

United International University Department of Computer Science and Engineering

CSE 2213/CSI 219: Discrete Mathematics Mid-term Examination : Summer 2024

Total Marks: 30 Time: 1 hour and 45 minutes

Any examinee found adopting unfair means will be expelled from the trimester / program as per UIU disciplinary rules.

Section A Answer all the five questions.

1. (a) Using propositional laws prove whether the proposition is a tautology, contradiction or neither of them. [2]

 $(p \land (p \to q)) \to q$

(b) Consider the following propositional variables:

p: "Carlsen does not make mistakes"

q: "Carlsen loses his queen"

r: "Vladimir wins the chess match"

s: "Vladimir retains his title"

Now consider the statement: "If Carlsen makes a mistake and loses his queen, then Vladimir will win the chess match and retain his title"

- i. Determine the contrapositive of the above statement.
- ii. Translate the above given statement and its contrapositive into connectives. Use the above propositional variables.
- 2. Consider the following predicates:

 $[1 \times 4 = 4]$

[2]

P(x): x is attentive.

Q(x): x does a good result in the examination.

Represent the following sentences using these predicates, appropriate quantifiers, and logical connectives. The domain of all the variables is the set of all students.

- (a) All students who are attentive do well in the examination.
- (b) There exists a student who is attentive and does well in the examination.
- (c) There is at least one student who does well in the examination but is not attentive.
- (d) For every student, if that student is attentive, then there exists a student who does well in the examination.
- 3. (a) Shade the following on a Venn diagram:

 $[1 \times 2 = 2]$

- i. $(A \cap B) \cap C$
- ii. $(A \cup C)' \cap B'$
- (b) Find out $P((A B) \cap B')$, where $U = \{1, 2, 3, x, y\}$, $A = \{1, 2, 3\}$, $U = A \cup B$ and A, B are disjoint sets. [2]
- 4. Determine whether each of the following functions is injective, surjective, bijective or neither. Hence, write down whether it is invertible or not. Give reasons for your answers. (The domains and codomains consist of all real numbers.)
 - (a) f(x) = 3x + 1
 - (b) g(y) = 37
- 5. (a) Using proof by contraposition, prove that "For all integer n, if $4n^2 + n + 6$ is even, then n is even"
 - (b) Use a direct proof to show that the product of two odd numbers is odd

[2] [2]

- 6. Determine whether the following two logical expressions are logically equivalent or not: [3]
 - i. $((p \to q) \to (r \to s)) \to ((t \to u) \to (p \to p))$
 - ii. $(p \land q) \land ((r \to s) \to t) \land ((t \to u) \to \neg p) \land \neg p$
- 7. The following are the definitions of two sets:

$$A = \{x | x \in Z^+, x = f(y) \text{ and } x < 10\}, \text{ where } f: Z^+ \to Z^+, f(y) = 2y - 1$$

$$B = \{x | x \in R \text{ and } x^2 - 7x + 12 = 0\}$$

- (a) Represent A and B by roaster method. [2]
- (b) Find out $|P(P(A \times P(B)))|$. [1]
- 8. Prove the following statement using an appropriate proof technique. Mention which technique you used. [4]

"If n is an integer such that $4n^2 + 6n + 5$ is divisible by 3, then n is odd."