

MTH26001 Assignment 3

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1. (Sec 4.2 - 3)

Given $a \equiv b \pmod{n}$ implies that $k * n = a - b$ for some k . Because $\gcd(b, n) \mid b$ and $\gcd(b, n) \mid n$, it's also true that $\gcd(b, n) \mid a = k*n + b$.

Since $\gcd(b, n) \mid a$, it should imply that $\gcd(b, n) \mid \gcd(a, n)$.

Then, we could also prove similar for $b = a - k*n$. Because $\gcd(a, n) \mid a$ and $\gcd(a, n) \mid n$, it's should follow that $\gcd(a, n) \mid b = a - k * n$.

Since $\gcd(a, n) \mid b$, it should imply that $\gcd(a, n) \mid \gcd(b, n)$.

Thus, $\gcd(a, n) \mid \gcd(b, n)$ and $\gcd(b, n) \mid \gcd(a, n)$ implies that $\gcd(a, n) = \gcd(b, n)$.

2. (Sec 4.2 - 5)

$$53^{103} \equiv 1^{103} \pmod{13} \text{ and } 103^{53} \equiv (-1)^{53} \pmod{13}.$$

$$\text{Thus, } 53^{103} + 103^{53} \equiv 1^{103} + (-1)^{53} \equiv 0 \pmod{13}.$$

$$\text{Also, } 53^{103} \equiv (-1)^{103} \pmod{3} \text{ and } 103^{53} \equiv 1^{53} \pmod{3}.$$

$$\text{Thus, } 53^{103} + 103^{53} \equiv (-1)^{103} + 1^{53} \equiv 0 \pmod{3}.$$

$$111^{333} + 4^{111} \equiv (-1)^{333} + 4^{111} \pmod{7}$$

$$\equiv (-1)^{333} + (4^3)^{37} \pmod{7}$$

$$\equiv (-1)^{333} + 1^{37} \equiv 0 \pmod{7} \text{ [as } 64 \equiv 1 \pmod{7}\text{]}.$$

3. (Sec 4.2 - 11)

$$0, 2^0, 2^1, \dots, 2^9 \equiv 0, 1, 2, 4, 8, 16, 32, 64, 128, 256, 512$$

$\equiv 0, 1, 2, 4, 8, 5, 10, 9, 7, 3, 6 \pmod{11}$, thus a complete residue set modulo 11.

But, $0, 1^2, 2^2, \dots, 10^2$ is not a complete residue set modulo 11 because $1^2 \equiv 10^2 \pmod{11}$.

4. (Sec 4.3 - 3)

Let's look for the period of $9^n \pmod{100}$:

so powers of nine modulo hundred are 1, 9, 81, 29, 61, 49, 41, 69, 21, 89, 1, with a period of 10.

Since $9^9 \equiv 9 \pmod{10}$ (here 10 is our period), $9^{99} \equiv 89 \pmod{100}$.

5. (Sec 4.3 - 10)

All numbers n with digits adding up to 15 are $n \equiv 6 \pmod{9}$. However, for all a , it's true that $a^3 \equiv 0, 1, 8 \pmod{9}$ and $a^2 \equiv 1, 4, 7 \pmod{9}$.

Thus, n can not be a cube or a square.

6. (Sec 4.3 - 11)

$495 = 5 * 9 * 11$, the resulting number should be divisible by all three 5, 9, and 11.

Since $273x49y5$ ends with 5, it already is divisible by 5.

In case of 9, sum of digits should be divisible by 9:

$$2 + 7 + 3 + x + 4 + 9 + y + 5 \equiv 0 \pmod{9}$$

$$\equiv 3 + x + y \equiv 0 \pmod{9}.$$

In case of 11, alternating sum of digits should be divisible by 11:

$$2 + 3 + 4 + y - (7 + x + 9 + 5) \equiv 0 \pmod{11}$$

$$x - y + 1 \equiv 0 \pmod{11}.$$

Thus, since $0 \leq x, y \leq 9$, there is only one solution:

$$x = 7, y = 8.$$

Indeed, $27374985 \equiv 0 \pmod{495}$.