CSE232: Discrete Mathematics
Assignment 2

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September 20, 2018

This homework assignment is due on Tuesday 10/02, 13:00, at the beginning of the lecture. **Please include your name and student ID.** Each question is worth 10 marks. So the total is 100 marks. You should follow the academic integrity rules for CSE232 that are described at the end of the slides of Lecture 0.

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1. Is the following compound proposition a tautology or a contradiction? Justify your answer.

$$(p \to q) \to (\neg q \to \neg p).$$

2. Is the following statement true? justify your answer.

$$\exists x \forall y (xy < 0 \to x < y)$$

where the domain for x and y is $\mathbb{R}$.

3. Let n be an integer. Prove or disprove the following: If $3n$ is odd, then n is odd.

4. Prove or disprove the following statement: If $g : A \to B$ and $f : B \to C$ are both one-to-one, the $f \circ g$ is also one-to-one.

5. Prove or disprove the following:

$$\forall n \in \mathbb{Z}\ (\lfloor n/2 \rfloor + \lceil n/2 \rceil = n)$$

6. How many integers in $\{1, 2, \ldots, 1000\}$ are divisible by 4 or 9? Justify your answer.

7. How many integers in $\{1, 2, \ldots, 1000\}$ are divisible by 4 or 6? Justify your answer.

8. How many two-to-one functions are there from $\{1, 2, 3, 4, 5, 6, \}$ to $\{a, b, c\}$? Justify your answer.

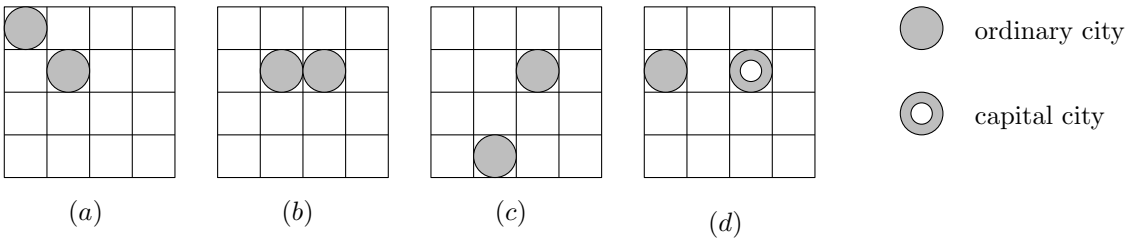

Figure 1: Examples where $n = 4$. (a) and (b) represent illegal placements because the two cities are adjacent. (c) and (d) represent legal placements.

9. In the game Civilization, you can build cities on a map consisting of square tiles. One city plays a special role—the capital city—and we assume that the other cities are identical. (See Figure 1.) The constraint is that no two cities can be in the same square, or in adjacent squares. (Two squares are adjacent if they touch along an edge, or diagonally at a corner.) Assume that you want to build cities on an $n \times n$ grid.

(a) How many different ways are there of placing your capital city and one ordinary city? Justify your answer.

(b) How many different ways are there to place two ordinary cities? Justify your answer.