CSE232 Assignment 1

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09/10/2018

1. (a) $A = \{x^2 | x \in N \wedge x \leq 5\}$
   (b) $B = \{(x, y) | x, y \in N \wedge (x + y) < 3\}$

2. (a) $A = \{1\}, B = \{1, 1\}$
   It is true that $A \subseteq B$ and $B \subseteq A$. Thus, $A = B$.
   (b) $A = \{1, 2\}, B = \{2, 1\}$
   It is true that $A \subseteq B$ and $B \subseteq A$. Thus, $A = B$.
   (c) $A = \{1, 2\}, B = (1, 2)$
   Obviously $A \neq B$ because A is a set. Whereas, B is not.

3. Prove that $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$

   Suppose $A = \{x | x \in A \wedge x \notin B\}$ and the size of it is n. Moreover, let's say that the size of the set B is m. Then it is obvious that the size of the set $A \cup B$ should equal $n + m$.
   Now, as the size of powerset of a set is always 2 to the power the size of that set, we can see that $\mathcal{P}(A \cup B)$ has $2^{(n+m)}$ elements. While, $\mathcal{P}(A) \cup \mathcal{P}(B)$ has at most $2^n + 2^m$ elements. Hence, the equation is not always true.

4. (a) Please refer to the handwritten sample attached to the paper.
   (b) $(A - B) \cup (B - A) = (A \cup B) \cap (\overline{A} \cup \overline{B})$
   $(A - B) \cup (B - A) = (A - B) \cup (B \cap \overline{A})$
   $= (A \cap \overline{B}) \cup (B \cap \overline{A})$
   $= ((A \cap \overline{B}) \cup B) \cap ((A \cap \overline{B}) \cup \overline{A})$
   $= (A \cup B) \cap (\overline{B} \cup B) \cap ((A \cup \overline{A}) \cap (\overline{A} \cup \overline{B})) = (A \cup B) \cap (\overline{A} \cup \overline{B})$
   q.e.d.

p	q	$p \wedge \neg q$	$\neg p \wedge q$	$p \vee q$	$\neg p \vee \neg q$	$(p \wedge \neg q) \vee (\neg p \wedge q)$	$(p \vee q) \wedge (\neg p \wedge \neg q)$
T	T	F	F	T	F	F	F
T	F	T	F	T	T	T	T
F	T	F	T	T	T	T	T
F	F	F	F	F	T	F	F

5. $A \times B = B \times A$ is true if one of the following is satisfied.
   (a) $A = B$
   (b) $A = \emptyset$ or $B = \emptyset$
6. $(p \wedge (q \oplus r)) \vee (q \wedge (p \oplus r)) \vee (r \wedge (q \oplus p))$
7. (a) Please refer to the table at the top of this page.
   (b) $(p \wedge \neg q) \vee (\neg p \wedge q) \equiv (p \vee q) \wedge (\neg p \vee \neg q)$
   Distributive [1]: $(p \wedge \neg q) \vee (\neg p \wedge q) \equiv (\neg p \vee (p \wedge \neg q)) \wedge (q \vee (p \wedge \neg q)) \equiv$
   Distributive [2]: $((\neg p \vee p) \wedge (\neg p \vee \neg q)) \wedge ((q \vee p) \wedge (q \vee \neg q)) \equiv$
   $(\neg p \vee \neg q) \wedge (p \vee q)$
   q.e.d.
8. $\forall x \forall y \exists z (P(x, z) \wedge P(y, z) \vee P(x, y))$
   as $\forall x P(x, x) \equiv True$, thus $\forall x \forall y \exists z (P(x, z) \wedge P(y, z))$ is sufficient for this problem.
9. $\neg \forall x \forall y (P(x, y) \vee Q(x, y)) \equiv \exists x \exists y \neg (P(x, t) \vee Q(x, y))$
   Via De Morgan's law it's $\equiv \exists x \exists y (\neg P(x, y) \wedge \neg Q(x, y))$