CSE232 Assignment 3

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1. Consider a graph with 5 edges in total:
   $e_1 = ab, e_2 = ac, e_3 = ad, e_4 = bc, e_5 = bd$
   where the vertices set is $\{a, b, c, d\}$.

2. No solution.

Proof 1. For any vertex x from a graph of 4 vertices to have a degree of 3, the vertex x should have an adjacent edge with the other (3) vertices. By the definition of this problem, we should find such 3 vertices with all having degrees of 3. That's why, every one of those 3 vertices should have an adjacent edge with the other 3, including the one we need to have the degree to equal 2. Which in turn means that this ($4th$) vertex should also have a degree of 3. q.e.d.

Proof 2. Consider the sum of the degrees of a graph G: $\sum\limits_{v \in V(G)} deg(v) = k$
One can prove that k is always even, because each edge in $E(G)$ will contribute to the degree of two different vertices - therefore, k should be exactly two times the number of edges on G.
Now, because $deg(a) + deg(b) + deg(c) + deg(d) = 11$ is *odd*, such a graph can not be constructed.

3. Suppose C_n is a cycle of length n. Now, assume that we were to color the vertices of this graph to either *black* or *white*. By definition, such a graph is *bipartite* if there exists such a coloring in which any adjacent vertices have different colors. For n is *odd* such a coloring is impossible. Because if we color each second vertex in the chain to *black*, we end up coloring the last vertex to *white* and as the first vertex is also *white*,

where in a cycle last and first vertices are adjacent, it is clear that cycle of *odd* length is not *bipartite*. By the same manner one could prove that cycle of *even* length *is* a *bipartite* graph.

4. We need to prove that that:
$$\sum_{k=1}^{n} 3k^2 - 3k + 1 = n^3 \text{ whenever } n \geq 1.$$

   - for $n = 1$: $3 * 1^2 - 3 * 1 + 1 = 1^3$ holds.
   - for $n > 1$:
     Assume that $\sum_{k=1}^{n} 3k^2 - 3k + 1 = n^3$ holds,
     then it should follow that $(n+1)^3 = \sum_{k=1}^{n+1} 3k^2 - 3k + 1$.
     Indeed,
     $$\begin{gathered}(n+1)^3 = 3(n+1)^2 - 3(n+1) + 1 + n^3 \\ = 3(n^2 + 2n + 1) - 3n - 3 + 1 + n^3 \\ n^3 + 3n^2 + 3n + 1 = (n+1)^3\end{gathered}$$
     q.e.d.