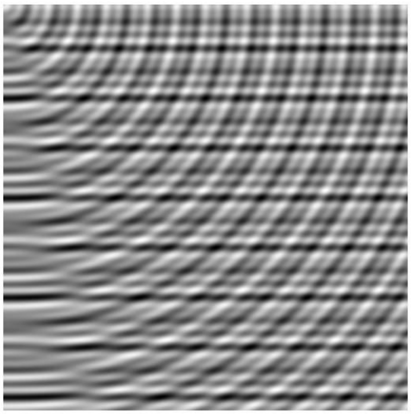
Code Link

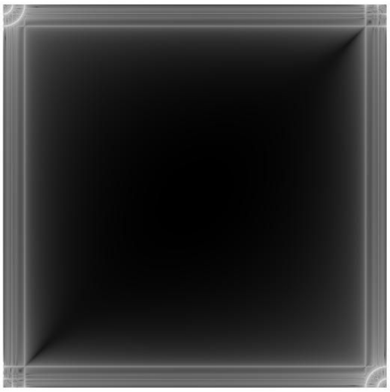
https://github.com/nuslarry/cs211-hw/tree/master/hw2

Part 1: Discrete Fourier Transform

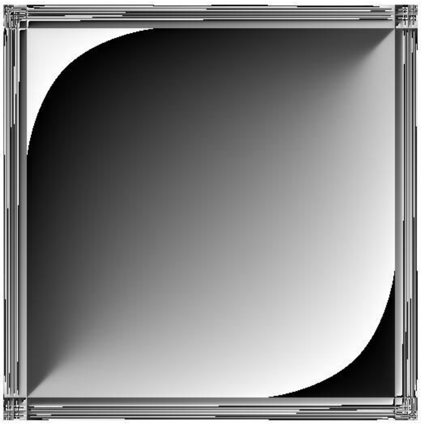
a)
Step1&2:
Creating a image.



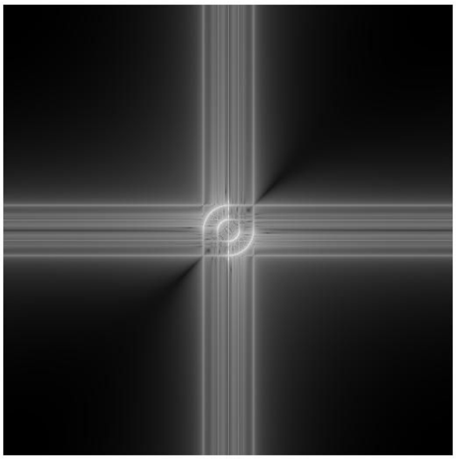
Step3:
Do the DFT and find its magnitude, phase and move the zero frequency to the center.
Magnitude(In order to easy distinguished its difference, we use log to enhance it.):



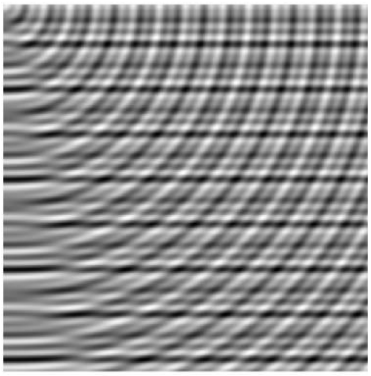
Phase:



Centered:(In order to easy distinguished its difference, we use log to enhance it.)

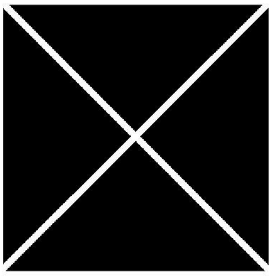


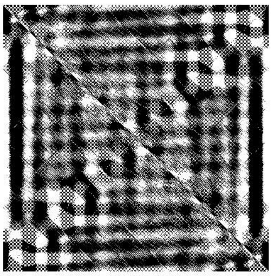
Step4:
Multiply magnitude of DFT by 2 and inverse maginitude of DFT



Difference:
We can easily look that the the left hand side image is the original image which is lighter. Another fact is that the right hand side image have another wave come from the lower right corner.

b)
Show the DFT of the cross image.





We can see that the edge(right hand side) is obviously perpendicular to the light cross(left hand side image) and the some of the blur straight line is come from that a diagonal line can only be approximated by the square pixels of the image, hence, additional frequencies are needed to compose the image.

Part 2: Notch Filter

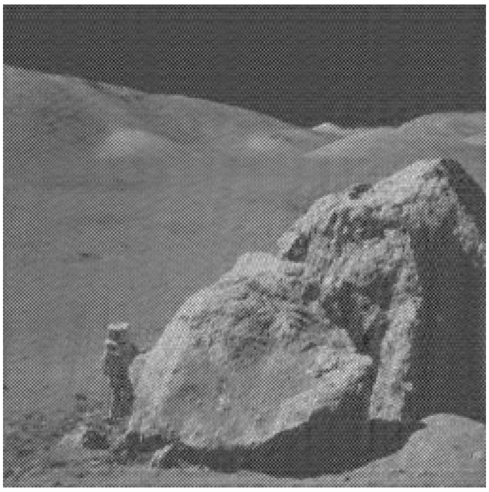
The main idea behind the notch filter is to zero out a small amount of frequencies in the FFT image, which causes noises.

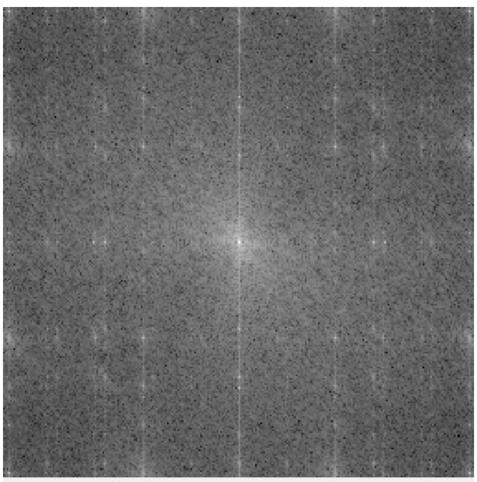
In order to find them, we can look at the FFT magnitude.

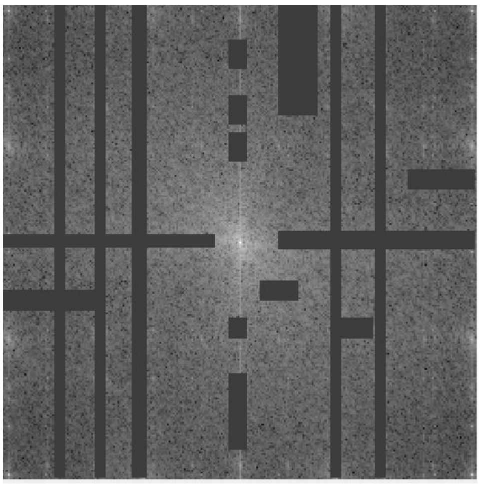
To filter noises , we use manual boxes that masks the components in the magnitude spectrum that are quite large relative to the other components as shown below.

(1)moonlanding.png


original image

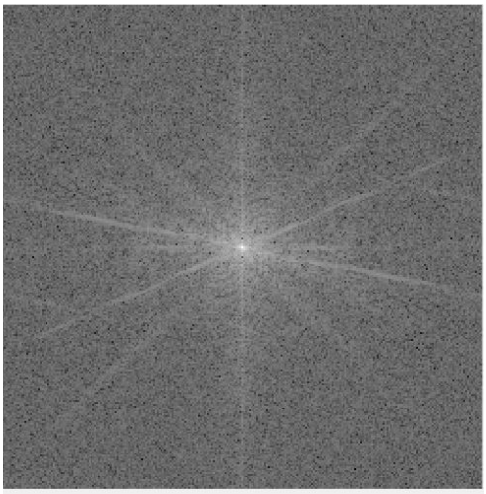

filted image

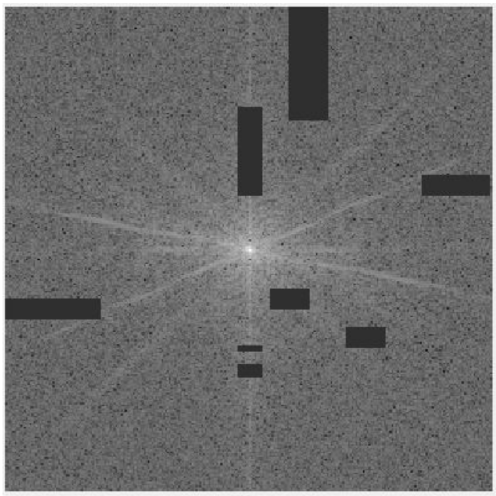

FFT magnitude


FFT magnitude with masked area

(2) psnr2.png

original image filted image





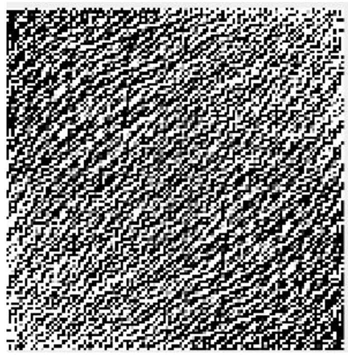
FFT magnitude FFT magnitude with masked area

Conclusion:

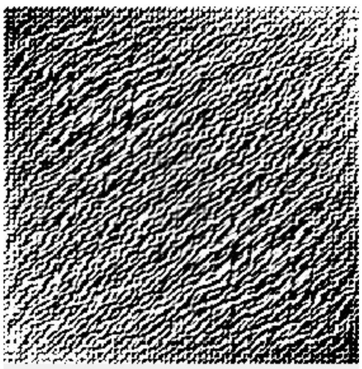
the components in the magnitude spectrum that are quite large relative to the other components are quite obvious in moonlanding.png. However, in psnr2.png, it is harder to observe strong magnitude. We also tried to do zero padding to the image but still cannot find obvious components with strong magnitude.

Part 3: Analyzing DFT

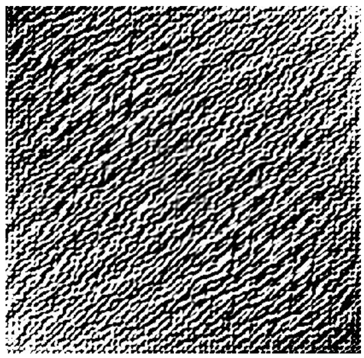
64x64



128x128



256x256



512x512

Zero padding allows one to use a longer FFT, which will produce a longer FFT result vector.

A longer FFT result has more frequency bins that are more closely spaced in frequency. But they will be essentially providing the same result as a high quality Sinc interpolation of a shorter non-zero-padded FFT of the original data.

This might result in a smoother looking spectrum when plotted without further interpolation, which matches the result of my program. The spectrum of our DFT images become smoother when we add more zero paddings. For example, the image of size 64x64 is the coarsest one and the image of size 512x512 is the smoothest one.

Although this interpolation won't help with resolving or the resolution of and/or between adjacent or nearby frequencies, it might make it easier to visually resolve the peak of a single isolated frequency that does not have any significant adjacent signals or noise in the spectrum. Statistically, the higher density of FFT result bins will probably make it more likely that the peak magnitude bin is closer to the frequency of a random isolated input frequency sinusoid, and without further interpolation (parabolic, et.al.).