$$\hat{\beta} = (X^T X)^{-1} X^T Y$$

$$\hat{\beta}_{(i)} = (X_{(i)}{}^T X_{(i)})^{-1} X_{(i)}{}^T Y_{(i)}$$

$$= (X^T X - x_{(i)} x_{(i)}{}^T)^{-1} (X^T Y - x_i y_i)$$

$$= \left[(X^T X)^{-1} + \frac{(X^T X)^{-1} x_{(i)} x_{(i)}{}^T (X^T X)^{-1}}{1 - x_{(i)} (X^T X)^{-1} x_{(i)}{}^T} \right] (X^T Y - x_i y_i)$$

$$= (X^T X)^{-1} X^T Y - (X^T X)^{-1} x_i y_i + \frac{(X^T X)^{-1} x_{(i)} x_{(i)}{}^T (X^T X)^{-1} X^T Y}{1 - w_{ii}} - \frac{(X^T X)^{-1} x_{(i)} x_{(i)}{}^T (X^T X)^{-1} x_i y_i}{1 - w_{ii}}$$

$$= \hat{\beta} - (X^T X)^{-1} x_i y_i + \frac{(X^T X)^{-1} x_{(i)} x_{(i)}{}^T \hat{\beta}}{1 - w_{ii}} - \frac{(X^T X)^{-1} x_{(i)} w_{ii} y_i}{1 - w_{ii}}$$

$$\Rightarrow \hat{\beta} - \hat{\beta}_{(i)} = (X^T X)^{-1} x_i y_i - \frac{(X^T X)^{-1} x_{(i)} x_{(i)}{}^T \hat{\beta}}{1 - w_{ii}} + \frac{(X^T X)^{-1} x_{(i)} w_{ii} y_i}{1 - w_{ii}}$$

$$= \left(1 + \frac{w_{ii}}{1 - w_{ii}}\right) (X^T X)^{-1} x_i y_i - \frac{(X^T X)^{-1} x_{(i)} x_{(i)}{}^T \hat{\beta}}{1 - w_{ii}}$$

$$= \frac{1}{1 - w_{ii}} \left[(X^T X)^{-1} x_i y_i - (X^T X)^{-1} x_{(i)} x_{(i)}{}^T \hat{\beta} \right]$$

$$= \frac{1}{1 - w_{ii}} \left[(X^T X)^{-1} x_i \left\{ y_i - x_{(i)}{}^T \hat{\beta} \right\} \right]$$

$$= \frac{(X^T X)^{-1} x_i e_i}{1 - w_{ii}}$$

$$\mathrm{CD}_i = \frac{\left(\hat{\beta}_{(i)} - \hat{\beta}\right)^T X^T X \left(\hat{\beta}_{(i)} - \hat{\beta}\right)}{pMS_{\mathrm{Res}}}$$

$$= \frac{e_i^T x_i^T (X^T X)^{-1} X^T X (X^T X)^{-1} x_i e_i}{(1 - w_{ii})(1 - w_{ii})} \frac{1}{pMS_{\mathrm{Res}}}$$

$$= \frac{e_i^T x_i^T (X^T X)^{-1} x_i e_i}{(1 - w_{ii})^2} \frac{1}{pMS_{\mathrm{Res}}}$$

$$= \frac{e_i^2}{(1 - w_{ii})^2} \frac{w_{ii}}{pMS_{\mathrm{Res}}}$$

$$= \left(\frac{e_i}{1 - w_{ii}}\right)^2 \frac{w_{ii}}{pMS_{\mathrm{Res}}}$$

$$= \frac{r_i^2}{p} \left(\frac{w_{ii}}{1 - w_{ii}}\right)$$