United International University
Department of Computer Science and Engineering

CSE 2213/CSI 219: Discrete Mathematics
Mid-term Examination — Spring 2025
Total Marks: 30 Time: 1 hour and 30 minutes
Any examinee found adopting unfair means will be expelled from the trimester / program as per UIU disciplinary rules.

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1. (a) Convert the given conditional statement into **if p then q** format. Then determine the converse, inverse and contrapositive version of the following statement. [2]

   "You will not be able to complete the syllabus unless you study 3 hours everyday."

   (b) Consider the following propositions: [2]

   p : You visit the park.
   z : You visit the zoo.
   v : You enjoy your vacation.

   Translate the following sentences using the above propositional variables:
   i. You can either visit the amusement park or the zoo, but you cannot visit both of the places.
   ii. You cannot enjoy your vacation if you do not visit the zoo, unless you go to the amusement park.

   (c) Prove that $(q \land (p \to \neg q)) \to (\neg p \lor r)$ is a tautology, using a sequence of logical equivalence laws. [2]

2. (a) Consider the following predicates: [$1 \times 4 = 4$]

   $S(x) \equiv x$ is Sherlock Holmes.
   $D(x) \equiv x$ is a detective.
   $M(x) \equiv x$ loves solving Mysteries.
   $F(x) \equiv x$ has many friends.

   Express the following statements using the given predicates and quantifiers. The domain of the variables is the set of all people of London.
   i. All detectives in London love solving mysteries.
   ii. Anyone who has many friends must be a detective or love solving mysteries.
   iii. Some mystery lovers are not detectives.
   iv. People who are detectives and have many friends are not Sherlock Holmes.

   (b) Explain with reasoning whether the following propositions are true or false. The domain of the variables is the set of real numbers. [$1 \times 2 = 2$]
   i. $\exists x \forall y (x \times y = x)$
   ii. $\forall x \exists y (x \times y = 1)$

3. Let the universal set, $U = \{x \in \mathbb{N} \mid 3x - 13 < 50\}$, $P = \{x \in \mathbb{Z}^+ \mid x \text{ is a multiple of 4 and } x \leq 20\}$, and $Q = \{2, 3, 5, 7, 11, 13, 17, 19\}$.

   (a) i. List the elements of set P. [1]
   ii. Write set Q using **set builder method**. [1]
   iii. Find the **cardinality and elements** of $Q \cap P$. [1]

   (b) Let $R = \{3, 4, 6, 9, 12, 15\}$.
   i. Find $\mathcal{P}((Q \cup R) - P)$. [1.5]
   ii. Determine the elements of $(Q' \cap R) \cup (P - R)$. [1.5]

4. (a) Consider two functions, $f(x) = x^2 - 5x + 1$ and $g(x) = 7x - 3$. Calculate $(f \circ g)(x)$ and $(g \circ f)(x)$ assuming that both the domain and the co-domain consist of real numbers. [2]

   (b) Is the following function invertible? If so, find its inverse function. [4]

   $$f(x) = \frac{x+1}{x+2}, f : \mathbb{R} - \{-2\} \to \mathbb{R} - \{-1\}$$

5. (a) Consider the statement: $\forall a \in \mathbb{R}\ \forall b \in \mathbb{R}\ ((a \leq b) \to (a^0 \leq b^0))$. Explain whether it can be proven using vacuous proof or trivial proof. [2]

   (b) Prove that, **"For all integers n, if $n^3 + 5$ is odd, then n is even."** [4]