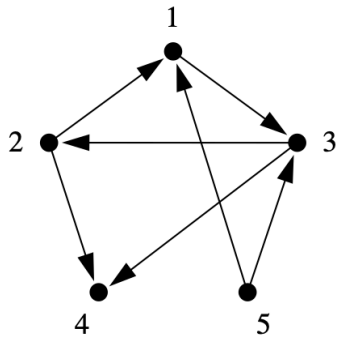
Homework Assignment # 2
Due on April 22(Wed), 18:00

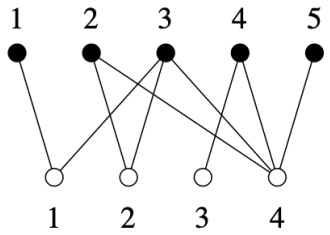
Direction: Write all relevant work clearly and concisely in English. Use the python for computational problems.

- Submit a PDF of your homework with all codes. Upload that files including all codes through BB with a file name in your name, for example, bongsoo-hw3.zip..

1. Consider the following two networks((a) a directed network and (b) an undirected network):



(a)



(b)

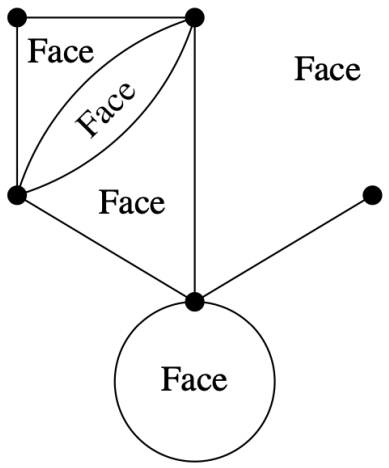
   - Find the adjacent matrix of network (a)
   - Find the incident matrix of network (b)
   - Find the adjacent matrix for the network generated when we project the network (b) into its black vertices.

2. Demonstrate or prove the followings for undirected networks:

   - A 3-regular graph must have an even number of nodes.
   - The average degree of a tree is strictly less than 2.

3. Consider a network which is simple (it contains no multiedges or self-edges) and consists of n nodes in a single component. What is the maximum possible number of edges it could have? What is the minimum possible edges if could have? Explain how you give the answer by providing the corresponding figures of networks.

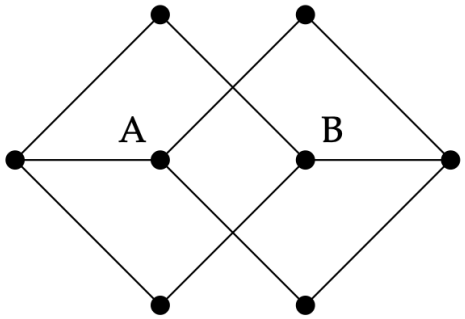
4. Consider a connected planar network with n vertices and m edges. Let f be the number of "faces" of the network, i.e., areas bounded by edges when the network is drawn in planar form. The "outside" of the network, the area extending to infinity on all sides, is also considered a face. The network can have multiedges and self-edges:



   - How do n, m, and f change when we add a single vertex to such a network along with a single edge attaching it to an existing vertex?
   - How do n, m, and f change when we add a single edge between two existing vertices (or a self-edge attached to just one vertex), in such a way as to maintain planarity of the network?
   - What are the values of n, m, and f for a network with a single vertex and no edges?
   - Hence by induction prove a general relation between n, m, and f for all connected planar networks.
   - Now suppose that our network is simple. Show that the mean degree c of a simple, connected, planar network is strictly less than six.

5. What is the difference between a 2-component and a 2-core? Draw a small network that has one 2-core but two 2-components.

6. Show that the edge connectivity of nodes A and B in the network is 2:



(Hint: A correct proof must show both that the connectivity is at least 2 and that it is no more than 2.)