# United International University

## Department of Computer Science and Engineering

CSE 2213/CSI 219: Discrete Mathematics (BSCSE and BSDS)

Mid-term Examination – Fall 2024

Total Marks: 30 Time: 1 hour and 30 minutes

Any examinee found adopting unfair means will be expelled from the trimester / program as per UIU disciplinary rules.

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1. (a) Consider the following propositions: [1 + 1 = 2]

   $p$ : You revised all discrete math lecture notes.
   $q$ : You practiced problems from the discrete math textbook.
   $r$ : You discussed challenging topics with your classmates.

   Now using the logical operators formulate the following compound propositions:

   i. You revised all discrete math lecture notes, but you did not practice problems from the textbook.

   ii. If you discussed challenging topics with your classmates, then practicing problems from the textbook is both necessary and sufficient for revising all lecture notes.

   (b) Show that $((p \wedge r) \wedge (p \rightarrow q) \wedge (q \rightarrow \neg r))$ is always false, by using logical equivalence laws. [2]

   (c) Express the following statement in propositional logic form and then determine its contrapositive, converse and inverse: [4 × 0.5 = 2]

   "You will finish the exam within the allotted time if you plan your answers carefully."

2. (a) Determine the truth value of each of these statements where domain consists of all real numbers. [2]

   i. $\forall x \exists y (x = y^2)$

   ii. $\exists x \forall y (xy = y)$

   (b) Let $F(x, y)$ be the statement "$x$ can fool $y$", where the domain consists of all people in the world. Use quantifiers to express each of these statements. [2]

   i. There is no one who can fool everybody

   ii. Everyone can be fooled by somebody.

   (c) Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers. [2]

   i. $\forall x (x - x = 0)$

   ii. $\forall x \exists y (x + y = 1)$

3. (a) Suppose you are given the following sets: [1 × 3 = 3]

   $$U = \{x \in \mathbb{N} \mid 2x + 5 < 36\}$$
   $$A = \{x \in \mathbb{Z}^+ \mid x \text{ is divisible by 3 and } x \leq 15\}$$
   $$B = \{1, 3, 5, 7, 9, 11, 13, 15\}$$

   i. Find out the elements of set A.

   ii. Express set B using Set Builder Method.

   iii. Determine the elements and cardinality of $B - A$.

   (b) Suppose you have another set, $C = \{2, 3, 5, 7\}$, along with the sets from question 3(a). [1.5 × 2 = 3]

   i. Find out $P((A \cup B) \cap C)$.

   ii. Find out $((A \cap B) - C) \cup (B' \cap C)$.

4. (a) If $f(x) = 2x^2 + 3x + 1$ and $g(x) = 4x + 2$, find $(f \circ g)(x)$ and $(g \circ f)(x)$, considering that both the domain and the co-domain consist of real numbers. [2]

   (b) Determine whether the function is invertible. If it is, find its inverse function. [4]

   $$f(x) = \frac{2x + 3}{x + 1}, \quad f : \mathbb{R} - \{-1\} \rightarrow \mathbb{R} - \{2\}$$

5. (a) Using an appropriate proof technique, prove that "If a rational number is divided by a nonzero rational number, then the result is rational". Mention which technique you used. [2 + 1 = 3]

   (b) Let $P(n, k)$ be the statement: [2 + 1 = 3]

   $$\text{If } a + a^n + (1 + b)^n > k, \text{ then } a^n = b^n, \text{where } a, b \text{ and } n \text{ are integers.}$$

   Using an appropriate proof technique, show that $P(0, 1)$ is true. Mention which technique you used.