

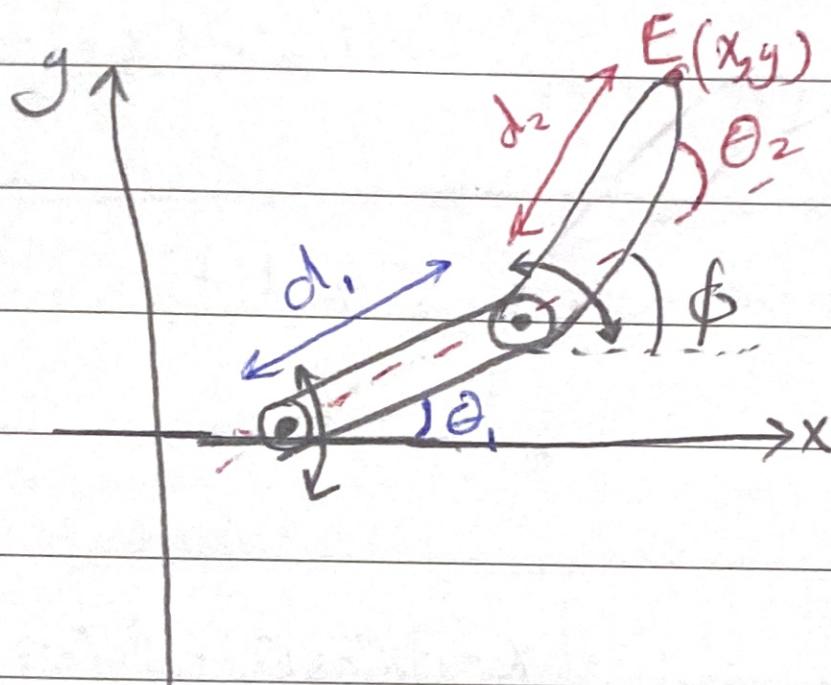
Forward Kinematics ~~one~~ degree of freedom (DOF)

Given joint angles $\rightarrow \theta_1, \theta_2, d_1, d_2$

We want to find a location of a point in different frame.

* What is forward kinematics?

It is a system can be 2D or 3D consist of two rods with a joint that can move in a specific motion specified by an angle.



NOTE:

θ : Anticlock $\rightarrow +ve$

θ : clockwise $\rightarrow -ve$

Joint Space

World Space

* $\theta_1, \theta_2 \Rightarrow$ it represents the coordinates attached to the joints.

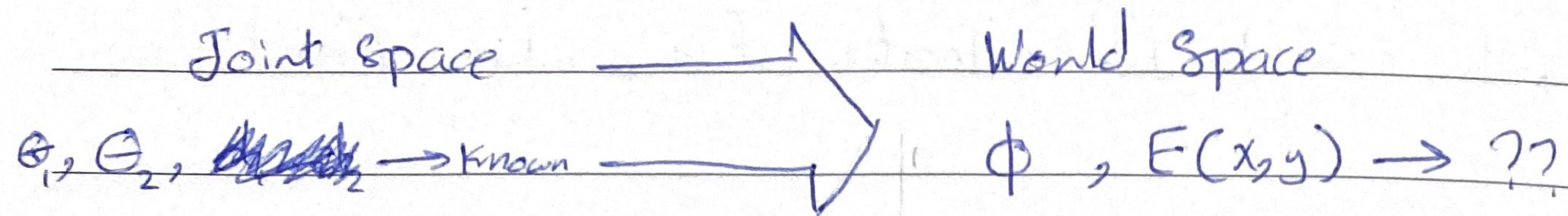
* $\phi, E(x,y) \Rightarrow$ it represents the absolute coordinates & position of end effector.

* $\phi \rightarrow$ represents the overall position of rod "d₂"

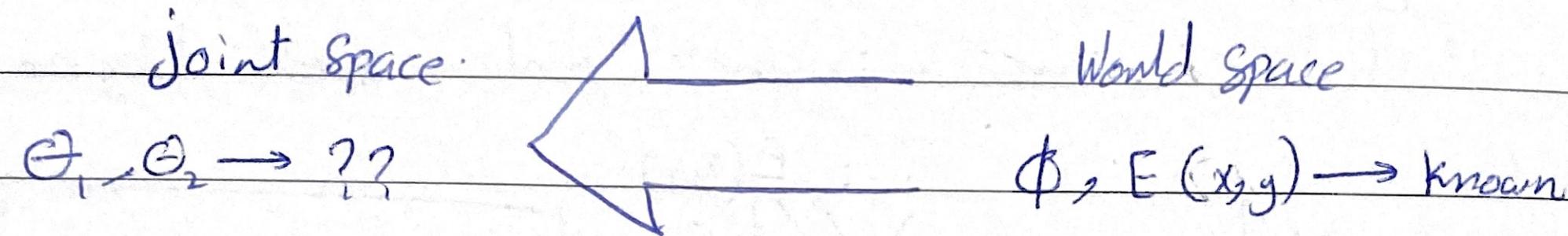
Forward Kinematics \rightarrow It is a system that goes from joint space to world space in order to find the unknowns.

Summary:

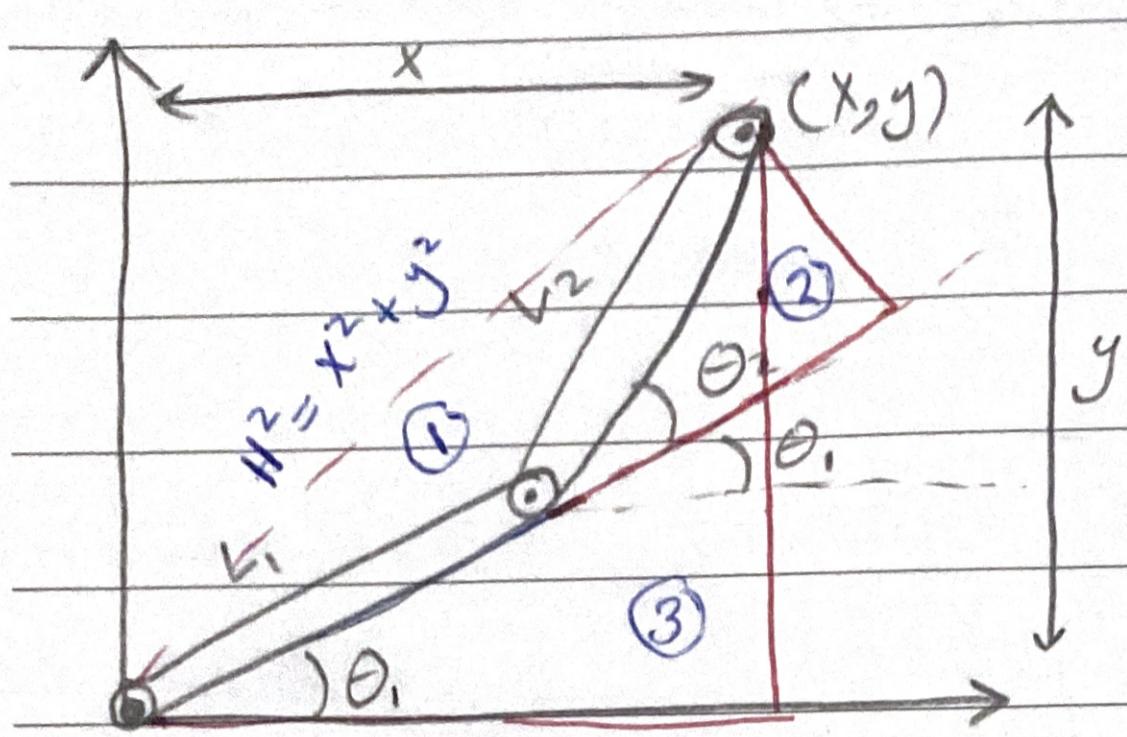
Forward / Direct Kinematics:



Reverse/Indirect Kinematics:



Inverse Kinematics: 2 DOF I need to find (θ_1, θ_2)



$$T_1 \rightarrow H^2 = x^2 + y^2$$

$$T_2 \rightarrow J = L_2 \sin \theta_2$$

$$T_3 \rightarrow K = L_1 + L_2 \cos \theta_2$$

$$T_4 \rightarrow H^2 = J^2 + K^2$$

$$x^2 + y^2 = J^2 + K^2$$

$$x^2 + y^2 = (L_2 \sin \theta_2)^2 + (L_1 + L_2 \cos \theta_2)^2$$

$$x^2 + y^2 = L_2^2 \sin^2 \theta_2 + L_1^2 + 2L_1 L_2 \cos \theta_2 + L_2^2 \cos^2 \theta_2$$

$$x^2 + y^2 = L_2^2 + L_1^2 + 2L_1 L_2 \cos \theta_2$$

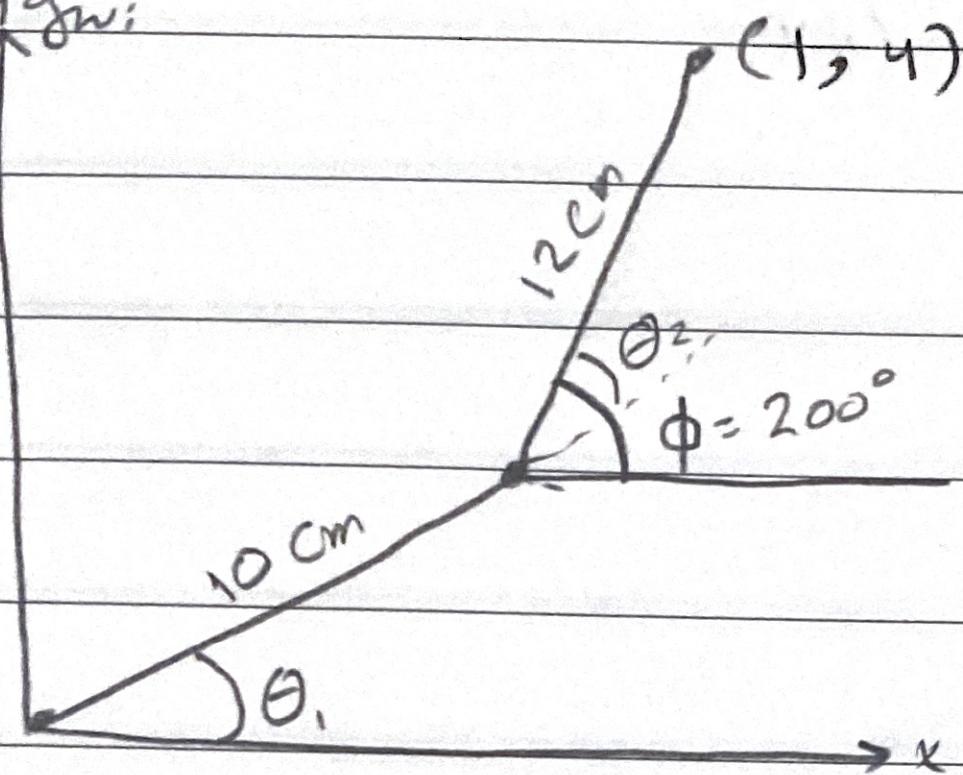
$$\cos \theta_2 = (x^2 + y^2 - L_1^2 - L_2^2) / 2L_1 L_2$$

$$\phi = \theta_1 + \theta_2 \rightarrow \theta_1 = \phi - \theta_2$$

calculation of 2DOF inverse kinematics assuming:

$$x = 16 \text{ cm}, y = 4 \text{ cm}, L_1 = 10 \text{ cm}, L_2 = 12 \text{ cm}, \phi = 200^\circ$$

① Design:



② Calculation:

→ find θ_2 :

$$\cos \theta_2 = (x^2 + y^2 - L_1^2 - L_2^2) / 2L_1 L_2$$

$$= (1^2 + 4^2 - 10^2 - 12^2) / 2(10)(12)$$

$$\cos \theta_2 = -0.8625 \rightarrow \theta_2 = \cos^{-1}(-0.8625)$$

$$\theta_2 = 149.5 \approx 150^\circ$$

→ find θ_1 :

$$\theta_1 = \phi - \theta_2$$

$$= 200 - 150 = 50^\circ$$