AMCS 251 - Numerical Linear Algebra: Homework #1

Due on 25/09/2022

Problem 1

Calculate the full and reduced singular value decompositions of the matrix

$$\begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$$

- Show all the steps and describe them.
- Add any comment you think it is relevant to explain the results you get (try to be critical).

Problem 2

Let A be a $m \times n$ matrix of full rank ($m \geq n$).

- What is the condition number of A in terms of the singular values of A? Use the 2-norm. (Hint: Use the SVD of A, i.e., $A = U\Sigma V^T$.)
- Using the SVD of A, compute the SVD of the following matrices in terms of U, Σ, and V:
  - $\left(A^T A\right)^{-1}$,
  - $\left(A^T A\right)^{-1} A^T$,
- Show all the steps and describe them.
- Add any comment you think it is relevant to explain the results you get (try to be critical).

Problem 3

Write an essay discussing the differences between the eigenvalue decomposition and the singular value decomposition. Include one or more examples to illustrate your points.