CSE232: Discrete Mathematics
Assignment 3

Antoine Vigneron

November 2, 2018

This homework assignment is due on Tuesday 11/13, 13:00, at the beginning of the lecture. **Please include your name and student ID.** Each question or subquestion is worth 10 marks. So the total is 100 marks. You should follow the academic integrity rules for CSE232 that are described at the end of the slides of Lecture 0.

---

1. Does there exist a graph with vertex set $\{a, b, c, d\}$ such that $\deg(a) = \deg(b) = 3$ and $\deg(c) = \deg(d) = 2$?

2. Does there exist a graph with vertex set $\{a, b, c, d\}$ such that $\deg(a) = \deg(b) = \deg(c) = 3$ and $\deg(d) = 2$?

3. For which values of n is the cycle C_n bipartite? Justify your answer.

4. Prove by induction that

$$\sum_{k=1}^{n} 3k^2 - 3k + 1 = n^3$$

whenever $n \geqslant 1$.

5. In Lecture 14, we proved by structural induction that $n(T) \leqslant 2^{h(T)+1} - 1$ for any full binary tree T, where $n(T)$ is the number of vertices of T and $h(T)$ is its height.

Give a different proof using strong induction on $h(T)$.

6. Let X be a random variable over a sample space S, such that $X(s) \geqslant 0$ for all $s \in S$. Prove that

$$p(X \geqslant a) \leqslant \frac{E(X)}{a}$$

for every $a > 0$.

7. The waiting room of Doctor Smith has 6 seats, numbered from 1 to 6. As Doctor Smith is popular, the waiting room is always full, so there is a patient seating on each seat. Every ten minutes, Doctor Smith rolls a die, thus obtaining a number $i \in \{1, 2, \ldots, 6\}$, and calls the patient seating on seat i. This patient is immediately replaced with a new patient.

(a) You just took a seat in Doctor Smith's waiting room. How much time should you expect to wait until he calls you? Justify your answer.

(b) Prove that the probability that you wait for more than 3 hours is at most 1/3.

8. During the midterm and the final exam of the Discrete Mathematics course, each student is assigned a seat at random. Suppose that there are n seats in the exam room, and that n students are enrolled. Let S be number of students who get the same seat at the midterm and the final exam.

(a) Determine $E(S)$, and justify your answer.

(b) Determine $V(S)$, and justify your answer.

(**Hint**: Introduce the random variable X_i such that $X_i = 1$ if student i does not change seat, and $X_i = 0$ otherwise.)