Due Week 11 Wednesday. Please submit the following documents in a single zip file through LumiNUS (only one submission per group):

- *Your typed answers in a single write-up file;*
- *Your source codes or Excel spreadsheets (if any).*

Please name your zip file in the format of "A@_G#.zip", where "@" is the assignment number and "#" is your group number on LumiNUS. For example, if you are submitting Assignment 2 for Group 1, please name your zip file as "A2_G1.zip".

References to resources that are not in the textbook or class handouts should be explicitly mentioned in the write-ups and source codes. Your submission will be graded based primarily on your reasoning and correctness. Presentation (e.g., clarity of writing, visualization of data, etc.) will also be taken into consideration.

1. A bank has a portfolio of financial derivatives on an asset. The bank has determined that relationship between the asset value S and the portfolio value P can be approximated by the following formula:

$$\Delta P \approx \delta \Delta S + \tfrac{1}{2}\gamma(\Delta S)^2.$$

In the expression above, ΔP and ΔS represent the dollar changes of the portfolio value and the asset value, respectively. The coefficients, δ and γ, are such that $\delta = -30$ and $\gamma = -5$. The current asset price is $S = \$20$. Suppose the net return of the asset follows a zero-mean normal distribution with standard deviation of 1%. Using the quadratic model, calculate the first three moments of the change in the portfolio value, i.e., $E[\Delta P], E[(\Delta P)^2]$, and $E[(\Delta P)^3]$. Using the Cornish-Fisher Expansion, calculate a one-day 99% VaR using: (a) the first two moments only (i.e., treating the third moment as zero) and (b) the first three moments.

Solution. Given the coefficients $\delta = -30, \gamma = -5, \sigma = 0.01$, and $S = 20$, we have that

$$E[\Delta P] = \big(0.5 * (20^2) * (-5) * (0.01)^2\big) = -0.1.$$

Similarly,

$$E[(\Delta P)^2] = S^2\,\delta^2\,\sigma^2 + 0.75\,S^4\,\gamma^2\,\sigma^4 = 20^2 * (-30)^2 * (0.01)^2 + 0.75 * (20)^4 * (-5)^2 * (0.01)^4 = 36.03,$$

and

$$\begin{aligned} E[(\Delta P)^3] =& 4.5\,S^4\,\delta^2\,\gamma\,\sigma^4 + 1.875\,S^6\,\gamma^3\,\sigma^6 \\ =& 4.5 * (20)^4 * (-30)^2 * (-5) * (0.01)^4 + 1.875 * (20)^6 * (-5)^3 * (0.01)^6 = -32.415. \end{aligned}$$

Therefore, the dollar change of the portfolio has a mean of $\mu_P = -0.1$, a standard deviation of $\sigma_P = \sqrt{36.03 - (-0.1)^2} = 6.002$, and a skewness of

$$\xi_P = \frac{E[(\Delta P)^3] - 3E[(\Delta P)^2]\mu_P + 2\mu_P^3}{\sigma_P^3} = \frac{-32.415 - 3 * 36.03 * (-0.1) + 2 * (-0.1)^3}{(6.002)^3} = -0.10.$$

Using only the first two moments, the one-day 99% value at risk is (note that when we convert from gain to loss, we need to flip the sign for mean and skewness)

$$(0.1) + 2.33 * 6.002 = \$14.08.$$

When three moments are considered in conjunction with a Cornish-Fisher expansion, the 99% VaR is

$$(0.1) + [2.33 + 1/6 * (2.33^2 - 1) * 0.1] * 6.002 = \$14.53.$$

2. Suppose that the price of an asset at close of trading the day before yesterday was \$300. The price at the close of trading yesterday was \$298 and its volatility was estimated as 1.3% per day. Estimate today's volatility using

   - The EWMA model with $\lambda = 0.94$;
   - The GARCH(1,1) model with $\omega = 0.000002, \alpha = 0.04$ and $\beta = 0.94$.

   You may assume that the expected daily return is zero. Also, what is the long-term volatility under the GARCH(1,1) model?

   Solution. The proportional change in the price of the asset is $-2/300 = 0.00667$ or -0.667%.

   - Using the EWMA model the variance is updated to

     $$0.94 * 0.013^2 + 0.06 * 0.00667^2 = 0.00016153$$

     so that the new daily volatility is $\sqrt{0.00016153} = 0.01271$ or 1.271% per day.
   - Using GARCH (1,1) the variance is updated to

     $$0.000002 + 0.94 * 0.013^2 + 0.04 * 0.00667^2 = 0.00016264$$

     so that the new daily volatility is $\sqrt{0.00016264} = 0.01275$ or 1.275% per day. The long-term volatility is $\sqrt{\omega/(1-\alpha-\beta)} = \sqrt{0.000002/(1-0.04-0.94)} = 0.01$ or 1 %.

3. The file *ps6_data.csv* contains values of the Euro- USD exchange rate data between July 27, 2005, and July 27, 2010. Estimate and summarize the parameters for the EWMA and GARCH(1,1) model. (When you estimate the GARCH model, please use both the basic method ("GARCH") and the variance targeting method ("GARCH-VT").) Compare and visualize the estimated dynamic volatilizes from the three different methods.

   Solution. Below is a time plot of the exchange rate; see Figure 1.

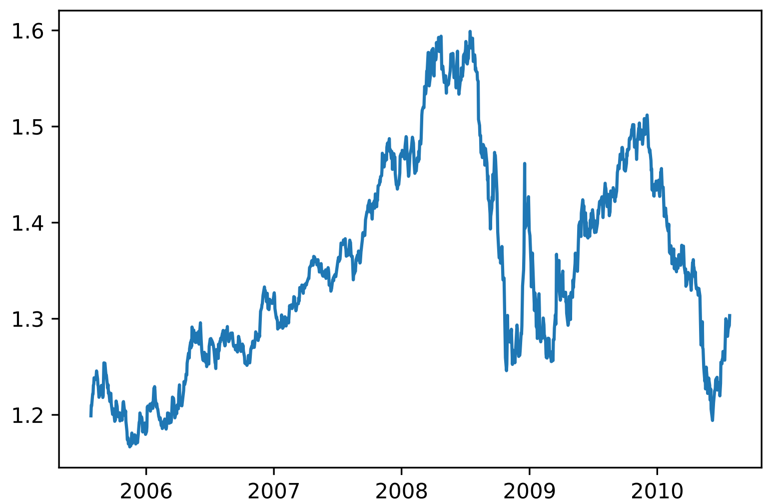

Figure 1: The Euro-USD exchange rate.

The estimated parameters are:

- EWMA: $\lambda = 0.958$ (Optimal objective function value: 11806.4767);
- GARCH(1,1): $\omega = 1.330, \alpha = 0.044$, and $\beta = 0.953$ (Optimal objective function value: 11811.1955);
- GARCH(1,1) with variance targeting: $\alpha = 0.043$ and $\beta = 0.953$ (Optimal objective function value: 11811.0559).

The estimated volatilizes could be summarized in Figure 2. As shown in Figure 2, the estimated volatility values are quite close to each other. Let us take a closer look by plotting the relative error of the other two models compared to GARCH(1,1); see Figure 3. As we can see, GARCH and GARCH-VT are almost identical. The relative error of EWMA compared to GARCH, defined as $(\sigma_i^{EWMA} - \sigma_i^{GARCH})/\sigma_i^{GARCH}$ is around -0.08, which is also quite small.

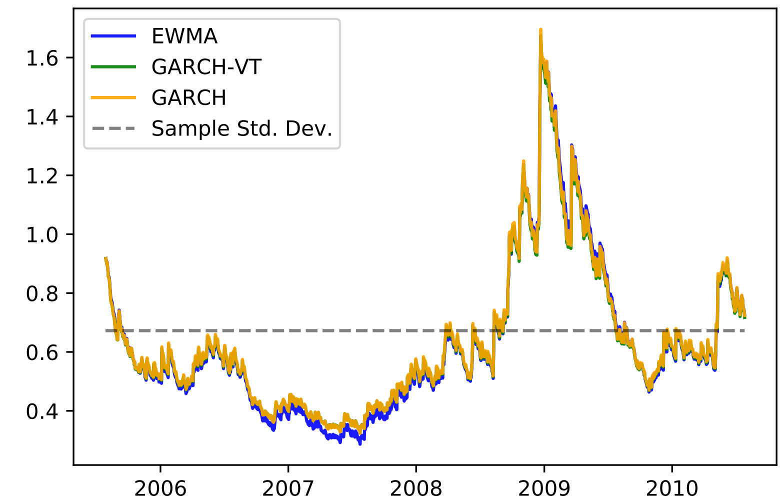

Figure 2: Estimated dynamic volatility models using three different methods.

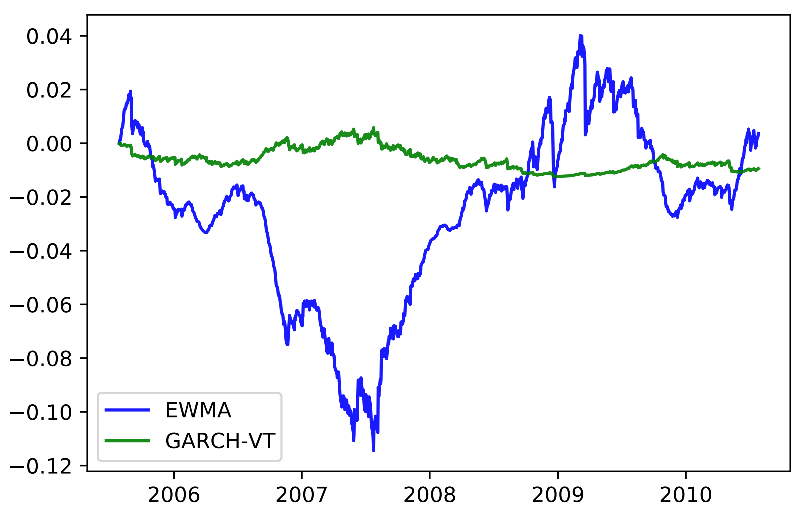

Figure 3: Relative errors of EWMA and GARCH(1,1) with variance targeting compared to the basic GARCH(1,1) method.