# 1 dim. Electrochemical Hydrodynamics for Shock Lightning 

$i, j, k$ … kinds of particles. $i, j, k \in \{e^-, H_2, H_2^+, H_2O^-, \text{Dust}, \text{Dust}^+, \text{Dust}^{2+}, \dots\}$

$m_i$ … mass of a particle of kind $i$ $\quad m_{H_2} = 3.34 \times 10^{-24}$ [g]

$Z_i$ … charge of a particle of kind $i$ $\quad Z_{e^-} = -1$ [Fr]

$c \in C$ … set of all collision events considered. $(H_2 \to e^- + H_2^+) \in C$

$\#_i(c)$ number of particle production of the kind in the collision.

$$\#_{H_2}(H_2 \to H_2^+ + e^-) = -1$$

$$\#_{e^-}(Z^{5-} \to Z^{3-} + 2e^-) = +2$$

$n_i(x)$ [/cm³] number density of particle of kind $i$

$\bar{v}_i(x)$ [cm/s] mean velocity of particles of kind $i$

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$Q(x)$ [Fr/cm³] total charge density

$E(x)$ [G] electric field.

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each collision satisfies the following:

$$\begin{rcases} \forall c. \quad \sum_i m_i \#_i(c) = 0 \\ \forall c. \quad \sum_i Z_i \#_i(c) = 0 \end{rcases} \text{elemental conservation of mass and charge.}$$

given event rate for each collision $\xi(c)$ [/cm³ s],

$$\begin{rcases} \frac{\partial n_i}{\partial t} = \frac{\partial}{\partial x}(n_i \bar{v}_i) + \sum_c \#_i(c)\,\xi(c) \\ \frac{\partial}{\partial t}(m_i n_i \bar{v}_i) = \frac{\partial}{\partial x}(m_i n_i \bar{v}_i) + \underset{[\text{dyn/cm}^3]}{F} \end{rcases} \text{on steady state. } \frac{\partial}{\partial t} = 0$$

where total conservation of charge and mass are satisfied.

$$\frac{d}{dt}\int \sum_i m_i n_i \, dx = \int \sum_i m_i \left( \underbrace{\frac{\partial}{\partial x}(n_i \bar{U}_i)}_{\substack{\{\text{integral by parts}\} \\ =0}} + \sum_c \#_i(c)\,\xi(c) \right) dx$$

$$= \int \sum_i m \sum_c \#_i(c)\,\xi(c)\, dx$$

change order of sum.

$$= \int \sum_c \xi(c) \underbrace{\sum_i m_i \#_i(c)}_{0} dx = 0$$

similarly.

$$\frac{d}{dt}\int \sum_i Z_i n_i \, dx = \int \sum_c \xi(c) \sum_i m_i \#_i(c)\, dx = 0$$

Maxwell eq.

$$\nabla \cdot \mathbb{E} = 4\pi Q \qquad \text{(Poisson)}$$
$$\nabla \cdot \mathbb{B} = 0 \qquad \text{(Divless)}$$
$$\nabla \times \mathbb{E} = -\frac{1}{c}\frac{\partial \mathbb{B}}{\partial t} \qquad \text{(Faraday)}$$
$$\nabla \times \mathbb{B} = \frac{1}{c}\left(4\pi \mathbb{J} + \frac{\partial \mathbb{E}}{\partial t}\right) \qquad \text{(Ampere)}$$

at the static condition

from Faraday

$$\nabla \times \mathbb{E} = 0.$$

trivial for 1dim.

From Ampere.

$$(\nabla \times \mathbb{B})_x = \frac{1}{c}\cdot 4\pi J_x$$
$$\partial_y B_z - \partial_z B_y = \frac{4\pi}{c} J_x$$

since 1dim. $\partial_y = \partial_z = 0$

$$\therefore J_x = 0$$

$J_x(x) = 0$ is automatically satisfied if $J_x(-\infty) = 0$

since $\frac{\partial Q}{\partial t} + \frac{\partial J}{\partial x} = 0$ and $\frac{\partial Q}{\partial t} = 0$ therefore $\frac{\partial J}{\partial x} = 0$.

2N+2 unknowns.

$$\left.\begin{array}{l} n_i(x) \\ \bar{v}_i(x) \end{array}\right\} \text{2N chemical.}$$

$$\left.\begin{array}{l} Q(x) \\ E(x) \end{array}\right\} \text{2 Electrical.}$$

2N+2 equations.

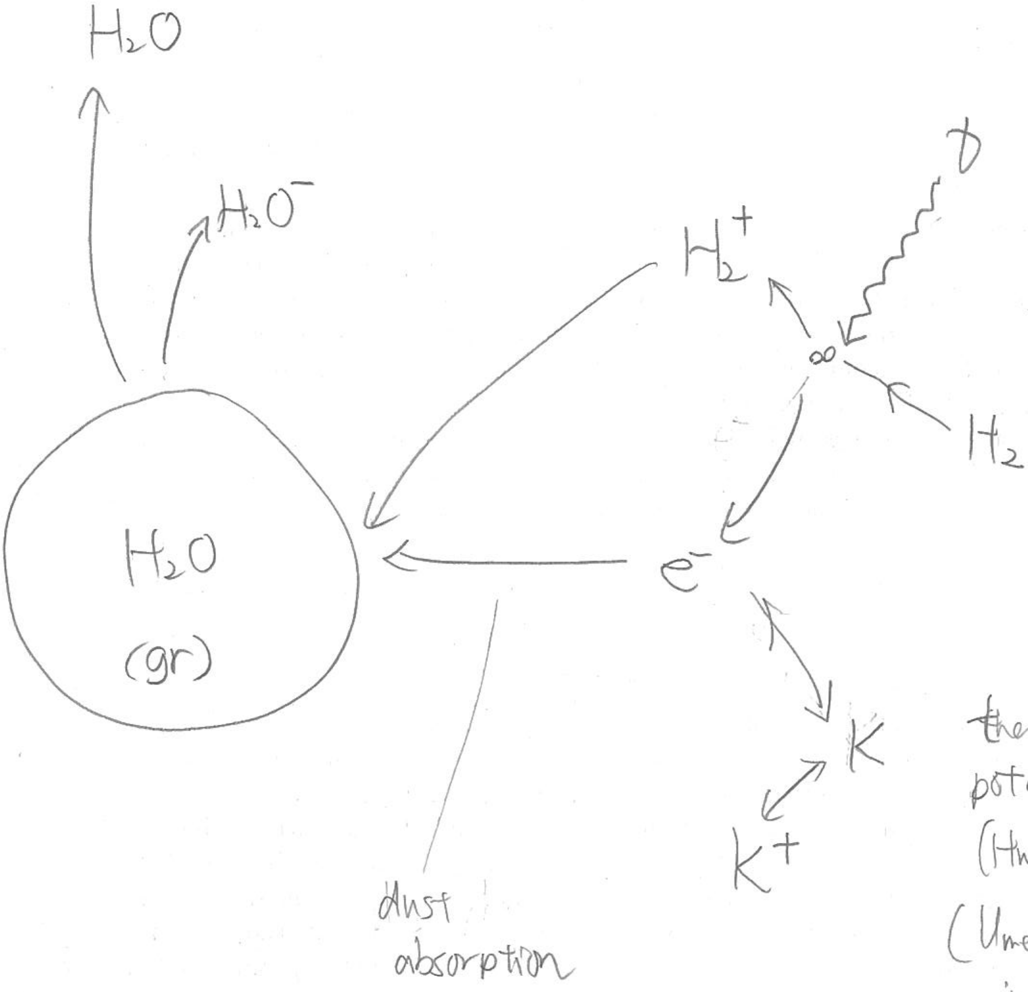
$$\left.\begin{array}{l} 0 = \frac{\partial}{\partial x}(n_i \bar{v}_i) + \sum_c \#_i(c)\, \xi(c) \\ 0 = \frac{\partial}{\partial x}(m_i n_i \bar{v}_i) + F_i^{(*)} \end{array}\right\} \text{2N chemical}$$

$$\left.\begin{array}{l} Q(x) = \sum_i Z_i n_i \\ \frac{\partial}{\partial x} E(x) = 4\pi Q(x) \end{array}\right\} \text{2 electrical.}$$

(*) Equation of motion

for neutral gas: $\bar{v}_{H_2}(x) = v_{post}$.

for charged particles: $Z_i n_i v_i = \nu_i E$ (Ohm's law)




ionization of $H_2$

thermal ionization of potassium
(Hubbard+. 2013)
(Umebayashi 1983)
in equilibrium.

( equilibrium can be obtained from Saha's eq. but dissociation ratio is unknown !! )