Computational Companion to “Quasi-Symmetric Nets: A Constructive Approach to the Equimodular Elliptic Type of Kokotsakis Polyhedra” — Example 2 Helper

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Tested on: Mathematica 14.0

In[1]:=
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(*=============================================================================
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 =====*)
(*Quit*)
(*All angle sets in degrees*)
anglesRad = {
   {ArcCos[1 / Sqrt[5]], ArcCos[7 / (5 Sqrt[2])],
    ArcCos[-1 / Sqrt[10]], ArcCos[0]}, (*Vertex 1*)
   {ArcCos[1 / (4 Sqrt[11])], ArcCos[7 Sqrt[7] / (4 Sqrt[22])],
    ArcCos[-1 / (2 Sqrt[2])], ArcCos[0]}, (*Vertex 2*)
   {ArcCos[1 / (4 Sqrt[11])], ArcCos[7 Sqrt[7] / (4 Sqrt[22])],
    ArcCos[1 / (2 Sqrt[2])], ArcCos[0]}, (*Vertex 3*)
   {ArcCos[1 / Sqrt[5]], ArcCos[7 / (5 Sqrt[2])],
    ArcCos[1 / Sqrt[10]], ArcCos[0]} (*Vertex 4*)};

anglesDeg = anglesRad * 180 / Pi;

(*---------------------------------------------*)
(*Function to compute sigma from 4 angles*)
computeSigma[{α_, β_, γ_, δ_}] := (α + β + γ + δ) / 2;
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(*-------------------------------------------*)
(*Function to compute a,b,c,d from angles*)
computeABCD[{α_, β_, γ_, δ_}] :=
  Module[{alpha = α Degree, beta = β Degree, gamma = γ Degree,
    delta = δ Degree, sigma}, sigma = computeSigma[{α, β, γ, δ} Degree];
   {Sin[alpha] / Sin[sigma - alpha], Sin[beta] / Sin[sigma - beta],
    Sin[gamma] / Sin[sigma - gamma], Sin[delta] / Sin[sigma - delta]}];

(*-------------------------------------------*)
(*Compute all σ values*)
sigmas = computeSigma /@ anglesDeg;

(*Compute all {a,b,c,d} values*)
results = computeABCD /@ anglesDeg;

(*Optional:extract individual values*)
{a1, b1, c1, d1} = results[[1]];
{a2, b2, c2, d2} = results[[2]];
{a3, b3, c3, d3} = results[[3]];
{a4, b4, c4, d4} = results[[4]];
{σ1, σ2, σ3, σ4} = FullSimplify[sigmas];


(*===========================================================================
 =====*)
(*================================
   CONDITION (N.0)=================================*)
(*===========================================================================
 =====*)
uniqueCombos = {{1, 1, 1, 1}, {1, 1, 1, -1}, {1, 1, -1, -1}, {1, 1, -1, 1},
   {1, -1, 1, 1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, -1, -1, -1}};

checkConditionN0Degrees[{α_, β_, γ_, δ_}] :=
  Module[{angles = {α, β, γ, δ}, results},
   results = Mod[uniqueCombos.angles, 360] // Chop;
   ! MemberQ[results, 0]];

conditionsN0 = checkConditionN0Degrees /@ anglesDeg;
allVerticesPass = And @@ conditionsN0;

Column[{TextCell[Style["================= CONDITION (N.0) =================",
    Darker[Green], Bold, 16], "Text"],
  If[allVerticesPass,
   Style["✓ All vertices satisfy (N.0).", Darker[Green], Bold],
   Style["✗ Some vertices fail (N.0).", Red, Bold]]}]
```

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(*===========================================================================
 =====*)
(*=============================
   CONDITION (N.3)=================================*)
(*===========================================================================
 =====*)
Ms = FullSimplify[Times @@@ results];
allEqualQ = Simplify[Equal @@ Ms];

Column[{TextCell[Style["================= CONDITION (N.3) =================",
     Blue, Bold, 16], "Text"], If[allEqualQ,
   Row[{Style["✔ M1 = M2 = M3 = M4 = ", Bold], Ms[[1]]}],
   Style["✘ M_i are not all equal.", Red, Bold]]}]


(*===========================================================================
 =====*)
(*==========================CONDITION (N.4)==========================*)
(*===========================================================================
 =====*)
aList = results[[All, 1]];
cList = results[[All, 3]];
dList = results[[All, 4]];

rList = FullSimplify /@ (aList * dList); {r1, r2, r3, r4} = rList;
sList = FullSimplify /@ (cList * dList); {s1, s2, s3, s4} = sList;

Column[{TextCell[Style["================= CONDITION (N.4) =================",
     Darker[Blue], Bold, 16], "Text"],
  If[r1 == r2 && r3 == r4 && s1 == s4 && s2 == s3, Column[
    {Row[{Style["✔ r1 = r2 = ", Bold], r1, Style["; ✔ r3 = r4 = ", Bold], r3}],
     Row[{Style["✔ s1 = s4 = ", Bold], s1, Style["; ✔ s2 = s3 = ", Bold],
       s2}]}], Style["✘ Condition (N.4) fails.", Red, Bold]]
 }]


(*===========================================================================
 =====*)
(*================================
  CONDITION (N.5)==============================*)
(*===========================================================================
 =====*)
fList = FullSimplify /@ (aList * cList); {f1, f2, f3, f4} = fList;
M1 = Ms[[1]];

m[M1_] := Piecewise[{{1 - M1, M1 < 1}, {(M1 - 1) / M1, M1 > 1}}] // N;
```

```
K = EllipticK[m[M1]] // N;
Kp = EllipticK[1 - m[M1]] // N;

computeTi[i_] :=
  Module[{sigma = sigmas[[i]], r = rList[[i]], s = sList[[i]], f = fList[[i]], base},
   base = I * Im[InverseJacobiDN[Piecewise[
         {{Sqrt[f], M1 < 1}, {1 / Sqrt[f], M1 > 1}}], m[M1]]] / Kp;
   Which[sigma < 180, Which[r > 1 && s > 1, base, (r < 1 && s > 1) || (r > 1 && s < 1),
     1 + base, r < 1 && s < 1, 2 + base], sigma > 180, Which[r > 1 && s > 1,
     2 + base, (r < 1 && s > 1) || (r > 1 && s < 1), 3 + base, r < 1 && s < 1, base]]];

tList = computeTi /@ Range[4];
RoundWithTolerance[x_, tol_ : 10 ^ -6] := Module[{nearest}, nearest = Round[x];
   If[Abs[x - nearest] ≤ tol, nearest, x]];

checkValidCombination[M1_, ε_ : 10 ^ -6] :=
  Module[{i, combo, dotProd, rePart, imPart, n2, expr, foundQ = False},
   Print[Style["⚠ Approximate validation using ε-tolerance. For rigorous
       proof, see the referenced paper.", Darker@Orange, Italic]];
   Do[combo = uniqueCombos[[i]];
    dotProd = tList.combo;
    rePart = Abs[Re[dotProd]];
    imPart = Abs[Im[dotProd]];
    If[M1 < 1,
     If[Mod[RoundWithTolerance[rePart], 4] < ε,
      If[Mod[RoundWithTolerance[imPart], 2] < ε, expr =
        tList[[1]] + combo[[2]] × tList[[2]] + combo[[3]] × tList[[3]] + combo[[4]] × tList[[4]];
       Print[Style["✅ Valid Combination Found (M < 1):",
         Darker[Green], Bold], "\n", Style["e1 = ", Bold], combo[[2]],
        Style[", e2 = ", Bold], combo[[3]], Style[", e3 = ", Bold], combo[[4]],
        "\n", Style["t1 = ", Bold], Re[tList[[1]]], "K + ", Im[tList[[1]]], "iK'",
        "\n", Style["t2 = ", Bold], Re[tList[[2]]], "K + ", Im[tList[[2]]], "iK'",
        "\n", Style["t3 = ", Bold], Re[tList[[3]]], "K + ", Im[tList[[3]]], "iK'",
        "\n", Style["t4 = ", Bold], Re[tList[[4]]], "K + ", Im[tList[[4]]],
        "iK'", "\n", Style["t1 + e1*t2 + e2*t3 + e3*t4 = ", Bold],
        Re[expr], "K + ", Im[expr], "iK'"];
       foundQ = True;
       Break[]]]];
    If[M1 > 1,
     If[Mod[RoundWithTolerance[imPart], 2] < ε,
      n2 = Quotient[RoundWithTolerance[imPart], 2];
      If[Mod[RoundWithTolerance[Abs[rePart - 2 n2]], 4] < ε, expr =
        tList[[1]] + combo[[2]] × tList[[2]] + combo[[3]] × tList[[3]] + combo[[4]] × tList[[4]];
       Print[Style["✅ Valid Combination Found (M > 1):",
         Darker[Green], Bold], "\n", Style["e1 = ", Bold], combo[[2]],
```

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          Style[", e2 = ", Bold], combo⟦3⟧, Style[", e3 = ", Bold], combo⟦4⟧,
          "\n", Style["t1 = ", Bold], Re[tList⟦1⟧], "K + ", Im[tList⟦1⟧], "iK'",
          "\n", Style["t2 = ", Bold], Re[tList⟦2⟧], "K + ", Im[tList⟦2⟧], "iK'",
          "\n", Style["t3 = ", Bold], Re[tList⟦3⟧], "K + ", Im[tList⟦3⟧], "iK'",
          "\n", Style["t4 = ", Bold], Re[tList⟦4⟧], "K + ", Im[tList⟦4⟧],
          "iK'", "\n", Style["t1 + e1*t2 + e2*t3 + e3*t4 = ", Bold],
          Re[expr], "K + ", Im[expr], "iK'"];
         foundQ = True;
         Break[]]]], {i, Length[uniqueCombos]}];
    If[! foundQ, Print[Style["❌ No valid combination found.", Red, Bold], "\n",
      Style["t1 = ", Bold], Re[tList⟦1⟧], "K + ", Im[tList⟦1⟧], "iK'", "\n",
      Style["t2 = ", Bold], Re[tList⟦2⟧], "K + ", Im[tList⟦2⟧], "iK'", "\n",
      Style["t3 = ", Bold], Re[tList⟦3⟧], "K + ", Im[tList⟦3⟧], "iK'", "\n",
      Style["t4 = ", Bold], Re[tList⟦4⟧], "K + ", Im[tList⟦4⟧], "iK'", "\n"]];];


Column[{TextCell[Style["================= CONDITION (N.5) =================",
    Purple, Bold, 16], "Text"]}]
res = checkValidCombination[M1];



(*===========================================================================
 =====*)
(*=================================
   OTHER PARAMETER==================================*)
(*===========================================================================
 =====*)
cosSigmaAlt[Mi_, fi_, ri_, si_] :=
  (Mi (fi + ri + si - 1) + ri fi si - ri fi - ri si - fi si) / (2 Sqrt[ri si fi] (1 - Mi));

Column[
 {TextCell[Style["================= OTHER PARAMETERS =================",
    Darker[Purple], Bold, 16], "Text"],
  Row[{Style["u = ", Bold], 1 - M1}],
  Row[{Style["σ1 = ", Bold], σ1 Degree, Style[", σ2 = ", Bold], σ2 Degree,
    Style[", σ3 = ", Bold], σ3 Degree, Style[", σ4 = ", Bold], σ4 Degree}],
  Row[{Style["σ1 ≈ ", Bold], N[σ1], Style["°", Bold], Style[", σ2 ≈ ", Bold],
    N[σ2], Style["°", Bold], Style[", σ3 ≈ ", Bold], N[σ3],
    Style["°", Bold], Style[", σ4 ≈ ", Bold], N[σ4], Style["°", Bold]}],
  Row[{Style["cosσ1 = ", Bold], Simplify@cosSigmaAlt[M1, f1, r1, s1],
    Style[", cosσ2 = ", Bold], Simplify@cosSigmaAlt[M1, f2, r2, s2],
    Style[", cosσ3 = ", Bold], Simplify@cosSigmaAlt[M1, f3, r3, s3],
    Style[", cosσ4 = ", Bold], Simplify@cosSigmaAlt[M1, f4, r4, s4]}],
  Row[{Style["f1 = ", Bold], f1, Style[", f2 = ", Bold],
    f2, Style[", f3 = ", Bold], f3, Style[", f4 = ", Bold], f4}],
  Row[{Style["z1 = ", Bold], FullSimplify[1 / (f1 - 1)], Style[", z2 = ", Bold],
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    FullSimplify[1 / (f2 - 1)], Style[", z3 = ", Bold], FullSimplify[1 / (f3 - 1)],
    Style[", z4 = ", Bold], FullSimplify[1 / (f4 - 1)]}],
  Row[{Style["x1 = ", Bold], FullSimplify[1 / (r1 - 1)], Style[", x2 = ", Bold],
    FullSimplify[1 / (r2 - 1)], Style[", x3 = ", Bold], FullSimplify[1 / (r3 - 1)],
    Style[", x4 = ", Bold], FullSimplify[1 / (r4 - 1)]}],
  Row[{Style["y1 = ", Bold], FullSimplify[1 / (s1 - 1)], Style[", y2 = ", Bold],
    FullSimplify[1 / (s2 - 1)], Style[", y3 = ", Bold], FullSimplify[1 / (s3 - 1)],
    Style[", y4 = ", Bold], FullSimplify[1 / (s4 - 1)]}],
  Row[{Style["p1 = ", Bold], Simplify[Sqrt[r1 - 1]],
    Style[", p2 = ", Bold], Simplify[Sqrt[r2 - 1]], Style[", p3 = ", Bold],
    Simplify[Sqrt[r3 - 1]], Style[", p4 = ", Bold], Simplify[Sqrt[r4 - 1]]}],
  Row[{Style["q1 = ", Bold], Simplify[Sqrt[s1 - 1]],
    Style[", q2 = ", Bold], Simplify[Sqrt[s2 - 1]], Style[", q3 = ", Bold],
    Simplify[Sqrt[s3 - 1]], Style[", q4 = ", Bold], Simplify[Sqrt[s4 - 1]]}],
  Row[{Style["p1·q1 = ", Bold], Simplify[Sqrt[r1 - 1] Sqrt[s1 - 1]],
    Style[", p2·q2 = ", Bold], Simplify[Sqrt[r2 - 1] Sqrt[s2 - 1]],
    Style[", p3·q3 = ", Bold], Simplify[Sqrt[r3 - 1] Sqrt[s3 - 1]],
    Style[", p4·q4 = ", Bold], Simplify[Sqrt[r4 - 1] Sqrt[s4 - 1]]}]
 }]


(*==============================================================================
 =====*)
(*==============================
  BRICARD's EQUATIONS===============================*)
(*==============================================================================
 =====*)
(*Bricard's equation as defined in terms of angles and variables*)
BricardsEquation[{α_, β_, γ_, δ_}, σ_, x_, y_] := Module[
   {c22, c20, c02, c11, c00},
   c22 = Sin[σ - δ] Sin[σ - δ - β];
   c20 = Sin[σ - α] Sin[σ - α - β];
   c02 = Sin[σ - γ] Sin[σ - γ - β];
   c11 = -Sin[α] Sin[γ];
   c00 = Sin[σ] Sin[σ - β];
   c22 x^2 y^2 + c20 x^2 + c02 y^2 + 2 c11 x y + c00
  ];

(*Step 1: Bricard's system of equations*)
Column[{(*Header*)TextCell[
   Style["================= Bricard's System of Equations =================",
    Red, Bold, 16], "Text"], (*Explanatory note*)
  Row[
   {TextCell[Style["We introduce new notation for the cotangents of half of
        the dihedral angles. Denote Z:= ", GrayLevel[0.3], 13],
     "Text"], TraditionalForm[cot[Subscript[θ, 1] / 2]], TextCell[
     Style[", W₂= ", GrayLevel[0.3], 13], "Text"],
```

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    TraditionalForm[cot[Subscript[θ, 2] / 2]],
    TextCell[Style[", U:= ", GrayLevel[0.3], 13], "Text"],
    TraditionalForm[cot[Subscript[θ, 3] / 2]],
    TextCell[Style[", and W₁= ", GrayLevel[0.3], 13], "Text"],
    TraditionalForm[cot[Subscript[θ, 4] / 2]]
   }], Spacer[12],
  (*Traditional form results*)Row[{"P₁(Z, W₁) = ",
    TraditionalForm[Collect[FullSimplify[BricardsEquation[anglesDeg⟦1⟧ Degree,
        sigmas⟦1⟧ Degree, Z, W₁]], W₁]], " = 0"}], Spacer[6],
  Row[{"P₂(Z, W₂) = ",
    TraditionalForm[Collect[FullSimplify[BricardsEquation[anglesDeg⟦2⟧ Degree,
        sigmas⟦2⟧ Degree, Z, W₂]], W₂]], " = 0"}], Spacer[6],
  Row[{"P₃(U, W₂) = ",
    TraditionalForm[Collect[FullSimplify[BricardsEquation[anglesDeg⟦3⟧ Degree,
        sigmas⟦3⟧ Degree, U, W₂]], W₂]], " = 0"}], Spacer[6],
  Row[{"P₄(U, W₁) = ", TraditionalForm[Collect[FullSimplify[BricardsEquation[
        anglesDeg⟦4⟧ Degree, sigmas⟦4⟧ Degree, U, W₁]], W₁]], " = 0"}]
 }]
(*==================================
 FLEXION 1=================================*)
Z[t_] := t;
```

$$W1[t\_] := \frac{6\,t - \sqrt{2\left(3\,t^2 - 1\right)\left(2 - 3\,t^2\right)}}{1 + 3\,t^2};$$

$$U[t\_] := \frac{1}{t};$$

$$W2[t\_] := \frac{5\sqrt{7}\,t - \sqrt{10\left(3\,t^2 - 1\right)\left(2 - 3\,t^2\right)}}{4 + 9\,t^2};$$

```
(*==================================
 FLEXION 2=================================*)
Z2[t_] := t;
```

$$W12[t\_] := \frac{6\,t + \sqrt{2\left(3\,t^2 - 1\right)\left(2 - 3\,t^2\right)}}{1 + 3\,t^2};$$

$$U2[t\_] := \frac{1}{t};$$

$$W22[t\_] := \frac{5\sqrt{7}\,t + \sqrt{10\left(3\,t^2 - 1\right)\left(2 - 3\,t^2\right)}}{4 + 9\,t^2};$$

```
(*Step 2: Formulas for flexions*)
Column[
 {(*Header*)TextCell[Style["================= FLEXIONS =================",
    Darker[Red], Bold, 16], "Text"], (*Explanatory note*)TextCell[Style[
```

```
    "Solutions to Bricard's equations under a free parameter t := Z ∈ ℂ:",
    GrayLevel[0.3], 13], "Text"], Spacer[12],
  (*Heading for results*)TextCell[Style["Solution 1:", Bold, 14], "Text"],
  Spacer[6], (*Traditional form results*)
  Row[{"Z(t) = ", TraditionalForm[FullSimplify[Z[t]]]}], Spacer[6],
  Row[{"W₂(t) = ", TraditionalForm[FullSimplify[W2[t]]]}], Spacer[6],
  Row[{"U(t) = ", TraditionalForm[FullSimplify[U[t]]]}], Spacer[6],
  Row[{"W₁(t) = ", TraditionalForm[FullSimplify[W1[t]]]}], Spacer[12],
  (*Heading for results*)TextCell[Style["Solution 2:", Bold, 14], "Text"],
  Spacer[6], (*Traditional form results*)
  Row[{"Z(t) = ", TraditionalForm[FullSimplify[Z2[t]]]}], Spacer[6],
  Row[{"W₂(t) = ", TraditionalForm[FullSimplify[W22[t]]]}], Spacer[6],
  Row[{"U(t) = ", TraditionalForm[FullSimplify[U2[t]]]}], Spacer[6],
  Row[{"W₁(t) = ", TraditionalForm[FullSimplify[W12[t]]]}]
 }]


(*Step 3: Compute and print all P_i for flexion 1*)
TextCell[
 Style["================= FLEXIBILITY (Double Checking) =================",
  Orange, Bold, 16], "Text"]
funcs = {{Z, W₁}, {Z, W₂}, {U, W₂}, {U, W₁}};
TextCell[Style["Solution 1:", Bold, 14], "Text"]
Do[angles = anglesDeg⟦i⟧ Degree;
  sigma = sigmas⟦i⟧ Degree;
  {α, β, γ, δ} = angles;
  poly = Which[i == 1, BricardsEquation[{α, β, γ, δ}, sigma, Z[t], W1[t]],
    i == 2, BricardsEquation[{α, β, γ, δ}, sigma, Z[t], W2[t]],
    i == 3, BricardsEquation[{α, β, γ, δ}, sigma, U[t], W2[t]],
    i == 4, BricardsEquation[{α, β, γ, δ}, sigma, U[t], W1[t]]];
  Print[Row[{Subscript["P", i], "(", funcs⟦i, 1⟧,
     ", ", funcs⟦i, 2⟧, ") = ", FullSimplify[poly]}]], {i, 1, 4}];

(*Compute and print all P_i for flexion 2*)
TextCell[Style["Solution 2:", Bold, 14], "Text"]
funcs = {{Z, W₁}, {Z, W₂}, {U, W₂}, {U, W₁}};
Do[angles = anglesDeg⟦i⟧ Degree;
  sigma = sigmas⟦i⟧ Degree;
  {α, β, γ, δ} = angles;
  poly = Which[i == 1, BricardsEquation[{α, β, γ, δ}, sigma, Z2[t], W12[t]],
    i == 2, BricardsEquation[{α, β, γ, δ}, sigma, Z2[t], W22[t]],
    i == 3, BricardsEquation[{α, β, γ, δ}, sigma, U2[t], W22[t]],
    i == 4, BricardsEquation[{α, β, γ, δ}, sigma, U2[t], W12[t]]];
  Print[Row[{Subscript["P", i], "(", funcs⟦i, 1⟧,
     ", ", funcs⟦i, 2⟧, ") = ", FullSimplify[poly]}]], {i, 1, 4}];
```

```
(*===========================================================================
 =====*)
(*=================== LINEAR COMPOUND ==================*)
(*===========================================================================
 =====*)
Column[{TextCell[Style["============= LINEAR COMPOUND =============",
    Darker[Orange], Bold, 16], "Text"],

   (*Explanatory text*)
  TextCell[Style["This configuration does belong to the linear compound class
      after switching the upper (or equivalently lower) boundary
      strip because Z U = 1 = const.", GrayLevel[0.3]], "Text"]
 }]

(*===========================================================================
 =====*)
(*===================================
 NOT TRIVIAL==================================*)
(*===========================================================================
 =====*)
(*Define domain limits for t*)
tMin = 1 / Sqrt[3];
tMax = Sqrt[2 / 3];

(*===================================
 FLEXION 1==================================*)
(*List of expressions to plot*)
expressions = {Z[t], W2[t], U[t], W1[t]};
labels = {"Z", "W₂", "U", "W₁"};

Column[{TextCell[
   Style["================= NOT TRIVIAL (FLEXION 1) =================",
    Darker[Cyan], Bold, 16], "Text"],

   (*Explanatory text*)
  TextCell[Style["This configuration does not belong to the trivial class –
      even after switching the boundary strips – since none of the
      functions Z, W₂, U, or W₁ is constant.", GrayLevel[0.3]], "Text"],
  Spacer[12],

   (*Plots in a light panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions〚i〛, {t, tMin, tMax},
       PlotLabel → Style[labels〚i〛, Bold, 14], PlotRange → All,
       AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
    Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]

 }]
```

```
(*=================================
 FLEXION 2=================================*)
(*List of expressions to plot*)
expressions = {Z2[t], W22[t], U2[t], W12[t]};
labels = {"Z", "W₂", "U", "W₁"};

Column[{TextCell[
   Style["================ NOT TRIVIAL (FLEXION 2) ================",
    Magenta, Bold, 16], "Text"],

  (*Explanatory text*)
  TextCell[Style["This configuration does not belong to the trivial class —
      even after switching the boundary strips — since none of the
      functions Z, W₂, U, or W₁ is constant.", GrayLevel[0.3]], "Text"],
  Spacer[12],

  (*Plots in a light panel*)
  Panel[GraphicsGrid[Partition[Table[Plot[expressions⟦i⟧, {t, tMin, tMax},
       PlotLabel → Style[labels⟦i⟧, Bold, 14], PlotRange → All,
       AxesLabel → {"t", None}, ImageSize → 250], {i, Length[expressions]}], 2],
    Spacings → {2, 2}], Background → Lighter[Gray, 0.95], FrameMargins → 15]

 }]


(*==========================================================================
 =====*)
(*==========================
 SWITCHING BOUNDARY STRIPS===========================*)
(*==========================================================================
 =====*)
SwitchingRightBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={1,2},{1,3},{4,2},{4,3}*)
  modified⟦1, 2⟧ = 180 - anglesDeg⟦1, 2⟧; (*β1*)
  modified⟦1, 3⟧ = 180 - anglesDeg⟦1, 3⟧; (*γ1*)
  modified⟦4, 2⟧ = 180 - anglesDeg⟦4, 2⟧; (*β4*)
  modified⟦4, 3⟧ = 180 - anglesDeg⟦4, 3⟧; (*γ4*)
  modified]

SwitchingLeftBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={2,2},{2,3},{3,2},{3,3}*)
  modified⟦2, 2⟧ = 180 - anglesDeg⟦2, 2⟧; (*β2*)
  modified⟦2, 3⟧ = 180 - anglesDeg⟦2, 3⟧; (*γ2*)
  modified⟦3, 2⟧ = 180 - anglesDeg⟦3, 2⟧; (*β3*)
  modified⟦3, 3⟧ = 180 - anglesDeg⟦3, 3⟧; (*γ3*)
  modified]
```

```
SwitchingLowerBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={1,1},{1,2},{2,1},{2,2}*)
  modified⟦1, 1⟧ = 180 - anglesDeg⟦1, 1⟧; (*α1*)
  modified⟦1, 2⟧ = 180 - anglesDeg⟦1, 2⟧; (*β1*)
  modified⟦2, 1⟧ = 180 - anglesDeg⟦2, 1⟧; (*α2*)
  modified⟦2, 2⟧ = 180 - anglesDeg⟦2, 2⟧; (*β2*)
  modified]

SwitchingUpperBoundaryStrip[anglesDeg_List] := Module[{modified = anglesDeg},
  (*Indices:{row,column}={3,1},{3,2},{4,1},{4,2}*)
  modified⟦3, 1⟧ = 180 - anglesDeg⟦3, 1⟧; (*α3*)
  modified⟦3, 2⟧ = 180 - anglesDeg⟦3, 2⟧; (*β3*)
  modified⟦4, 1⟧ = 180 - anglesDeg⟦4, 1⟧; (*α4*)
  modified⟦4, 2⟧ = 180 - anglesDeg⟦4, 2⟧; (*β4*)
  modified]


(*==========================================================================
 =====*)
(*==================================NOT CONIC & NOT CHIMERA & NOT LINEAR
     CONJUGATE & NOT ISOGONAL==================================*)
(*==========================================================================
 =====*)
Column[{TextCell[Style[
    "================= NOT CONIC & NOT CHIMERA & NOT LINEAR CONJUGATE & NOT
      ISOGONAL=================", Darker[Magenta], Bold, 16], "Text"],
  TextCell[Style[
    "Condition (N.0) is satisfied for all i=1,...,4 ⇒ NOT equimodular-conic,
      NOT chimera, NOT isogonal and NOT linear conjugate.
      Applying any boundary-strip switch still preserves
      (N.0), so no conic, no chimera, no isogonal and no
      linear conjugate form emerges.", GrayLevel[0.3]], "Text"]
 }]

(*Now the exact same Module for checking all switch combinations…*)
Module[{angles = anglesDeg, switchers, combinations, results},
 (*Define switch functions*)
 switchers = <|"Right" → SwitchingRightBoundaryStrip,
   "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
   "Upper" → SwitchingUpperBoundaryStrip|>;
 (*Generate all combinations of switches (from size 1 to 4)*)
 combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
 (*Evaluate condition after each combination of switches*)results = Table[
   Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];
    passQ = And @@ (checkConditionN0Degrees /@ switched);
```

```
    (*Print the result after switching*)
    (*Print["\nSwitch combination: ",name];
    Print["Switched anglesDeg:"];
    Print[MatrixForm[switched]];*)
    {name, passQ}], {combo, combinations}];
 (*Display results*)
 Column[Prepend[Table[Module[{comboName = res⟦1⟧, passQ = res⟦2⟧},
     Row[{Style[comboName <> ": ", Bold],
       If[passQ,
        Style["Condition (N.0) is still satisfied.", Darker[Green]],
        Style["Condition (N.0) fails.", Red, Bold]
       ]
      }
     ]], {res, results}], TextCell[
    Style["CONDITION (N.0) AFTER SWITCHING BOUNDARY STRIPS", 14], "Text"]]]]


(*=============================================================================
 =====*)
(*================================
 NOT ORTHODIAGONAL================================*)
(*=============================================================================
 =====*)

(*Column[
  {TextCell[Style["================= NOT ORTHODIAGONAL =================",
     Darker[Blue],Bold,16],"Text"],
   TextCell[Style[
     "cos(αi)·cos(γi) ≠ cos(βi)·cos(δi) for each i = 1 ⇒ NOT orthodiagonal.
       Switching boundary strips does not
       correct this.",  GrayLevel[0.3]],"Text"]
  }]

Module[{angles=anglesDeg,switchers,combinations,results},
  (*Define switch functions*)switchers=<|"Right"→SwitchingRightBoundaryStrip,
    "Left"→SwitchingLeftBoundaryStrip,"Lower"→SwitchingLowerBoundaryStrip,
    "Upper"→SwitchingUpperBoundaryStrip|>;
  (*Helper function:compute and print difference only*)
  formatOrthodiagonalCheck[quad_List]:=
   Module[{vals},vals=Table[Module[{a,b,c,d,lhs,rhs,diff},{a,b,c,d}=quad⟦i⟧;
       lhs=FullSimplify[Cos[a Degree] Cos[c Degree]];
       rhs=FullSimplify[Cos[b Degree] Cos[d Degree]];
       diff=Chop[lhs-rhs];
       Style[Row[{"cos(α"<>ToString[i]<>")·cos(γ"<>ToString[i]<>") - ",
          "cos(β"<>ToString[i]<>")·cos(δ"<>ToString[i]<>") = ",
          NumberForm[diff,{5,3}]}],If[diff==0,Red,Black]]],{i,Length[quad]}];
    Column[vals]];
```

```
  (*Orthodiagonal check for anglesDeg before any switching*)
  Print[TextCell[Style["\nInitial anglesDeg (no switches):",Bold]]];
  Print[MatrixForm[angles]];
  Print[TextCell[Style[
     "Orthodiagonal check: cos(αi)·cos(γi) - cos(βi)·cos(δi) for i = 1..4",
     Italic]]];
  Print[formatOrthodiagonalCheck[angles]];
  (*Generate all combinations of switches (from size 1 to 4)*)
  combinations=Subsets[Keys[switchers],{1,Length[switchers]}];
  (*Evaluate condition after each combination of switches*)results=
   Table[Module[{switched=angles,name,passQ},name=StringRiffle[combo," + "];
     Do[switched=switchers[sw][switched],{sw,combo}];
     passQ=And@@(checkConditionN0Degrees/@switched);
     Print[Style["\nSwitch combination: ", Bold],name];
     Print[Style["Switched anglesDeg:", Italic]];
     Print[MatrixForm[switched]];
     Print[
      TextCell[Style["Orthodiagonal check: cos(αi)·cos(γi) - cos(βi)·cos(δi)
          for i = 1..4",Italic]]];
     Print[formatOrthodiagonalCheck[switched]];
     {name,passQ}],{combo,combinations}];]*)


Column[
 {TextCell[Style["================= ORTHOGONALITY CHECK =================",
    Brown, Bold, 16], "Text"],
  TextCell[Style["cos(αi)·cos(γi) ≠ cos(βi)·cos(δi) for at least
      one i = 1,..., 4 ⇒ NOT orthodiagonal. Switching boundary
      strips does not correct this.", GrayLevel[0.3]], "Text"]}]

(*Helper
 function:Returns True if at least one cosine product difference is non-
   zero.Returns False if all differences are zero.*)
isNotOrthodiagonal[quad_List] :=
  Or @@ Table[Module[{a, b, c, d, diff}, {a, b, c, d} = quad[[i]];
     diff = Chop[Cos[a Degree] Cos[c Degree] - Cos[b Degree] Cos[d Degree]];
     diff ≠ 0], {i, Length[quad]}];

(*First,check the initial,unswitched angles*)
Print[TextCell[Style["\nInitial anglesDeg (no switches):", Bold]]];
If[isNotOrthodiagonal[anglesDeg], Print[Style[
    " -> Condition met: At least one difference is non-zero.", Darker@Green]],
  Print[Style[" -> Condition NOT met: All differences are zero.", Red]]];

(*Now,use your desired module to check all switch combinations*)
Module[{angles = anglesDeg, switchers, combinations, results},
 (*Define switch functions*)
```

```
switchers = <|"Right" → SwitchingRightBoundaryStrip,
   "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
   "Upper" → SwitchingUpperBoundaryStrip|>;
 (*Generate all combinations of switches (from size 1 to 4)*)
 combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
 (*Evaluate condition after each combination
  of switches and store in'results'*)results = Table[
   Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];
    (* ***THIS IS THE KEY CHANGE****)(*Set passQ using our
     new helper function*)passQ = isNotOrthodiagonal[switched];
    {name, passQ}], {combo, combinations}];
 (*Display results in the specified column format*)
 Column[Prepend[Table[Module[{comboName = res〚1〛, passQ = res〚2〛},
     Row[{Style[comboName <> ": ", Bold], If[passQ,
        Style["Condition met (at least one difference is non-zero).", Darker[
          Green]], Style["Condition NOT met (all differences are zero).",
         Red, Bold]]}]], {res, results}], TextCell[
    Style["\nNON-ORTHOGONALITY CHECK AFTER SWITCHING BOUNDARY STRIPS", 14],
    "Text"]]]]



(*==========================================================================
 =====*)
(*============================
   NOT CONJUGATE-MODULAR=============================*)
(*==========================================================================
 =====*)
(*Column[{TextCell[
     Style["================= NOT CONJUGATE-MODULAR =================",
      Purple,Bold,16],"Text"],
    TextCell[Style["M1 = M2 = M3 = M4 = M
        and M ≠ 2 ⇒ NOT conjugate-modular. Boundary-strip
        switches preserve this.",GrayLevel[0.3]],"Text"]
   }]
  Ms=FullSimplify[Times@@@results];
allEqualQ=Simplify[Equal@@Ms];

Module[{angles=anglesDeg,switchers,combinations,results,
  computeConjugateModularInfo},(*Define switch functions*)
 switchers=<|"Right"→SwitchingRightBoundaryStrip,
   "Left"→SwitchingLeftBoundaryStrip,"Lower"→SwitchingLowerBoundaryStrip,
   "Upper"→SwitchingUpperBoundaryStrip|>;
 (*Computes Mi and pi and prints them,
 with classification*)computeConjugateModularInfo[quad_List]:=
  Module[{abcdList,Ms,summary},abcdList=computeABCD/@quad;
```

```
    Ms=FullSimplify[Times@@@abcdList];
    summary=If[Simplify[Equal@@Ms]&&Ms[[1]]=!=2,
      Style["M1 = M2 = M3 = M4 = M and M ≠ 2",Bold],
      Style["M1 = M2 = M3 = M4 = M and M = 2",Red,Bold]];
    Column[{Style["Mi values:",Bold],Row[{"M1 = ",Ms[[1]],
        ", M2 = ",Ms[[2]],", M3 = ",Ms[[3]],", M4 = ",Ms[[4]]}],summary}]];
  (*Original anglesDeg check*)
  Print[
   TextCell[Style["\nInitial configuration (no switches applied):",Bold]]];
  Print[MatrixForm[angles]];
  Print[computeConjugateModularInfo[angles]];
  (*Generate all switch combinations (from size 1 to 4)*)
  combinations=Subsets[Keys[switchers],{1,Length[switchers]}];
  (*Evaluate each switched configuration*)results=
   Table[Module[{switched=angles,name,passQ},name=StringRiffle[combo," + "];
     Do[switched=switchers[sw][switched],{sw,combo}];
     passQ=And@@(checkConditionN0Degrees/@switched);
     Print[Style["\nSwitch combination: ", Bold],name];
     Print[Style["Switched anglesDeg:", Italic]];
     Print[MatrixForm[switched]];
     Print[computeConjugateModularInfo[switched]];
     {name,passQ}],{combo,combinations}];]*)


Column[{TextCell[
    Style["================= CONJUGATE-MODULAR CHECK =================",
     Darker[Brown], Bold, 16], "Text"],
   TextCell[Style["M1 = M2 = M3 = M4 = M and M ≠ 2 ⇒ NOT conjugate-modular.
       Boundary-strip switches preserve this.", GrayLevel[0.3]], "Text"]}]

(*Helper Function:Returns True if all M_i values are equal
    AND their common value is not 2. Returns False otherwise.*)
isNotConjugateModular[quad_List] :=
   Module[{abcdList, Ms}, abcdList = computeABCD /@ quad;
    Ms = FullSimplify[Times @@@ abcdList];
    (*The condition is met if they are all equal AND the value isn't 2*)
    Simplify[Equal @@ Ms] && (Ms[[1]] ≠ 2)];

(*First,check the initial,unswitched angles*)
Print[TextCell[Style["\nInitial anglesDeg (no switches):", Bold]]];
If[isNotConjugateModular[anglesDeg], Print[
    Style[" -> Condition met: All Mi are equal and M ≠ 2.", Darker@Green]],
   Print[Style[" -> Condition NOT met.", Red]]];

(*Now,use the clean module to check all switch combinations*)
Module[{angles = anglesDeg, switchers, combinations, results},
  (*Define switch functions*)
```

```
switchers = <|"Right" → SwitchingRightBoundaryStrip,
   "Left" → SwitchingLeftBoundaryStrip, "Lower" → SwitchingLowerBoundaryStrip,
   "Upper" → SwitchingUpperBoundaryStrip|>;
(*Generate all combinations of switches (from size 1 to 4)*)
combinations = Subsets[Keys[switchers], {1, Length[switchers]}];
(*Evaluate condition after each combination
 of switches and store the result*)results = Table[
   Module[{switched = angles, name, passQ}, name = StringRiffle[combo, " + "];
    Do[switched = switchers[sw][switched], {sw, combo}];
    (*Set passQ using our new helper function for this check*)
    passQ = isNotConjugateModular[switched];
    {name, passQ}], {combo, combinations}];
(*Display results in the specified column format*)
Column[Prepend[Table[Module[{comboName = res[[1]], passQ = res[[2]]},
     Row[{Style[comboName <> ": ", Bold], If[passQ,
        Style["Condition met (All Mᵢ are equal and M ≠ 2).", Darker[Green]],
        Style["Condition NOT met.", Red, Bold]]}]], {res, results}],
   TextCell[Style["\nCONJUGATE-MODULAR CHECK AFTER SWITCHING BOUNDARY STRIPS",
     14], "Text"]]]
```

Out[16]=

================= CONDITION (N.0) =================

✔ All vertices satisfy (N.0).

Out[19]=

================= CONDITION (N.3) =================

✔ M1 = M2 = M3 = M4 = $\frac{1}{2}$

Out[25]=

================= CONDITION (N.4) =================

✔ r1 = r2 = $\frac{4}{3}$**; ✔ r3 = r4 =** $\frac{5}{2}$

✔ s1 = s4 = 3**; ✔ s2 = s3 =** $\frac{11}{6}$

Out[35]=

================= CONDITION (N.5) =================

⚠ Approximate validation using
ε-tolerance. For rigorous proof, see the referenced paper.

✅ Valid Combination Found (M < 1):

e1 = 1, **e2** = 1, **e3** = 1

t1 = 0.K + 0.554485iK'

t2 = 0.K + 0.509302iK'

t3 = 0.K + 0.490698iK'

t4 = 0.K + 0.445515iK'

t1 + e1*t2 + e2*t3 + e3*t4 = 0.K + 2.iK'

Out[38]=

================== OTHER PARAMETERS ==================

$$u = \frac{1}{2}$$

$$\sigma 1 = 135\,{}^\circ,\ \sigma 2 = {}^\circ\left(135 + \frac{90\,\mathrm{ArcCos}\left[\frac{15\sqrt{7}}{44}\right]}{\pi}\right)$$

$$,\ \sigma 3 = \frac{90\,{}^\circ\left(\pi + \mathrm{ArcTan}\left[\frac{4\sqrt{7}}{3}\right]\right)}{\pi},\ \sigma 4 = {}^\circ\left(135 - \frac{45\,\mathrm{ArcTan}\left[\frac{24}{7}\right]}{\pi}\right)$$

$$\sigma 1 \approx 135.{}^\circ,\ \sigma 2 \approx 147.792{}^\circ,\ \sigma 3 \approx 127.087{}^\circ,\ \sigma 4 \approx 116.565{}^\circ$$

$$\cos\sigma 1 = -\frac{1}{\sqrt{2}},\ \cos\sigma 2 = -\frac{3\sqrt{\frac{7}{22}}}{2},\ \cos\sigma 3 = -\frac{2}{\sqrt{11}},\ \cos\sigma 4 = -\frac{1}{\sqrt{5}}$$

$$f1 = 2,\ f2 = \frac{7}{4},\ f3 = \frac{5}{3},\ f4 = \frac{3}{2}$$

$$z1 = 1,\ z2 = \frac{4}{3},\ z3 = \frac{3}{2},\ z4 = 2$$

$$x1 = 3,\ x2 = 3,\ x3 = \frac{2}{3},\ x4 = \frac{2}{3}$$

$$y1 = \frac{1}{2},\ y2 = \frac{6}{5},\ y3 = \frac{6}{5},\ y4 = \frac{1}{2}$$

$$p1 = \frac{1}{\sqrt{3}},\ p2 = \frac{1}{\sqrt{3}},\ p3 = \sqrt{\frac{3}{2}},\ p4 = \sqrt{\frac{3}{2}}$$

$$q1 = \sqrt{2},\ q2 = \sqrt{\frac{5}{6}},\ q3 = \sqrt{\frac{5}{6}},\ q4 = \sqrt{2}$$

$$p1\cdot q1 = \sqrt{\frac{2}{3}},\ p2\cdot q2 = \frac{\sqrt{\frac{5}{2}}}{3},\ p3\cdot q3 = \frac{\sqrt{5}}{2},\ p4\cdot q4 = \sqrt{3}$$

Out[40]=

================== Bricard's System of Equations ==================

We introduce new notation for the cotangents of half of the dihedral angles. Denote $Z := \cot\left(\frac{\theta_1}{2}\right)$, $W_2 = \cot\left(\frac{\theta_2}{2}\right)$, $U := \cot\left(\frac{\theta_3}{2}\right)$, and $W_1 = \cot\left(\frac{\theta_4}{2}\right)$

$$P_1(Z, W_1) = \frac{W_1^2\left(3Z^2+1\right)}{5\sqrt{2}} - \frac{6}{5}\sqrt{2}\,W_1 Z + \frac{6Z^2+4}{5\sqrt{2}} = 0$$

$$P_2(Z, W_2) = \frac{1}{8}\sqrt{\frac{7}{22}}\,W_2^2\left(9Z^2+4\right) - \frac{35 W_2 Z}{4\sqrt{22}} + \frac{5}{8}\sqrt{\frac{7}{22}}\left(2Z^2+1\right) = 0$$

$$P_3(U, W_2) = \frac{1}{8}\sqrt{\frac{7}{22}}\left(4U^2+9\right)W_2^2 + \frac{5}{8}\sqrt{\frac{7}{22}}\left(U^2+2\right) - \frac{35\,U W_2}{4\sqrt{22}} = 0$$

$$P_4(U, W_1) = \frac{\left(U^2+3\right)W_1^2}{5\sqrt{2}} + \frac{4U^2+6}{5\sqrt{2}} - \frac{6}{5}\sqrt{2}\,U W_1 = 0$$

Out[49]=

================= **FLEXIONS** =================

Solutions to Bricard's equations under a free parameter t := Z ∈ ℂ:

Solution 1:

$Z(t) = t$

$W_2(t) = \frac{5\sqrt{7}\,t - \sqrt{10}\,\sqrt{-9t^4+9t^2-2}}{9t^2+4}$

$U(t) = \frac{1}{t}$

$W_1(t) = \frac{6t - \sqrt{2}\,\sqrt{-9t^4+9t^2-2}}{3t^2+1}$

Solution 2:

$Z(t) = t$

$W_2(t) = \frac{\sqrt{10}\,\sqrt{-9t^4+9t^2-2} + 5\sqrt{7}\,t}{9t^2+4}$

$U(t) = \frac{1}{t}$

$W_1(t) = \frac{\sqrt{2}\,\sqrt{-9t^4+9t^2-2} + 6t}{3t^2+1}$

Out[50]=

================= **FLEXIBILITY (Double Checking)** =================

Out[52]=

Solution 1:

$P_1(Z, W_1) = 0$

$P_2(Z, W_2) = 0$

$P_3(U, W_2) = 0$

$P_4(U, W_1) = 0$

Out[54]=

Solution 2:

$P_1(Z, W_1) = 0$

$P_2(Z, W_2) = 0$

$P_3(U, W_2) = 0$

$P_4(U, W_1) = 0$

Out[57]=

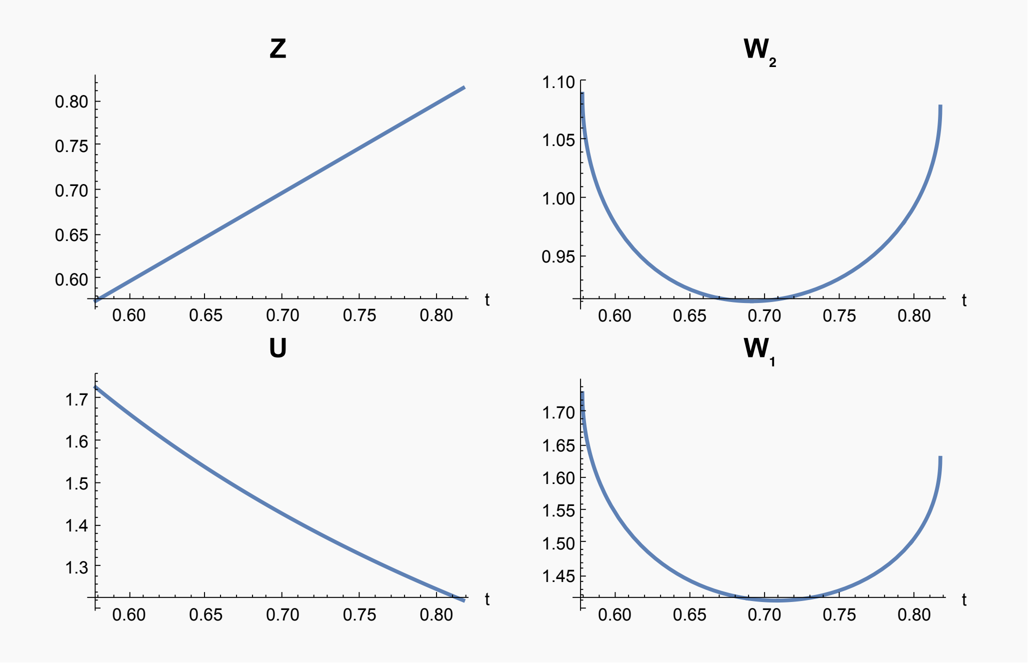
============= **LINEAR COMPOUND** =============

This configuration does belong to the linear compound class after switching the upper (or equivalently lower) boundary strip because Z U = 1 = const.

Out[62]=

================= **NOT TRIVIAL (FLEXION 1)** =================

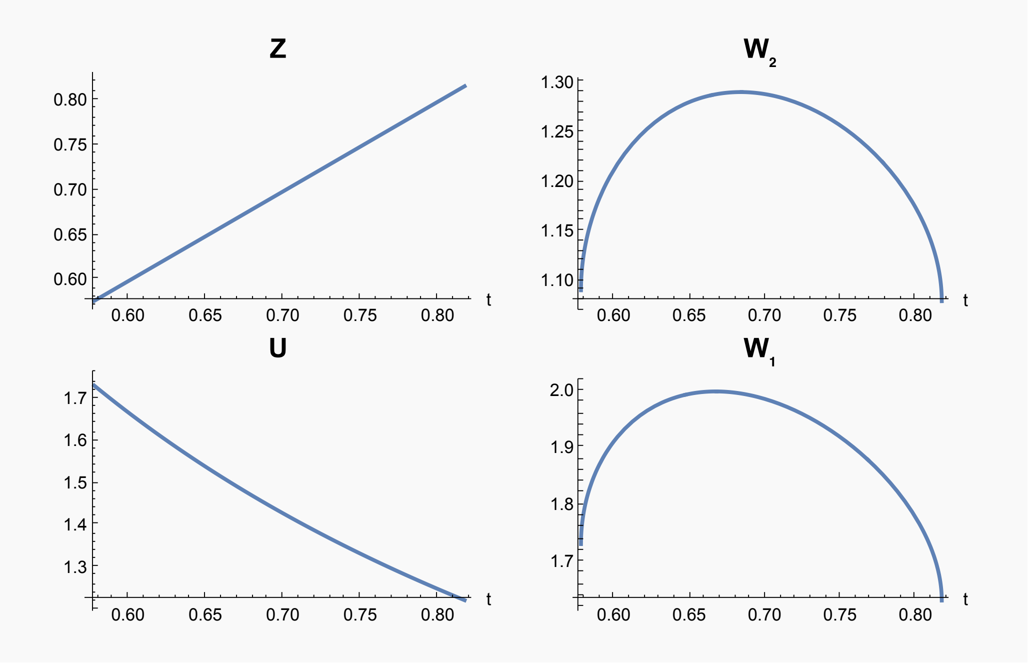
This configuration does not belong to the trivial class – even after switching the boundary strips – since none of the functions Z, W_2, U, or W_1 is constant.



Out[65]=

================= NOT TRIVIAL (FLEXION 2) =================

This configuration does not belong to the trivial class — even after switching the boundary strips — since none of the functions Z, W_2, U, or W_1 is constant.



Out[70]=

================= NOT CONIC & NOT CHIMERA & NOT LINEAR CONJUGATE & NOT ISOGONAL=================

Condition (N.0) is satisfied for all i=1,...,4 ⇒ NOT equimodular-conic, NOT chimera, NOT isogonal and NOT linear conjugate. Applying any boundary-strip switch still preserves (N.0), so no conic, no chimera, no isogonal and no linear conjugate form emerges.

Out[71]=

CONDITION (N.0) AFTER SWITCHING BOUNDARY STRIPS
Right: Condition (N.0) is still satisfied.
Left: Condition (N.0) is still satisfied.
Lower: Condition (N.0) is still satisfied.
Upper: Condition (N.0) is still satisfied.
Right + Left: Condition (N.0) is still satisfied.
Right + Lower: Condition (N.0) is still satisfied.
Right + Upper: Condition (N.0) is still satisfied.
Left + Lower: Condition (N.0) is still satisfied.
Left + Upper: Condition (N.0) is still satisfied.
Lower + Upper: Condition (N.0) is still satisfied.
Right + Left + Lower: Condition (N.0) is still satisfied.
Right + Left + Upper: Condition (N.0) is still satisfied.
Right + Lower + Upper: Condition (N.0) is still satisfied.
Left + Lower + Upper: Condition (N.0) is still satisfied.
Right + Left + Lower + Upper: Condition (N.0) is still satisfied.

Out[72]=

================== **ORTHOGONALITY CHECK** ==================

$\cos(\alpha_i)\cdot\cos(\gamma_i) \neq \cos(\beta_i)\cdot\cos(\delta_i)$ for at least one i = 1,..., 4 ⇒ NOT orthodiagonal. Switching boundary strips does not correct this.

Initial anglesDeg (no switches):

-> Condition met: At least one difference is non-zero.

Out[76]=

NON-ORTHOGONALITY CHECK AFTER SWITCHING BOUNDARY STRIPS
Right: Condition met (at least one difference is non-zero).
Left: Condition met (at least one difference is non-zero).
Lower: Condition met (at least one difference is non-zero).
Upper: Condition met (at least one difference is non-zero).
Right + Left: Condition met (at least one difference is non-zero).
Right + Lower: Condition met (at least one difference is non-zero).
Right + Upper: Condition met (at least one difference is non-zero).
Left + Lower: Condition met (at least one difference is non-zero).
Left + Upper: Condition met (at least one difference is non-zero).
Lower + Upper: Condition met (at least one difference is non-zero).
Right + Left + Lower: Condition met (at least one difference is non-zero).
Right + Left + Upper: Condition met (at least one difference is non-zero).
Right + Lower + Upper: Condition met (at least one difference is non-zero).
Left + Lower + Upper: Condition met (at least one difference is non-zero).
Right + Left + Lower + Upper:
Condition met (at least one difference is non-zero).

Out[77]=

================== **CONJUGATE-MODULAR CHECK** ==================

M1 = M2 = M3 = M4 = M and M ≠ 2 ⇒ NOT conjugate-modular. Boundary-strip switches preserve this.

Initial anglesDeg (no switches):

-> Condition met: All M_i are equal and $M \neq 2$.

Out[81]=

CONJUGATE−MODULAR CHECK AFTER SWITCHING BOUNDARY STRIPS

Right: Condition met (All M_i are equal and $M \neq 2$).
Left: Condition met (All M_i are equal and $M \neq 2$).
Lower: Condition met (All M_i are equal and $M \neq 2$).
Upper: Condition met (All M_i are equal and $M \neq 2$).
Right + Left: Condition met (All M_i are equal and $M \neq 2$).
Right + Lower: Condition met (All M_i are equal and $M \neq 2$).
Right + Upper: Condition met (All M_i are equal and $M \neq 2$).
Left + Lower: Condition met (All M_i are equal and $M \neq 2$).
Left + Upper: Condition met (All M_i are equal and $M \neq 2$).
Lower + Upper: Condition met (All M_i are equal and $M \neq 2$).
Right + Left + Lower: Condition met (All M_i are equal and $M \neq 2$).
Right + Left + Upper: Condition met (All M_i are equal and $M \neq 2$).
Right + Lower + Upper: Condition met (All M_i are equal and $M \neq 2$).
Left + Lower + Upper: Condition met (All M_i are equal and $M \neq 2$).
Right + Left + Lower + Upper: Condition met (All M_i are equal and $M \neq 2$).