q1

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a) Disproving algorithm using counter example

$S = \{5, 10, 15, 20\}$
$h = \{1, 6, 11, 16\}$

applying greedy algorithm

s_i	5	10	15	20
	6	11	16	1
$\lvert s_i - h_i \rvert$	1	1	1	19

$$\frac{\sum \lvert s_i - h_i \rvert}{n} = \frac{22}{4} = 5.5$$

However there is a better solution

s_i	5	10	15	20
h_i	1	6	11	16
$\lvert s_i - h_i \rvert$	4	4	4	4

$$\frac{\sum \lvert s_i - h_i \rvert}{n} = \frac{16}{4} = 4$$

Therefore the greedy algorithm is incorrect

b) Proving algorithm is correct

$$D(X) = \sum_{i=1}^{n} \frac{|h_i - s_{a_i}|}{n}$$

where $X = \{(s_1, h_1) \cdots (s_n, h_n)\}$

1) Proving Optimal substructure
Let O_i be the optimal solution for s_i and h_i shirts. Assume optimal substructure is not followed so O_{n-1} is not the optimal solution for s_{i-1} and h_{i-1} shirts. There must exist M_{i-1} that better assigns h_k to s_m ($h_k \in h$, $s_m \in S$) which is better than O_{i-1}.
So M_i should be better than O_i which is a contradiction.
Thus the problem follows optimal substructure.

$O_i = \{(s_1, h_1), (s_2, h_2) \cdots (s_n, h_n)\}$

where s_i and h_i ... sorted

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$O_n = \{(s_1, h_1), (s_2, h_2) \cdots (s_n, h_n)\}$

where s_i and h_i are sorted in ascended order

$O_{n-1} = \{(s_1, h_1) \cdots (s_{n-1}, h_{n-1})\}$

$O_{n-1} \neq M_{n-1}$ and

$S(O_{n-1}) \not> O(M_{n-1})$

2) Proving Greedy Property

Lemma 1

If S and H of size n and sorted in ascending order and mapped completely to each other, either all s_i were mapped to h_i or atleast 2 or more s_i were not mapped to h_i.

set of all possible mapping =

$\forall (s_i$ mapped to $h_i) \cup$

2 or more s_i not mapped to h_i

let $A = \{(s_1, h_1) \cdots (s_n, h_n)\}$

where s_k, h_k are sorted ascending -

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let $A = \{(s_1, h_1) \cdots (s_n, h_n)\}$
where s_n, h_n are sorted ascending -
Any other combination can be formed by swapping h_n, h_m
Atleast 1 swap is needed thus atleast 2 or more s_n will not be mapped to h_n.

If O is the optimal solution and G is the Greedy solution, we assume G is not the Optimal solution. The greedy solution has all s_i mapped to h_i so O must have atleast 2 s_i not mapped to h_i. If the mapping of O can changed such that s_i is mapped to h_i, $\Sigma|s_i - h_i|$ would be reduced. This is a contradiction since O was the optimal solution.
Thus the greedy solution is the optimal solution.

If s_i and h_n sorted in ascending.
$G = \{(s_1, h_1) \cdots (s_n, h_n)\}$

If s_i and h_u sorted in ascending.

$G = \{(s_1, h_1) \cdots (s_n, h_n)\}$

$O = \{(S_1, h_1) \cdots (s_k, h_m), (h_m, s_k) \cdots (s_n, h_n)\}$

$O' = \{(s_1, h_1) \cdots (s_u, h_u), (h_m, s_m) \cdots (s_n, h_n)\}$

$S(O) \geq S(O')$