



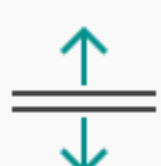
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Sunday, 10 May 2020

3:43 PM

a) Disproving algorithm using counter example

$$S = \{5, 10, 15, 20\}$$

$$h = \{1, 6, 11, 16\}$$

applying greedy algorithm

$s_i$	5	10	15	20
$h_i$	6	11	16	1
$ s_i - h_i $	1	1	1	19

$$\frac{\sum |s_i - h_i|}{n} = \frac{22}{4} = 5.5$$

However there is a better solution

$s_i$	5	10	15	20
$h_i$	1	6	11	16
$ s_i - h_i $	4	4	4	4

$$\frac{\sum |s_i - h_i|}{n} = \frac{16}{4} = 4$$



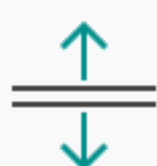
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Therefore the greedy algorithm is incorrect

b) Proving algorithm is correct

$$D(X) = \sum_{i=1}^n \frac{|s_i - s_{a_i}|}{n}$$

where  $X = \{(s_1, h_1) \dots (s_n, h_n)\}$

1) Proving Optimal substructure

Let  $O_i$  be the optimal solution for  $s_i$  and  $h_i$  shirts.

Assume optimal substructure is not followed

so  $O_{i-1}$  is not the optimal solution for  $s_{i-1}$  and  $h_{i-1}$  shirts.

There must exist  $M_{i-1}$

that better assigns  $h_k$  to  $s_m$  ( $h_k \in h, s_m \in S$ ) which is better than  $O_{i-1}$ .

So  $M_i$  should be better than  $O_i$  which is a contradiction.

Thus the problem follows optimal substructure.

$$O_n = \{(s_1, h_1), (s_2, h_2) \dots (s_n, h_n)\}$$

where  $s_i$  and  $h_i$  are sorted





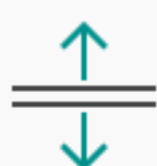
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$O_n = \{(s_1, h_1), (s_2, h_2) \dots (s_n, h_n)\}$   
 where  $s_i$  and  $h_i$  are sorted  
 in ascending order

$O_{n-1} = \{(s_1, h_1) \dots (s_{n-1}, h_{n-1})\}$

$O_{n-1} \neq M_{n-1}$  and  
 $S(O_{n-1}) \neq O(M_{n-1})$

## 2) Proving Greedy Property

### Lemma 1

If  $S$  and  $H$  of size  $n$  and sorted in ascending order and mapped completely to each other, either all  $s_i$  were mapped to  $h_i$  or at least 2 or more  $s_i$  were not mapped to  $h_i$ .

Set of all possible mapping =  
 $\forall (s_i \text{ mapped to } h_i) \cup$   
 2 or more  $s_i$  not mapped to  $h_i$

Let  $A = \{(s_1, h_1) \dots (s_n, h_n)\}$   
 where  $s_k, h_k$  are sorted ascending -





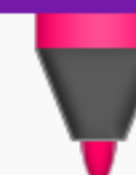
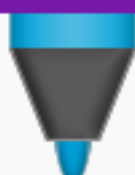
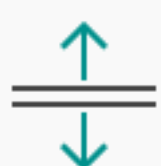
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let  $A = \{(s_1, h_1) \dots (s_n, h_n)\}$

where  $s_k, h_k$  are sorted ascending -

Any other combination can be formed by swapping  $h_u, h_m$

At least 1 swap is needed thus at least 2 or more  $s_n$  will not be mapped to  $s_n$ .

If  $O$  is the optimal solution and  $G$  is the Greedy solution, we assume  $G$  is not the optimal solution. The greedy solution has all  $s_i$  mapped to  $h_i$  so  $O$  must have at least 2  $s_i$  not mapped to  $h_i$ . If the mapping of  $O$  can be changed such that  $s_i$  is mapped to  $h_i$ ,  $\sum |s_i - h_i|$  would be reduced. This is a contradiction since  $O$  was the optimal solution.

Thus the greedy solution is the optimal solution.

If  $s_i$  and  $s_k$  sorted in ascending.

$G = \{(s_1, h_1) \dots (s_n, h_n)\}$



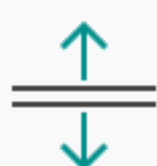
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gf  $s_i$  and  $s_n$  sorted in ascending.

$$C = \{(s_1, h_1) \dots (s_n, h_n)\}$$

$$O = \{(s_1, h_1) \dots (s_k, h_m), (h_m, s_k) \dots (s_n, h_n)\}$$

$$O' = \{(s_1, h_1) \dots (s_n, h_n), (h_m, s_m) \dots (s_n, h_n)\}$$

$$S(O) \geq S(O')$$