Lecture 15:
Red-Black Trees

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Red-Black Trees

- Another self-balancing binary search tree
- Guarantee O(log n) insertion, search, delete
- Definition
  - Binary search tree that every node is colored either red or black
  - Leaf nodes do not contain data
    - External nodes
    - i.e., every node has either 0 or 2 children.

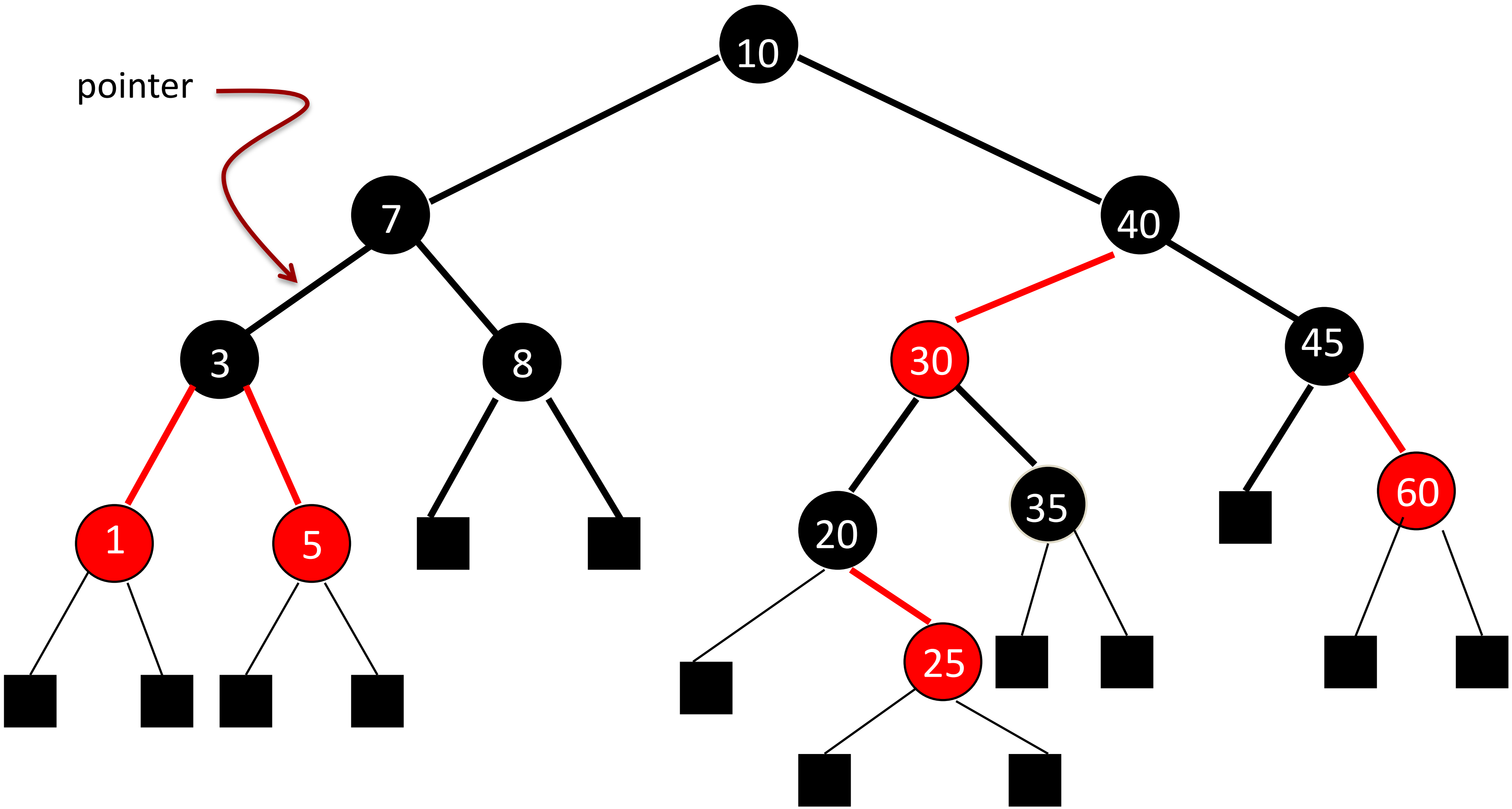
Red-Black Tree Properties

- The root and all external nodes are <u>black</u>
- No root-to-external-node path has two consecutive red nodes
  - (=) Red node must have two black children
- All root-to-external-node paths have the same number of black nodes

Red-Black Tree Properties (Pointer)

- Pointers from an internal node to an external node are black
- No root-to-external-node path has two consecutive red pointers
- All root-to-external-node paths have the same number of black pointers

Example Red-Black Tree



Properties

- If P and Q are two root-to-external-node paths in a red-black tree, then

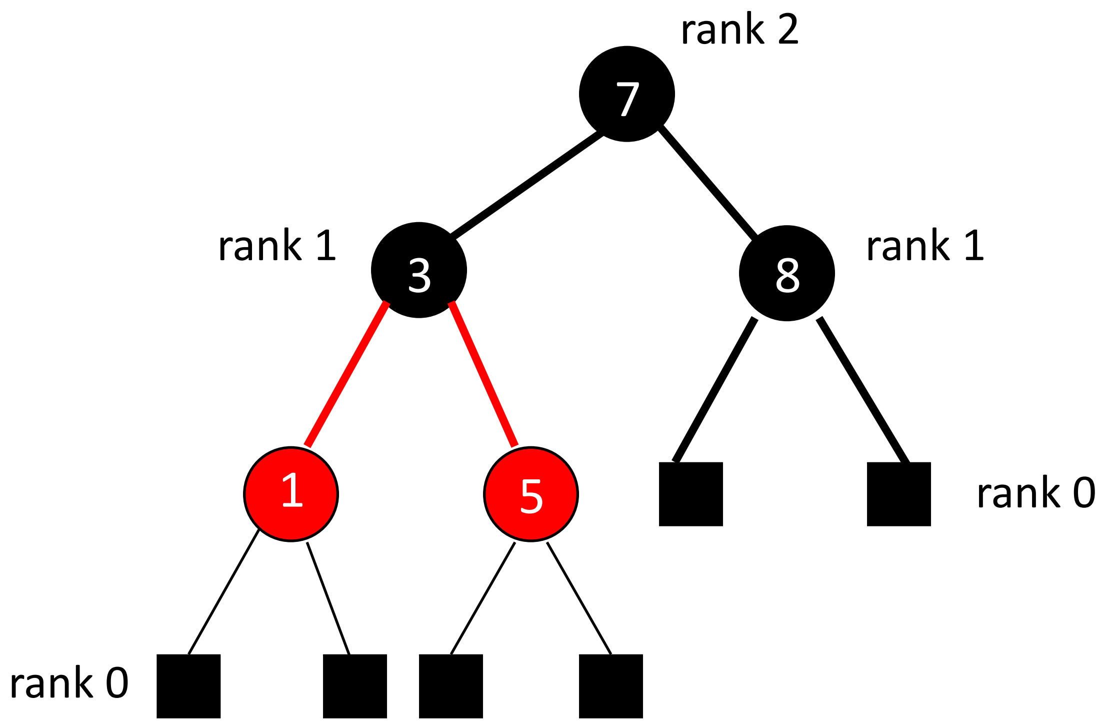
$$Length_P \leq 2Length_Q$$

i.e., $Length_{Longest} \leq 2Length_{Shortest}$

  – Shortest path : B-B-B-....-B
  – Longest path : B-R-B-R...-B
  – Number of B must be same for all paths by definition

Properties

- Rank : # of black edges on any path from a node to any external node ($= L_{Shortest}$)



Properties: $h = O(\log n)$

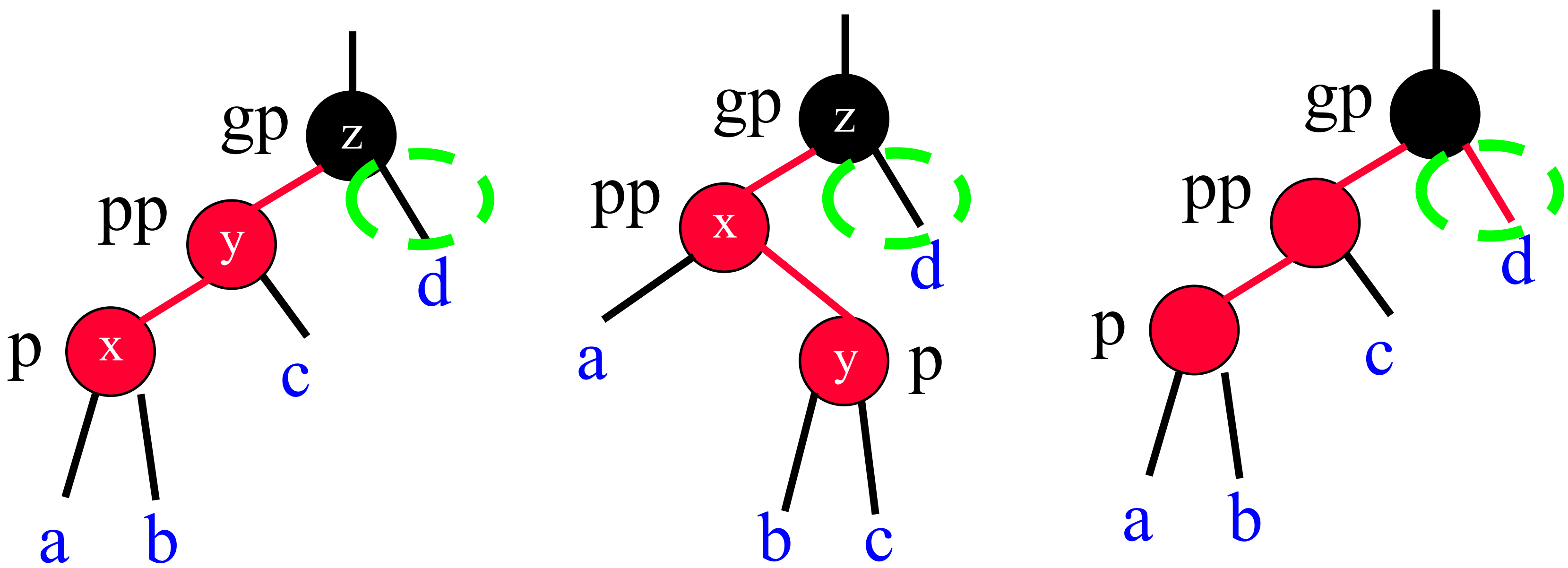
- h : height, r : rank of the root, n : # of nodes
- $h \leq 2r$
  - Discussed earlier.
- $n \geq 2^r - 1$
  - When all nodes are black.

- $h \leq 2\log_2(n+1) \Rightarrow h = O(\log n)$

Inserting

- Just insert and then color
- How to color a new node?
  - If the tree was <u>empty</u>, new node is root so assign <u>black</u>
  - If the tree was not empty, assign black causes increase one black node in the path : NO!
    - b/c violate same # of black nodes for all paths, difficult resolve
  - If the tree was <u>not empty</u>, assign <u>red</u> may cause two consecutive red nodes in the path : OK!
    - Can be resolved by <u>rotation and color flips</u>

Possible consecutive reds

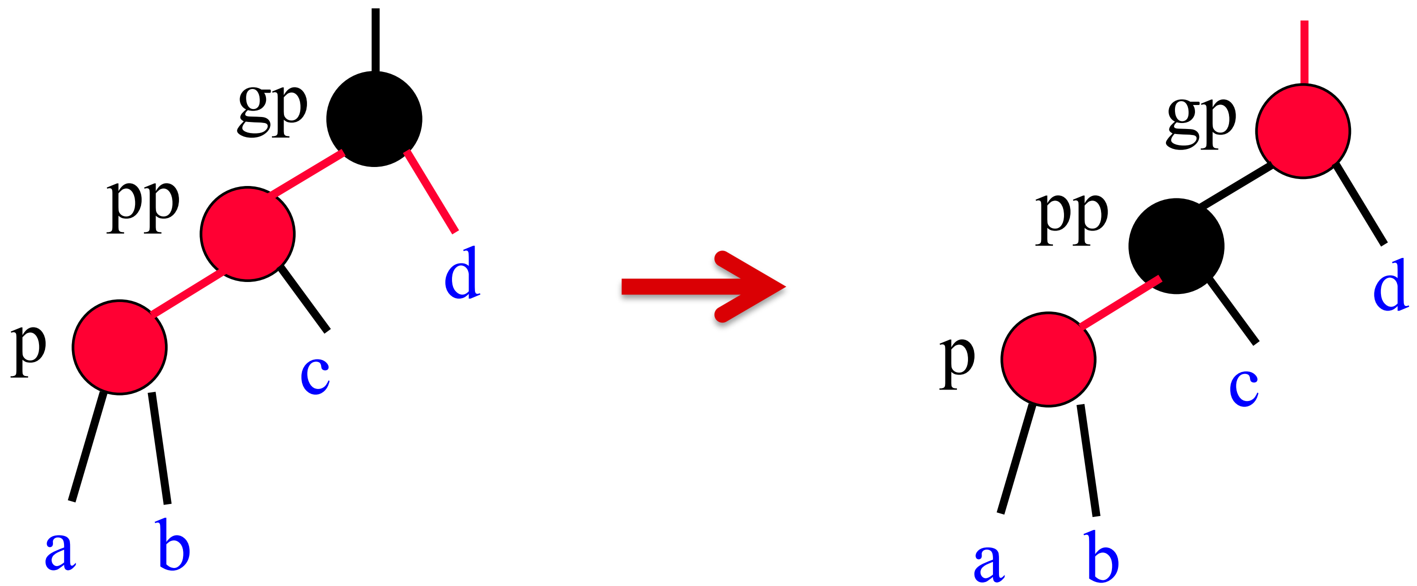


LL(RR)b

LL(RR)b

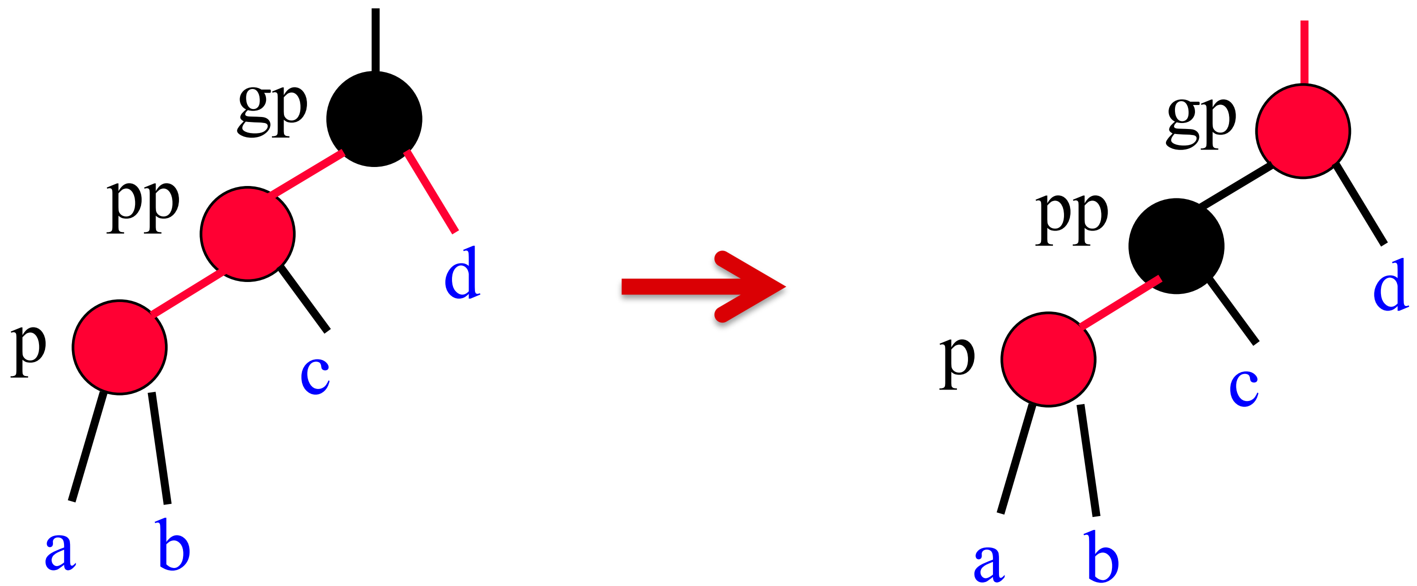
XYr

XYr → color flip



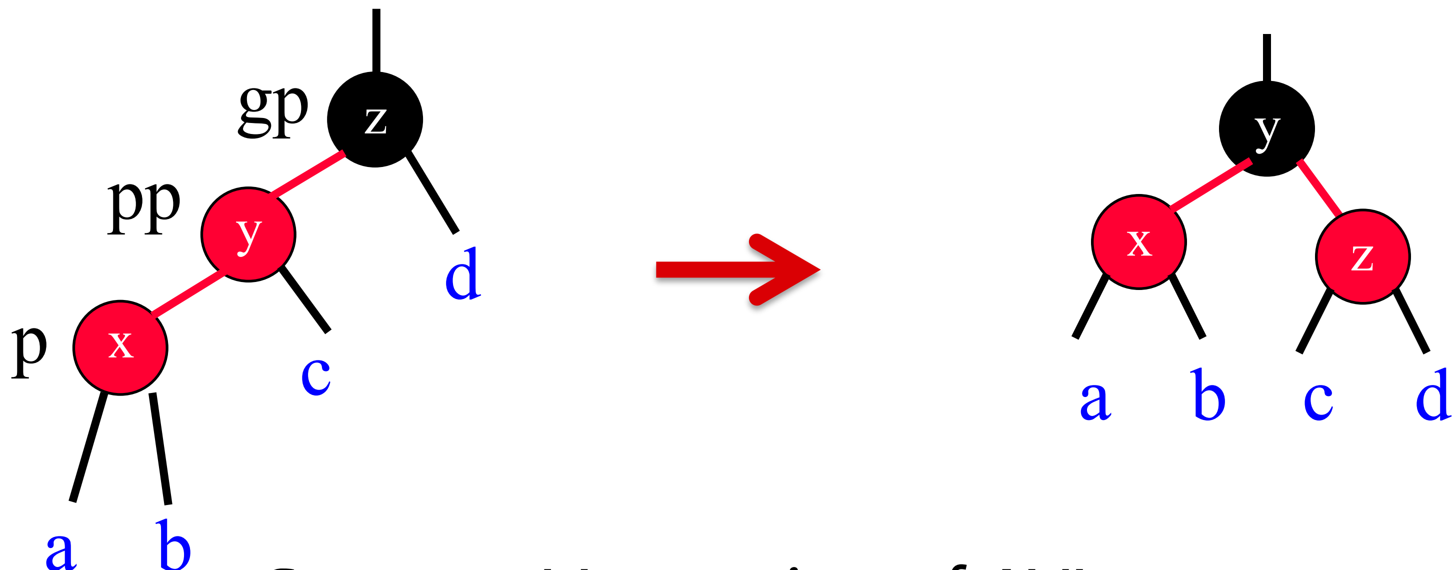
- Flip color of pp, gp, d and pointers of gp
- Flip color d to black is ok b/c gp is also flipped
- Reapply transformation to gp by p = gp

XYr → color flip



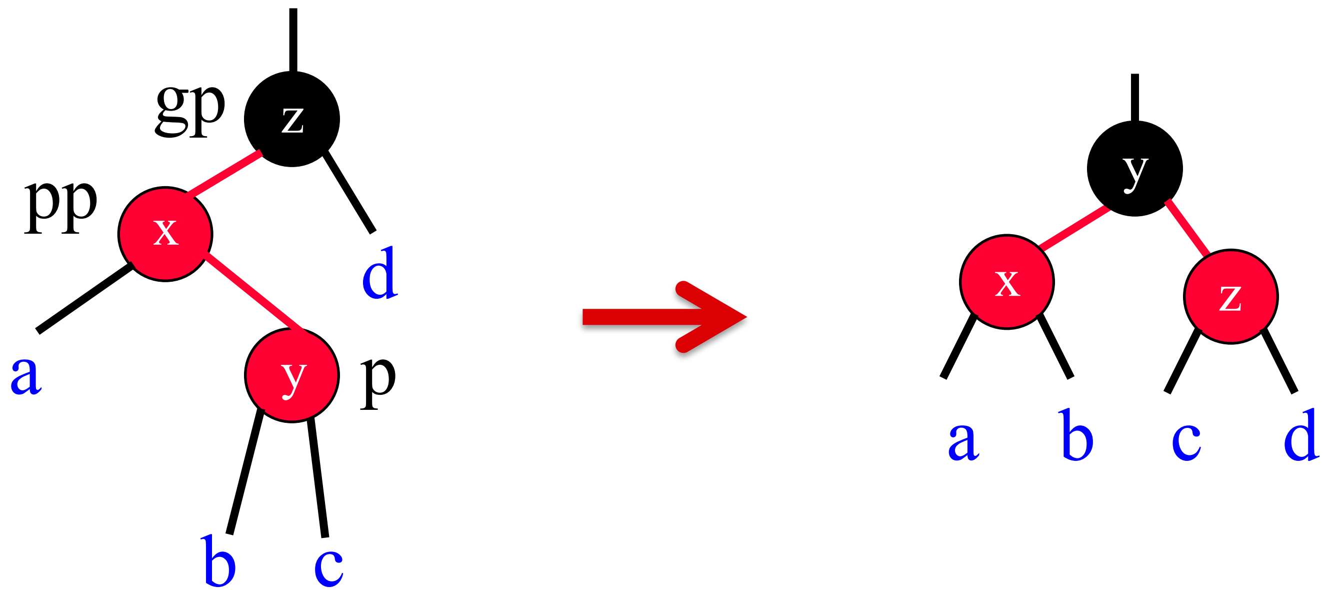
- The number of blacks in each path is preserved.
- No two consecutive reds (in this part).
- Need to flip recursively.

LLb → rotate and flip



- Same as LL rotation of AVL tree
- Filp color of pp and gp after rotation
- No need to check parent; root color is not changed

LRb → rotate twice and flip

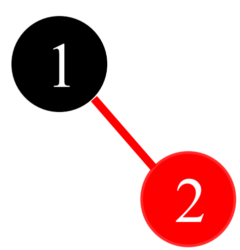


- Same as LR rotation of AVL tree
- Flip color of p and gp
- RRb and RLb are symmetric

Insert Example

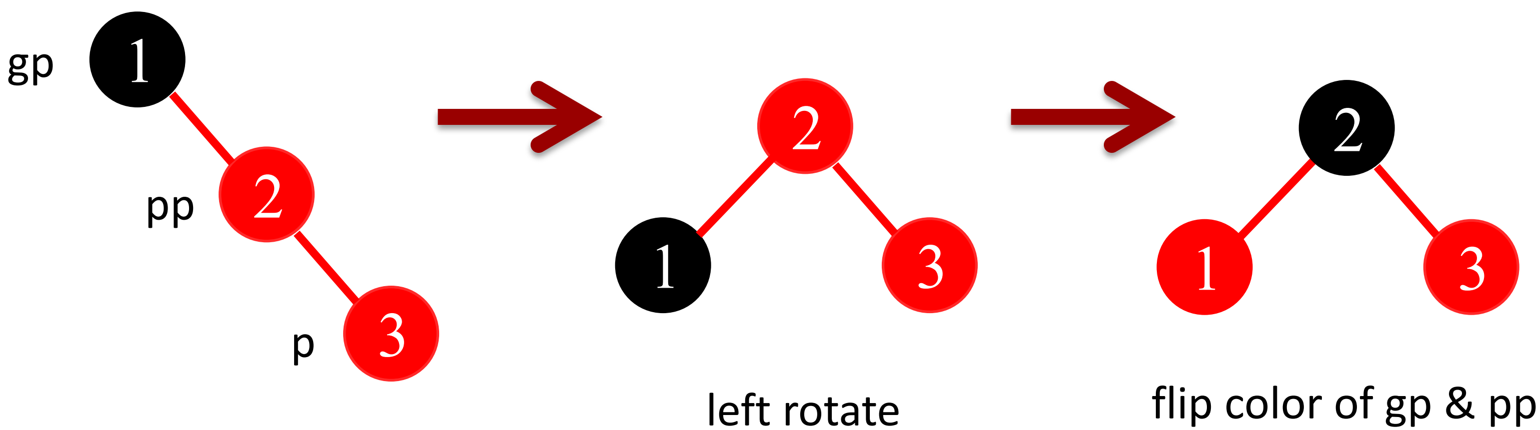
- Insert 1
  - Root

- Insert 2
  - Red



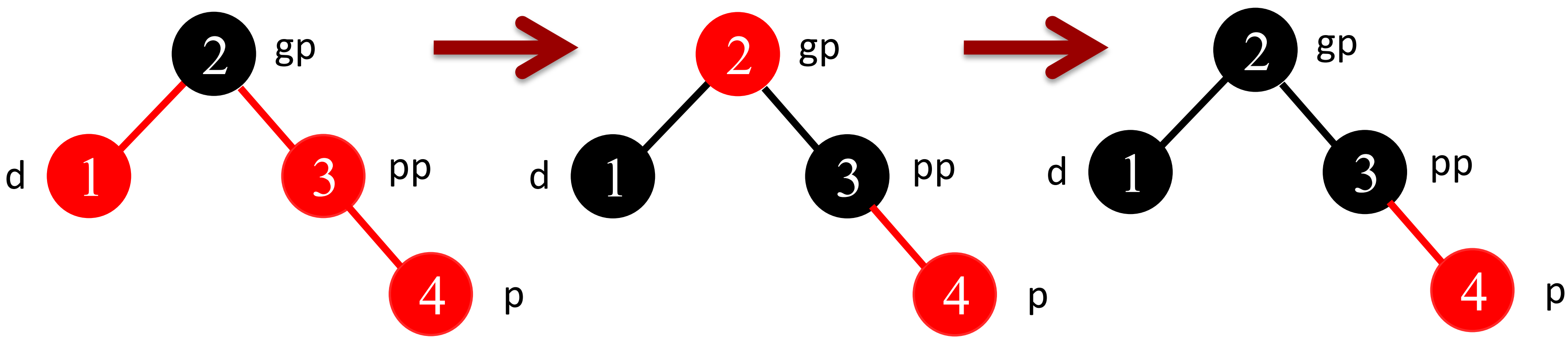
Insert Example

- Insert 3
  - RRb



Insert Example

- Insert 4
  - RRr

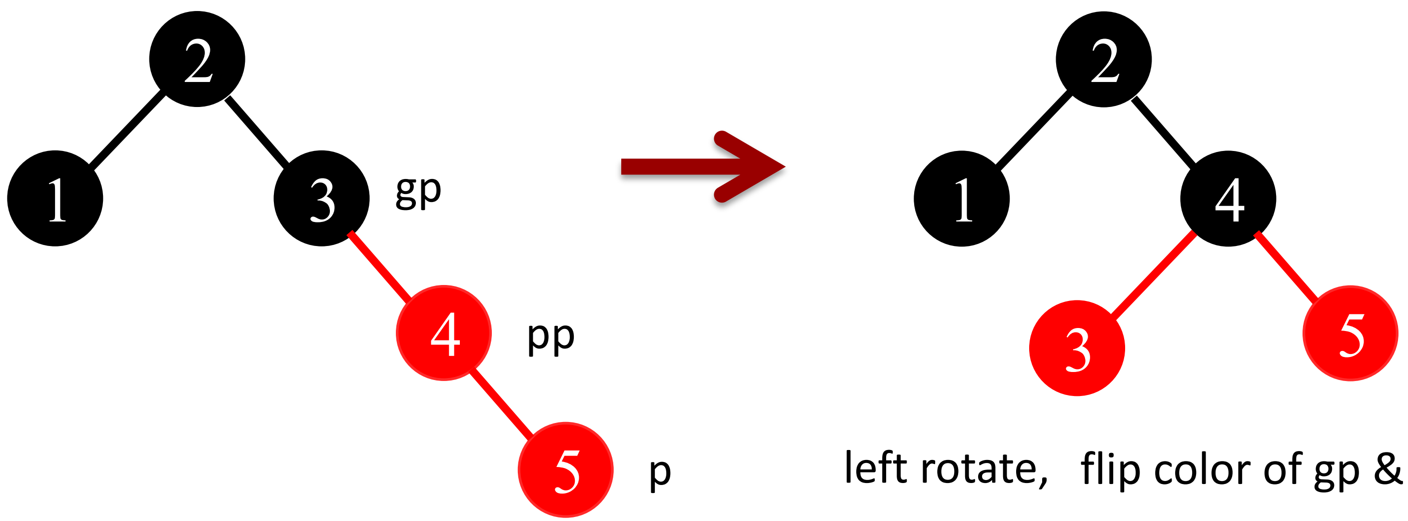


flip gp, pp, d color

flip gp back to black (root)

Insert Example

- Insert 5
  - RRb

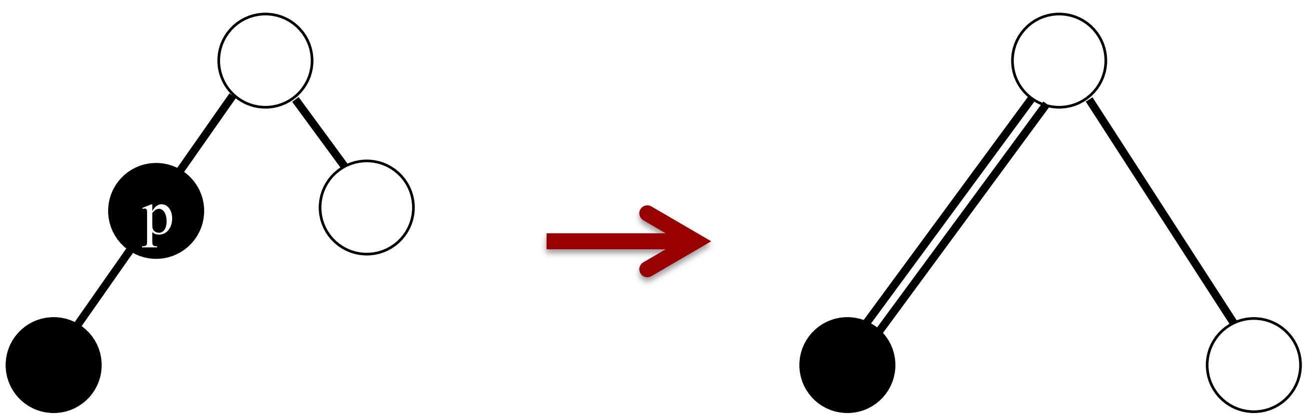


left rotate, flip color of gp & pp

Delete

- Similar to insert, but more complicated
- Delete black will violate red-black property
  - Path passing through deleted node will have less number of black nodes
  - Double black pointer

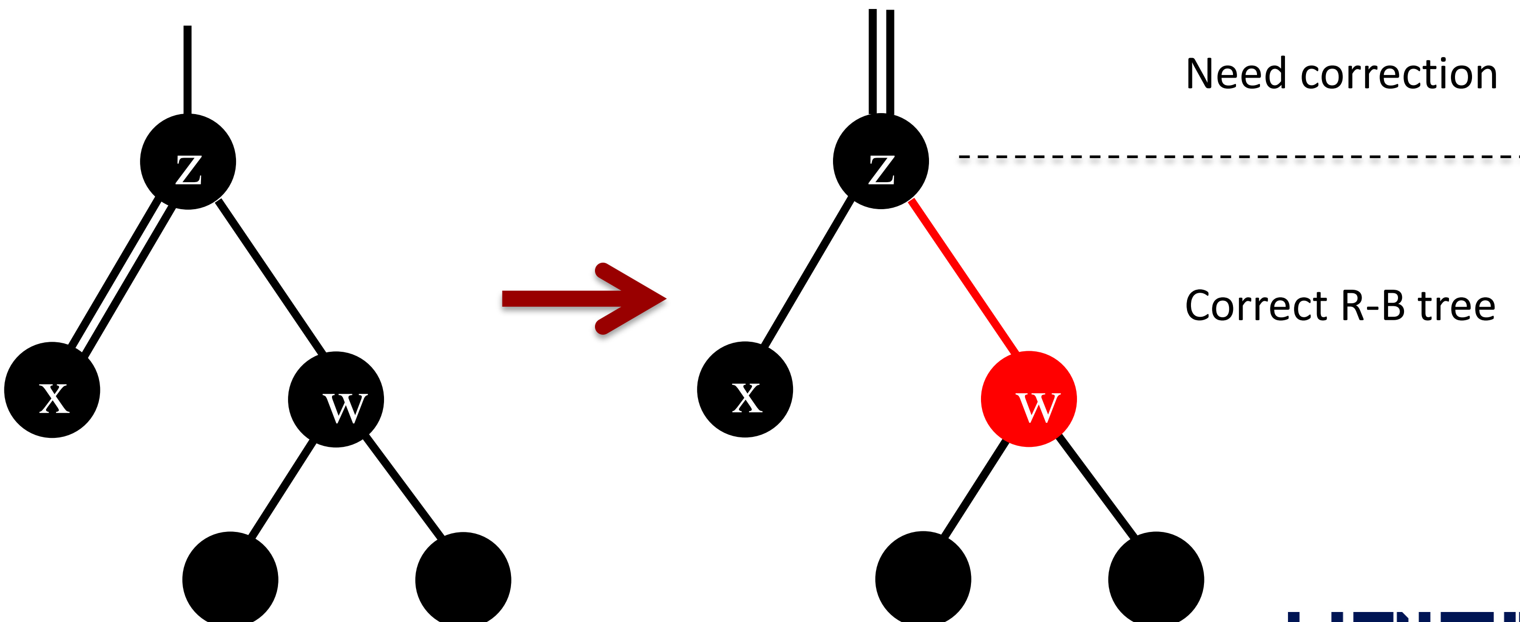
white circle : any color can be placed



delete p

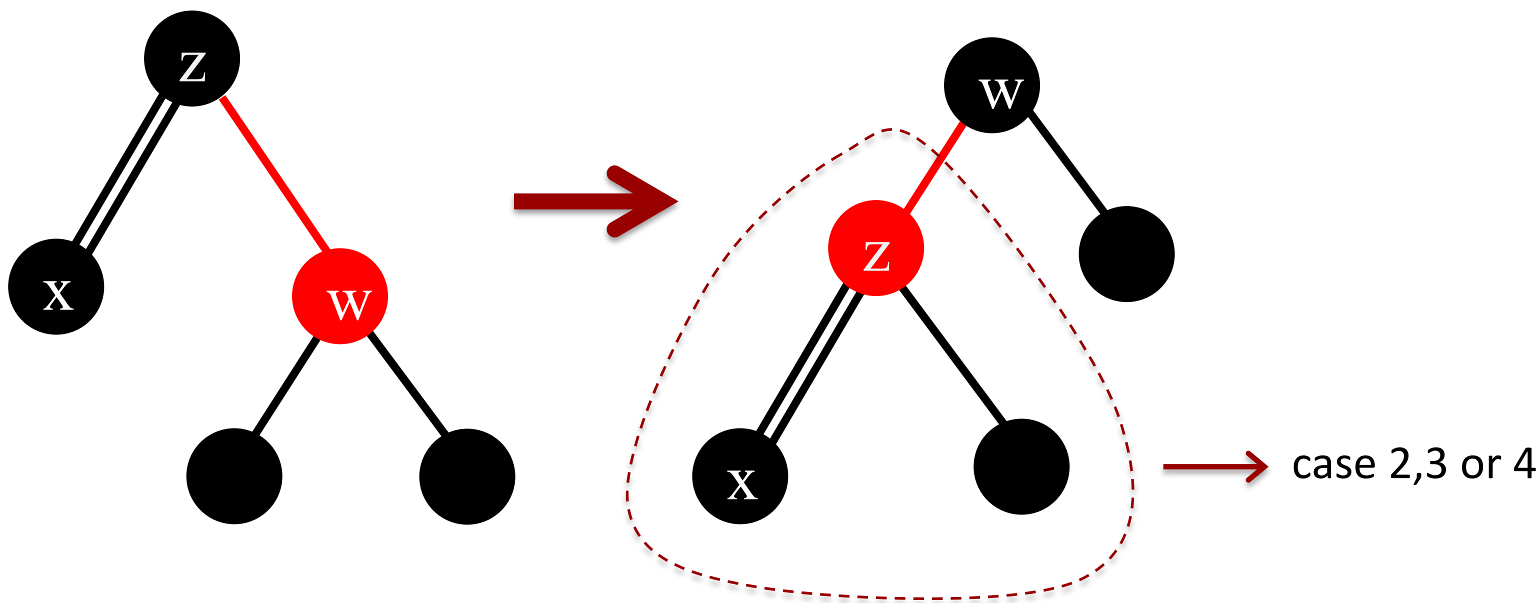
Delete

- Case 0 : w (sibling) and its children are black
  - Make w red
  - If z is black, then move up double black pointer. x = z, z = z->parent and restart (below z is ok)



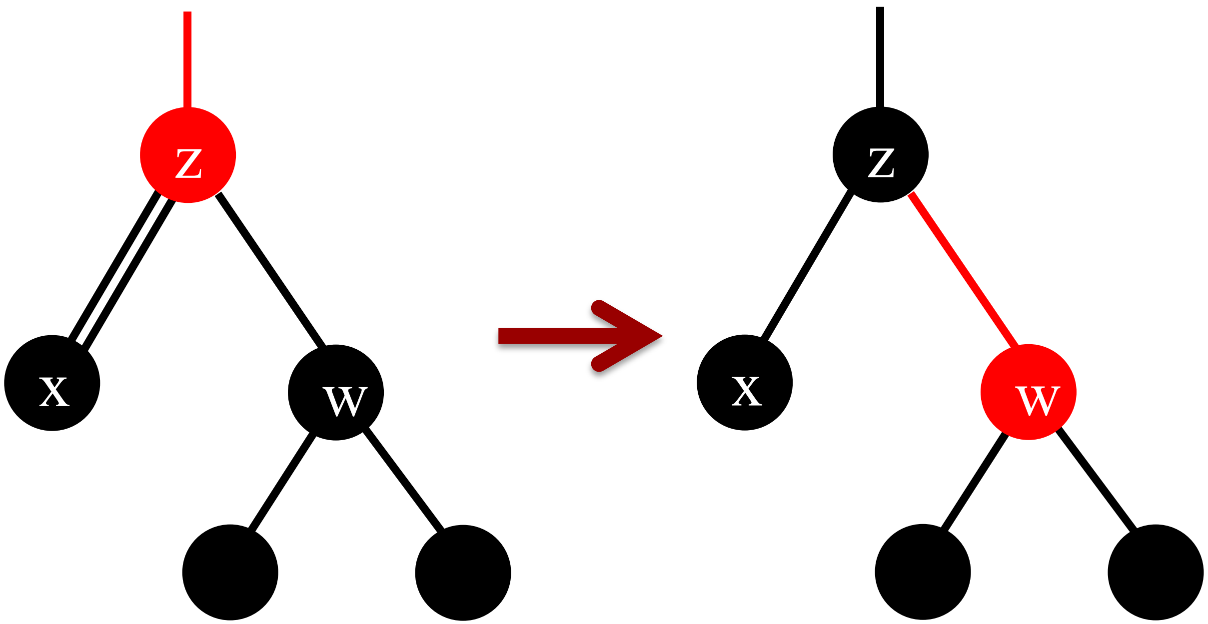
Delete

- Case 1 : z is black and w is red
  - Left-rotate at z and exchange colors of z & w
  - Go to case 2, 3, or 4 for subtree of z



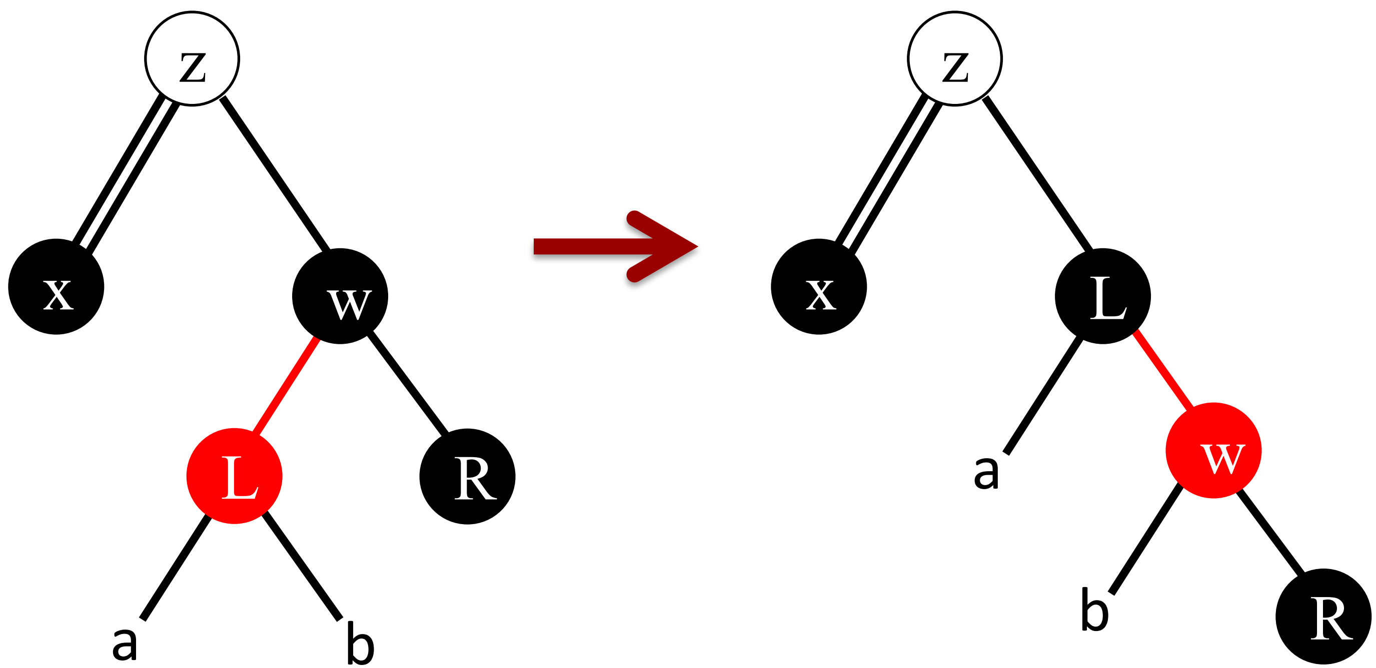
Delete

- Case 2 : w and its two children are black
  - Make w red
  - If z is red, then make z black and remove double pointer. Done.



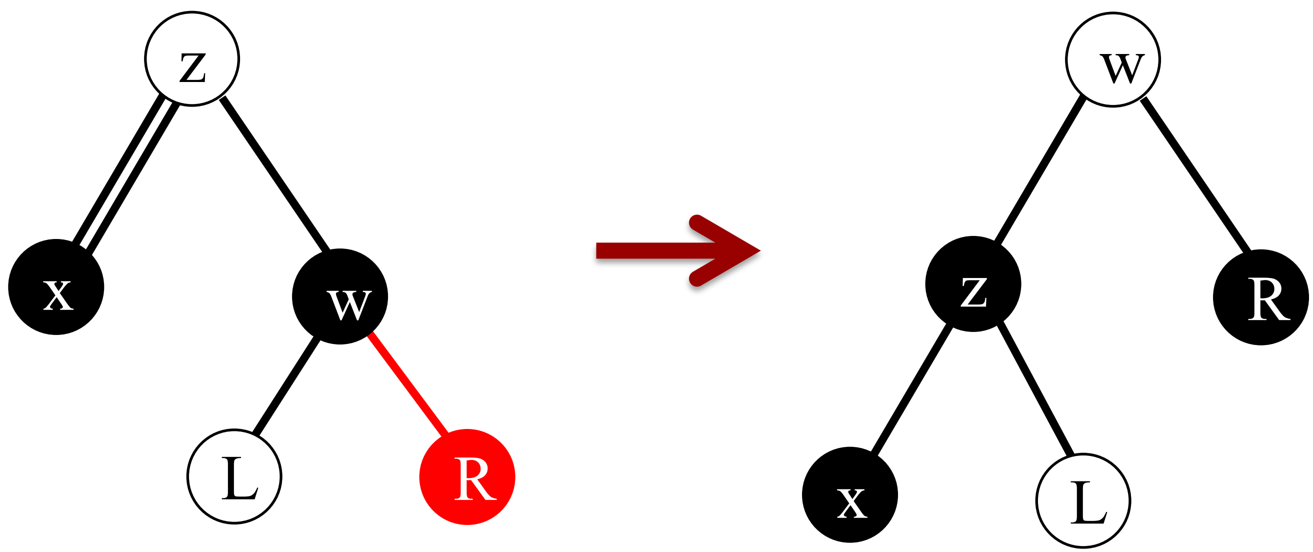
Delete

- Case 3 : w and its right child are black while its left child is red
  - Right-rotate at w and exchange colors of w and its left child. Go to case 4.



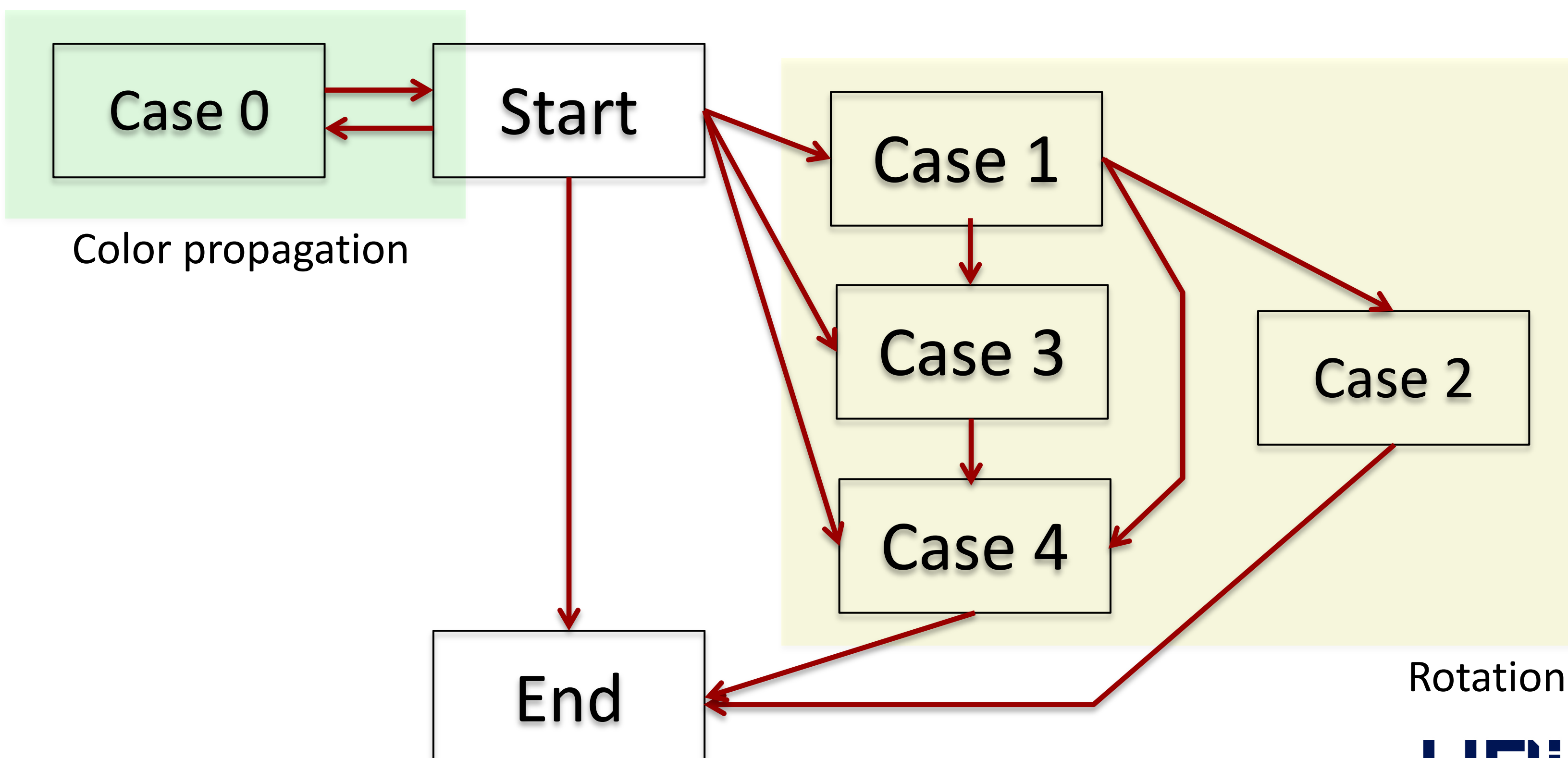
Delete

- Case 4 : w is black and its right child is red
  - Left rotate at z, exchange colors of z & w
  - Remove double black pointer, change R to black
  - Done



Delete Workflow

- At most 3 rotations are needed
- Color exchange may propagate $\log n$ times



Discussion

- Red-Black trees use color as balancing information instead of height as in AVL trees
- Insertion/deletion may cause a perturbation (if two consecutive red nodes exist)
- Perturbation is either
  - resolved locally (rotations), or
  - propagated to a higher level in the tree by recoloring (color flip)
- O(1) for a rotation or O(log n) color flips
- Total time: O(log n)

Questions?