## CSE232 Assignment 4

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- 1. a = 6, b = 2, c = 3. So,  $6 \mid (2 * 3), 6 \nmid 2, 6 \nmid 3$ .
- 2. Because 7 and 123 are coprime, we have by Euler's Theorem:  $\phi(7) = 6$  and  $123^{456} \equiv 123^{456 \mod \phi(7)} \pmod{7}$ . So,  $123^{456} \equiv 123^{456 \mod 6} \equiv 123^0 \equiv 1 \pmod{7}$ .
- 3. It's sufficient to check for 6 congruent clasess.

For 
$$n \equiv 0, 0^2 \mod 6 \equiv 0 \neq 2$$
. ( mod 6)

For 
$$n \equiv 1, 1^2 \mod 6 \equiv 1 \neq 2$$
. (  $\mod 6$ )

For 
$$n \equiv 2, 2^2 \mod 6 \equiv 4 \neq 2$$
. (  $\mod 6$ )

For 
$$n \equiv 3, 3^2 \mod 6 \equiv 3 \neq 2$$
. ( mod 6)

For 
$$n \equiv 4, 4^2 \mod 6 \equiv 4 \neq 2$$
. ( mod 6)

For 
$$n \equiv 5, 5^2 \mod 6 \equiv 1 \neq 2$$
. ( mod 6)

4. The strongly connected components are:

$$\{i\}$$

$$\{a,b,c\}$$

$$\{d,e,g,h\}$$

- $\{f\}$
- 5. (a) Has an Euler path because it has exactly two vertices with odd degree.  $\{d,f\}$ 
  - (b) Doesn't have Euler circuit not all vertices have even degree.
  - (c) It has a Hamilton Path as:  $\{a,b,e,f,g,c,d\}$

- (d) By Dirac's Theorem we know that in a graph with  $3 \le n$  vertices, if each vertex has  $n/2 \le deg(v)$ , then the graph has a Hamilton circuit. However, this theorem is not neccessary but it is sufficient. A quick manual check gives us that this graph doesn't have a Hamilton circuit.
- 6. We know that every graph that doesn't have a cycle of odd length is bipartite.

All trees are acyclic so we might say that they have a cycle of length 0 which is even.

Thus, all trees are bipartite.