Parallel Computing Systems: Exercise Sheet 2 Solutions

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Task 1: LogP Model Analysis

Parameters:

- $L = 3$ cycles (Latency)
- $o = 1$ cycle (Overhead, assumed $o_s = o_r = o$)
- $g = 2$ cycles (Gap)
- $P = 6$ units (P_0 and $P_1 \dots P_5$)

1. Bus Network

In a Bus network, the communication medium is shared and serialized. P_0 sends 5 requests to $P_1 \dots P_5$, and each sends one response back.

- **Traffic:** 5 Requests + 5 Responses = 10 Messages total.
- **Constraint:** The bus can only transmit one message every g cycles.
- **Total Bus Time:** 10 messages $\times g = 20$ cycles.
- **Critical Path (Last Response):**
  1. The bus is busy continuously transmitting messages $1 \dots 10$.
  2. The 10th message (the last response) starts transmission at $t = 9 \times g = 18$ cycles.
  3. It takes L cycles to traverse the network and o_r to be received.
  4. Arrival Time $= 18 + L + o_r = 18 + 3 + 1 = 22$.

Result: $T_{run} \approx$ **22** cycles.

2. Star Network

In a Star network with P_0 as the center, P_0 is the bottleneck for both sending requests and receiving responses.

- **Sending Phase ($P_0 \to P_{1..5}$):**
  - P_0 sends 5 requests sequentially.
  - Gap $g = 2$ dominates overhead $o = 1$.
  - P_0 finishes sending the 5th request at $t = 4 \times g = 8$.
- **Round Trip (Critical Path for one node, e.g., P_5):**
  - Request 5 sent at $t = 8$.
  - Arrives at P_5: $8 + L + o_r = 12$.
  - P_5 processes (0 assumed) and sends response at $t = 12$.
  - Response arrives at P_0's input port: $12 + L = 15$.
- **Receiving Phase (Bottleneck at P_0):**
  - P_0 receives 5 responses.
  - Responses from $P_1, P_2, \ldots$ arrive earlier but are serialized by P_0's receiving gap g.
  - If the first response (P_1) arrives around $t \approx 0 + L + L = 6$, P_0 starts receiving.
  - P_0 must receive 5 items: $5 \times g = 10$ cycles.
  - However, the last response (P_5) arrives at the port at $t = 15$.
  - P_0 might process previous responses while waiting, but let's look at the stream: Arrivals at Port: $P_1(7), P_2(9), P_3(11), P_4(13), P_5(15)$. P_0 Receive Start Times: $7, 9, 11, 13, 15$. (Since gap $g = 2$ matches the arrival interval).
  - Last response (P_5) handling starts at 15.
  - Completion: $15 + o_r = 16$ (or 17 if strictly waiting for gap to finish).

Result: $T_{run} \approx \mathbf{17}$ cycles.

Task 2: BSP Task Graph Analysis

Parameters: $g = 3$ flops, $L = 10$ flops. The execution proceeds in 4 Supersteps corresponding to the layers of the graph.

Superstep 1 (Nodes A, B, C)

- **Computation:** $W_{max} = \max(10, 20, 15) = 20$.
- **Communication:** The graph shows a butterfly-like connection (All-to-All between layers). Each processor sends data to 2 other processors (based on edges A→E,F etc.). Edge weight = 5. $h = 2$ messages $\times$ 5 flops = 10. Cost = $h \cdot g = 10 \cdot 3 = 30$.
- **Sync:** $L = 10$.
- **Total:** $20 + 30 + 10 = 60$.

Superstep 2 (Nodes D, E, F)

- **Computation:** $W_{max} = \max(10, 10, 10) = 10$.
- **Communication:** Each processor broadcasts to others. Edge weight = 10. $h = 2$ messages $\times$ 10 flops = 20. Cost = $h \cdot g = 20 \cdot 3 = 60$.
- **Sync:** $L = 10$.
- **Total:** $10 + 60 + 10 = 80$.

Superstep 3 (Nodes G, H, I)

- **Computation:** $W_{max} = \max(15, 10, 20) = 20$.
- **Communication:** Nodes G, H, I all send to node J (located on P_1). P_1 receives from P_2 and P_3. Edge weight = 5. $h_{recv} = 5 + 5 = 10$. (Max h determines the cost). Cost = $10 \cdot 3 = 30$.
- **Sync:** $L = 10$.
- **Total:** $20 + 30 + 10 = 60$.

Superstep 4 (Node J)

- **Computation:** $J = 5$.
- **Total:** 5.

Total Execution Time: $60 + 80 + 60 + 5 =$ **205** flops.

Task 3: Multi-BSP Model (Ryzen)

Unit Conversion: The problem specifies:

- **Time Unit:** 100 clock cycles (cc).
- **Data Unit:** KB (1024 Bytes).

We must convert the Bandwidth (B) and Latency (L_{cyc}) from Table 1 into these model units.

Derivation of Parameters:

1. **Level 3 (L3 Cache - Shared by CCD tasks):**
   - $B = 32$ B/cycle.
   - Time to transfer 1 KB: $1024/32 = 32$ cycles.
   - In model units: $32/100 = \mathbf{0.32}$. So, $g_{L3} = 0.32$.
   - Latency: $L = 40$ cycles $\rightarrow \mathbf{0.4}$ units.
2. **Level 4 (RAM - Between CCDs):**
   - $B = 16$ B/cycle.
   - Time to transfer 1 KB: $1024/16 = 64$ cycles.
   - In model units: $64/100 = \mathbf{0.64}$. So, $g_{RAM} = 0.64$.
   - Latency: $L = 300$ cycles $\rightarrow \mathbf{3.0}$ units.

Scenario 1: All Tasks on First CCD (Single CCD)

Communication occurs via L3 ($g = 0.32, L = 0.4$). Assumed Mapping: p_i, p_j, p_k are cores within CCD1.

- **Superstep 1 (A, B $\rightarrow$ D, E):**
  - Comp: $A = 5, B = 7$. $W = 7$.
  - Comm: $A \rightarrow D$ (32 KB), $A \rightarrow C$ (32 KB). $h = 32 + 32 = 64$.
  - Cost: $7 + 64(0.32) + 0.4 = 7 + 20.48 + 0.4 = \mathbf{27.88}$.
- **Superstep 2 (C, D, E $\rightarrow$ F, G):**
  - Comp: $C = 7, D = 5, E = 3$. $W = 7$.
  - Comm: $D \rightarrow G$ (1024 KB), $C \rightarrow F$ (1024 KB).

- $h = 1024$.
- Cost: $7 + 1024(0.32) + 0.4 = 7 + 327.68 + 0.4 = \mathbf{335.08}$.

- **Superstep 3 (F, G $\rightarrow$ H):**
  - Comp: $F = 3, G = 5$. $W = 5$.
  - Comm: $F \rightarrow H$ (32 KB), $G \rightarrow H$ (1024 KB).
  - Receiver (p_H) gets $32 + 1024 = 1056$ KB.
  - $h = 1056$.
  - Cost: $5 + 1056(0.32) + 0.4 = 5 + 337.92 + 0.4 = \mathbf{343.32}$.
- **Superstep 4 (H):** Cost $= 2$.
- **Total:** $27.88 + 335.08 + 343.32 + 2 \approx \mathbf{708}$ units.

Scenario 2: Split Across CCDs

Mapping: p_i, p_j on CCD1; p_k on CCD2. Local comm uses g_{L3}, Remote uses g_{RAM}.

- **Superstep 1 ($A \rightarrow D$):**
  - $A(p_i) \rightarrow D(p_j)$: Local (CCD1).
  - Cost same as above: **17.64** (only 32KB moved here).
- **Superstep 2 ($D \rightarrow G$):**
  - $D(p_j) \in$ CCD1 $\rightarrow G(p_k) \in$ CCD2.
  - **Remote Transfer!** Uses RAM parameters.
  - $h = 1024$.
  - Cost: $7(\text{comp}) + 1024(0.64) + 3.0 = 7 + 655.36 + 3.0 = \mathbf{665.36}$.
- **Superstep 3 ($F \rightarrow H, G \rightarrow H$):**
  - $F(p_i) \in$ CCD1 $\rightarrow H(p_j) \in$ CCD1. (Local: $h = 32$).
  - $G(p_k) \in$ CCD2 $\rightarrow H(p_j) \in$ CCD1. (Remote: $h = 1024$).
  - The step duration is determined by the slower Remote communication.
  - Cost: $5(\text{comp}) + 1024(0.64) + 3.0 = \mathbf{663.36}$.
- **Superstep 4 (H):** Cost $= 2$.
- **Total:** $17.64 + 665.36 + 663.36 + 2 \approx \mathbf{1348}$ units.