

# **CS4247**

# **Graphics Rendering Techniques**

Semester 2, 2015/2016

## **Lecture 1**

## **Raster Graphics Pipeline**

**School of Computing**  
**National University of Singapore**

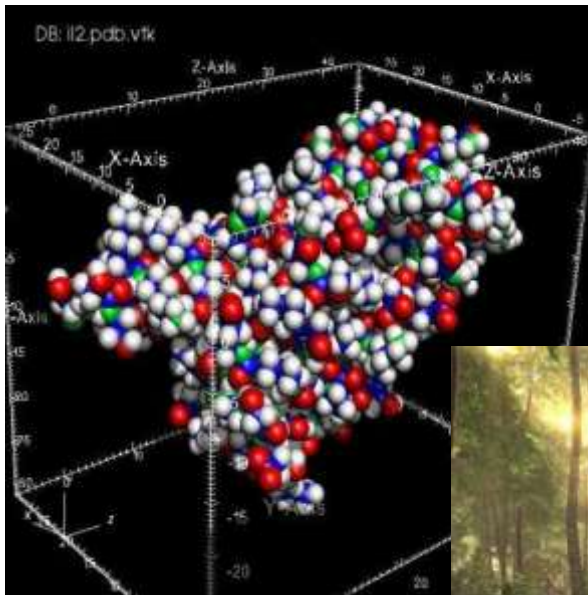
# Lecture Outline

- Polygonal representation of 3D objects
  - (Watt 2000 Sec 2.1)
- Raster graphics pipeline
  - (Watt 2000 Chapter 5 & 6)

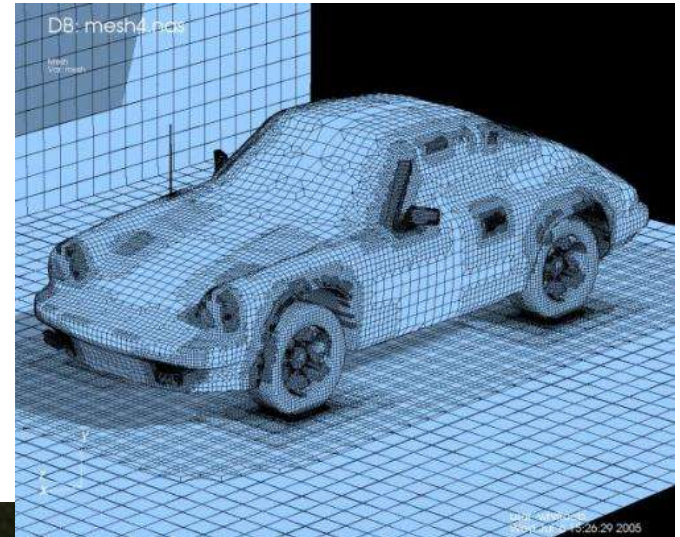
# Example Applications

- Real-time interactive 3D graphics

Scientific Visualization



3D design



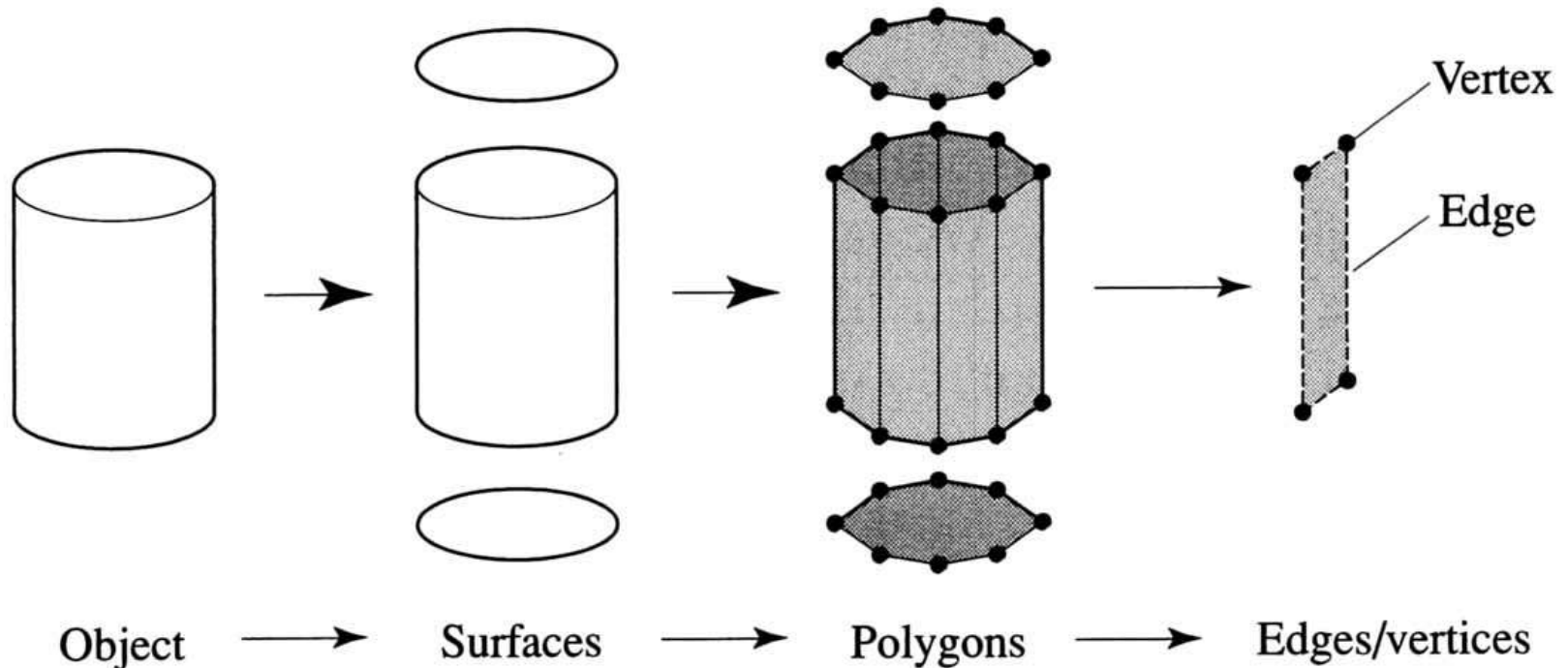
(Image of model Adrienne Curry rendered on a GeForce 8800 GTX GPU)

3D games

# **Polygonal Representation of 3D Objects**

# Polygonal Representation of 3D Objects

- 3D objects are approximated by a net or mesh of planar polygonal facets



# Polygonal Representation of 3D Objects

- Polygonal representation and rendering is currently the mainstream for real-time 3D graphics
- Advantages
  - Simplicity & generality
  - Simple & effective shading algorithms available to reduce faceted appearance
  - Well-supported by current commodity graphics hardware
    - Video processors (GPUs) made by NVidia and AMD (ATI)
    - Popular game consoles, e.g. Sony PlayStation 1/2/3/4, MS XBox/360/One, Nintendo Wii

# Polygonal Representation of 3D Objects

## ■ Disadvantages

- Inaccuracy of representation
- Complex objects (esp. with curved surfaces) need large number of polygons, but is wasteful when objects are projected to only a few pixels on screen
- Not suitable for interactive manipulation and free-form sculpting of models

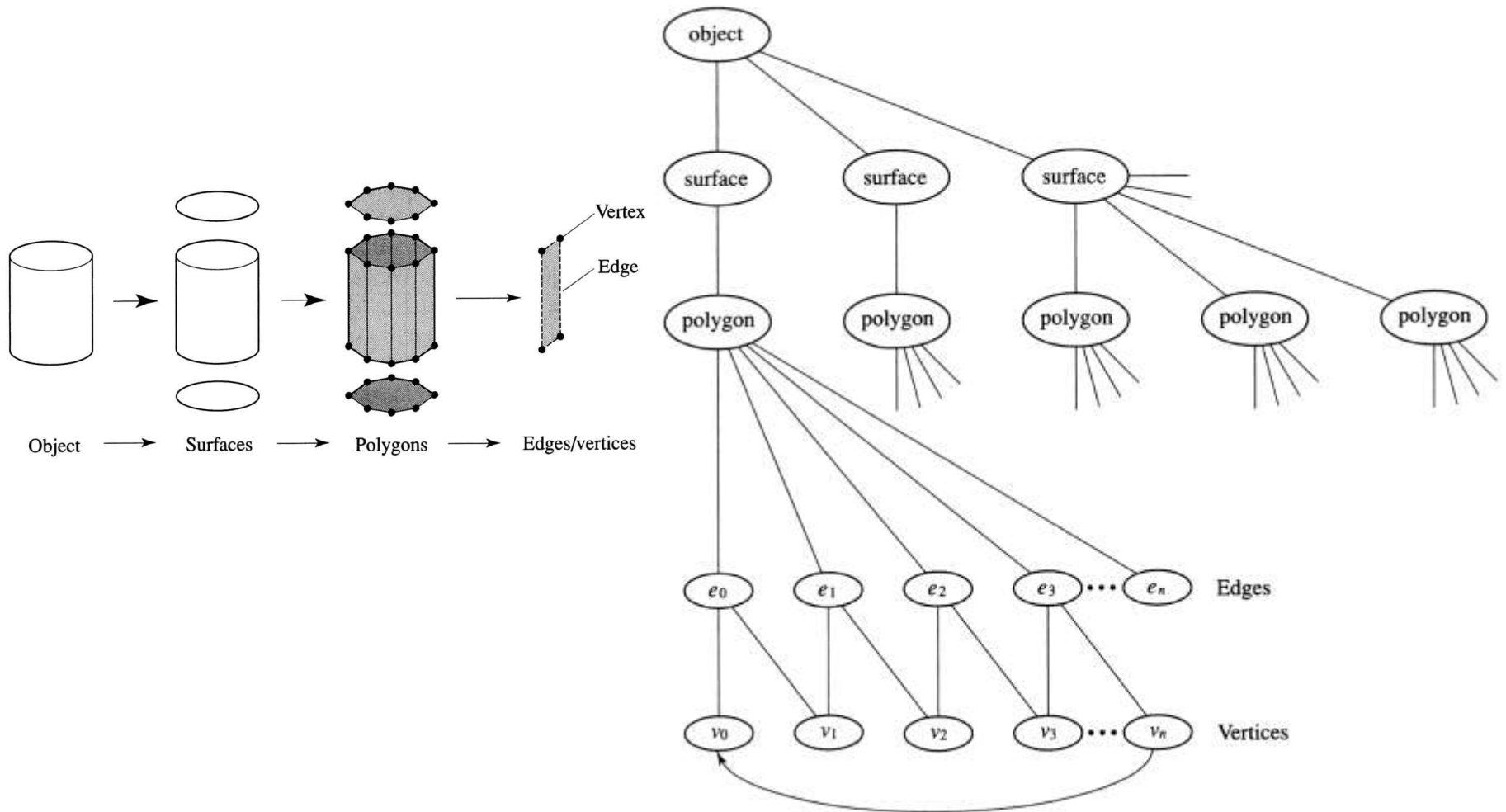
# Data Structures for Polygonal Models

- The simplest is stored each polygon independently with a list of  $(x, y, z)$  coordinates that are the polygon vertices
  - Wasteful because vertices shared by multiple polygons are stored multiple times
  - Inefficient for geometric queries and manipulations
    - Find all faces adjacent to a face/vertex
    - Find all vertices adjacent to a face/vertex

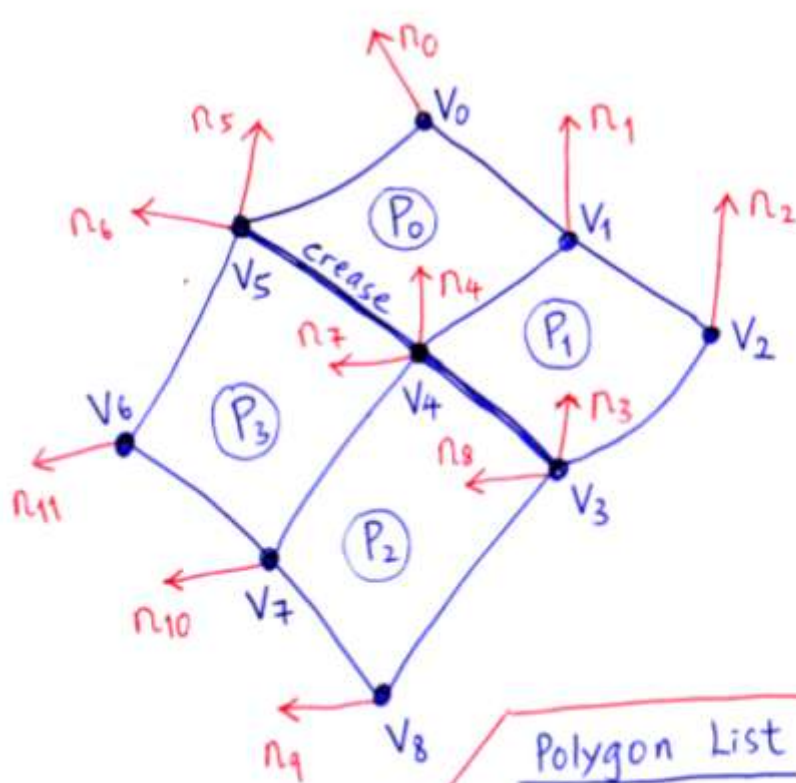


# Data Structures for Polygonal Models

- Represent as a hierarchical structure



# Data Structures for Polygonal Models



Vertex List  
 0:  $x_0 \ y_0 \ z_0$   
 $\vdots$   
 8:  $x_8 \ y_8 \ z_8$

Normal List  
 0:  $i_0 \ j_0 \ k_0$   
 $\vdots$   
 11:  $i_{11} \ j_{11} \ k_{11}$

Polygon List

0:	4	1	0	5	4	1	0	5	4
$\vdots$	$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$
	# sides	vertex indices				normal indices			
3:	4	5	6	7	4	6	11	10	7
		$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$	$\underbrace{\hspace{1cm}}$

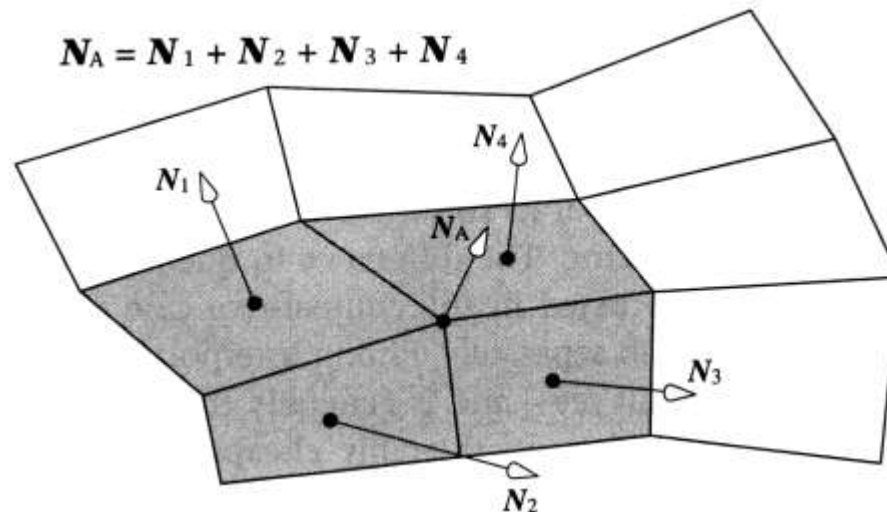
Vertex-list  
data structure

# Data Structures for Polygonal Models

- Winged-edge data structure
  - Can represent holes
  - Efficient for many geometric queries and manipulations, e.g.
    - Find all faces adjacent to a face/vertex
    - Find all vertices adjacent to a face/vertex

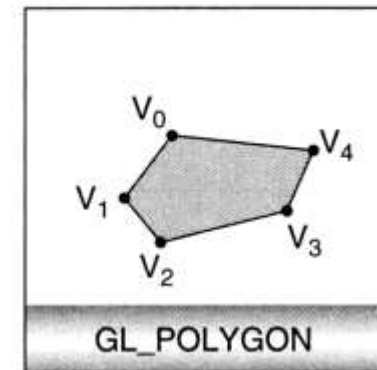
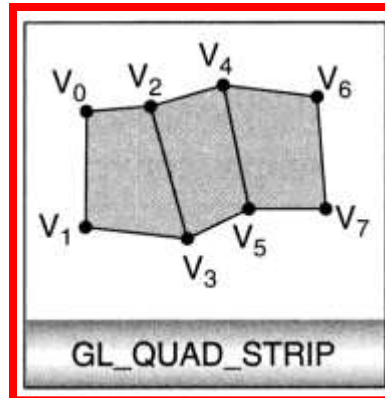
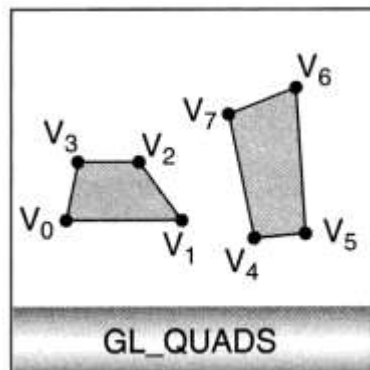
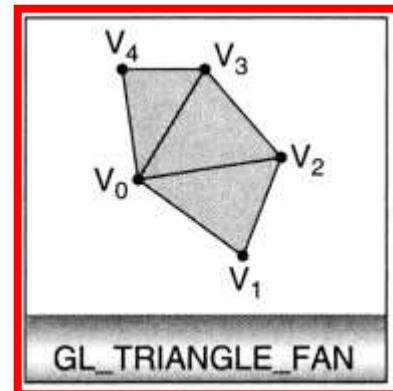
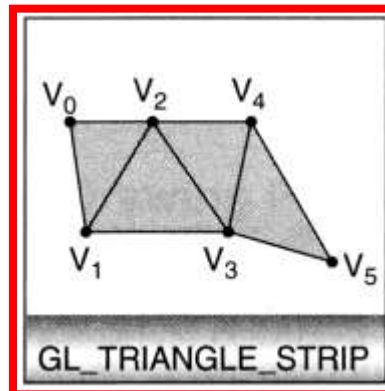
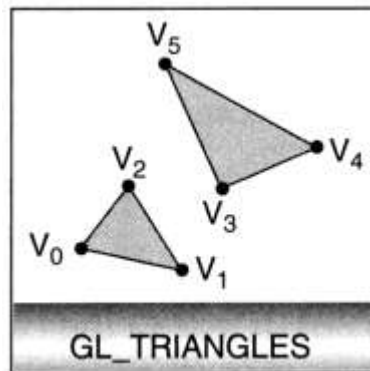
# Vertex Normals

- Each polygon vertex is associated a vertex normal
  - For computation of light reflection intensity at the vertex
  - For “flat” shading, use polygon normal as vertex normal
  - For “smooth” shading,
    - use “true” surface normal (analytical) at the vertex, or
    - use average polygon normals of polygons sharing the vertex



# Sending Polygons for Rendering

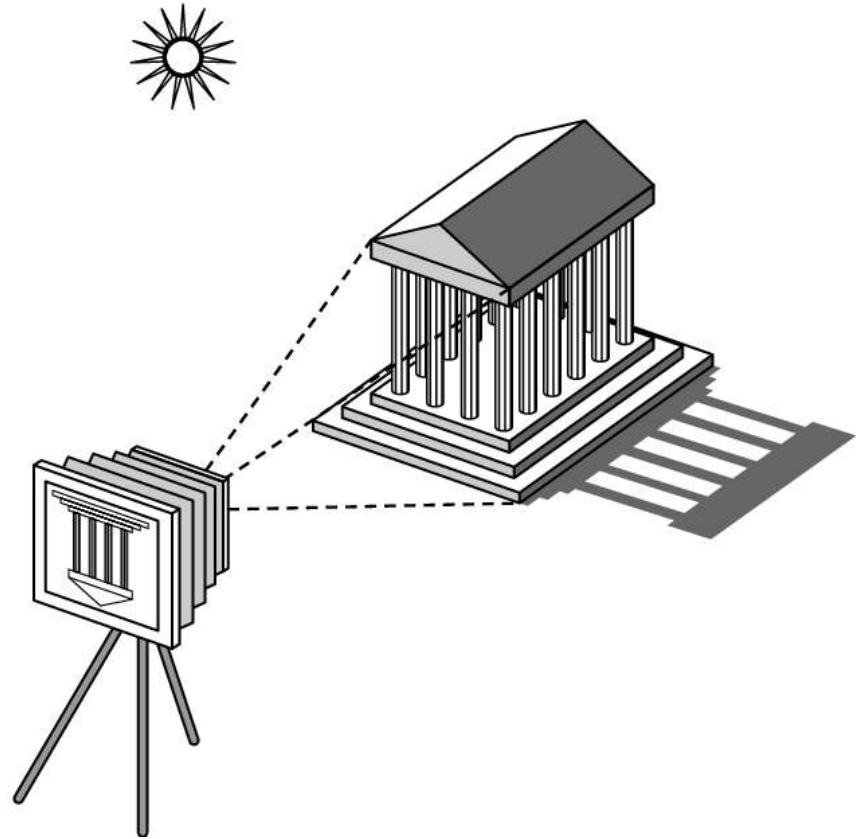
- It is inefficient to send polygons independently to the graphics pipeline for rendering
  - OpenGL provides functions to reduce sending duplicate data



# **OpenGL Rendering Pipeline**

# Elements of Image Formation

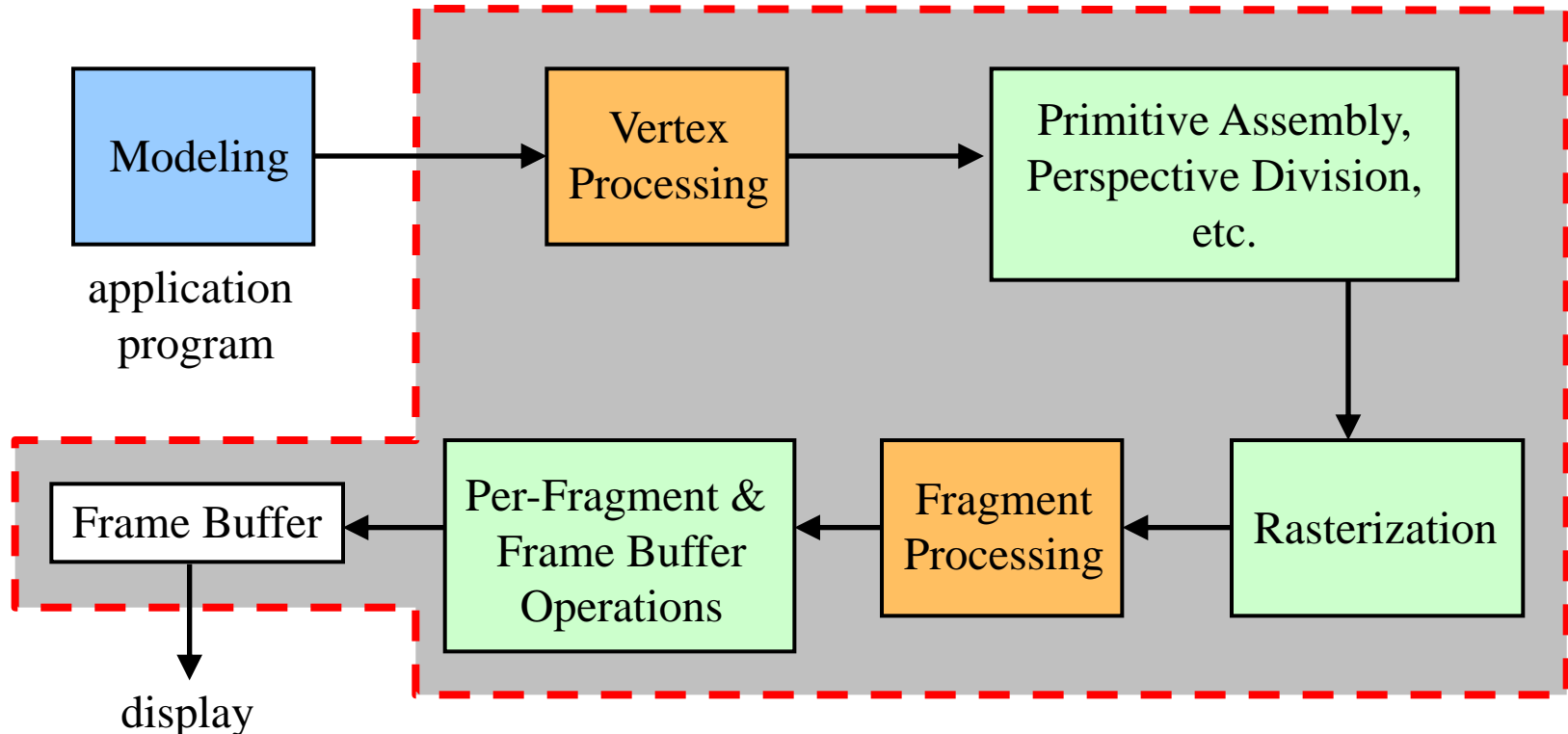
- Objects
- Viewer
- Light source(s)
- Materials
  - Attributes that govern how light interacts with the materials in the scene



# Basic OpenGL 3D Rendering Pipeline

- To render a primitive using OpenGL, the primitive goes through the following main stages
  - Turning primitive into pixels

Important





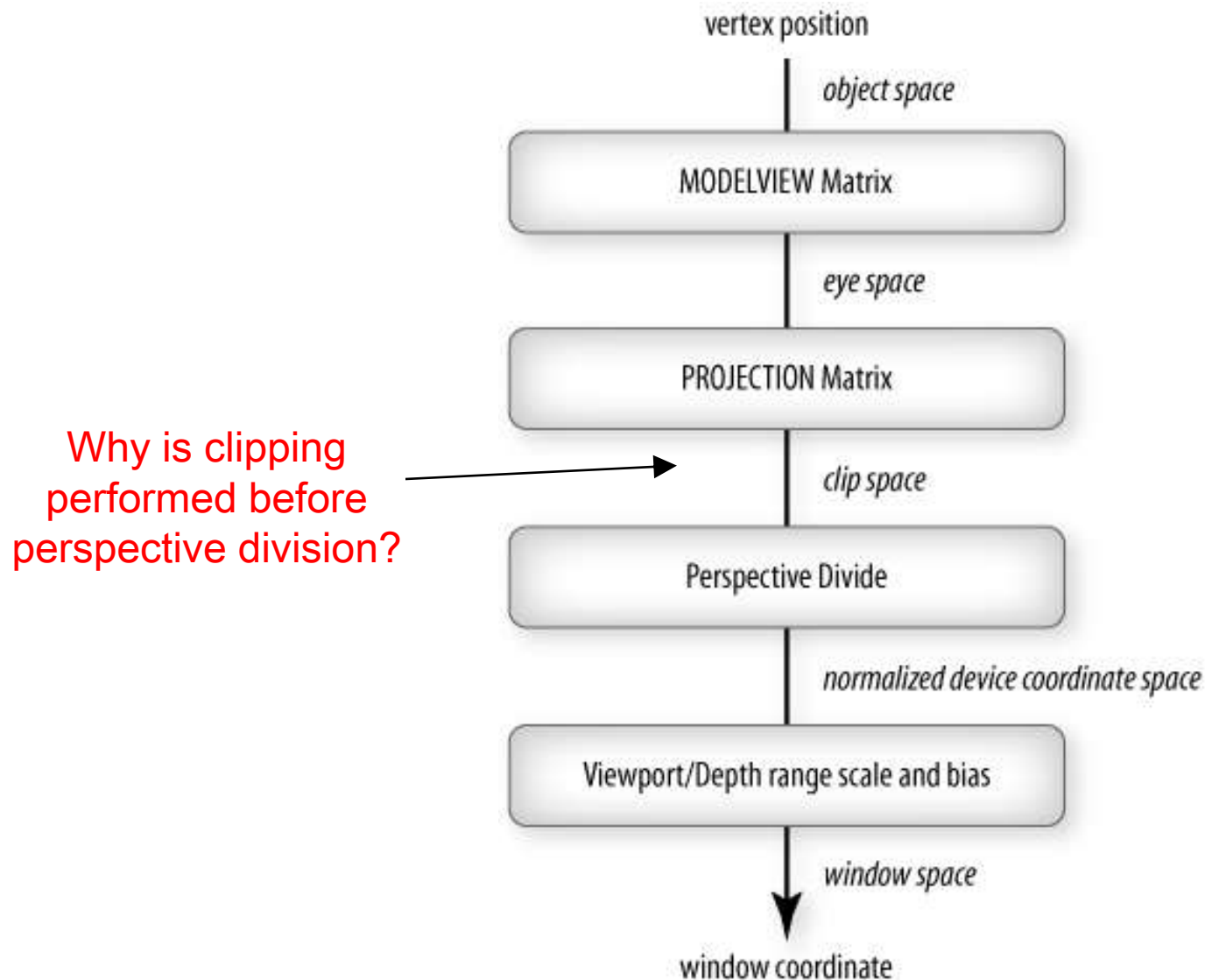
# Modeling

- Performed in the application program
- Sets up the OpenGL states (graphics/render context)
  - Light sources, materials, textures, view/projection, etc.
- Provides vertices to specify geometric primitives
  - Each vertex can be associated with attributes
    - Color, vertex normal, texture coordinates, etc.
  - Vertices and attributes can be provided using vertex array or display list
- May perform geometric processing to reduce amount of geometric data passed to rendering pipeline
  - E.g., view-frustum culling, occlusion culling

# Vertex Processing

- The fixed-functioned stage performs
  - Vertex transformation
    - View transformation and projection
  - Normal transformation and normalization
    - Into camera/view/eye space
  - Texture coordinate generation
  - Texture coordinate transformation
  - Lighting computation
    - Computed in eye space
    - Resulting color replaces input vertex color attribute
  - Color material application
- Programmable by GLSL (vertex shader)

# OpenGL Geometric Transformation Stages



# Primitive Assembly, etc.

- Primitive assembly

- Vertex data is collected into complete primitives
- Necessary for clipping and back-face culling

- Clipping

- Perspective division

- To normalized device coordinate (NDC) space

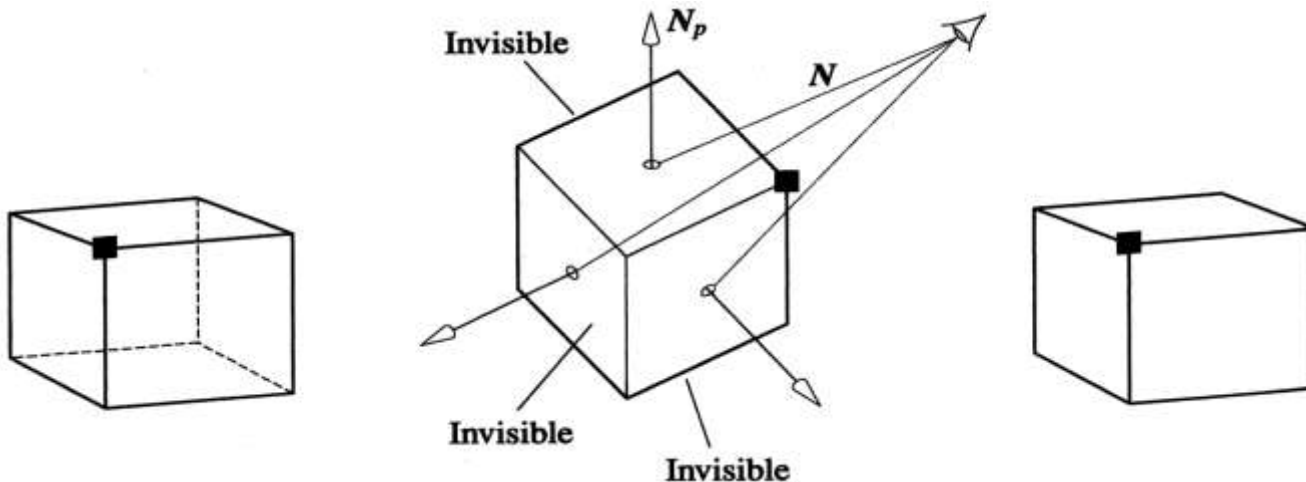
- Viewport transformation

- To windows space
- Include depth range scaling

- Back-face culling

# Back-Face Culling / Elimination

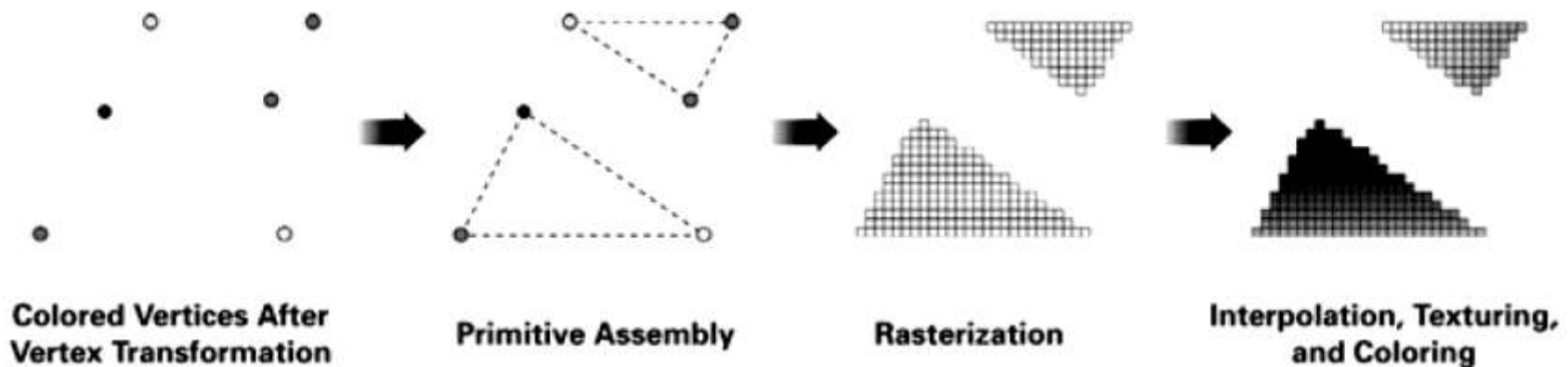
- Eliminate a polygon if it is back-facing and invisible
  - Polygon is back-facing if  $\mathbf{N}_p \cdot \mathbf{N} < 0$



- By convention, when front-face of a polygon is visible, its vertices are listed in counter-clockwise order
- In OpenGL, back-face culling is performed in the window space by computing the signed 2D area of the polygon

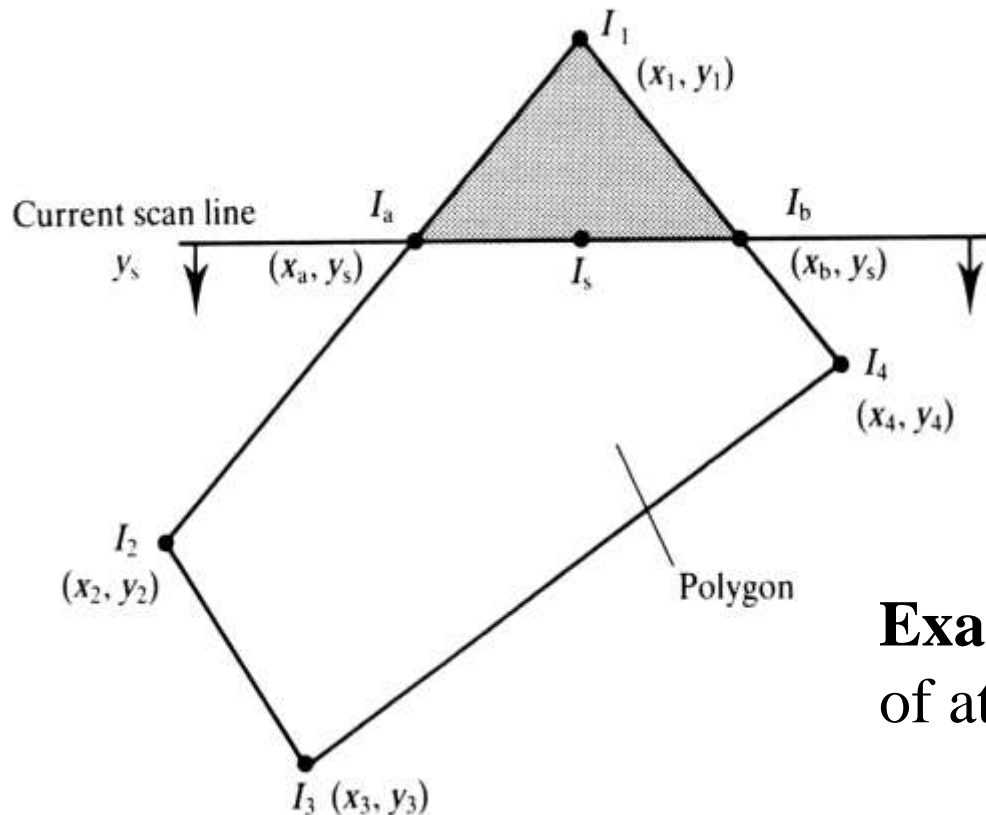
# Rasterization (Scan Conversion)

- If geometric primitive is not clipped out, the appropriate pixels in the frame buffer must be assigned colors
- Rasterizer produces a set of fragments for each primitive
  - Fragments are “potential pixels”
    - Has a pixel/fragment location (in window space)
    - Has color and depth attributes (and others)
- Vertex attributes are interpolated over the primitive



# Interpolation of Vertex Attributes

- Attribute values at fragments are computed by interpolating attribute values assigned to vertices
  - Interpolation is performed in window space (2D)



$$I_a = \frac{1}{y_1 - y_2} [I_1 (y_s - y_2) + I_2 (y_1 - y_s)]$$

$$I_b = \frac{1}{y_1 - y_4} [I_1 (y_s - y_4) + I_4 (y_1 - y_s)]$$

$$I_s = \frac{1}{x_b - x_a} [I_a (x_b - x_s) + I_b (x_s - x_a)]$$

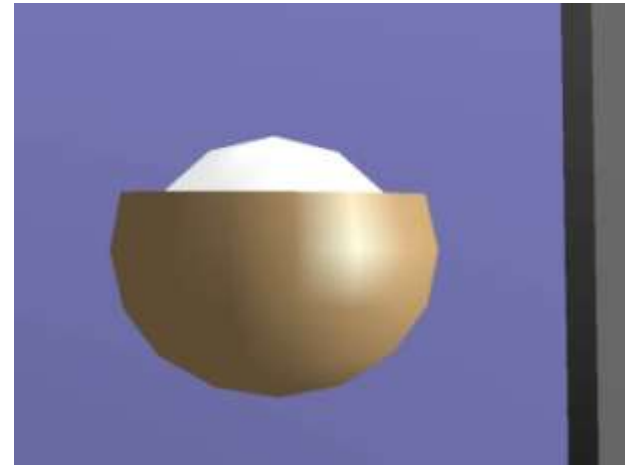
**Example:** Bilinear interpolation  
of attribute values

# Example: Color Interpolation



**Flat Shading**

No interpolation of vertex colors



**Smooth (Gouraud) Shading**

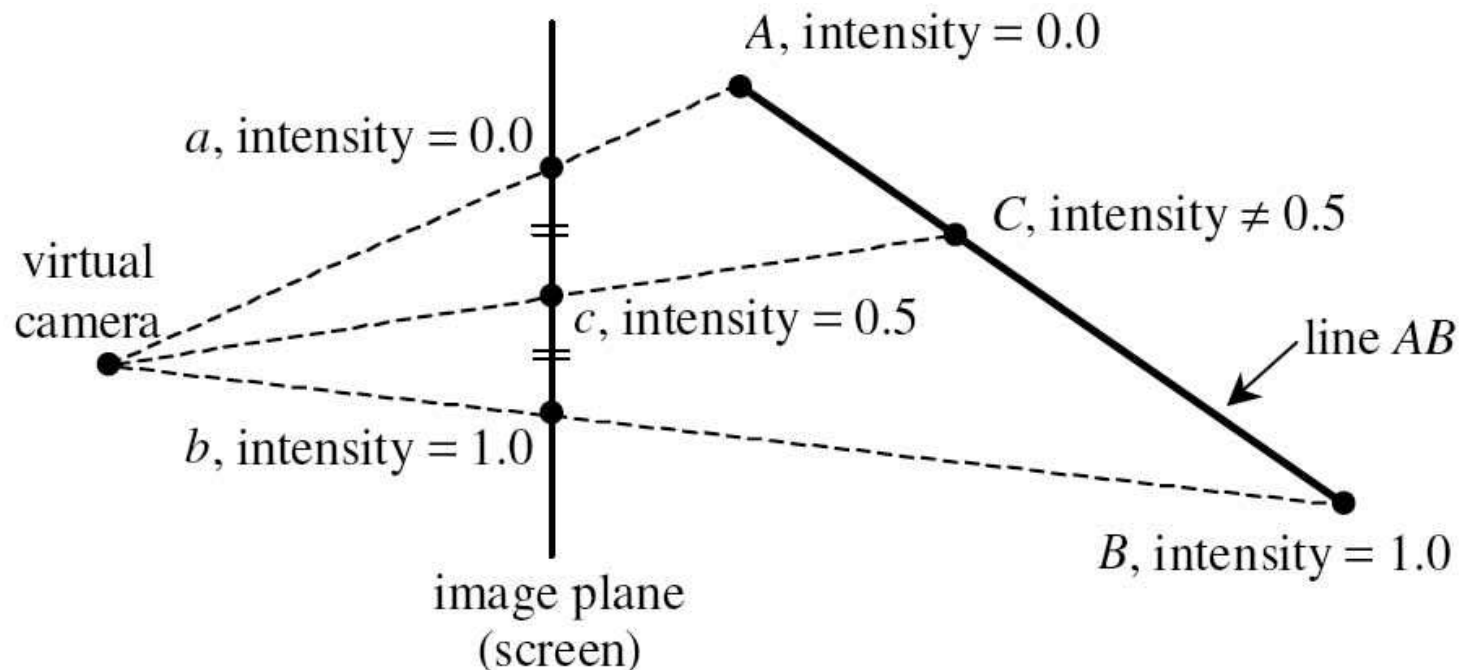
Interpolation of vertex colors

- When flat shading, only vertex color and secondary color are “flat”, all other attributes are still smooth
  - Flat shading uses color of last vertex of the primitive
- Other vertex attributes that are interpolated
  - Z-value (depth), texture coordinates

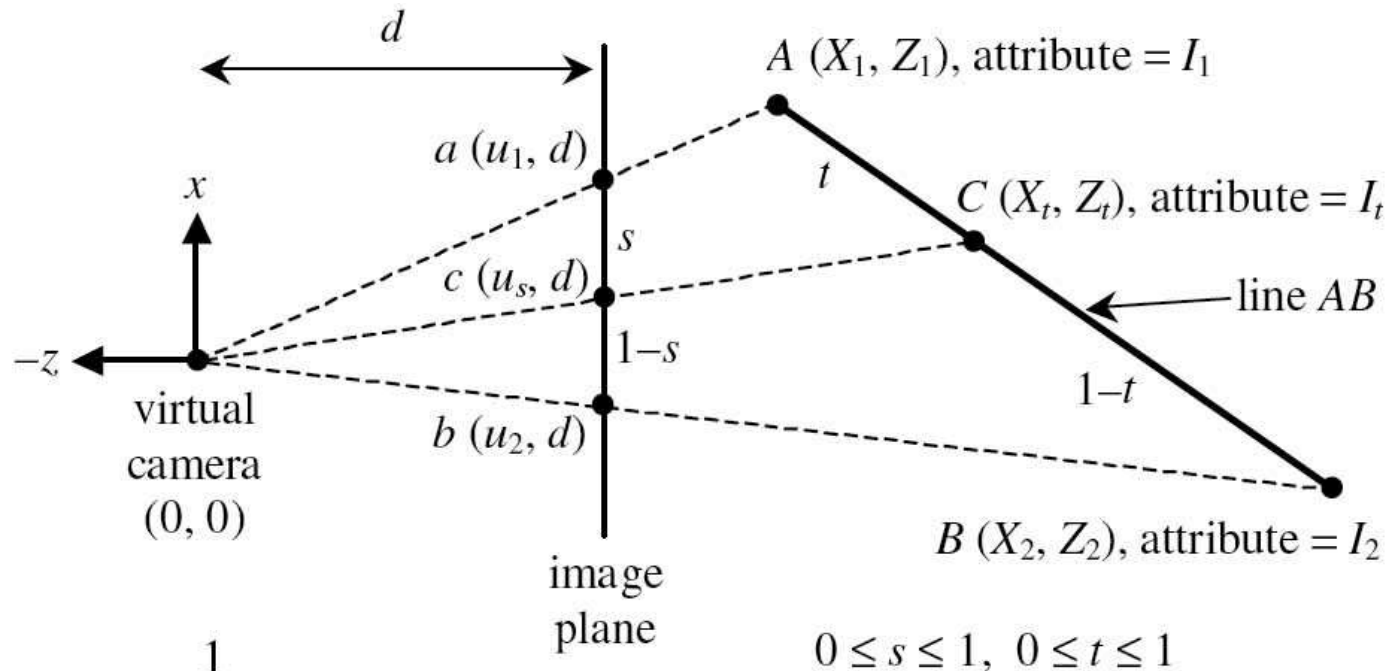


# Perspective-Correct Interpolation

- Note that direct bilinear interpolation in window space does not produce perspective-correct result
  - Bilinearly varying attribute value in 3D space in general does not vary bilinearly in 2D window space



# Perspective-Correct Interpolation



$$Z_t = \frac{1}{\frac{1}{Z_1} + s \left( \frac{1}{Z_2} - \frac{1}{Z_1} \right)}$$

$$I_t = \left( \frac{I_1}{Z_1} + s \left( \frac{I_2}{Z_2} - \frac{I_1}{Z_1} \right) \right) \bigg/ \frac{1}{Z_t}$$

Refer to technical report “Perspective-Correct Interpolation” at

[http://www.comp.nus.edu.sg/~lowkl/publications/lowk\\_persp\\_interp\\_techrep.pdf](http://www.comp.nus.edu.sg/~lowkl/publications/lowk_persp_interp_techrep.pdf)

# Fragment Processing

Q: What is a fragment?  
Where are they from?

- Each generated fragment is processed to determine the color of the corresponding pixel in the frame buffer
- The fixed-functioned stage performs
  - Texture access (using interpolated texture coordinates)
    - May access multiple texture maps using multiple sets of texture coordinates (multi-texture)
  - Texture application
    - Texture color can be combined with the fragment color of the primitive (or with result from applying previous layer of texture)
    - Example: replace, modulate, decal, blend
  - Color sum
    - Combining primary and secondary colors
- Programmable by GLSL (fragment shader)

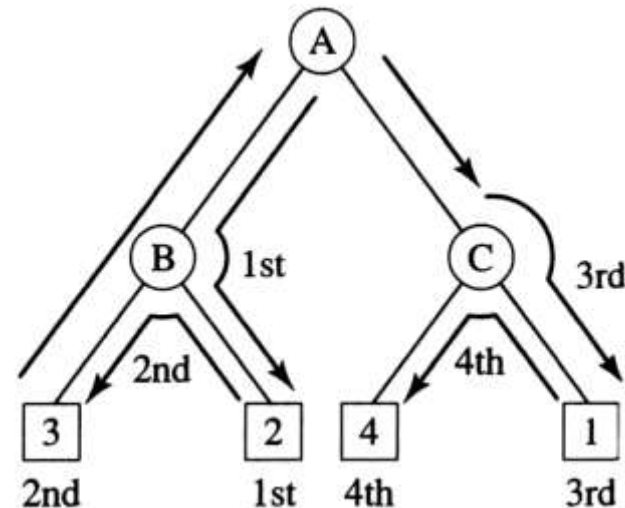
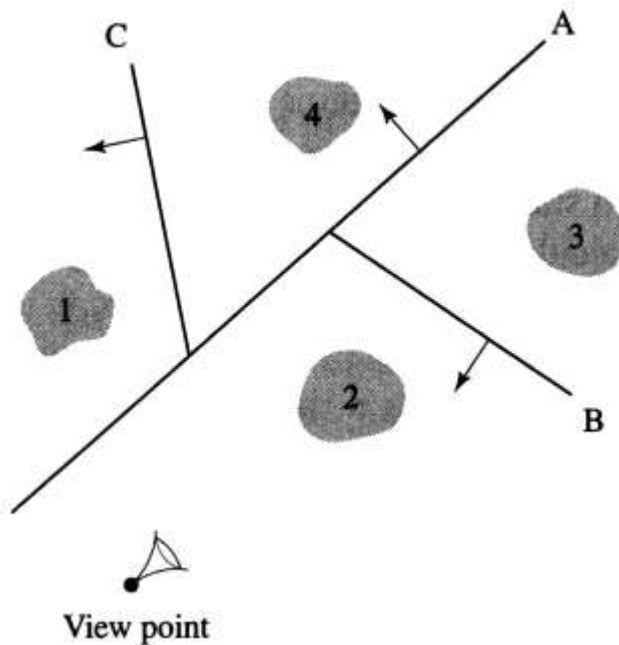
- After fragment processing, each resulting fragment is submitted to a set of simple operations before reaching the frame buffer



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# Hidden Surface Removal Using BSP Trees

- A binary space partitioning (BSP) tree partitions the scene (polygons) into hierarchical spatial regions
- Can be used to find the visibility order of the polygons from a given viewpoint
  - Allow back-to-front rendering of the polygons (no need z-buffer)

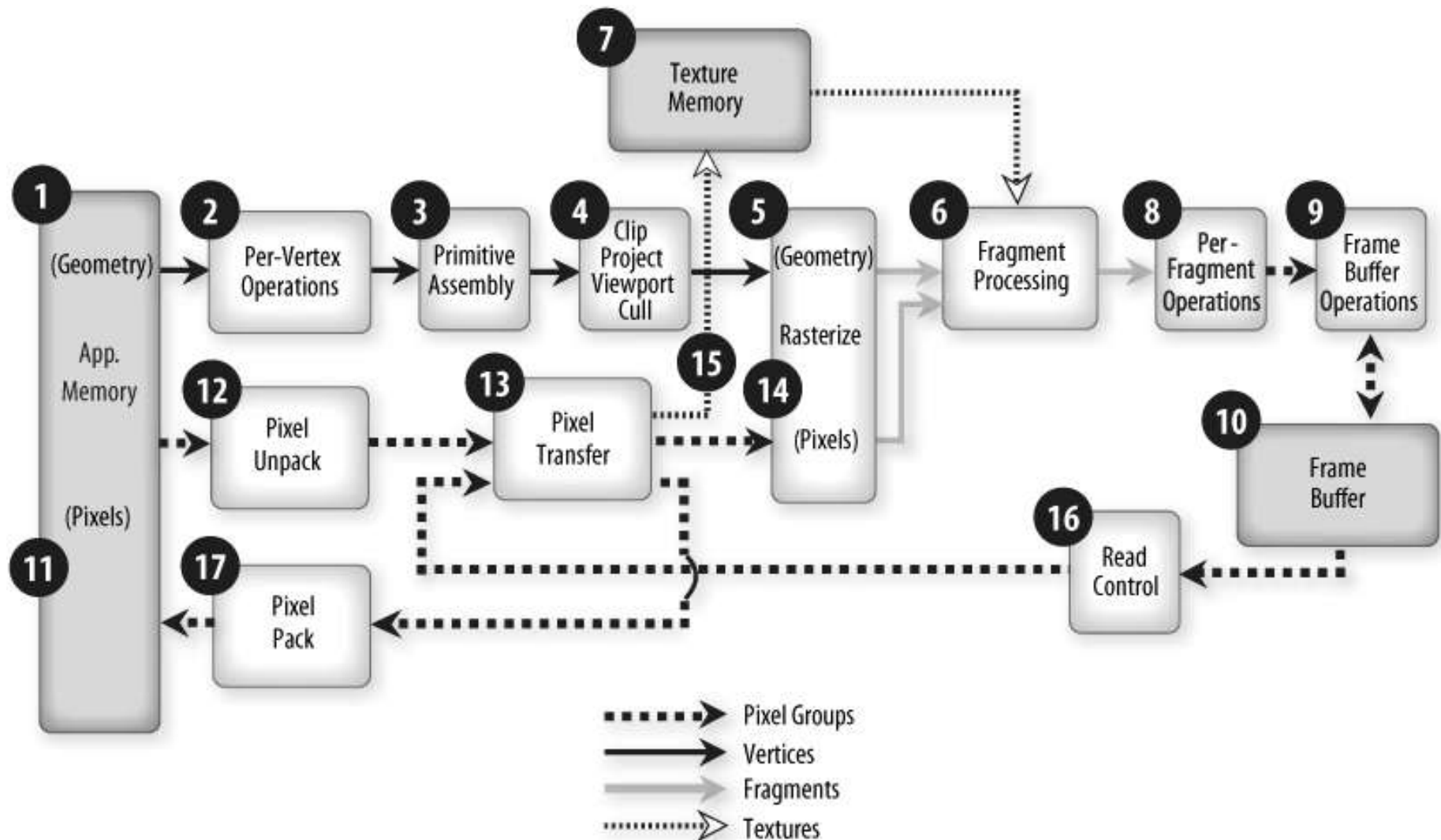


# Frame Buffer Operations

- Operations that affect or control whole frame buffer
  - Example
    - Target color buffer (e.g. front or back buffer)
    - Enable/disable writes to color channels
    - Enable/disable writes to depth buffer
    - Enable/disable writes to stencil buffer bitplanes

Important for some rendering algorithms. E.g. shadow volume algorithm enables/disables writes to depth and color buffers.

# More Detailed OpenGL Pipeline



# Images and Bitmaps

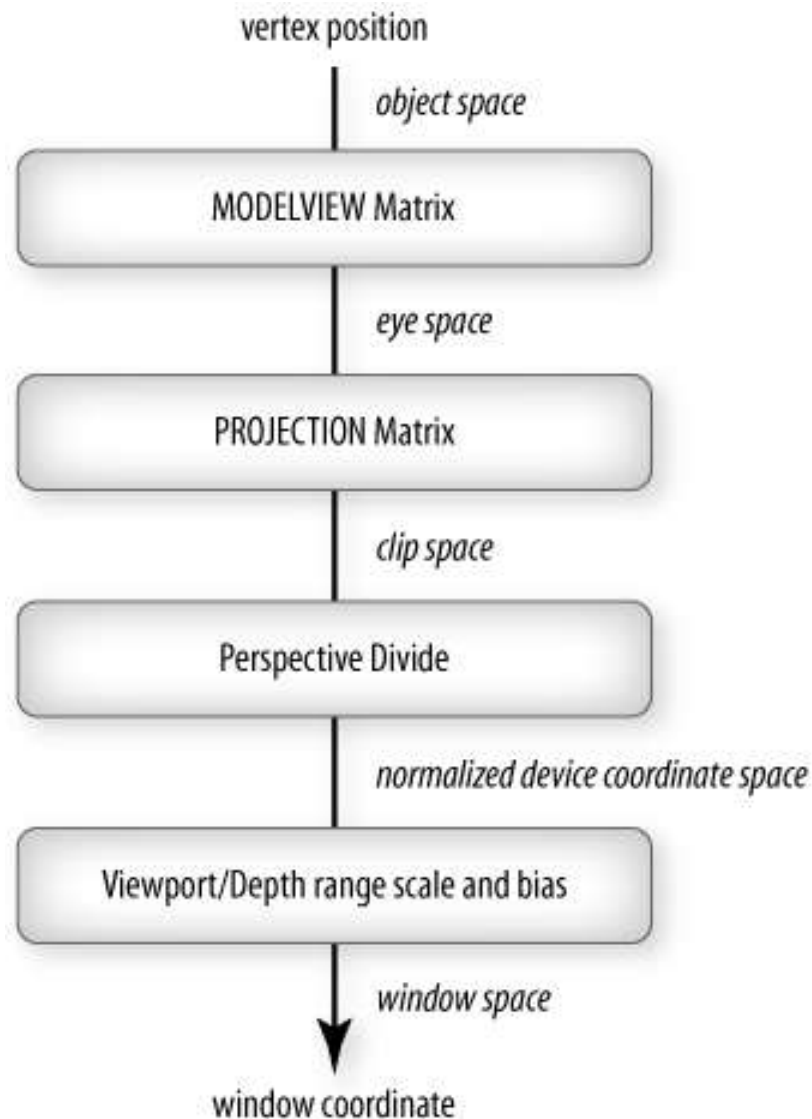
- OpenGL also has support for color/gray images (pixel rectangles) and bitmaps
  - Images and bitmaps from application can be rasterized to the frame buffer
  - Images in frame buffer can be read by application
  - Images from application can be read into texture memory
  - Images in frame buffer can be read into texture memory
- Read OpenGL “Redbook”, Chapter 8 for more details



# **OpenGL**

## **Geometric Transformations**

# More About OpenGL Geometric Transformations



# OpenGL Code – Example 1

```
void display( void ) {  
    ...  
    glviewport( 0, 0, 800, 600 );  
    glClear( GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT );  
  
    glMatrixMode( GL_PROJECTION );  
    glLoadIdentity();  
    gluPerspective( viewerFovY, viewerAspect, viewerNear, viewerFar );  
  
    glMatrixMode( GL_MODELVIEW );  
    glLoadIdentity();  
    gluLookAt( viewerPosX, viewerPosY, viewerPosZ,  
               lookAtX, lookAtY, lookAtZ, upX, upY, upZ);  
  
    glTranslate3d( 100.0, 200.0, 300.0 );  
    glScale3d( 3.0, 3.0, 3.0 );  
  
    glBegin( GL_QUADS );  
        glColor3d( 1.0, 0.0, 0.0 );    glVertex3d( 0.0, 0.0, 0.0 );  
        glColor3d( 0.0, 1.0, 0.0 );    glVertex3d( 1.0, 0.0, 0.0 );  
        glColor3d( 0.0, 0.0, 1.0 );    glVertex3d( 1.0, 1.0, 0.0 );  
        glColor3d( 1.0, 1.0, 1.0 );    glVertex3d( 0.0, 1.0, 0.0 );  
    glEnd();  
  
    glutSwapBuffers();  
}
```

# OpenGL Code – Example 2

```
void display( void ) {  
    ...  
    glViewport( 0, 0, 800, 600 );  
    glClear( GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT );  
  
    glMatrixMode( GL_PROJECTION );  
    glLoadIdentity();  
    gluPerspective( viewerFovY, viewerAspect, viewerNear, viewerFar );  
  
    glMatrixMode( GL_MODELVIEW );  
    glLoadIdentity();  
    gluLookAt( viewerPosX, viewerPosY, viewerPosZ,  
              lookAtX, lookAtY, lookAtZ, upX, upY, upZ);  
  
    glPushMatrix();  
        glTranslate3d( 100.0, 200.0, 300.0 );  
        glScale3d( 3.0, 3.0, 1.0 );  
        glColor3d( 1.0, 0.0, 0.0 ); // Red  
        drawUnitSquare();  
    glPopMatrix();  
  
    glPushMatrix();  
        glTranslate3d( 400.0, 400.0, 500.0 );  
        glScale3d( 6.0, 6.0, 1.0 );  
        glColor3d( 0.0, 1.0, 0.0 ); // Green  
        drawUnitSquare();  
    glPopMatrix();  
  
    glutSwapBuffers();  
}
```

```
void drawUnitSquare( void ) {  
    glBegin(GL_QUADS);  
        glVertex3d( 0.0, 0.0, 0.0 );  
        glVertex3d( 1.0, 0.0, 0.0 );  
        glVertex3d( 1.0, 1.0, 0.0 );  
        glVertex3d( 0.0, 1.0, 0.0 );  
    glEnd();  
}
```

# Local / Modeling / Object Coordinates

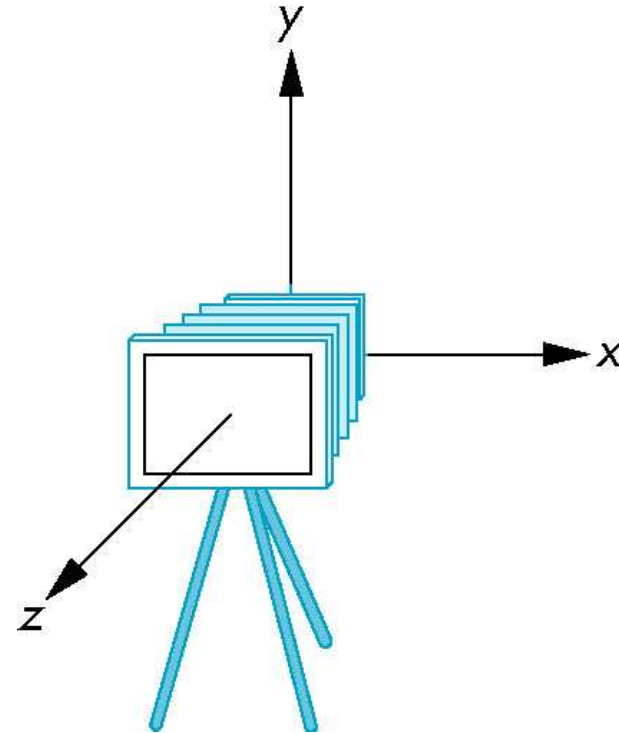
- Each object model has its own local coordinate frame
  - The coordinates of the vertices and vertex normals are specified with respect to the local coordinate frame
- Convenient for modeling of the object, e.g.
  - A sphere is centered at the origin
  - A cube with a corner located at the origin and edges parallel to the coordinate axes and

# World Coordinates

- A common coordinate frame for all objects to form the scene to be rendered
  - Each object is transformed from its local space to a common world space
  - Vertex normals must also be transformed
- Lights are defined in this space
  - Positions and directions
- Camera pose is defined in this space

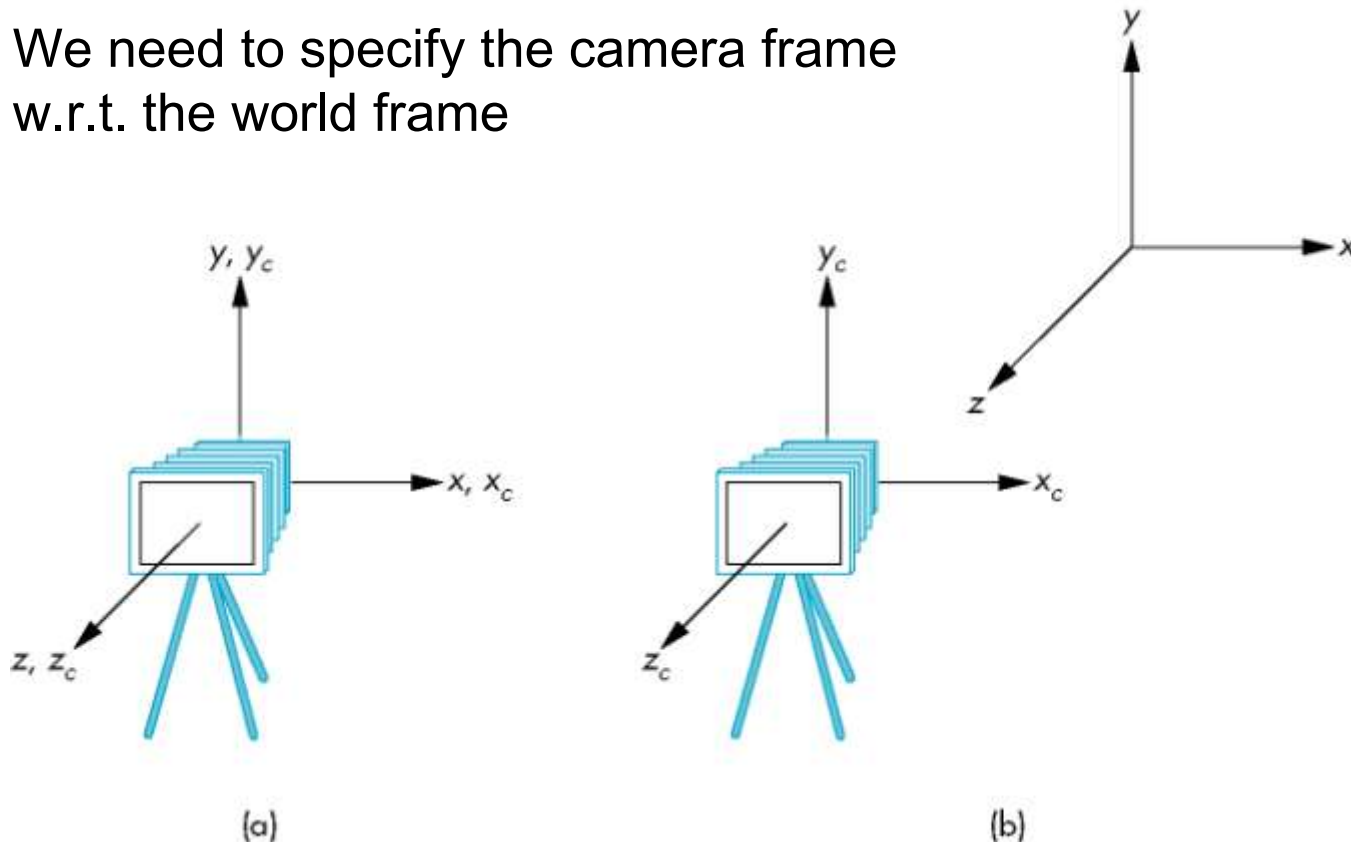
# Camera Coordinates

- The camera has a local coordinate frame, called the *camera coordinate frame*
  - Camera is located at the origin
  - Looking in negative  $z$  direction
  - $+y$ -axis is the “*up-vector*”
- All projections are w.r.t. the camera frame
- Initially the world and camera frames are the same
  - Default model-view matrix is an identity
- To specify camera pose, we need to specify the camera coordinate frame w.r.t. the world coordinate frame



# Specifying Camera Pose

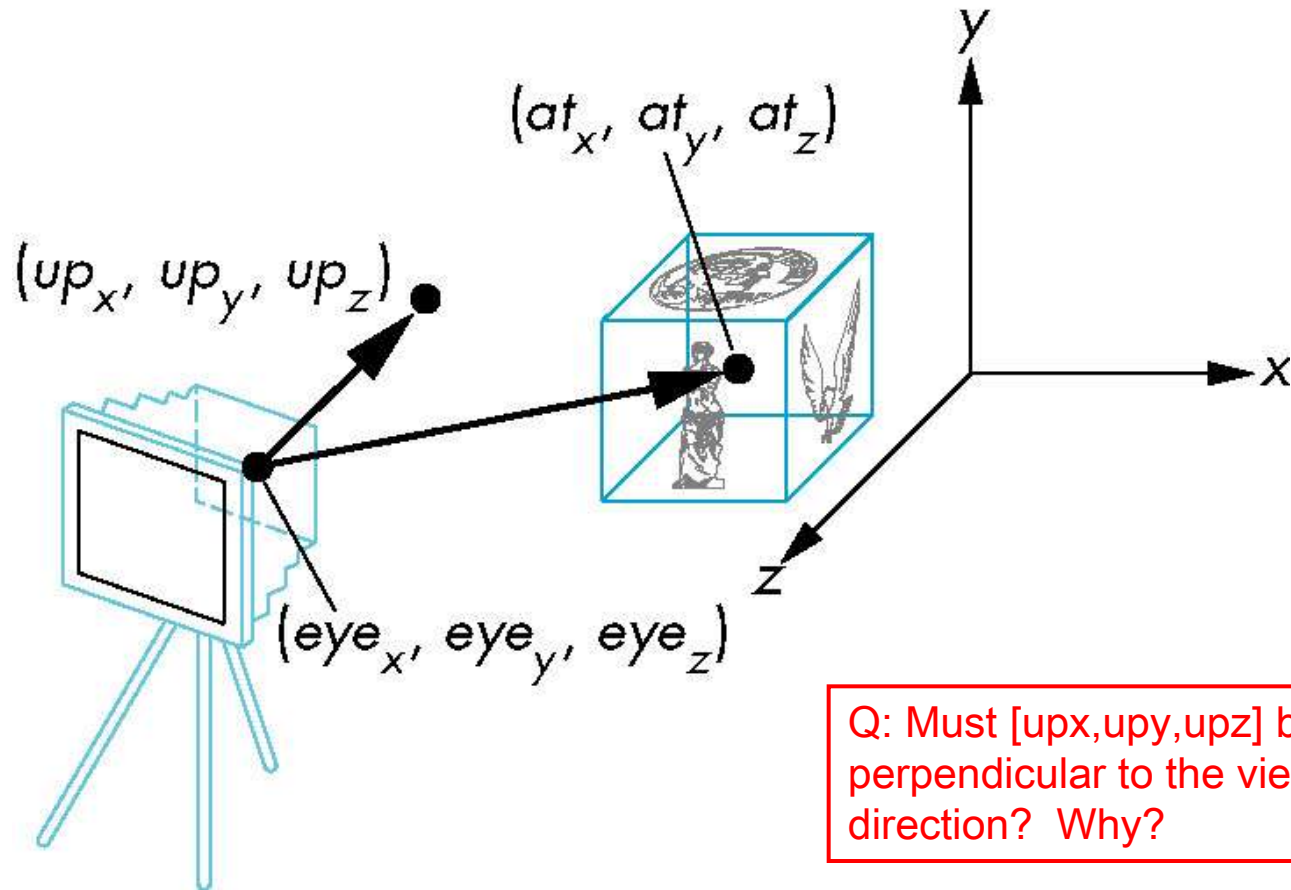
- By default, the camera frame coincides with the world frame
- Often, we want to put the camera at other location and orientation
  - We need to specify the camera frame w.r.t. the world frame





# Using the `gluLookAt()` Function

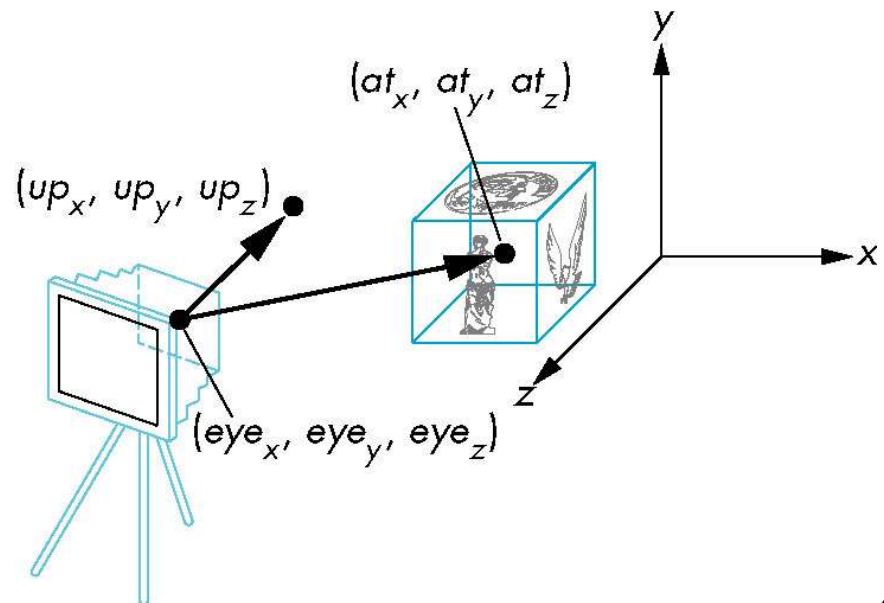
```
gluLookAt( eyex, eyey, eyez,  
           atx, aty, atz,  
           upx, upy, upz );
```



Q: Must  $[up_x, up_y, up_z]$  be perpendicular to the view direction? Why?

# The `gluLookAt()` Function

- Note that it does not directly specify the camera frame axes vectors **u**, **v**, **n** (correspond to  $x_c$ ,  $y_c$ ,  $z_c$  axes)
- The “up-vector” may not be perpendicular to the view direction
- The vectors **u**, **v**, **n** can be derived as follows
  - $\mathbf{n} = \text{normalize}(\mathbf{eye} - \mathbf{at})$
  - $\mathbf{u} = \text{normalize}(\mathbf{up}) \times \mathbf{n}$
  - $\mathbf{v} = \mathbf{n} \times \mathbf{u}$



# The `gluLookAt()` Function

- Conceptually, it positions the camera at the required location and orientation (w.r.t. the world frame)
- Internally, it generates a transformation matrix that can be used to express all points in the world frame w.r.t. the camera frame
- This is called *view transformation*
- This *view transformation matrix* is post-multiplied to the current model-view matrix
- In OpenGL, the view transformation matrix is normally the last transformation in the model-view matrix

```
glMatrixMode( GL_MODELVIEW );  
glLoadIdentity();  
// specify view transformation matrix here;  
...
```

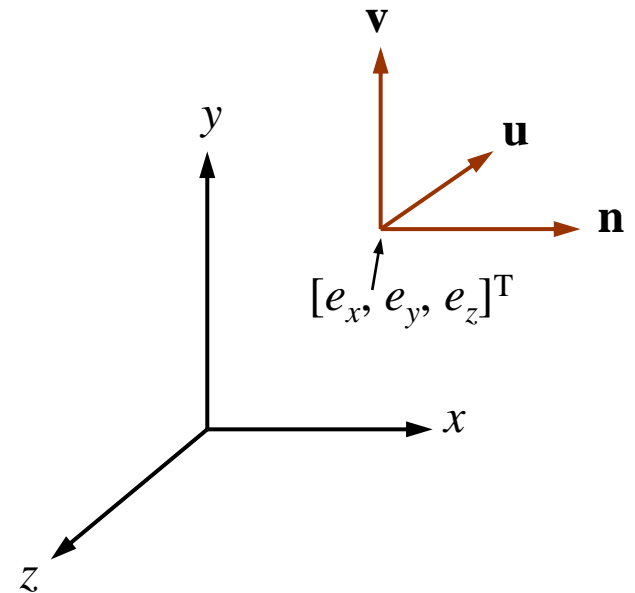
# View Transformation

- All points in the world frame are expressed w.r.t. the camera frame
  - Can be performed using a 4x4 matrix
  - It is made up of a translation first, then a rotation
    - $\mathbf{M}_{\text{view}} = \mathbf{R} \mathbf{T}$
  - The translation  $\mathbf{T}$  moves the camera position back to the world origin
  - The rotation  $\mathbf{R}$  rotates the axes of the camera frame to coincide with the corresponding axes of the world frame
  - Multiply all points in the world frame by  $\mathbf{M}_{\text{view}}$  and they will be expressed w.r.t. to the camera frame
  - Why view transformation?

# Deriving View Transformation Matrix

- Suppose the camera has been moved to the location  $[e_x, e_y, e_z]^T$ , and its  $x_c, y_c, z_c$  axes are the unit vectors  $\mathbf{u}, \mathbf{v}, \mathbf{n}$ , respectively, then

$$\mathbf{M}_{\text{view}} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



- Note that  $[e_x, e_y, e_z]^T$  and  $\mathbf{u}, \mathbf{v}, \mathbf{n}$  are all specified w.r.t. to the world frame

Q: Why use homogeneous coordinates in computer graphics?

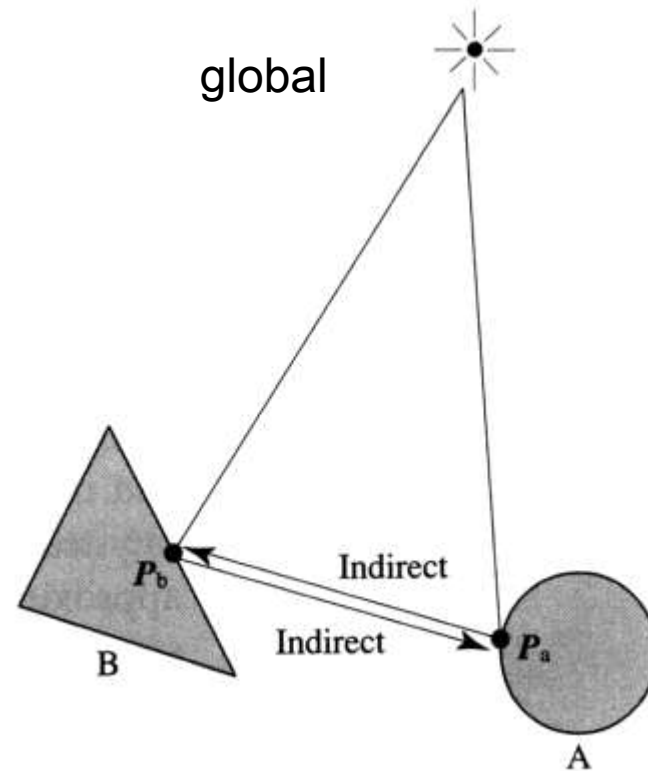
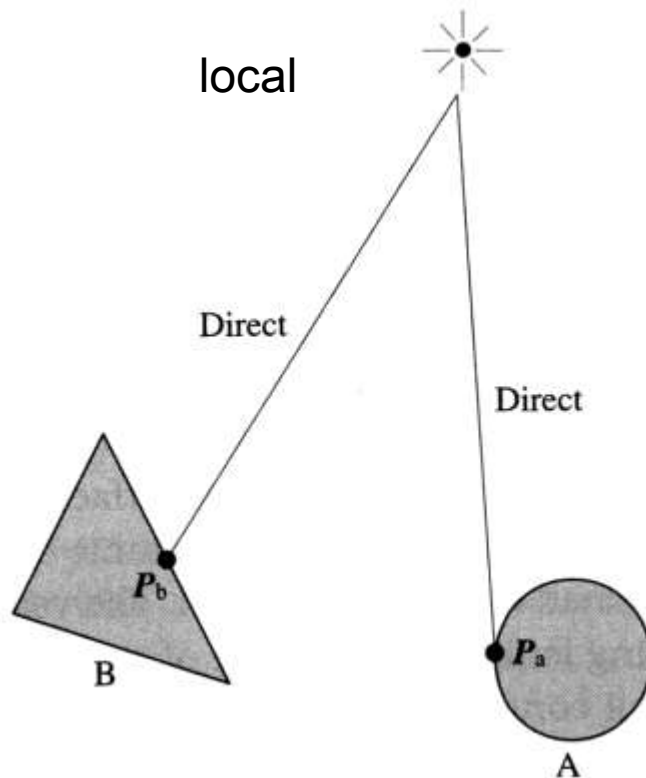
# Transformation of Normal Vectors

- When vertices are transformed by a transformation  $\mathbf{M}$ , how should the normal vectors be transformed?
  - $\mathbf{M}_n = (\mathbf{M}_t^{-1})^T$ 
    - where  $\mathbf{M}_t$  is the upper-left 3x3 submatrix of  $\mathbf{M}$
  - What is  $\mathbf{M}_n$  if  $\mathbf{M}_t$  is just a rotation?

Note that the matrix  $\mathbf{M}_n$  is computed by OpenGL automatically.

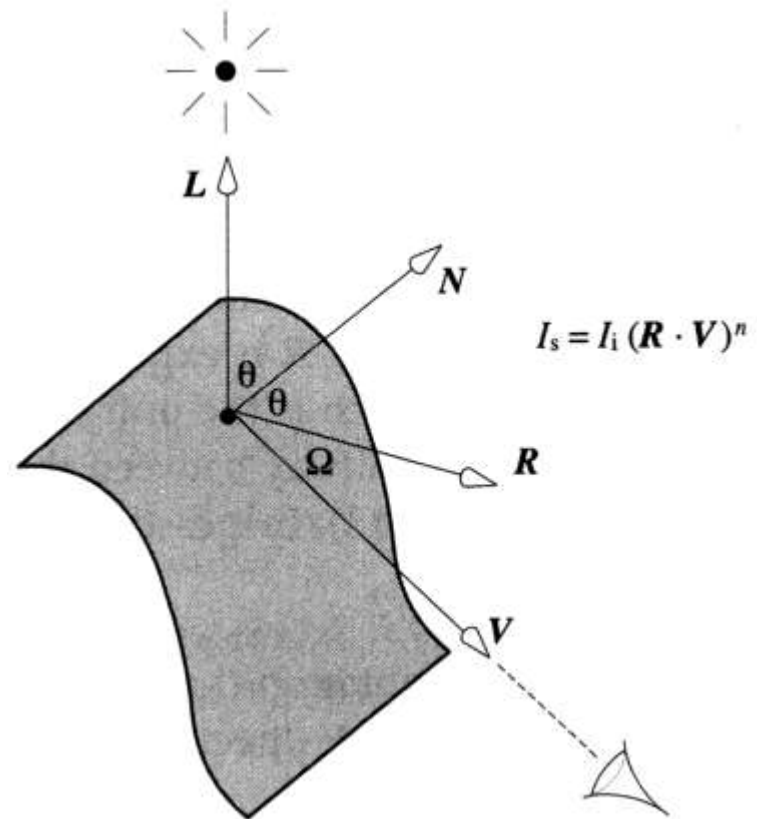
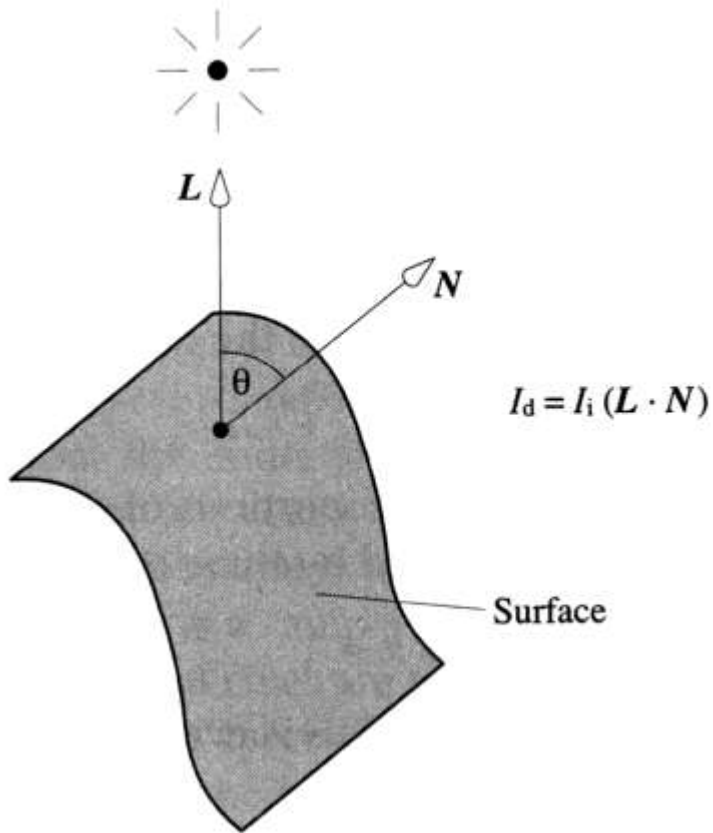
# Lighting Computation in Eye Space

- Calculate reflected light intensity at each vertex
  - Using a local reflection model, e.g. Phong Model



# Phong Reflection Model

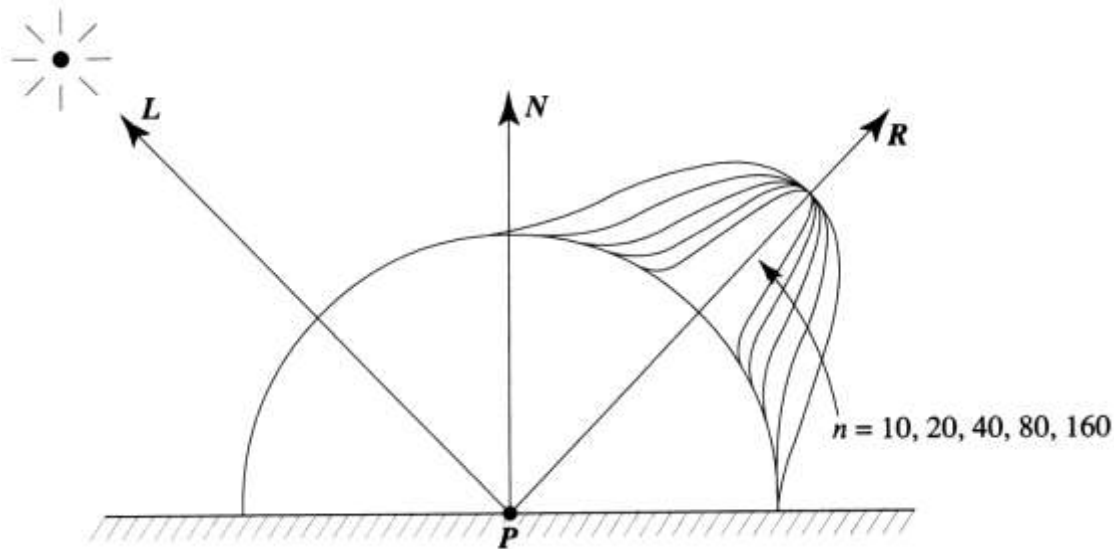
$$I = k_a I_a + I_i (k_d (\mathbf{L} \cdot \mathbf{N}) + k_s (\mathbf{R} \cdot \mathbf{V})^n)$$



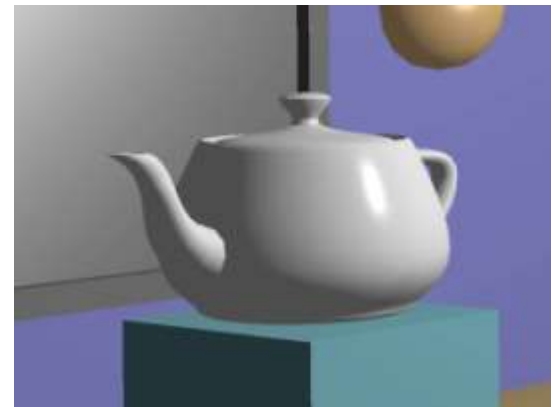


# Phong Reflection Model

$$I = k_a I_a + I_i (k_d (\mathbf{L} \cdot \mathbf{N}) + k_s (\mathbf{R} \cdot \mathbf{V})^n)$$

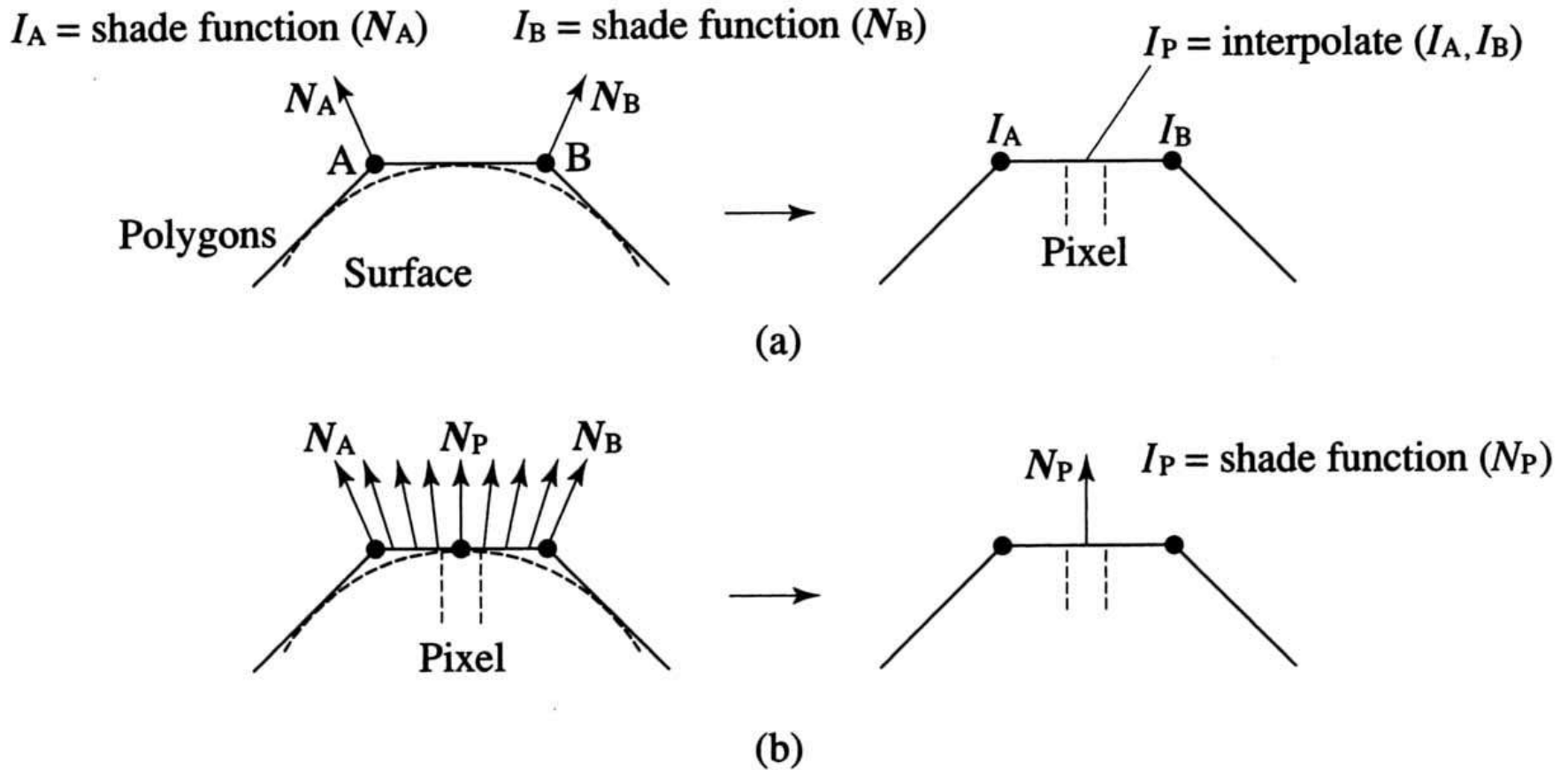


small exponent  $n$



large exponent  $n$

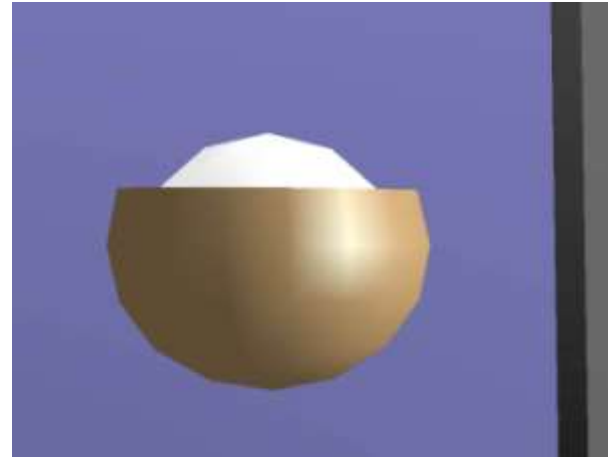
# Gouraud & Phong Shading



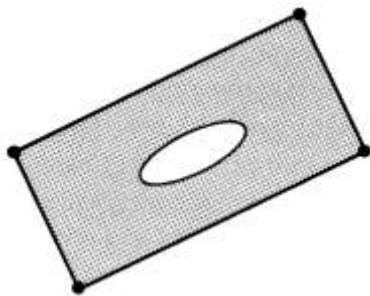
# Gouraud & Phong Shading



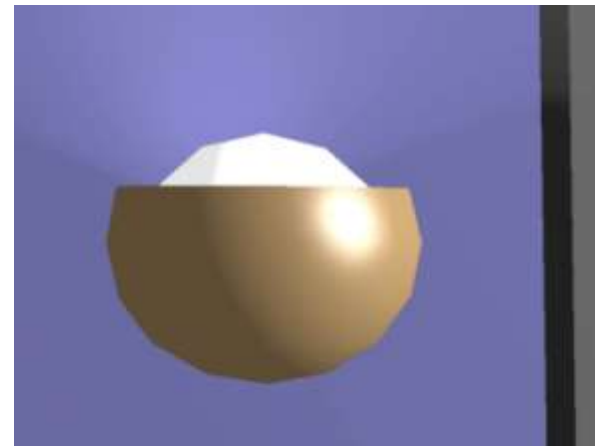
Flat Shaded



Gouraud Shaded



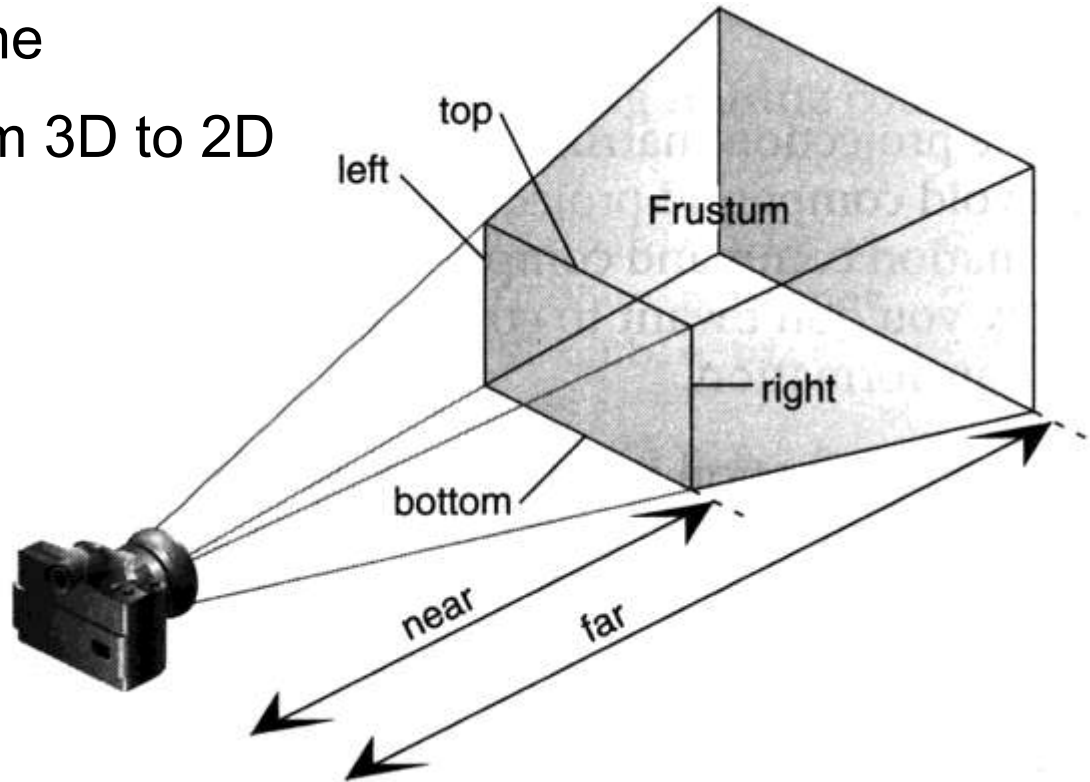
Gouraud shading may miss highlight  
in the interior of polygon.



Phong Shaded

# Projections

- Specifying a view volume for projection
  - Specified w.r.t. the eye space
  - 3D region of the scene to appear in the rendered image
  - A clipping volume
  - A projection from 3D to 2D



# Projections

- In OpenGL, after a vertex is multiplied by the model-view matrix, it is then multiplied by the projection matrix
- The projection matrix is a 4x4 matrix that defines the type of projection
- The projection matrix can be specified by first defining a *view volume* (or *clipping volume*) in the camera frame
  - For orthographic projection, use `glOrtho()`
  - For perspective projection, use `glFrustum()`

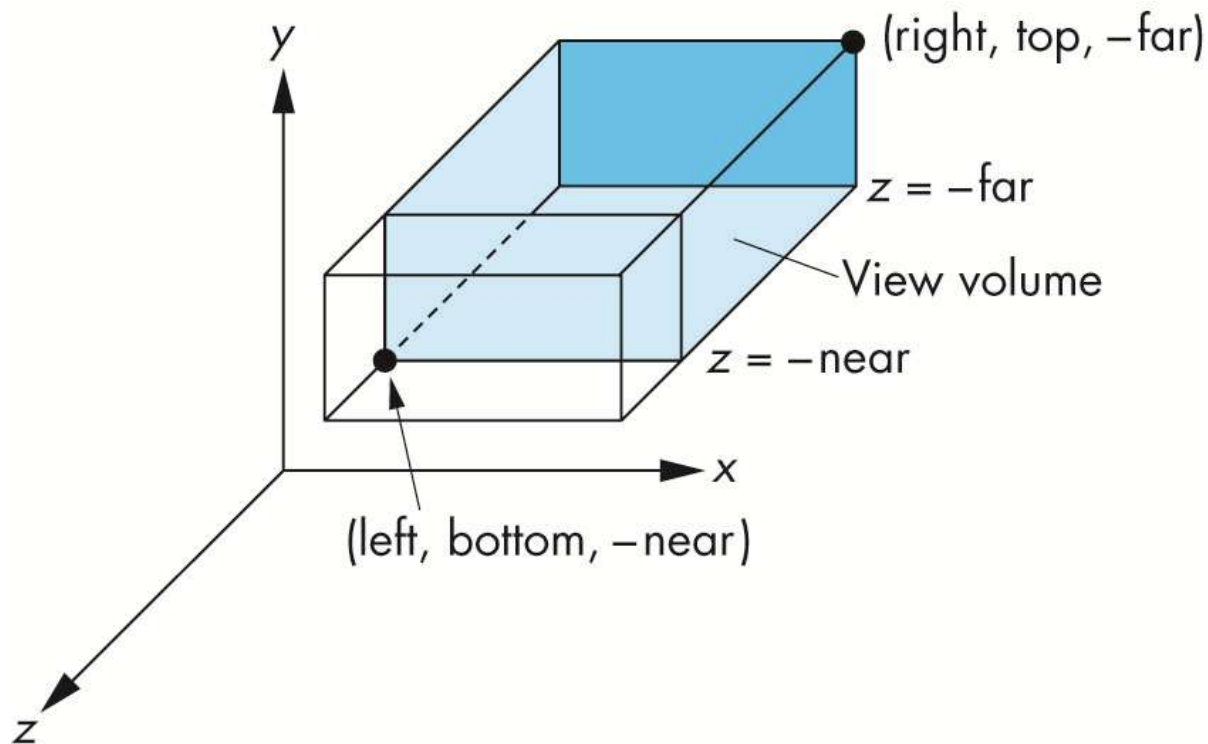
# Projections

- A projection matrix is then computed such that it maps points in the view volume to a *canonical view volume*
  - The canonical view volume is the 2 x 2 x 2 cube defined by the planes  $x = \pm 1$ ,  $y = \pm 1$ ,  $z = \pm 1$
  - Also called the *Normalized Device Coordinates* (NDC)
- Some criteria
  - Preserve depth order (in  $z$  coordinate)
  - Preserve lines
- The canonical view volume is then mapped to the viewport (*viewport transformation*)

# OpenGL Orthographic Projection

- Can be specified by defining a view volume (in the camera frame) using

```
glOrtho( left, right, bottom, top, near, far );
```

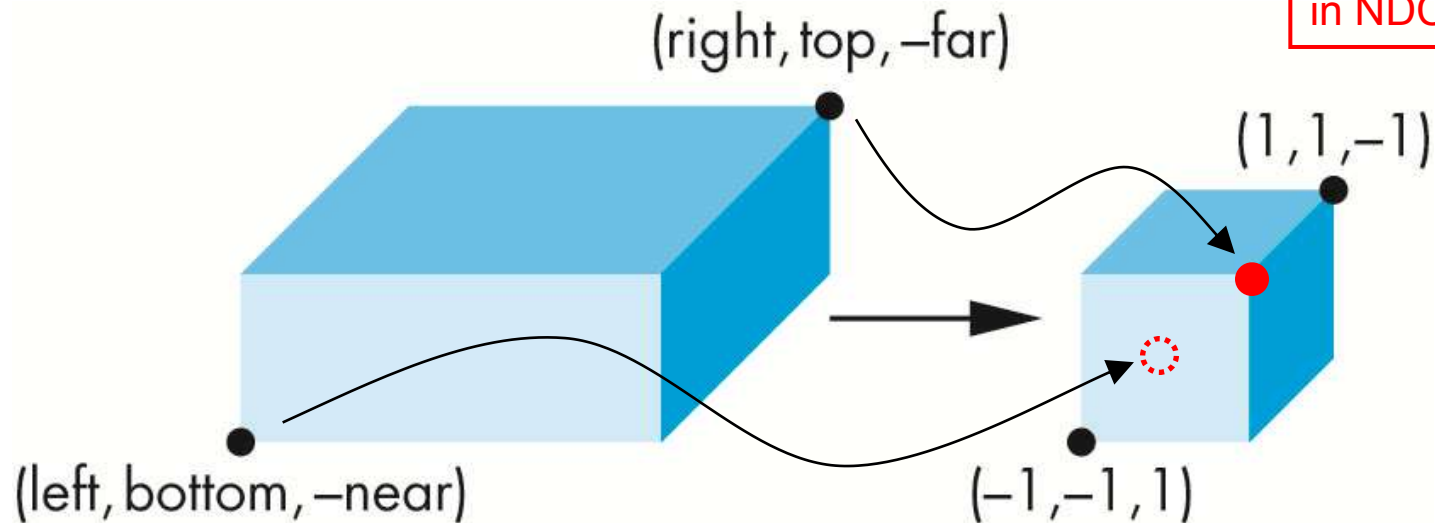


# OpenGL Orthographic Projection

- The `glOrtho()` function then generates a matrix that linearly maps the view volume to the canonical view volume, where

- (left, bottom, -near) is mapped to  $(-1, -1, -1)$
- (right, top, -far) is mapped to  $(1, 1, 1)$

Q: Why are the z values inverted in NDC?





# Orthographic Projection Matrix

- The mapping can be found by
  - First, translating the view volume to the origin
  - Then, scaling the view volume to the size of the canonical view volume

$$\mathbf{M}_{\text{ortho}} = \mathbf{S} \left( \frac{2}{\text{right} - \text{left}}, \frac{2}{\text{top} - \text{bottom}}, \frac{2}{\text{near} - \text{far}} \right) \cdot \mathbf{T} \left( \frac{-(\text{right} + \text{left})}{2}, \frac{-(\text{top} + \text{bottom})}{2}, \frac{(\text{far} + \text{near})}{2} \right)$$

- Note that  $z = -\text{near}$  is mapped to  $z = -1$ ,  
and  $z = -\text{far}$  to  $z = +1$

# Orthographic Projection Matrix

$$\mathbf{M}_{\text{ortho}} = \begin{bmatrix} \frac{2}{\text{right} - \text{left}} & 0 & 0 & \frac{-(\text{right} + \text{left})}{\text{right} - \text{left}} \\ 0 & \frac{2}{\text{top} - \text{bottom}} & 0 & \frac{-(\text{top} + \text{bottom})}{\text{top} - \text{bottom}} \\ 0 & 0 & \frac{-2}{\text{far} - \text{near}} & \frac{-(\text{far} + \text{near})}{\text{far} - \text{near}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Viewport Transformation

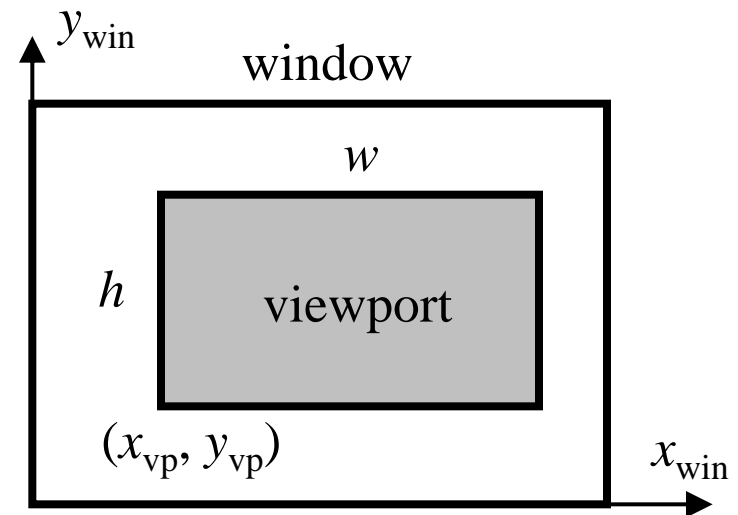
- The canonical view volume is then mapped to the viewport (from NDC to window coordinates)

$$\frac{x_{\text{NDC}} - (-1)}{2} = \frac{x_{\text{win}} - x_{\text{vp}}}{w} \Rightarrow x_{\text{win}} = x_{\text{vp}} + \frac{w(x_{\text{NDC}} + 1)}{2}$$

$$\frac{y_{\text{NDC}} - (-1)}{2} = \frac{y_{\text{win}} - y_{\text{vp}}}{h} \Rightarrow y_{\text{win}} = y_{\text{vp}} + \frac{h(y_{\text{NDC}} + 1)}{2}$$

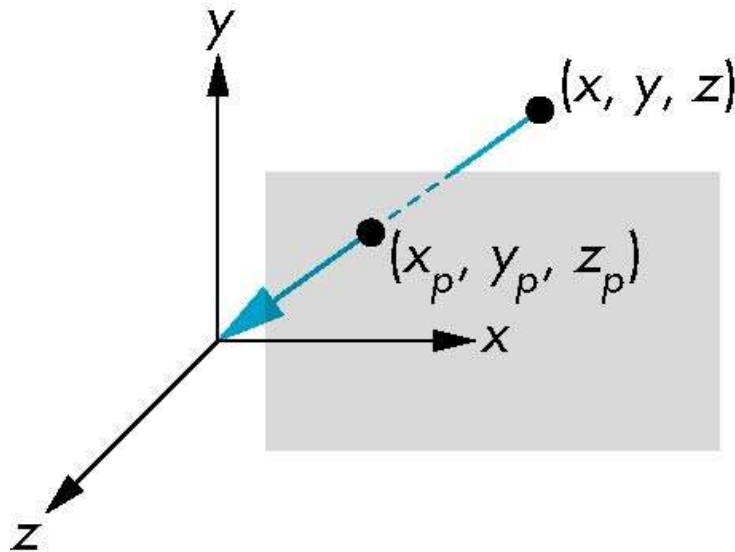
$$z_{\text{win}} = \frac{z_{\text{NDC}} + 1}{2}$$

- By default,  $z_{\text{win}}$  is between 0 and 1
  - set using `glDepthRange()`
- It is needed for z-buffer hidden surface removal



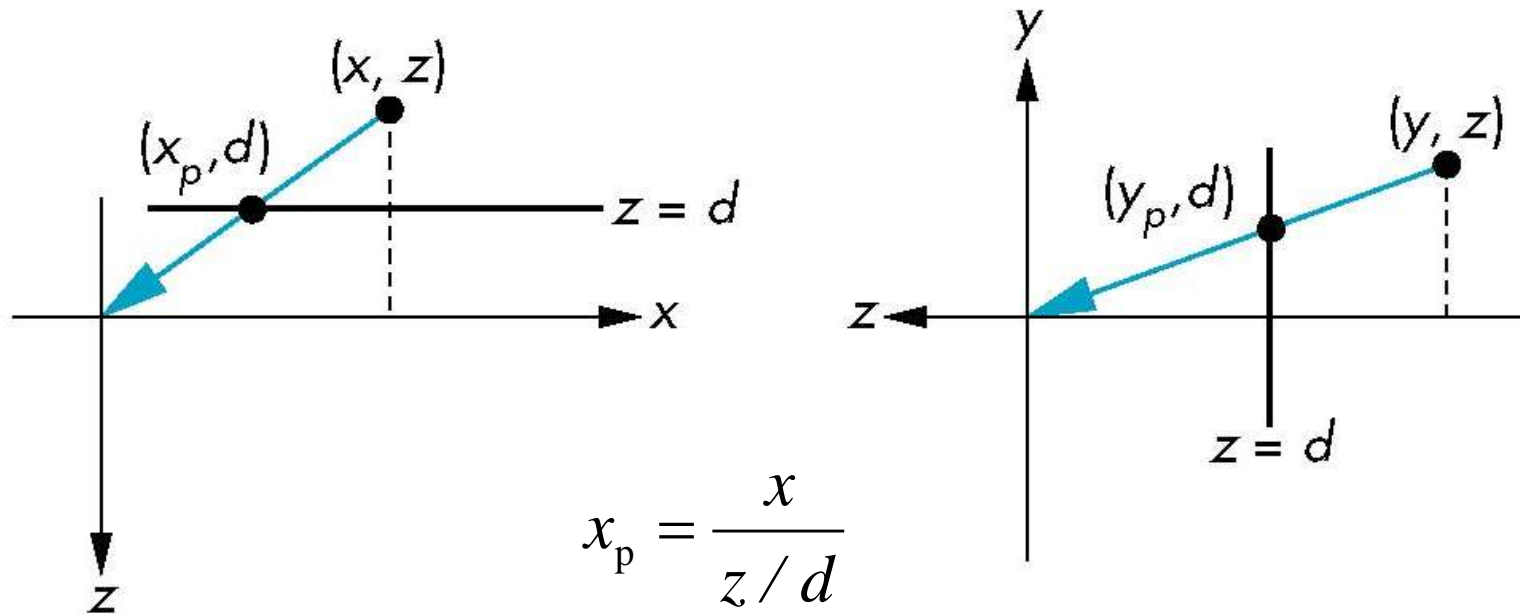
# Simple Perspective Projection

- Center of projection at the origin
- Projection plane is  $z = d$ ,  $d < 0$



# Perspective Equations

- Consider top and side views



$$x_p = \frac{x}{z/d}$$

$$y_p = \frac{y}{z/d}$$

$$z_p = d$$

# Using Matrix Multiplication

- Consider  $\mathbf{p} = \mathbf{M}\mathbf{q}$  where

$$\mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} \quad \mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \quad \mathbf{q} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Perspective Division

- However  $w \neq 1$ , so we must divide by  $w$  to return from homogeneous coordinates
- This *perspective division* yields

$$\mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} \xrightarrow{\text{perspective division}} \mathbf{p}' = \begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \\ 1 \end{bmatrix}$$

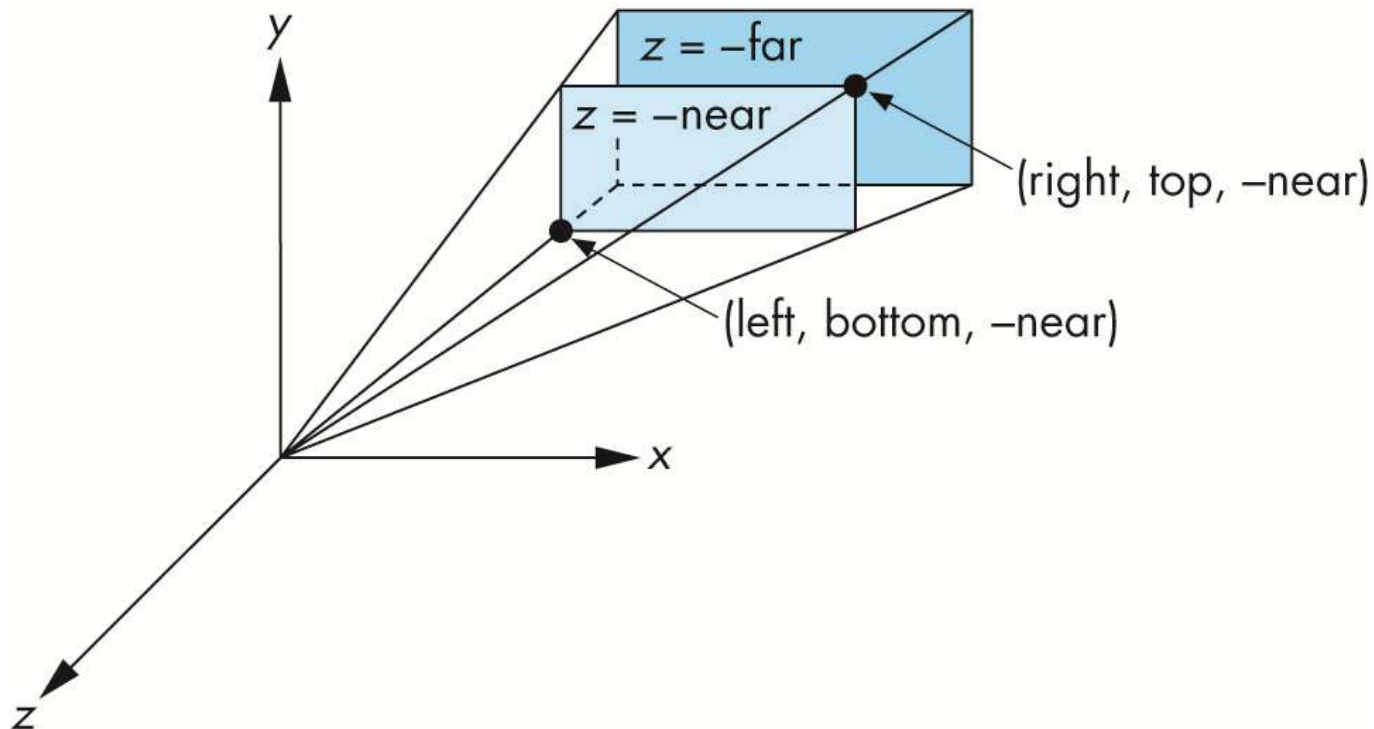
the desired perspective equations

- This is one reason why 3D graphics API uses homogeneous coordinates

# OpenGL Perspective Projection

- Can be specified by defining a view volume (*view frustum*) in the camera frame using

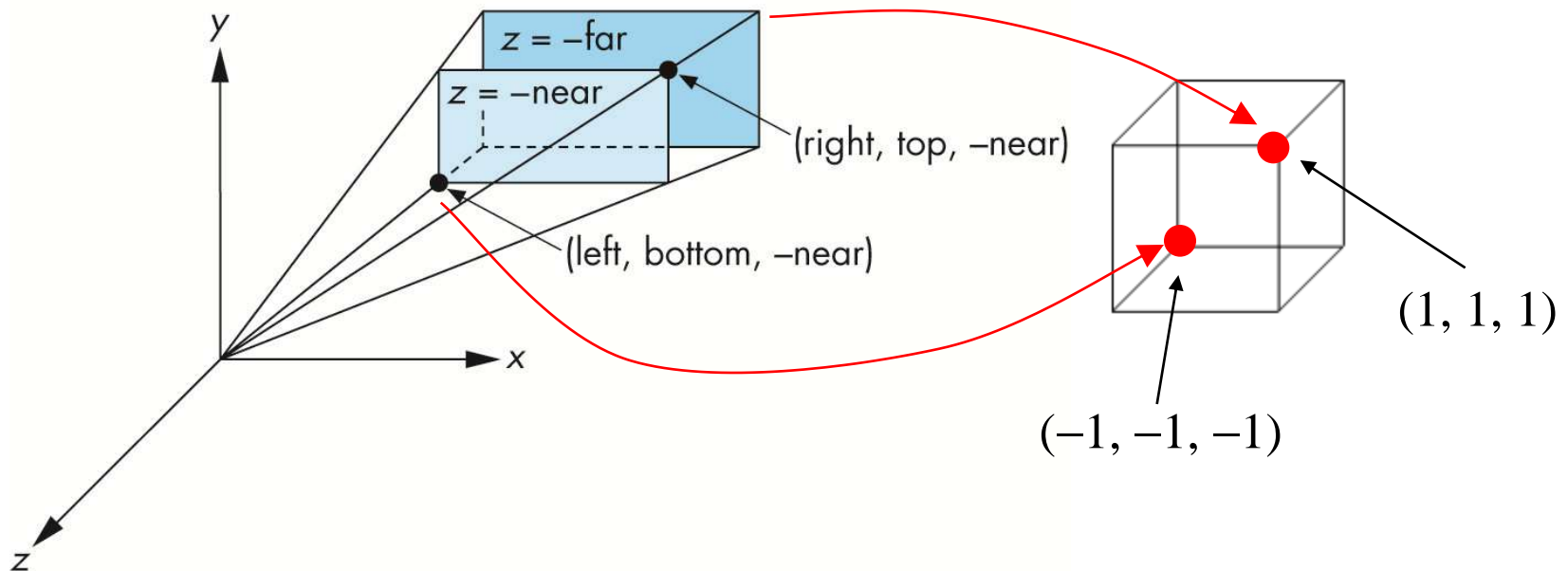
```
glFrustum( left, right, bottom, top, near, far );
```





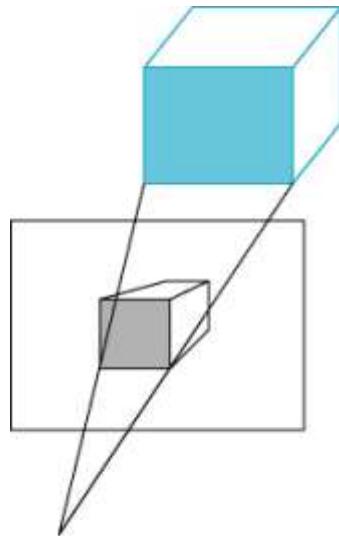
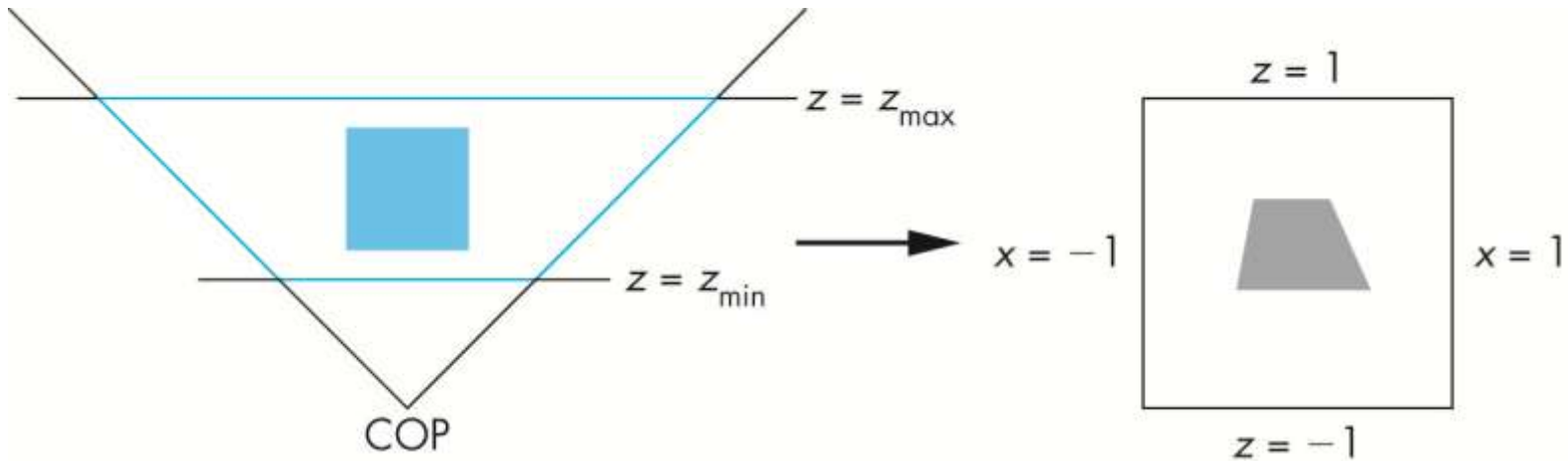
# OpenGL Perspective Projection

- The `glFrustum()` function then generates a matrix that maps the view frustum to the canonical view volume, where

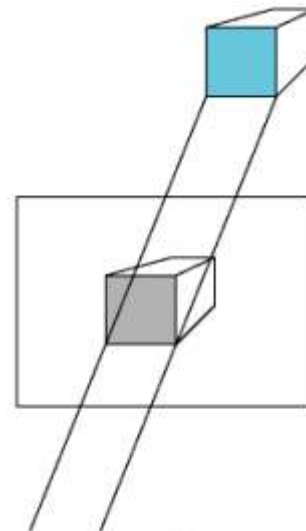


Q: Why transform view volume to canonical cube?

# OpenGL Perspective Projection



(a)



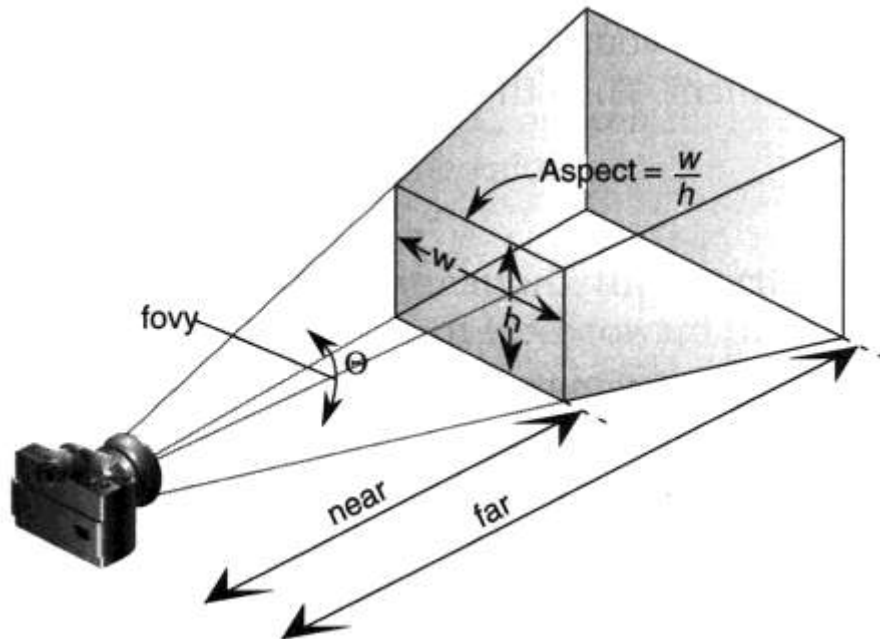
(b)

# Perspective Projection Matrix

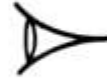
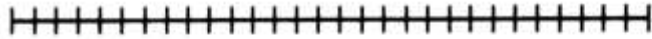
$$\mathbf{M}_{\text{persp}} = \begin{bmatrix} \frac{2 \cdot \text{near}}{\text{right} - \text{left}} & 0 & \frac{\text{right} + \text{left}}{\text{right} - \text{left}} & 0 \\ 0 & \frac{2}{\text{top} - \text{bottom}} & \frac{\text{top} + \text{bottom}}{\text{top} - \text{bottom}} & 0 \\ 0 & 0 & \frac{-(\text{far} + \text{near})}{\text{far} - \text{near}} & \frac{-2 \cdot \text{far} \cdot \text{near}}{\text{far} - \text{near}} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

# Perspective Projection Using Field of View

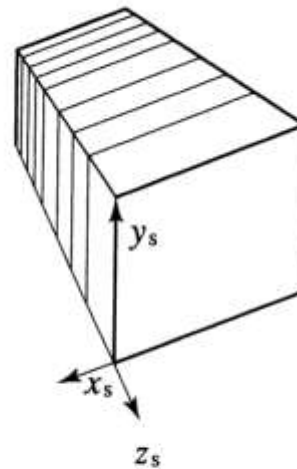
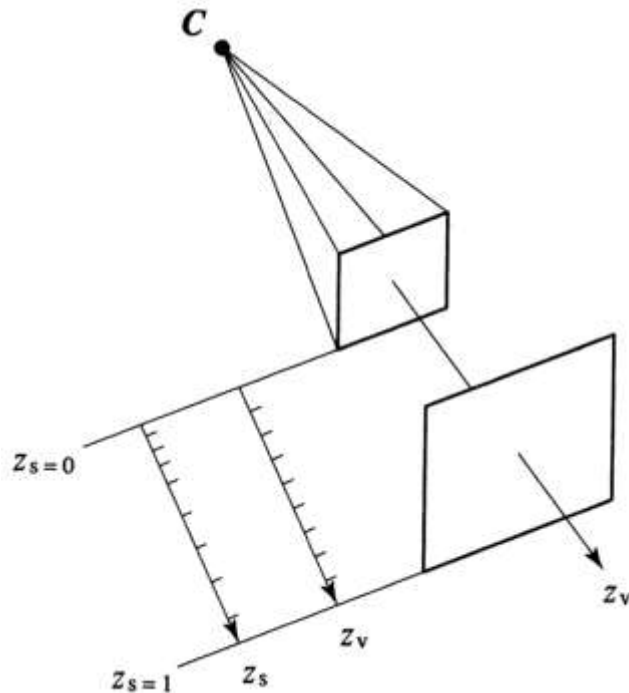
- The `glFrustum()` function allows (off-center) asymmetric view volume
  - What are the uses of off-center view volume?
- For symmetric view volume, we can also use  
`gluPerspective( fovy, aspect, near, far );`



# Distortion in Z Values



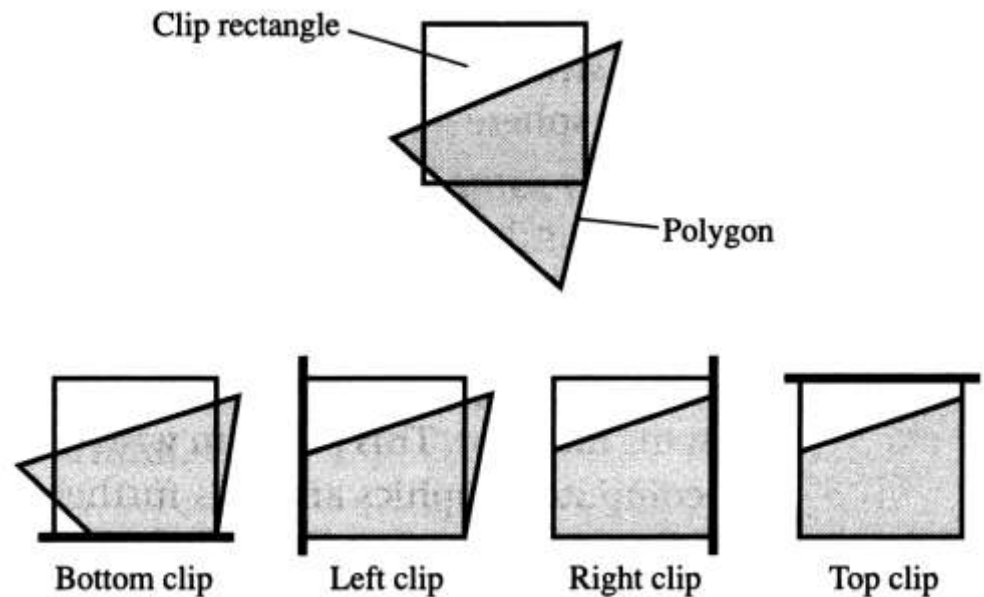
Distortion in  $z$  values after perspective projection.



Precision in  $z$  values.

# Clipping Space

- It is the space after perspective transformation  $\mathbf{M}_{\text{persp}}$  and before the perspective division by  $w = -z_v$
- Polygons partially inside the view volume are clipped
- The clipping limits
  - $-w \leq x \leq w$
  - $-w \leq y \leq w$
  - $-w \leq z \leq w$



# Framebuffer

- “Standard” window-system-provided framebuffer
  - Up to 4 color buffers
    - Front-left, front-right, back-left, back-right
  - A depth buffer
  - A stencil buffer
  - An accumulation buffer
  - A multisample buffer
  - One or more auxiliary buffers
    - Offscreen memory buffers
- Other offscreen memory and texture images can be used as framebuffer through framebuffer object
  - Render-to-texture
  - Multiple rendering targets (MRT)

# OpenGL Execution Model

## ■ Client-server

- Client – application program
- Server – OpenGL implementation (driver and hardware)
- Majority of OpenGL states stored at server side

## ■ OpenGL commands always processed in order

- Out-of-order execution of OpenGL commands not allowed
- Applies to queries of state and frame buffer read operations

## ■ Data binding occurs when commands are issued

- Data copied into OpenGL memory
- Subsequent changes to this data by the application have no effect



**End of Lecture 1**