600
\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= -32.963 \\
\alpha_2 - \gamma_2 &= -144.980 \\
\alpha_2 - \delta_2 &= -98.834 \\
\beta_2 - \gamma_2 &= -112.020 \\
\beta_2 - \delta_2 &= -65.871 \\
\gamma_2 - \delta_2 &= 46.148 \\
\alpha_2 + \beta_2 - 180 &= -114.700 \\
\gamma_2 + \delta_2 - 180 &= 96.148 \\
\alpha_2 + \gamma_2 - 180 &= -2.686 \\
\beta_2 + \delta_2 - 180 &= -15.871 \\
\alpha_2 + \delta_2 - 180 &= -48.834 \\
\beta_2 + \gamma_2 - 180 &= 30.277 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -210.850 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 13.185 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -79.110
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= -10.900 \\
\alpha_3 - \gamma_3 &= 100.020 \\
\alpha_3 - \delta_3 &= 54.655 \\
\beta_3 - \gamma_3 &= 110.930 \\
\beta_3 - \delta_3 &= 65.556 \\
\gamma_3 - \delta_3 &= -45.369 \\
\alpha_3 + \beta_3 - 180 &= 100.210 \\
\gamma_3 + \delta_3 - 180 &= -65.369 \\
\alpha_3 + \gamma_3 - 180 &= -10.714 \\
\beta_3 + \delta_3 - 180 &= 45.556 \\
\alpha_3 + \delta_3 - 180 &= 34.655 \\
\beta_3 + \gamma_3 - 180 &= 0.186 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 165.580 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -56.270 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 34.469
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= -12.521 \\
\alpha_4 - \gamma_4 &= 86.979 \\
\alpha_4 - \delta_4 &= 12.951 \\
\beta_4 - \gamma_4 &= 99.499 \\
\beta_4 - \delta_4 &= 25.472 \\
\gamma_4 - \delta_4 &= -74.028 \\
\alpha_4 + \beta_4 - 180 &= 68.423 \\
\gamma_4 + \delta_4 - 180 &= -44.028 \\
\alpha_4 + \gamma_4 - 180 &= -31.076 \\
\alpha_4 + \delta_4 - 180 &= 55.472
\end{aligned}$$

$$\begin{aligned}\mu_4 + \nu_4 &= 180 = 55.712 \\ \alpha_4 + \delta_4 &= 180 = 42.951 \\ \beta_4 + \gamma_4 &= 180 = -18.556 \\ \alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 112.450 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -86.548 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 61.507\end{aligned}$$

Switch combination: Right + Lower

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 82.2408 & 158.051 & 60 \\ 163.834 & 49.129 & 18.8525 & 115 \\ 134.655 & 34.4444 & 145.369 & 80 \\ 117.951 & 130.472 & 30.9725 & 105 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}\alpha_1 - \beta_1 &= 71.551 \\ \alpha_1 - \gamma_1 &= -4.260 \\ \alpha_1 - \delta_1 &= 93.791 \\ \beta_1 - \gamma_1 &= -75.810 \\ \beta_1 - \delta_1 &= 22.241 \\ \gamma_1 - \delta_1 &= 98.051 \\ \alpha_1 + \beta_1 - 180 &= 56.032 \\ \gamma_1 + \delta_1 - 180 &= 38.051 \\ \alpha_1 + \gamma_1 - 180 &= 131.840 \\ \beta_1 + \delta_1 - 180 &= -37.759 \\ \alpha_1 + \delta_1 - 180 &= 33.791 \\ \beta_1 + \gamma_1 - 180 &= 60.292 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 17.981 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 169.600 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -26.500\end{aligned}$$

Vertex 2

$$\begin{aligned}\alpha_2 - \beta_2 &= 114.700 \\ \alpha_2 - \gamma_2 &= 144.980 \\ \alpha_2 - \delta_2 &= 48.834 \\ \beta_2 - \gamma_2 &= 30.277 \\ \beta_2 - \delta_2 &= -65.871 \\ \gamma_2 - \delta_2 &= -96.148 \\ \alpha_2 + \beta_2 - 180 &= 32.963 \\ \gamma_2 + \delta_2 - 180 &= -46.148 \\ \alpha_2 + \gamma_2 - 180 &= 2.686 \\ \beta_2 + \delta_2 - 180 &= -15.871 \\ \alpha_2 + \delta_2 - 180 &= 98.834 \\ \beta_2 + \gamma_2 - 180 &= -112.020 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 79.110 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 18.557 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 210.850\end{aligned}$$

Vertex 3

$$\begin{aligned}\alpha_3 - \beta_3 &= 100.210 \\ \alpha_3 - \gamma_3 &= -10.714 \\ \alpha_3 - \delta_3 &= 54.655 \\ \beta_3 - \gamma_3 &= -110.930 \\ \beta_3 - \delta_3 &= -45.556 \\ \gamma_3 - \delta_3 &= 65.369 \\ \alpha_3 + \beta_3 - 180 &= -10.900 \\ \gamma_3 + \delta_3 - 180 &= 45.369 \\ \alpha_3 + \gamma_3 - 180 &= 100.020 \\ \beta_3 + \delta_3 - 180 &= -65.556 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 100.020\end{aligned}$$

$$\begin{aligned}\alpha_3 + \delta_3 - 180 &= 34.033 \\ \beta_3 + \gamma_3 - 180 &= -0.186 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -56.270 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= 165.580 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 34.841\end{aligned}$$

Vertex 4

$$\begin{aligned}\alpha_4 - \beta_4 &= -12.521 \\ \alpha_4 - \gamma_4 &= 86.979 \\ \alpha_4 - \delta_4 &= 12.951 \\ \beta_4 - \gamma_4 &= 99.499 \\ \beta_4 - \delta_4 &= 25.472 \\ \gamma_4 - \delta_4 &= -74.028 \\ \alpha_4 + \beta_4 - 180 &= 68.423 \\ \gamma_4 + \delta_4 - 180 &= -44.028 \\ \alpha_4 + \gamma_4 - 180 &= -31.076 \\ \beta_4 + \delta_4 - 180 &= 55.472 \\ \alpha_4 + \delta_4 - 180 &= 42.951 \\ \beta_4 + \gamma_4 - 180 &= -18.556 \\ \alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 112.450 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -86.548 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 61.507\end{aligned}$$

Switch combination: Right + Upper*Switched anglesDeg:*

$$\begin{pmatrix} 26.2086 & 97.7592 & 158.051 & 60 \\ 16.1662 & 130.871 & 18.8525 & 115 \\ 45.3447 & 145.556 & 145.369 & 80 \\ 62.0488 & 49.5283 & 30.9725 & 105 \end{pmatrix}$$

*Angle relation checks for $i = 1..4$:***Vertex 1**

$$\begin{aligned}\alpha_1 - \beta_1 &= -71.551 \\ \alpha_1 - \gamma_1 &= -131.840 \\ \alpha_1 - \delta_1 &= -33.791 \\ \beta_1 - \gamma_1 &= -60.292 \\ \beta_1 - \delta_1 &= 37.759 \\ \gamma_1 - \delta_1 &= 98.051 \\ \alpha_1 + \beta_1 - 180 &= -56.032 \\ \gamma_1 + \delta_1 - 180 &= 38.051 \\ \alpha_1 + \gamma_1 - 180 &= 4.260 \\ \beta_1 + \delta_1 - 180 &= -22.241 \\ \alpha_1 + \delta_1 - 180 &= -93.791 \\ \beta_1 + \gamma_1 - 180 &= 75.810 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -94.083 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 26.500 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -169.600\end{aligned}$$

Vertex 2

$$\begin{aligned}\alpha_2 - \beta_2 &= -114.700 \\ \alpha_2 - \gamma_2 &= -2.686 \\ \alpha_2 - \delta_2 &= -98.834 \\ \beta_2 - \gamma_2 &= 112.020 \\ \beta_2 - \delta_2 &= 15.871 \\ \gamma_2 - \delta_2 &= -96.148 \\ \alpha_2 + \beta_2 - 180 &= -32.963 \\ \gamma_2 + \delta_2 - 180 &= -46.148 \\ \alpha_2 + \gamma_2 - 180 &= -144.980 \\ \beta_2 + \delta_2 - 180 &= 65.871 \\ \alpha_2 + \delta_2 - 180 &= -48.834 \\ \beta_2 + \gamma_2 - 180 &= 38.277\end{aligned}$$

$$\begin{aligned}\beta_2 + \gamma_2 - 180 &= -30.211 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 13.185 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= -210.850 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -18.557\end{aligned}$$

Vertex 3

$$\begin{aligned}\alpha_3 - \beta_3 &= -100.210 \\ \alpha_3 - \gamma_3 &= -100.020 \\ \alpha_3 - \delta_3 &= -34.655 \\ \beta_3 - \gamma_3 &= 0.186 \\ \beta_3 - \delta_3 &= 65.556 \\ \gamma_3 - \delta_3 &= 65.369 \\ \alpha_3 + \beta_3 - 180 &= 10.900 \\ \gamma_3 + \delta_3 - 180 &= 45.369 \\ \alpha_3 + \gamma_3 - 180 &= 10.714 \\ \beta_3 + \delta_3 - 180 &= 45.556 \\ \alpha_3 + \delta_3 - 180 &= -54.655 \\ \beta_3 + \gamma_3 - 180 &= 110.930 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -34.469 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -34.841 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -165.580\end{aligned}$$

Vertex 4

$$\begin{aligned}\alpha_4 - \beta_4 &= 12.521 \\ \alpha_4 - \gamma_4 &= 31.076 \\ \alpha_4 - \delta_4 &= -42.951 \\ \beta_4 - \gamma_4 &= 18.556 \\ \beta_4 - \delta_4 &= -55.472 \\ \gamma_4 - \delta_4 &= -74.028 \\ \alpha_4 + \beta_4 - 180 &= -68.423 \\ \gamma_4 + \delta_4 - 180 &= -44.028 \\ \alpha_4 + \gamma_4 - 180 &= -86.979 \\ \beta_4 + \delta_4 - 180 &= -25.472 \\ \alpha_4 + \delta_4 - 180 &= -12.951 \\ \beta_4 + \gamma_4 - 180 &= -99.499 \\ \alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -24.395 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -61.507 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 86.548\end{aligned}$$

Switch combination: Left + Lower*Switched anglesDeg:*

$$\begin{pmatrix} 153.791 & 97.7592 & 21.9491 & 60 \\ 163.834 & 130.871 & 161.148 & 115 \\ 134.655 & 145.556 & 34.6305 & 80 \\ 117.951 & 49.5283 & 149.028 & 105 \end{pmatrix}$$

*Angle relation checks for $i = 1..4$:***Vertex 1**

$$\begin{aligned}\alpha_1 - \beta_1 &= 56.032 \\ \alpha_1 - \gamma_1 &= 131.840 \\ \alpha_1 - \delta_1 &= 93.791 \\ \beta_1 - \gamma_1 &= 75.810 \\ \beta_1 - \delta_1 &= 37.759 \\ \gamma_1 - \delta_1 &= -38.051 \\ \alpha_1 + \beta_1 - 180 &= 71.551 \\ \gamma_1 + \delta_1 - 180 &= -98.051 \\ \alpha_1 + \gamma_1 - 180 &= -4.260 \\ \beta_1 + \delta_1 - 180 &= -22.241 \\ \alpha_1 + \delta_1 - 180 &= 33.791 \\ \beta_1 + \gamma_1 - 180 &= -60.292\end{aligned}$$

$$\begin{aligned}\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 169.600 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 17.981 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 94.083\end{aligned}$$

Vertex 2

$$\begin{aligned}\alpha_2 - \beta_2 &= 32.963 \\ \alpha_2 - \gamma_2 &= 2.686 \\ \alpha_2 - \delta_2 &= 48.834 \\ \beta_2 - \gamma_2 &= -30.277 \\ \beta_2 - \delta_2 &= 15.871 \\ \gamma_2 - \delta_2 &= 46.148 \\ \alpha_2 + \beta_2 - 180 &= 114.700 \\ \gamma_2 + \delta_2 - 180 &= 96.148 \\ \alpha_2 + \gamma_2 - 180 &= 144.980 \\ \beta_2 + \delta_2 - 180 &= 65.871 \\ \alpha_2 + \delta_2 - 180 &= 98.834 \\ \beta_2 + \gamma_2 - 180 &= 112.020 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 18.557 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 79.110 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -13.185\end{aligned}$$

Vertex 3

$$\begin{aligned}\alpha_3 - \beta_3 &= -10.900 \\ \alpha_3 - \gamma_3 &= 100.020 \\ \alpha_3 - \delta_3 &= 54.655 \\ \beta_3 - \gamma_3 &= 110.930 \\ \beta_3 - \delta_3 &= 65.556 \\ \gamma_3 - \delta_3 &= -45.369 \\ \alpha_3 + \beta_3 - 180 &= 100.210 \\ \gamma_3 + \delta_3 - 180 &= -65.369 \\ \alpha_3 + \gamma_3 - 180 &= -10.714 \\ \beta_3 + \delta_3 - 180 &= 45.556 \\ \alpha_3 + \delta_3 - 180 &= 34.655 \\ \beta_3 + \gamma_3 - 180 &= 0.186 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 165.580 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -56.270 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 34.469\end{aligned}$$

Vertex 4

$$\begin{aligned}\alpha_4 - \beta_4 &= 68.423 \\ \alpha_4 - \gamma_4 &= -31.076 \\ \alpha_4 - \delta_4 &= 12.951 \\ \beta_4 - \gamma_4 &= -99.499 \\ \beta_4 - \delta_4 &= -55.472 \\ \gamma_4 - \delta_4 &= 44.028 \\ \alpha_4 + \beta_4 - 180 &= -12.521 \\ \gamma_4 + \delta_4 - 180 &= 74.028 \\ \alpha_4 + \gamma_4 - 180 &= 86.979 \\ \beta_4 + \delta_4 - 180 &= -25.472 \\ \alpha_4 + \delta_4 - 180 &= 42.951 \\ \beta_4 + \gamma_4 - 180 &= 18.556 \\ \alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -86.548 \\ \alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= 112.450 \\ \alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 24.395\end{aligned}$$

Switch combination: Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 82.2408 & 21.9491 & 60 \\ 16.1662 & 49.129 & 161.148 & 115 \\ 45.3447 & 34.4444 & 34.6305 & 80 \\ 62.0488 & 130.472 & 149.028 & 105 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned} \alpha_1 - \beta_1 &= -56.032 \\ \alpha_1 - \gamma_1 &= 4.260 \\ \alpha_1 - \delta_1 &= -33.791 \\ \beta_1 - \gamma_1 &= 60.292 \\ \beta_1 - \delta_1 &= 22.241 \\ \gamma_1 - \delta_1 &= -38.051 \\ \alpha_1 + \beta_1 - 180 &= -71.551 \\ \gamma_1 + \delta_1 - 180 &= -98.051 \\ \alpha_1 + \gamma_1 - 180 &= -131.840 \\ \beta_1 + \delta_1 - 180 &= -37.759 \\ \alpha_1 + \delta_1 - 180 &= -93.791 \\ \beta_1 + \gamma_1 - 180 &= -75.810 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 26.500 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= -94.083 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -17.981 \end{aligned}$$

Vertex 2

$$\begin{aligned} \alpha_2 - \beta_2 &= -32.963 \\ \alpha_2 - \gamma_2 &= -144.980 \\ \alpha_2 - \delta_2 &= -98.834 \\ \beta_2 - \gamma_2 &= -112.020 \\ \beta_2 - \delta_2 &= -65.871 \\ \gamma_2 - \delta_2 &= 46.148 \\ \alpha_2 + \beta_2 - 180 &= -114.700 \\ \gamma_2 + \delta_2 - 180 &= 96.148 \\ \alpha_2 + \gamma_2 - 180 &= -2.686 \\ \beta_2 + \delta_2 - 180 &= -15.871 \\ \alpha_2 + \delta_2 - 180 &= -48.834 \\ \beta_2 + \gamma_2 - 180 &= 30.277 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -210.850 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 13.185 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -79.110 \end{aligned}$$

Vertex 3

$$\begin{aligned} \alpha_3 - \beta_3 &= 10.900 \\ \alpha_3 - \gamma_3 &= 10.714 \\ \alpha_3 - \delta_3 &= -34.655 \\ \beta_3 - \gamma_3 &= -0.186 \\ \beta_3 - \delta_3 &= -45.556 \\ \gamma_3 - \delta_3 &= -45.369 \\ \alpha_3 + \beta_3 - 180 &= -100.210 \\ \gamma_3 + \delta_3 - 180 &= -65.369 \\ \alpha_3 + \gamma_3 - 180 &= -100.020 \\ \beta_3 + \delta_3 - 180 &= -65.556 \\ \alpha_3 + \delta_3 - 180 &= -54.655 \\ \beta_3 + \gamma_3 - 180 &= -110.930 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -34.841 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -34.469 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 56.270 \end{aligned}$$

Vertex 4

$$\begin{aligned} \alpha_4 - \beta_4 &= -68.423 \\ \alpha_4 - \gamma_4 &= -86.979 \\ \alpha_4 - \delta_4 &= -42.951 \\ \beta_4 - \gamma_4 &= -18.556 \end{aligned}$$

```

 $\beta_4 - \delta_4 = -10.950$ 
 $\beta_4 - \delta_4 = 25.472$ 
 $\gamma_4 - \delta_4 = 44.028$ 
 $\alpha_4 + \beta_4 - 180 = 12.521$ 
 $\gamma_4 + \delta_4 - 180 = 74.028$ 
 $\alpha_4 + \gamma_4 - 180 = 31.076$ 
 $\beta_4 + \delta_4 - 180 = 55.472$ 
 $\alpha_4 + \delta_4 - 180 = -12.951$ 
 $\beta_4 + \gamma_4 - 180 = 99.499$ 
 $\alpha_4 + \beta_4 - \gamma_4 - \delta_4 = -61.507$ 
 $\alpha_4 + \gamma_4 - \beta_4 - \delta_4 = -24.395$ 
 $\alpha_4 + \delta_4 - \beta_4 - \gamma_4 = -112.450$ 

```

Switch combination: Lower + Upper

Switched anglesDeg:

```

( 153.791 97.7592 21.9491 60 )
( 163.834 49.129 18.8525 115 )
( 45.3447 145.556 145.369 80 )
( 62.0488 130.472 149.028 105 )

```

Angle relation checks for i = 1..4:

Vertex 1

```

 $\alpha_1 - \beta_1 = 56.032$ 
 $\alpha_1 - \gamma_1 = 131.840$ 
 $\alpha_1 - \delta_1 = 93.791$ 
 $\beta_1 - \gamma_1 = 75.810$ 
 $\beta_1 - \delta_1 = 37.759$ 
 $\gamma_1 - \delta_1 = -38.051$ 
 $\alpha_1 + \beta_1 - 180 = 71.551$ 
 $\gamma_1 + \delta_1 - 180 = -98.051$ 
 $\alpha_1 + \gamma_1 - 180 = -4.260$ 
 $\beta_1 + \delta_1 - 180 = -22.241$ 
 $\alpha_1 + \delta_1 - 180 = 33.791$ 
 $\beta_1 + \gamma_1 - 180 = -60.292$ 
 $\alpha_1 + \beta_1 - \gamma_1 - \delta_1 = 169.600$ 
 $\alpha_1 + \gamma_1 - \beta_1 - \delta_1 = 17.981$ 
 $\alpha_1 + \delta_1 - \beta_1 - \gamma_1 = 94.083$ 

```

Vertex 2

```

 $\alpha_2 - \beta_2 = 114.700$ 
 $\alpha_2 - \gamma_2 = 144.980$ 
 $\alpha_2 - \delta_2 = 48.834$ 
 $\beta_2 - \gamma_2 = 30.277$ 
 $\beta_2 - \delta_2 = -65.871$ 
 $\gamma_2 - \delta_2 = -96.148$ 
 $\alpha_2 + \beta_2 - 180 = 32.963$ 
 $\gamma_2 + \delta_2 - 180 = -46.148$ 
 $\alpha_2 + \gamma_2 - 180 = 2.686$ 
 $\beta_2 + \delta_2 - 180 = -15.871$ 
 $\alpha_2 + \delta_2 - 180 = 98.834$ 
 $\beta_2 + \gamma_2 - 180 = -112.020$ 
 $\alpha_2 + \beta_2 - \gamma_2 - \delta_2 = 79.110$ 
 $\alpha_2 + \gamma_2 - \beta_2 - \delta_2 = 18.557$ 
 $\alpha_2 + \delta_2 - \beta_2 - \gamma_2 = 210.850$ 

```

Vertex 3

```

 $\alpha_3 - \beta_3 = -100.210$ 
 $\alpha_3 - \gamma_3 = -100.020$ 
 $\alpha_3 - \delta_3 = -34.655$ 
 $\beta_3 - \gamma_3 = 0.186$ 
 $\beta_3 - \delta_3 = 66.556$ 

```

```

 $\mu_3 - \nu_3 = 65.550$ 
 $\gamma_3 - \delta_3 = 65.369$ 
 $\alpha_3 + \beta_3 - 180 = 10.900$ 
 $\gamma_3 + \delta_3 - 180 = 45.369$ 
 $\alpha_3 + \gamma_3 - 180 = 10.714$ 
 $\beta_3 + \delta_3 - 180 = 45.556$ 
 $\alpha_3 + \delta_3 - 180 = -54.655$ 
 $\beta_3 + \gamma_3 - 180 = 110.930$ 
 $\alpha_3 + \beta_3 - \gamma_3 - \delta_3 = -34.469$ 
 $\alpha_3 + \gamma_3 - \beta_3 - \delta_3 = -34.841$ 
 $\alpha_3 + \delta_3 - \beta_3 - \gamma_3 = -165.580$ 

```

Vertex 4

```

 $\alpha_4 - \beta_4 = -68.423$ 
 $\alpha_4 - \gamma_4 = -86.979$ 
 $\alpha_4 - \delta_4 = -42.951$ 
 $\beta_4 - \gamma_4 = -18.556$ 
 $\beta_4 - \delta_4 = 25.472$ 
 $\gamma_4 - \delta_4 = 44.028$ 
 $\alpha_4 + \beta_4 - 180 = 12.521$ 
 $\gamma_4 + \delta_4 - 180 = 74.028$ 
 $\alpha_4 + \gamma_4 - 180 = 31.076$ 
 $\beta_4 + \delta_4 - 180 = 55.472$ 
 $\alpha_4 + \delta_4 - 180 = -12.951$ 
 $\beta_4 + \gamma_4 - 180 = 99.499$ 
 $\alpha_4 + \beta_4 - \gamma_4 - \delta_4 = -61.507$ 
 $\alpha_4 + \gamma_4 - \beta_4 - \delta_4 = -24.395$ 
 $\alpha_4 + \delta_4 - \beta_4 - \gamma_4 = -112.450$ 

```

Switch combination: Right + Left + Lower

Switched anglesDeg:

```

( 153.791 82.2408 158.051 60 )
( 163.834 130.871 161.148 115 )
( 134.655 145.556 34.6305 80 )
( 117.951 130.472 30.9725 105 )

```

Angle relation checks for $i = 1..4$:

Vertex 1

```

 $\alpha_1 - \beta_1 = 71.551$ 
 $\alpha_1 - \gamma_1 = -4.260$ 
 $\alpha_1 - \delta_1 = 93.791$ 
 $\beta_1 - \gamma_1 = -75.810$ 
 $\beta_1 - \delta_1 = 22.241$ 
 $\gamma_1 - \delta_1 = 98.051$ 
 $\alpha_1 + \beta_1 - 180 = 56.032$ 
 $\gamma_1 + \delta_1 - 180 = 38.051$ 
 $\alpha_1 + \gamma_1 - 180 = 131.840$ 
 $\beta_1 + \delta_1 - 180 = -37.759$ 
 $\alpha_1 + \delta_1 - 180 = 33.791$ 
 $\beta_1 + \gamma_1 - 180 = 60.292$ 
 $\alpha_1 + \beta_1 - \gamma_1 - \delta_1 = 17.981$ 
 $\alpha_1 + \gamma_1 - \beta_1 - \delta_1 = 169.600$ 
 $\alpha_1 + \delta_1 - \beta_1 - \gamma_1 = -26.500$ 

```

Vertex 2

```

 $\alpha_2 - \beta_2 = 32.963$ 
 $\alpha_2 - \gamma_2 = 2.686$ 
 $\alpha_2 - \delta_2 = 48.834$ 
 $\beta_2 - \gamma_2 = -30.277$ 
 $\beta_2 - \delta_2 = 15.871$ 

```

$$\begin{aligned}
\gamma_2 - \alpha_2 &= 46.148 \\
\alpha_2 + \beta_2 - 180 &= 114.700 \\
\gamma_2 + \delta_2 - 180 &= 96.148 \\
\alpha_2 + \gamma_2 - 180 &= 144.980 \\
\beta_2 + \delta_2 - 180 &= 65.871 \\
\alpha_2 + \delta_2 - 180 &= 98.834 \\
\beta_2 + \gamma_2 - 180 &= 112.020 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 18.557 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 79.110 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -13.185
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= -10.900 \\
\alpha_3 - \gamma_3 &= 100.020 \\
\alpha_3 - \delta_3 &= 54.655 \\
\beta_3 - \gamma_3 &= 110.930 \\
\beta_3 - \delta_3 &= 65.556 \\
\gamma_3 - \delta_3 &= -45.369 \\
\alpha_3 + \beta_3 - 180 &= 100.210 \\
\gamma_3 + \delta_3 - 180 &= -65.369 \\
\alpha_3 + \gamma_3 - 180 &= -10.714 \\
\beta_3 + \delta_3 - 180 &= 45.556 \\
\alpha_3 + \delta_3 - 180 &= 34.655 \\
\beta_3 + \gamma_3 - 180 &= 0.186 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= 165.580 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -56.270 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 34.469
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= -12.521 \\
\alpha_4 - \gamma_4 &= 86.979 \\
\alpha_4 - \delta_4 &= 12.951 \\
\beta_4 - \gamma_4 &= 99.499 \\
\beta_4 - \delta_4 &= 25.472 \\
\gamma_4 - \delta_4 &= -74.028 \\
\alpha_4 + \beta_4 - 180 &= 68.423 \\
\gamma_4 + \delta_4 - 180 &= -44.028 \\
\alpha_4 + \gamma_4 - 180 &= -31.076 \\
\beta_4 + \delta_4 - 180 &= 55.472 \\
\alpha_4 + \delta_4 - 180 &= 42.951 \\
\beta_4 + \gamma_4 - 180 &= -18.556 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= 112.450 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -86.548 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 61.507
\end{aligned}$$

Switch combination: Right + Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 97.7592 & 158.051 & 60 \\ 16.1662 & 49.129 & 161.148 & 115 \\ 45.3447 & 34.4444 & 34.6305 & 80 \\ 62.0488 & 49.5283 & 30.9725 & 105 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= -71.551 \\
\alpha_1 - \gamma_1 &= -131.840 \\
\alpha_1 - \delta_1 &= -33.791 \\
\beta_1 - \gamma_1 &= -60.292 \\
\beta_1 - \delta_1 &= 37.759 \\
\gamma_1 - \delta_1 &= 98.051 \\
&\dots
\end{aligned}$$

$$\begin{aligned}
\alpha_1 + \beta_1 - 180 &= -56.032 \\
\gamma_1 + \delta_1 - 180 &= 38.051 \\
\alpha_1 + \gamma_1 - 180 &= 4.260 \\
\beta_1 + \delta_1 - 180 &= -22.241 \\
\alpha_1 + \delta_1 - 180 &= -93.791 \\
\beta_1 + \gamma_1 - 180 &= 75.810 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= -94.083 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 26.500 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -169.600
\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= -32.963 \\
\alpha_2 - \gamma_2 &= -144.980 \\
\alpha_2 - \delta_2 &= -98.834 \\
\beta_2 - \gamma_2 &= -112.020 \\
\beta_2 - \delta_2 &= -65.871 \\
\gamma_2 - \delta_2 &= 46.148 \\
\alpha_2 + \beta_2 - 180 &= -114.700 \\
\gamma_2 + \delta_2 - 180 &= 96.148 \\
\alpha_2 + \gamma_2 - 180 &= -2.686 \\
\beta_2 + \delta_2 - 180 &= -15.871 \\
\alpha_2 + \delta_2 - 180 &= -48.834 \\
\beta_2 + \gamma_2 - 180 &= 30.277 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= -210.850 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 13.185 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -79.110
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= 10.900 \\
\alpha_3 - \gamma_3 &= 10.714 \\
\alpha_3 - \delta_3 &= -34.655 \\
\beta_3 - \gamma_3 &= -0.186 \\
\beta_3 - \delta_3 &= -45.556 \\
\gamma_3 - \delta_3 &= -45.369 \\
\alpha_3 + \beta_3 - 180 &= -100.210 \\
\gamma_3 + \delta_3 - 180 &= -65.369 \\
\alpha_3 + \gamma_3 - 180 &= -100.020 \\
\beta_3 + \delta_3 - 180 &= -65.556 \\
\alpha_3 + \delta_3 - 180 &= -54.655 \\
\beta_3 + \gamma_3 - 180 &= -110.930 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -34.841 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -34.469 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 56.270
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= 12.521 \\
\alpha_4 - \gamma_4 &= 31.076 \\
\alpha_4 - \delta_4 &= -42.951 \\
\beta_4 - \gamma_4 &= 18.556 \\
\beta_4 - \delta_4 &= -55.472 \\
\gamma_4 - \delta_4 &= -74.028 \\
\alpha_4 + \beta_4 - 180 &= -68.423 \\
\gamma_4 + \delta_4 - 180 &= -44.028 \\
\alpha_4 + \gamma_4 - 180 &= -86.979 \\
\beta_4 + \delta_4 - 180 &= -25.472 \\
\alpha_4 + \delta_4 - 180 &= -12.951 \\
\beta_4 + \gamma_4 - 180 &= -99.499 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -24.395 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -61.507 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 86.548
\end{aligned}$$

Switch combination: Right + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 82.2408 & 158.051 & 60 \\ 163.834 & 49.129 & 18.8525 & 115 \\ 45.3447 & 145.556 & 145.369 & 80 \\ 62.0488 & 49.5283 & 30.9725 & 105 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned} \alpha_1 - \beta_1 &= 71.551 \\ \alpha_1 - \gamma_1 &= -4.260 \\ \alpha_1 - \delta_1 &= 93.791 \\ \beta_1 - \gamma_1 &= -75.810 \\ \beta_1 - \delta_1 &= 22.241 \\ \gamma_1 - \delta_1 &= 98.051 \\ \alpha_1 + \beta_1 - 180 &= 56.032 \\ \gamma_1 + \delta_1 - 180 &= 38.051 \\ \alpha_1 + \gamma_1 - 180 &= 131.840 \\ \beta_1 + \delta_1 - 180 &= -37.759 \\ \alpha_1 + \delta_1 - 180 &= 33.791 \\ \beta_1 + \gamma_1 - 180 &= 60.292 \\ \alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 17.981 \\ \alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 169.600 \\ \alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -26.500 \end{aligned}$$

Vertex 2

$$\begin{aligned} \alpha_2 - \beta_2 &= 114.700 \\ \alpha_2 - \gamma_2 &= 144.980 \\ \alpha_2 - \delta_2 &= 48.834 \\ \beta_2 - \gamma_2 &= 30.277 \\ \beta_2 - \delta_2 &= -65.871 \\ \gamma_2 - \delta_2 &= -96.148 \\ \alpha_2 + \beta_2 - 180 &= 32.963 \\ \gamma_2 + \delta_2 - 180 &= -46.148 \\ \alpha_2 + \gamma_2 - 180 &= 2.686 \\ \beta_2 + \delta_2 - 180 &= -15.871 \\ \alpha_2 + \delta_2 - 180 &= 98.834 \\ \beta_2 + \gamma_2 - 180 &= -112.020 \\ \alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 79.110 \\ \alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 18.557 \\ \alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= 210.850 \end{aligned}$$

Vertex 3

$$\begin{aligned} \alpha_3 - \beta_3 &= -100.210 \\ \alpha_3 - \gamma_3 &= -100.020 \\ \alpha_3 - \delta_3 &= -34.655 \\ \beta_3 - \gamma_3 &= 0.186 \\ \beta_3 - \delta_3 &= 65.556 \\ \gamma_3 - \delta_3 &= 65.369 \\ \alpha_3 + \beta_3 - 180 &= 10.900 \\ \gamma_3 + \delta_3 - 180 &= 45.369 \\ \alpha_3 + \gamma_3 - 180 &= 10.714 \\ \beta_3 + \delta_3 - 180 &= 45.556 \\ \alpha_3 + \delta_3 - 180 &= -54.655 \\ \beta_3 + \gamma_3 - 180 &= 110.930 \\ \alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -34.469 \\ \alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -34.841 \\ \alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= -165.580 \end{aligned}$$

.. .

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= 12.521 \\
\alpha_4 - \gamma_4 &= 31.076 \\
\alpha_4 - \delta_4 &= -42.951 \\
\beta_4 - \gamma_4 &= 18.556 \\
\beta_4 - \delta_4 &= -55.472 \\
\gamma_4 - \delta_4 &= -74.028 \\
\alpha_4 + \beta_4 - 180 &= -68.423 \\
\gamma_4 + \delta_4 - 180 &= -44.028 \\
\alpha_4 + \gamma_4 - 180 &= -86.979 \\
\beta_4 + \delta_4 - 180 &= -25.472 \\
\alpha_4 + \delta_4 - 180 &= -12.951 \\
\beta_4 + \gamma_4 - 180 &= -99.499 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -24.395 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -61.507 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 86.548
\end{aligned}$$

Switch combination: Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 97.7592 & 21.9491 & 60 \\ 163.834 & 130.871 & 161.148 & 115 \\ 45.3447 & 34.4444 & 34.6305 & 80 \\ 62.0488 & 130.472 & 149.028 & 105 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= 56.032 \\
\alpha_1 - \gamma_1 &= 131.840 \\
\alpha_1 - \delta_1 &= 93.791 \\
\beta_1 - \gamma_1 &= 75.810 \\
\beta_1 - \delta_1 &= 37.759 \\
\gamma_1 - \delta_1 &= -38.051 \\
\alpha_1 + \beta_1 - 180 &= 71.551 \\
\gamma_1 + \delta_1 - 180 &= -98.051 \\
\alpha_1 + \gamma_1 - 180 &= -4.260 \\
\beta_1 + \delta_1 - 180 &= -22.241 \\
\alpha_1 + \delta_1 - 180 &= 33.791 \\
\beta_1 + \gamma_1 - 180 &= -60.292 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 169.600 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 17.981 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= 94.083
\end{aligned}$$

Vertex 2

$$\begin{aligned}
\alpha_2 - \beta_2 &= 32.963 \\
\alpha_2 - \gamma_2 &= 2.686 \\
\alpha_2 - \delta_2 &= 48.834 \\
\beta_2 - \gamma_2 &= -30.277 \\
\beta_2 - \delta_2 &= 15.871 \\
\gamma_2 - \delta_2 &= 46.148 \\
\alpha_2 + \beta_2 - 180 &= 114.700 \\
\gamma_2 + \delta_2 - 180 &= 96.148 \\
\alpha_2 + \gamma_2 - 180 &= 144.980 \\
\beta_2 + \delta_2 - 180 &= 65.871 \\
\alpha_2 + \delta_2 - 180 &= 98.834 \\
\beta_2 + \gamma_2 - 180 &= 112.020 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 18.557 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 79.110 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -13.185
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= 10.900 \\
\alpha_3 - \gamma_3 &= 10.714 \\
\alpha_3 - \delta_3 &= -34.655 \\
\beta_3 - \gamma_3 &= -0.186 \\
\beta_3 - \delta_3 &= -45.556 \\
\gamma_3 - \delta_3 &= -45.369 \\
\alpha_3 + \beta_3 - 180 &= -100.210 \\
\gamma_3 + \delta_3 - 180 &= -65.369 \\
\alpha_3 + \gamma_3 - 180 &= -100.020 \\
\beta_3 + \delta_3 - 180 &= -65.556 \\
\alpha_3 + \delta_3 - 180 &= -54.655 \\
\beta_3 + \gamma_3 - 180 &= -110.930 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -34.841 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -34.469 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 56.270
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= -68.423 \\
\alpha_4 - \gamma_4 &= -86.979 \\
\alpha_4 - \delta_4 &= -42.951 \\
\beta_4 - \gamma_4 &= -18.556 \\
\beta_4 - \delta_4 &= 25.472 \\
\gamma_4 - \delta_4 &= 44.028 \\
\alpha_4 + \beta_4 - 180 &= 12.521 \\
\gamma_4 + \delta_4 - 180 &= 74.028 \\
\alpha_4 + \gamma_4 - 180 &= 31.076 \\
\beta_4 + \delta_4 - 180 &= 55.472 \\
\alpha_4 + \delta_4 - 180 &= -12.951 \\
\beta_4 + \gamma_4 - 180 &= 99.499 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -61.507 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -24.395 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= -112.450
\end{aligned}$$

Switch combination: Right + Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 82.2408 & 158.051 & 60 \\ 163.834 & 130.871 & 161.148 & 115 \\ 45.3447 & 34.4444 & 34.6305 & 80 \\ 62.0488 & 49.5283 & 30.9725 & 105 \end{pmatrix}$$

Angle relation checks for $i = 1..4$:

Vertex 1

$$\begin{aligned}
\alpha_1 - \beta_1 &= 71.551 \\
\alpha_1 - \gamma_1 &= -4.260 \\
\alpha_1 - \delta_1 &= 93.791 \\
\beta_1 - \gamma_1 &= -75.810 \\
\beta_1 - \delta_1 &= 22.241 \\
\gamma_1 - \delta_1 &= 98.051 \\
\alpha_1 + \beta_1 - 180 &= 56.032 \\
\gamma_1 + \delta_1 - 180 &= 38.051 \\
\alpha_1 + \gamma_1 - 180 &= 131.840 \\
\beta_1 + \delta_1 - 180 &= -37.759 \\
\alpha_1 + \delta_1 - 180 &= 33.791 \\
\beta_1 + \gamma_1 - 180 &= 60.292 \\
\alpha_1 + \beta_1 - \gamma_1 - \delta_1 &= 17.981 \\
\alpha_1 + \gamma_1 - \beta_1 - \delta_1 &= 169.600 \\
\alpha_1 + \delta_1 - \beta_1 - \gamma_1 &= -26.500
\end{aligned}$$

Vertex 2

$$\alpha_2 - \beta_2 = 32.963$$

$$\begin{aligned}
\alpha_2 - \gamma_2 &= 2.686 \\
\alpha_2 - \delta_2 &= 48.834 \\
\beta_2 - \gamma_2 &= -30.277 \\
\beta_2 - \delta_2 &= 15.871 \\
\gamma_2 - \delta_2 &= 46.148 \\
\alpha_2 + \beta_2 - 180 &= 114.700 \\
\gamma_2 + \delta_2 - 180 &= 96.148 \\
\alpha_2 + \gamma_2 - 180 &= 144.980 \\
\beta_2 + \delta_2 - 180 &= 65.871 \\
\alpha_2 + \delta_2 - 180 &= 98.834 \\
\beta_2 + \gamma_2 - 180 &= 112.020 \\
\alpha_2 + \beta_2 - \gamma_2 - \delta_2 &= 18.557 \\
\alpha_2 + \gamma_2 - \beta_2 - \delta_2 &= 79.110 \\
\alpha_2 + \delta_2 - \beta_2 - \gamma_2 &= -13.185
\end{aligned}$$

Vertex 3

$$\begin{aligned}
\alpha_3 - \beta_3 &= 10.900 \\
\alpha_3 - \gamma_3 &= 10.714 \\
\alpha_3 - \delta_3 &= -34.655 \\
\beta_3 - \gamma_3 &= -0.186 \\
\beta_3 - \delta_3 &= -45.556 \\
\gamma_3 - \delta_3 &= -45.369 \\
\alpha_3 + \beta_3 - 180 &= -100.210 \\
\gamma_3 + \delta_3 - 180 &= -65.369 \\
\alpha_3 + \gamma_3 - 180 &= -100.020 \\
\beta_3 + \delta_3 - 180 &= -65.556 \\
\alpha_3 + \delta_3 - 180 &= -54.655 \\
\beta_3 + \gamma_3 - 180 &= -110.930 \\
\alpha_3 + \beta_3 - \gamma_3 - \delta_3 &= -34.841 \\
\alpha_3 + \gamma_3 - \beta_3 - \delta_3 &= -34.469 \\
\alpha_3 + \delta_3 - \beta_3 - \gamma_3 &= 56.270
\end{aligned}$$

Vertex 4

$$\begin{aligned}
\alpha_4 - \beta_4 &= 12.521 \\
\alpha_4 - \gamma_4 &= 31.076 \\
\alpha_4 - \delta_4 &= -42.951 \\
\beta_4 - \gamma_4 &= 18.556 \\
\beta_4 - \delta_4 &= -55.472 \\
\gamma_4 - \delta_4 &= -74.028 \\
\alpha_4 + \beta_4 - 180 &= -68.423 \\
\gamma_4 + \delta_4 - 180 &= -44.028 \\
\alpha_4 + \gamma_4 - 180 &= -86.979 \\
\beta_4 + \delta_4 - 180 &= -25.472 \\
\alpha_4 + \delta_4 - 180 &= -12.951 \\
\beta_4 + \gamma_4 - 180 &= -99.499 \\
\alpha_4 + \beta_4 - \gamma_4 - \delta_4 &= -24.395 \\
\alpha_4 + \gamma_4 - \beta_4 - \delta_4 &= -61.507 \\
\alpha_4 + \delta_4 - \beta_4 - \gamma_4 &= 86.548
\end{aligned}$$

Out[1582]=

===== NOT CONJUGATE-MODULAR =====

$M_1 = M_2 = M_3 = M_4 = M$ and $M \neq 2 \Rightarrow$ NOT

conjugate-modular. Boundary-strip switches preserve this.

Initial configuration (no switches applied):

$$\begin{pmatrix} 26.2086 & 82.2408 & 21.9491 & 60 \\ 16.1662 & 130.871 & 18.8525 & 115 \\ 134.655 & 34.4444 & 145.369 & 80 \\ 117.951 & 49.5283 & 149.028 & 105 \end{pmatrix}$$

M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Right

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 97.7592 & 158.051 & 60 \\ 16.1662 & 130.871 & 18.8525 & 115 \\ 134.655 & 34.4444 & 145.369 & 80 \\ 117.951 & 130.472 & 30.9725 & 105 \end{pmatrix}$$
M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Left

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 82.2408 & 21.9491 & 60 \\ 16.1662 & 49.129 & 161.148 & 115 \\ 134.655 & 145.556 & 34.6305 & 80 \\ 117.951 & 49.5283 & 149.028 & 105 \end{pmatrix}$$
M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Lower

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 97.7592 & 21.9491 & 60 \\ 163.834 & 49.129 & 18.8525 & 115 \\ 134.655 & 34.4444 & 145.369 & 80 \\ 117.951 & 49.5283 & 149.028 & 105 \end{pmatrix}$$
M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Upper

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 82.2408 & 21.9491 & 60 \\ 16.1662 & 130.871 & 18.8525 & 115 \\ 45.3447 & 145.556 & 145.369 & 80 \\ 62.0488 & 130.472 & 149.028 & 105 \end{pmatrix}$$
M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Right + Left

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 97.7592 & 158.051 & 60 \\ 16.1662 & 49.129 & 161.148 & 115 \\ 134.655 & 145.556 & 34.6305 & 80 \\ 117.951 & 130.472 & 30.9725 & 105 \end{pmatrix}$$

M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Right + Lower

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 82.2408 & 158.051 & 60 \\ 163.834 & 49.129 & 18.8525 & 115 \\ 134.655 & 34.4444 & 145.369 & 80 \\ 117.951 & 130.472 & 30.9725 & 105 \end{pmatrix}$$
M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Right + Upper

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 97.7592 & 158.051 & 60 \\ 16.1662 & 130.871 & 18.8525 & 115 \\ 45.3447 & 145.556 & 145.369 & 80 \\ 62.0488 & 49.5283 & 30.9725 & 105 \end{pmatrix}$$
M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 97.7592 & 21.9491 & 60 \\ 163.834 & 130.871 & 161.148 & 115 \\ 134.655 & 145.556 & 34.6305 & 80 \\ 117.951 & 49.5283 & 149.028 & 105 \end{pmatrix}$$
M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 82.2408 & 21.9491 & 60 \\ 16.1662 & 49.129 & 161.148 & 115 \\ 45.3447 & 34.4444 & 34.6305 & 80 \\ 62.0488 & 130.472 & 149.028 & 105 \end{pmatrix}$$
M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 97.7592 & 21.9491 & 60 \\ 163.834 & 49.129 & 18.8525 & 115 \\ 45.3447 & 145.556 & 145.369 & 80 \\ 62.0488 & 130.472 & 149.028 & 105 \end{pmatrix}$$

M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Right + Left + Lower

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 82.2408 & 158.051 & 60 \\ 163.834 & 130.871 & 161.148 & 115 \\ 134.655 & 145.556 & 34.6305 & 80 \\ 117.951 & 130.472 & 30.9725 & 105 \end{pmatrix}$$
M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Right + Left + Upper

Switched anglesDeg:

$$\begin{pmatrix} 26.2086 & 97.7592 & 158.051 & 60 \\ 16.1662 & 49.129 & 161.148 & 115 \\ 45.3447 & 34.4444 & 34.6305 & 80 \\ 62.0488 & 49.5283 & 30.9725 & 105 \end{pmatrix}$$
M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Right + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 82.2408 & 158.051 & 60 \\ 163.834 & 49.129 & 18.8525 & 115 \\ 45.3447 & 145.556 & 145.369 & 80 \\ 62.0488 & 49.5283 & 30.9725 & 105 \end{pmatrix}$$
M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 97.7592 & 21.9491 & 60 \\ 163.834 & 130.871 & 161.148 & 115 \\ 45.3447 & 34.4444 & 34.6305 & 80 \\ 62.0488 & 130.472 & 149.028 & 105 \end{pmatrix}$$
M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Switch combination: Right + Left + Lower + Upper

Switched anglesDeg:

$$\begin{pmatrix} 153.791 & 82.2408 & 158.051 & 60 \\ 163.834 & 130.871 & 161.148 & 115 \\ 45.3447 & 34.4444 & 34.6305 & 80 \\ 62.0488 & 49.5283 & 30.9725 & 105 \end{pmatrix}$$

M_i values:

M1 = 1.22593, M2 = 1.22593, M3 = 1.22593, M4 = 1.22593

M1 = M2 = M3 = M4 = M and M ≠ 2

Out[1586]=

===== NOT CHIMERA =====

Fails conic, orthodiagonal & isogonal tests for all
 $i=1, \dots, 4 \Rightarrow$ NOT chimera. Boundary-strip switches
 preserve these failures as demonstrated in the NOT
 CONIC, NOT ORTHODIAGONAL, and NOT ISOGONAL sections.