DATA SCIENCE

DREAM JOB

Logistic Regression

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1a. Overview

Logistic Regression is derived from the Logit Transformation

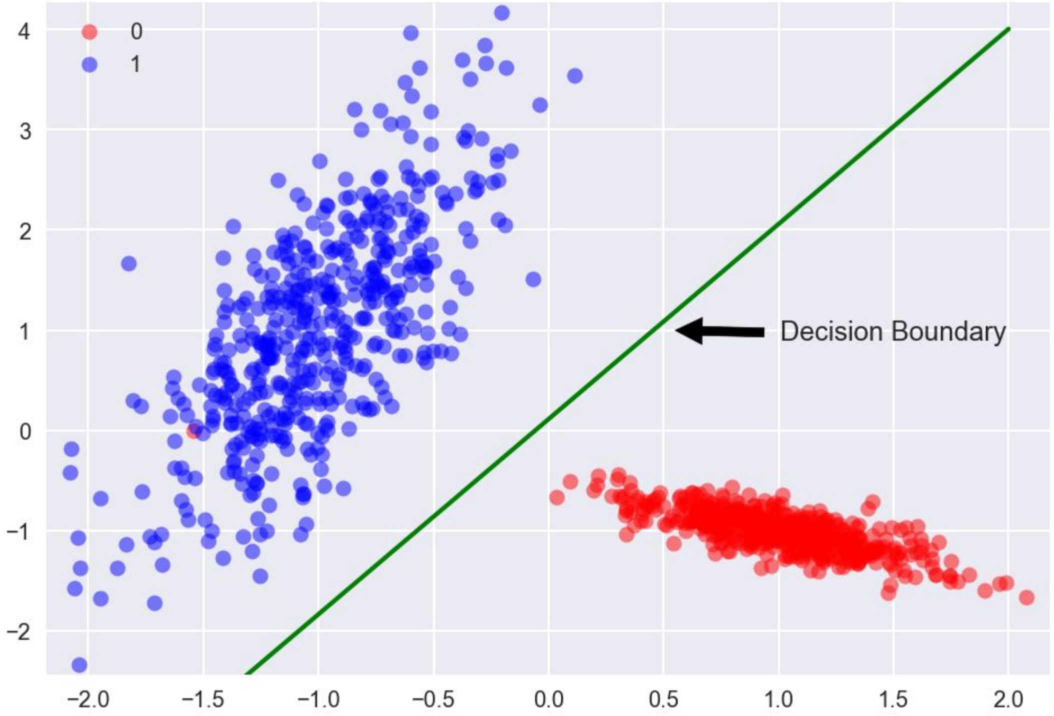
And is an algorithm that learns a model for binary classification

Logistic Regression Concepts:

- Predicts binary outcomes
- Predicts probability scores
- DV must have discrete values (i.e. True/False)
- Uses a Logit Transformation on the DV to fit linear model

Example

Fit a line that **best separates** the classes



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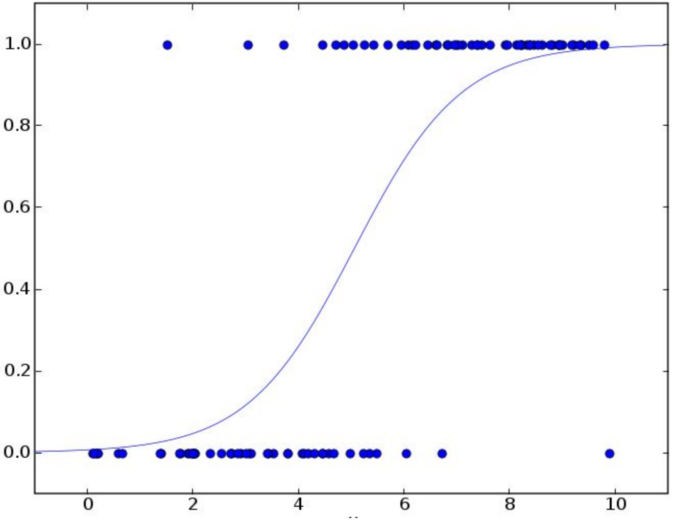
Example (2)

Logistic Regression allows you to make:

i. Soft Predictions (% Probability)

ii. Hard Predictions (0’s, 1’s)



Predicting Credit Default:

Name	Predicted Default Probability	
Bob	0.85	
Henry	0.76	
Mary	0.53	
Paul	0.32	**Threshold**: 0.15
Maria	0.14	
Sonny	0.12	
Ryan	0.07	
Howard	0.04	

Custom Threshold

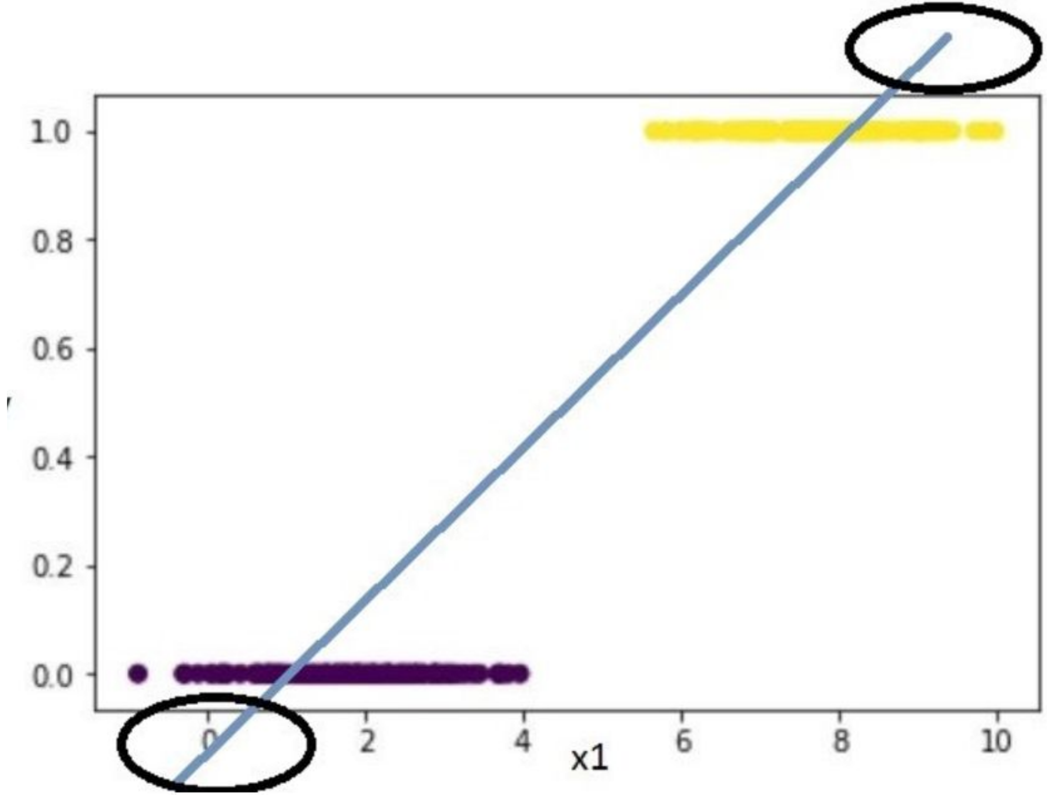
1b. Logistic Regression Model

- Uses **Logit Transformation** on DV to fit a regression model
- DV has two (binary) outcomes

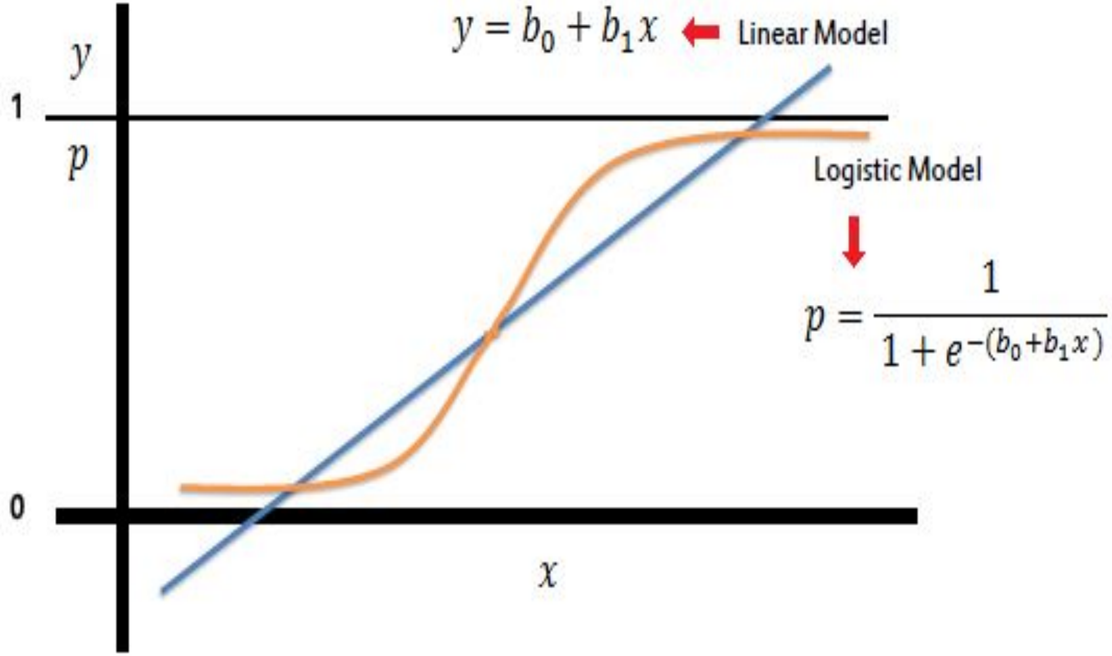
$$Ln\left(\frac{P}{1-P}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k$$

Logit Transformation

The Problem with Linear Regression…



Logistic Regression vs Linear Regression



1c. Important Concepts & Terminologies

Logistic Regression models the **probability** of an event - rather than a measure.

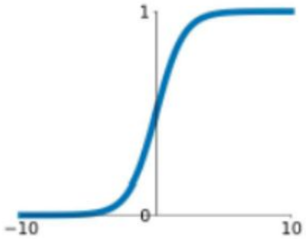
1. Probability
2. Odds
3. Logit/Log Odds
4. Sigmoid
5. Log Loss

- Probability = $\dfrac{\#\text{ events}}{\#\text{ subjects}}$
- Odds = $\dfrac{\dfrac{\#\text{ events}}{\#\text{ subjects}}}{\dfrac{\#\text{ non-events}}{\#\text{ subjects}}} = \dfrac{\text{probability}}{(1 - \text{probability})}$
- **Odds = p / (1 - p)**

  [ratio of two probabilities]

Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



$$\text{LogLoss} = -\frac{1}{n}\sum_{i=1}^{n}\left[y_i \log(\hat{y}_i) + (1 - y_i)\log(1 - \hat{y}_i)\right],$$

2. Understanding the Algorithm

- Logistic Regression models the **PROBABILITY** of an event
- Probabilities range from (0’s to 1’s)
- Requires a **LOGIT** transformation on DV (hence the name Logistic Regression)
- Still considered a Linear Model because of Input Parameters

The Logistic Function

$$\text{Log}\left[\frac{Y}{(1-Y)}\right] = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + \dots + b_nX_n$$

Log(Likelihood)

diet score (0-15)

age group (0/1)

sex (0/1)

2. Understanding the Algorithm

Linear Regression Equation:

Y = B0 + B1X1 + B2X2 +… + BkXk

Logit Regression Equation:

Log(Odds) = Ln(Y/1-Y) = B0 + B1X1 + B2X2 +… + BkXk

Why not just measure the probability?

EX) P = B0 + B1X1 + B2X2 +… + BkXk ????

How does it predict probability scores?

$$\ln\left(\frac{P}{1-P}\right) = a + bX$$

$$\frac{P}{1-P} = e^{a+bX}$$

$$P = \frac{e^{a+bX}}{1+e^{a+bX}}$$

2b. Cost Function - Log Loss

$$J(\theta) = -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)}))]$$

Objective: Minimize the Log Loss Error

Logistic Regression uses the Log Loss Function (as opposed to OLS) to find optimal parameters

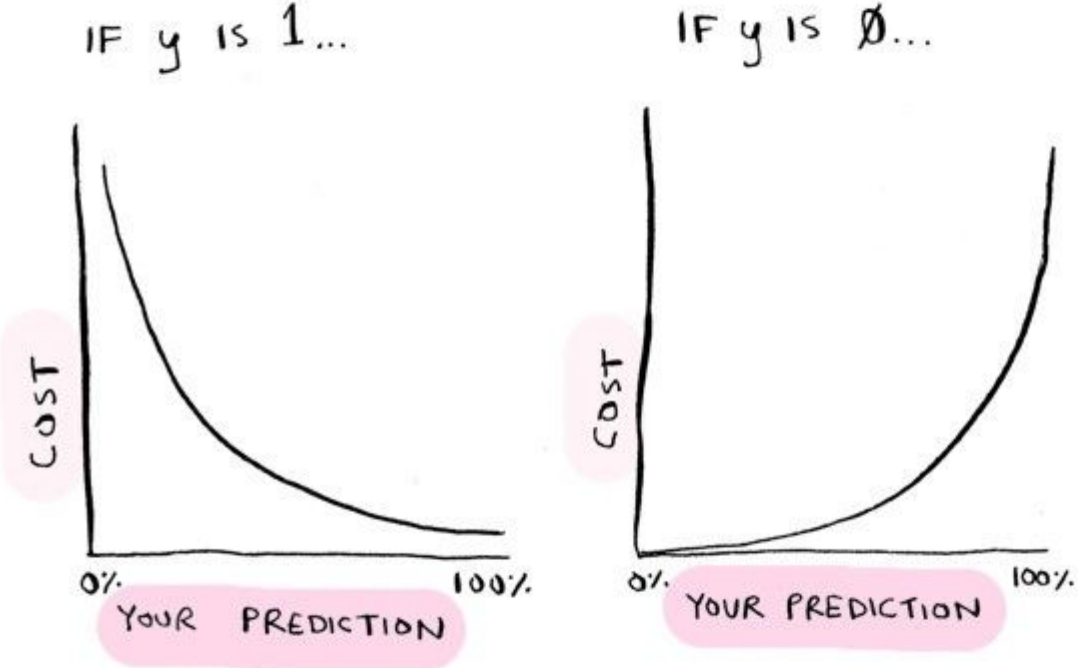
- **y(i)**: the DV (0 or 1)
- **h0(xi)**: predicted probability

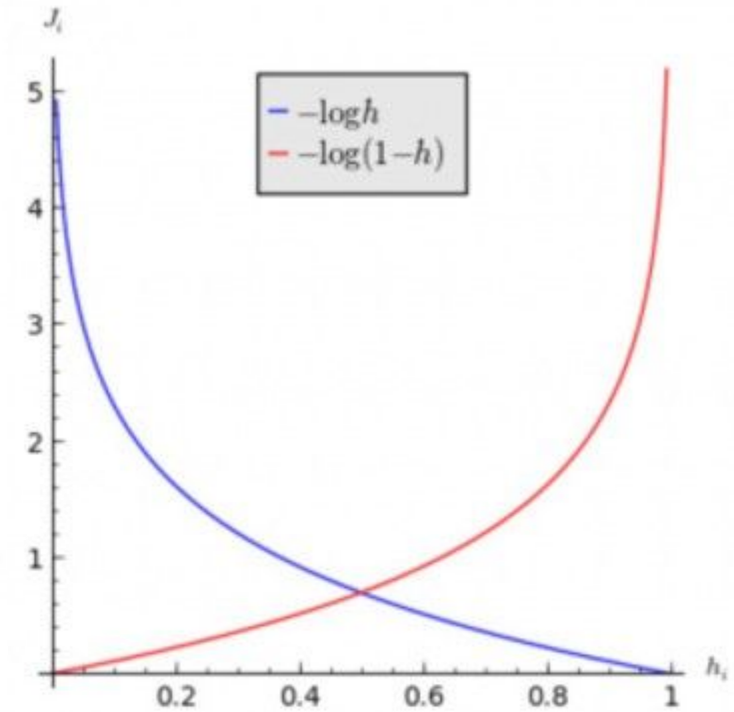
When **y=0** and probability is **LOW** → then low error

When **y=1** and probability is **HIGH** → then low error

Log Loss

$$J(\theta) = -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)}))]$$





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3. Strengths and Limitations

💪 **Strengths:**

- Highly interpretable
- Fast training & predictions
- No model tuning required (except for regularization)
- No need to scale features
- Outputs probability scores
- Good baseline model

Limitations:

- Assumes a linear relationship
- Performance is not as great compared to other models
- Sensitive to outliers
- Can’t automatically learn feature interactions like tree-based methods

Summary

$$Ln\left(\frac{P}{1-P}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k$$

Logistic Regression is used for **binary classification** problem:

- It's fast, simple, powerful, and is a great baseline model
- Uses a Logit Transformation on the DV to fit linear model
- Can make probability predictions (Pos class if p>0.5, Neg class if p<0.5)

It takes a weighted combination of input features

⇩

Passes it through a sigmoid function

⇩

Which maps any real number to a probability (0~1)