MTH 361, Homework Assignment 2

Nurseiit Abdimomyn – 20172001

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1.
   - Find the adjacent matrix of network (a).

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

   - Find the incident matrix of network (b).

$$B = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

   - Find the adjacent matrix for the network generated when we project the network (b) into its black vertices.

$$A' = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

2.
   - A 3-regular graph must have an even number of nodes.

Proof. By Handshaking lemma we have sum of all degrees:

$$\sum_{v \in V} deg(v) = 2 * |E|$$

and by the defition of regular graphs, 3-regular graph has sum of all degrees

$$\sum_{v \in V} deg(v) = 3 * |V|.$$

Thus, we have

$$3 * |V| = 2 * |E|$$

which implies that $|V| = 2 * k$ for some k. □

- The average degree of a tree is strictly less than 2.

  Proof. Via Handshaking lemma, the average degree of a graph is as follows:

  $$c = \frac{1}{|V|} * \sum_{v \in V} deg(v) = \frac{2 * |E|}{|V|}.$$

  By defition, the following holds true for all trees:

  $$|E| = |V| - 1.$$

  Thus, by substituting $|V|$:

  $$c = \frac{2 * (|V| - 1)}{|V|} = 2 - \frac{1}{|V|} < 2$$

  □

3. Consider a network which is simple (it contains no multiedges or self-edges) and consists of n nodes in a single component.
   (i) What is the maximum possible number of edges it could have?
   (ii) What is the minimum possible edges if could have?
   Explain how you give the answer by providing the corresponding figures of networks.

   (a) Lorem ipsum...