AMCS 251 - Numerical Linear Algebra: Homework #2

Due on 19/10/2023

Problem 1

Write a code that implements the naive Gaussian elimination and the variants with column pivoting and total pivoting. The goal is to have a code that solve the small linear system of algebraic equations.

- Construct a clean and readable code.
- The code does not need to be complex and extremely general; focus on solving the aforementioned linear systems.
- Add any comment you think it is relevant to explain the results you get (try to be critical).

Linear systems:

1. Fixing a value of n, define the polynomial

$$p(t) = 1 + t + t^2 + \ldots + t^{n-1} = \sum_{j=1}^{n} t^{j-1}$$

The coefficients in this polynomial are equal to 1. We want to recover these coefficients from n values of the polynomial. We use the values of $p(t)$ at the integers $t = i + 1$ for $i = 1, 2, \ldots, n$. If we denote the coefficients of the polynomial as $x_1, x_2, \ldots, x_n$, we have:

$$\sum_{j=1}^{n} (1+i)^{j-1} x_j = \frac{1}{i}\left[(1+i)^n - 1\right], \qquad (1 \le i \le n)$$

Letting $a_{ij} = (1+i)^{j-1}$ and $b_i = \frac{1}{i}\left[(1+i)^n - 1\right]$ we have the linear system

$$\sum_{j=1}^{n} a_{ij} x_j = b_i, \qquad (1 \le i \le n).$$

Solve the system for $n = 4, 5, 6, 7, 8, 9$ and compute the error in the coefficients.

2. Consider the system

$$\begin{cases} \epsilon x_1 + x_2 = 1 \\ x_1 + x_2 = 2. \end{cases}$$

Solve the system for $\epsilon = 10^2, 10^{-1}$, and 10^{-9} using double precision but only looking at the solution with 8-digit.

Could you please try to present a small example where complete pivoting is needed?