SP3175-Modelling 3: The Daisyworld

Biota and Earth's habitability & stability

The Daisyworld model was created by Lovelock and Watson[1] in 1983 in order to support Lovelock's earlier theory (1979): the Gaia hypothesis[2]. The Gaia hypothesis states that Earth is a self-regulating system controlled by the interactions of the biota and the surrounding environment (atmosphere, ocean and land). Those interactions are an elaborate feedback system which maintains the conditions for life on Earth. The Daisyworld model is very simple and far from reflecting the complexity of real Earth, however it does help us to get some insight on how life on Earth contributed to stabilize the climate. The objective is to find out whether steady states are reached despite the continuous forcing the planet is subjected to and if so, under which conditions it happens.

Go through the paper and take some time to get familiar with the modelling (section1). We will build (section 2) the computer models with Python:

1. for a planet only filled with black daisies
2. for a planet only filled with white daisies
3. for a planet filled with black and white daisies together

The section 3 is on the analysis of the modelling's results. You can actually go through this section and answer the questions just by looking at the paper.

Supporting materials:

- Lovelock and Watson's publication: Biological homeostasis of the global environment: the parable of Daisyworld. (1983)
- Your own model

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1 General understanding of Lovelock and Watson's publication

4. What is the objective of the Daisyworld model?
5. Describe briefly all the constants and variables present in the model
6. List and explain the equations for:
   a) Daisy's growth (black or white)
   b) Daisy's local temperature
   c) Daisy's coverage
   d) Available ground
   e) Planet's albedo
   f) Planet's temperature
7. List all the constants
8. What is the main characteristic of the black and white daisies?
9. What is the constant p? How does it vary?
10. What does q represent? How does it vary?

2 Building-up your own model

The aim of the activity is to create a computer model based on Lovelock and Watson's mathematical model. This will help to understand the evolution of life and the planet's temperature.

2.1 Plotting the daisy's coverage (A_b) vs time (t)

Parameters: $L = 1$, $R = 0.2$, S_o=917 W.m^{-2} and death rate is $\gamma = 0.3$.

We will assume first that only **one type of daisy** exists on the planet. For example we are looking at the **black daisy** planet. We want to vizualise how the daisy's area (A) changes with time on the planet. We do not have a simple equation linking both variables. However we can make this possible by using the Euler's method.

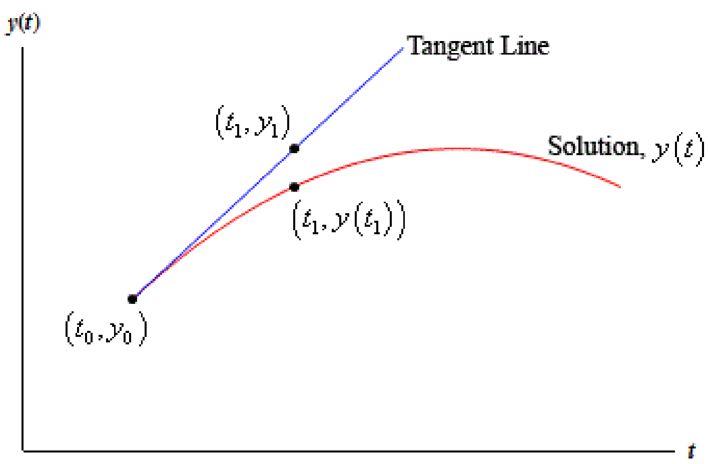
Euler's method

The Euler's method is used to solve differential equations:

$$\frac{dy}{dt} = f(t, y)$$

With inital values known:

$$t = t_0, \quad y(t_0) = y_0, \quad \left.\frac{dy}{dt}\right|_{t_0,y_0} = f(t_0, y_0)$$



If t_1 is close to t_0 we can write:

$$y_1 = y_0 + \left.\frac{dy}{dt}\right|_{t_0,y_0} (t_1 - t_0) = y_0 + f(t_0, y_0)\Delta t$$

In general:

$$y_{n+1} = y_n + f(t_n, y_n)\Delta t$$

Use the Euler's method to express the black daisy area $A_{b(n+1)}$ as:

$$A_{b(n+1)} = A_{b(n)} + f(x, \beta, \gamma)\Delta t$$

Assuming the initial area (at t_0) is known ($A_{b0} = 0.01$) express the black daisy area A_{b1} at t_1(area of daisy after one increment of time).

Discuss with you group: Come up with a step by step method (algorithm) in the form of a flowchart to calculate the data points (values of A_b at different time) needed to plot the evolution of daisies?

$$A_b = f(t)$$

Start with a time step of $\Delta t = 0.9$ and adjust according to the model results.

2.2 Plot the planet temperature (T_p) vs time (t)

Now we are interested to look at the change in the planet temperature. However like for the area we do not have a direct simple equation that link the planet temperature with time. **Identify the main equation in the paper that shows the change in temperature.** Looking at the equation what variable(s) are changing with time?

Discuss with you group: How can you calculate the planet temperature (T_p) at different time?

2.3 Evolution of a planet with only white daisies

Apply the same method used previously to understand the evolution of a planet only covered with white daisies. Plot the evolution of the white daisies area (A_w) and the planet temperature (T_p), i.e **plot T_p vs time and A_w vs time**.

2.4 Evolution of a planet with black and white daisies

Now we have a planet where both types of daisies coexist. Adapt the equations you have previously used to take into account both daisies at the same time. **Plot A_w and A_b vs time on the same graph and on another graph, plot the planet temperature T_e vs time.**

2.5 Temperature and daisy's coverage with change in luminosity

Find the steady states values of A_w and A_b and T_p (in ^{o}C) for L increasing from [0.5-1.6] and decreasing [1.6-0.5]. You can decide on the best increment (ΔL).

Plot A_w and A_b vs L and T_p vs L.

Programming note: Don't let the daisies die completely (set a little threshold) otherwise you cannot make them grow again when L is decreasing.

3 Analysis of the Daisyworld model

3.1 Black Daisy planet versus White Daisy planet

Let's start playing with a planet hosting only black daisies and a planet hosting only white daisies. Fix the luminosity (L) at 1 and let the death rate (χ) be 0.3.

1. Vary the amount of initial black daisies (A_{bo} is the area of black daisies at t=0), can you reach a steady state for the planet temperature (T_p) without killing all daisies? if so give the final steady state conditions (T_p and A_b).
2. Repeat the same but now ONLY WHITE daisies. Same question as 1.
3. Now compare the result of the two types of daisies (Temperature vs time and the change in daisy population vs time for the same luminosity), do they reach the same result? And behave the same way to moderate T? Explain.
4. Now we will vary the sun luminosity but still look at the results for black and white daisies separately.
   a) Black Daisies only: If you increase L, how is the temperature changing? Give the range of luminosity when the temperature is stable.
   b) White Daisies only: same as above.

3.2 A planet with black and white daisies

We are now looking at a planet with white and black daisies growing together.

5. What happens if you change the albedo of the daisies to be the same as the barren soil? Why?
6. Back to initial albedos. What happens to the planet when you vary the parameter q (letting all the other parameters the same) and L increasing (you can compare the situations when q is minimum, maximum and average).
7. Explain how the planet reaches steady state in the two situations:

a) $A_w = 0.2$, $A_b = 0.2$ and $L = 0.9$

b) $A_w = 0.2$, $A_b = 0.2$ and $L = 1$

8. How do the two types of daisies coexist and remain stable at fixed L?
9. Looking at the plot versus L. What do you notice about the planet temperature when the white daisies appear despite L increasing? Why does the black daisy domination only last for a short time vs the white daisies?
10. Explain the plot versus L in terms of feedback mechanisms.
11. There is one interesting phenomenon called hysteresis with the white daisy planet when the L is decreasing (from 1.6 back to 0.8), see figure 1.c in the paper. By carefully observing the T and associated white area can you explain why this happens?
12. Give your conclusion about the effect of the daisies (versus one species and no daisy at all) on the planet T with increasing luminosity.
13. From an Earth perspective, can you relate to those conditions to the real world?
14. What external forcing could you add to your model, when L is high, to help the remaining daisies to survive? How would this change impact the environment?

3.3 Alternative method to vizualise equilibria: The state space picture

In this section instead of looking how the area varies with time, we are looking at how $\frac{dA}{dt}$ varies with area.

Run the Python code written by Park Kun Hee (SPS Batch 2015) available in the Daisyworld folder.

3.3.1 Steady State Solutions and Stability

We first look at how we can find steady state solutions and their stabilities with this new picture.

a) What is the value of $\frac{dA}{dt}$ for steady state (equilibrium) solutions? Can you identify them in the plot?

b) Are all steady states reachable from other positions? What does it say about their stabilities? What is the physical significance of the equilibrium solutions and their stabilities?

c) Pick several different starting positions and trace how the points move in the state space. Where do they end up? Compare the results with those from the time-evolution plot.

d) How do the time-evolution plot and state-space plot compare in terms of the information they display?

3.3.2 2D Generalisation

For the two-daisy case, the state of the system is totally characterised by two variables, $A_b \in [0;\ 1]$ and $A_w \in [0;\ 1]$, constrained by $Ab + Aw \leq 1$ Therefore, the state space, where all possible states reside, is a region in two-dimensional space bounded by a triangle, whose vertices are at (0,0), (0,1) and (1,0). Justas the one-daisy case, a state of the system is represented by a point in the triangular region, and its time evolution is represented by a continuous slide across the region. Unlike the one-daisy case however, there are now two different rates of change of area that direct how to evolve the system, namely dA_b/dt and dA_w/dt. These instruct how to evolve the system in the respective directions; dA_b/dt tells us where to go in the A_b-direction (taken to be the horizontal axis), and dA_w/dt tells us where to go in the A_w-direction (taken to be the vertical axis). We may therefore define a vector $\vec{v}$ that consists of two components: magnitude of dA_b/dt in the A_b-direction, and magnitude of dA_w/dt in the A_w-direction.

a) What do $v_b = dA_b/dt$ and $v_w = dA_w/dt$ represent?

b) What is the difference between the previous plot (with one daisy) and this 2D plot (stream plot)?

c) Pick several different starting positions and trace how the points move in the state space. Where do they end up? Compare the results with those from the time-evolution plot.

d) What is the value of v for steady state (equilibrium) solutions? Can you identify them in the plot?

e) Are all steady states reachable from other positions? What does it say about their stabilities?

f) What is the physical significance of the equilibrium solutions and their stabilities?

g) How do time-evolution plot and state-space plot compare in terms of the information they display?

3.3.3 Bifurcation

The bifurcation plot shows how steady states change with respect to a parameter. In this example, we will choose the parameter to be luminosity L , you may try other parameters as well. Can you observe the hysteresis loops? What can you learn from this plot?

References

[1] ANDREW J. WATSON and JAMES E. LOVELOCK. Biological homeostasis of the global environment: the parable of daisyworld. *Tellus B*, 35B(4):284–289, 1983.

[2] JAMES E. LOVELOCK. *Gaia: A New Look at Life on Earth*. Oxford University Press, 1979.