Theoretical Analysis of Forces on Materials Inspired by ‘The Three Little Pigs’

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Aim

To investigate the theoretical tensile strengths of different materials inspired by the fable of ‘The Three Little Pigs’.

Introduction – Overview of the materials and forces being analysed.

Construction has leveraged the various properties of unique materials to build the foundation and structure of houses. Hay bales, as inspired by the fable, are not widely used for this purpose. However, they have the inherent benefit of being renewable, are easily available, naturally fire-resistant, and provide adequate insulation. The individual straw that haybales consist of are not known for strength and are yet to be proved in this report.

Continuing in the chronological order to the fable, Western white pine wood is light in colour, straight grained and nonporous softwood with a fine and uniform texture. Western white pine is generally sawn into lumber, which finds its main use in windows, doors, as well as construction lumber. Western White Pine wood is the most used wood for construction as it grows rapidly and is relatively easy to work and has good machining qualities. All of which will be investigated in theoretical tensile stress tests within this report.

Diverging from the anthropomorphic pigs, Aluminium 6061 is a variant alloy heat-treatable composite, containing magnesium, silicon, manganese as primary alloying elements. It exhibits good mechanical and weldability properties. The metal is used in transportation, reinforced plastic structures, aircraft, architectural, and other industrial applications. As a ductile metal, Aluminium 6061 exhibits phenomenal tensile strengths – acting as a control against the former materials tested.

The materials discussed will have metrics of theoretical stress analysed to compare the overall strengths of materials against one another. This report will determine which material would best serve for the construction of houses.

Theoretical Behaviour of Different Materials Under Various Forces

Mechanical Physics Concepts

Deformation, in terms of physics, is a change in shape due to the application of force. These forces can be categorised as stress and strain.

Stress is the representation of the magnitude of forces that cause deformation. Stress is derived as the force over the surface area of an object, measured in the standard international units' pascals (Pa). The forces of stress can be exerted as elongation, compression, or bulk constriction against an object. This report primarily focused on the elongation, or tensile strengths, of a material. Tensile strength being the maximum stress that a material can withstand before breaking when stretched or pulled. Tensile strength is a general indicator of a material's ability to withstand loads or forces of in supporting the construction of large projects, such as buildings or houses. Similarly, yield strength is the measurement that determines the maximum stress before deformation is irreversible in a material. Importantly, the moments after surpassing the yield strength of the material is referred to as necking. And before which is considered the elastic region of a material's elongation

To provide further insight on the strength of each material, this report observes stress in tandem with strain, which is a descriptor of the amount of deformation in an object – measured by the length gained from elongation. Information was collected, regarding the strain of a material at the points of tensile and yield strength, respectively known as tensile strain and yield strain. The stress and strain properties of a material revealed what is known as Young's Modulus or modulus of elasticity, a property of the material that tells us how easily it can stretch and deform – a ratio without units.

Ultimately, these forces and values represent the usefulness of a material when load bearing, in any construction. Those individual materials included Hay Bales, Western White Pine wood, and Aluminium 6061, all of which are used in house construction and closely resembled the source of inspiration.

Theoretical Background

Due to the limited resources available, only the mechanical stress and strain of each material was examined. Therefore, the values to simulate such theoretical behaviour must be calculated, not measured. To do so, the variables described previously can derive a formula known as Ramberg-Osgood Equation, which can be used to calculate total strain as a function of stress. The formula closely approximates true stress-strain curves, however, is overcomplicated for such accuracy

$$\boldsymbol{\varepsilon = \frac{\sigma}{E} + 0.002\left(\frac{\sigma}{S_{ty}}\right)^{\frac{1}{n}}}$$

$\boldsymbol{\varepsilon}$: Outputted Strain Value
$\boldsymbol{\sigma}$: Inputted Stress Value
$\boldsymbol{E}$: Young's Modulus/Modulus of Elasticity
$\boldsymbol{S_{ty}}$: Tensile Yield Strength
$\boldsymbol{n}$: Constant per Material

(Figure A: Ramberg-Osgood Equation)

For this equation to be useful when generating data for mechanical stress-strain curves, a similar approximation known as the H. N. Hill Equation acts interchangeably to Ramberg-Osgood Equation without added complexity:

$$\boldsymbol{\varepsilon = \frac{\sigma}{E} + 0.002\left(\frac{\sigma}{S_{ty}}\right)^{n}}$$

$\boldsymbol{\varepsilon}$: Outputted Strain Value
$\boldsymbol{\sigma}$: Inputted Stress Value
$\boldsymbol{E}$: Young's Modulus/Modulus of Elasticity

S_{ty}: Tensile Yield Strength

n: Constant per Material

(Figure B: H. N. Hill Equation)

Distinctly, the parameter ‘n’ is not inversed to become its reciprocal. The parameter ‘n’ also does not represent any specific force of physics; however, it is an independent constant per material, which can be derived using known variables

$$n = \frac{\ln\left(\frac{\varepsilon_{max} - \frac{\sigma_{UTS}}{E}}{0.002}\right)}{\ln\left(\frac{\sigma_{UTS}}{\sigma_{YS}}\right)}$$

n: Constant per Material

ε_{max}: Maximum Elongation

σ_{UTS}: Ultimate Tensile Strength

σ_{YS}: Yield Strength

E: Modulus of Elasticity

(Figure C: Parameter ‘n’ Equation)

With these formulas, individual strain values were generated per stress value, up to the ultimate tensile strength of a material. This was simulated through an originally created Python script that converted the above formulas into executable code:

```python
def H_N_Hill_Equation(self, o):
    """
    Returns strain value (ε/e (float): represents total strain), given varying stress

    Parameters:
        o (float): signifies stress (σ)
        E (float): stands for Young's Modulus
        oYS (float): yield strength (σYS)
        n (float): material-dependent constants
    """
    try:
        return (o / self.E) + (0.002 * math.pow(o / self.oYS, self.n))
    except ValueError:
        print("(math domain error) Ultimate strength must be greater than yield strength!")
```

```python
def Parameter_n(self):
    """
    Returns parameter n, to be fed into H_N_Hill_Equation

    oUTS (float): Ultimate Tensile Strength (σUTS)
    eMAX (float): Maximum Elongation (εmax)
    oYS (float): yield strength (σYS)
    """
    try:
        return (math.log((self.eMAX - (self.oUTS/self.E)) / 0.002)) / (math.log(self.oUTS / self.oYS))
    except ValueError:
        print("(math domain error) Elastic modulus is too small compared to the strength!")
```

These functions were run in a while loop to evaluate, with a given resolution for data, all mechanical strain values per stress input, inserted into a value table. The data was then fed into an excel spreadsheet to allow easy access to outputted data and graphing for visualisation of trends:

```python
    self.n = self.Parameter_n()

    self.value_table = {
    # strain : stress,
    }

    self.calculate_value_table()

def calculate_value_table(self):
    interval = self.oUTS / self.resolution
    stress = 0
    while stress <= self.oUTS:
        try:
            strain = self.H_N_Hill_Equation(o=stress)

            stress += interval

            self.value_table[round(strain,7)] = round(stress, 3)
        except:
            pass # some values may suffer from rounding errors, and are uncomputable
```

Results

Tables

Strain	Stress	Strain Ultimate Strength	Stress Ultimate Strength	Strain Yield Strength	Stress Yield Strength	
0	320	0.2	320000	0.0095	15000	
0.0002063	640					
0.0004112	960	**Young's Modulus**				
0.0006156	1280	1250000				
0.0008198	1600					
0.0010236	1920					
0.0012273	2240					
0.0014309	2560					
0.0016343	2880					

(Figure 1: stress-strain data of Hay Bale)

Strain	Stress	Strain Ultimate Strength	Stress Ultimate Strength	Stress Yield Strength	Strain Yield Strength	
0	39000	0.1	39000000	1790000	0.002215663	
0.0000212	78000					
0.0000487	117000	**Young's Modulus**				
0.0000795	156000	8200000000				
0.0001126	195000					
0.0001476	234000					
0.0001841	273000					
0.0002221	312000					
0.0002613	351000					

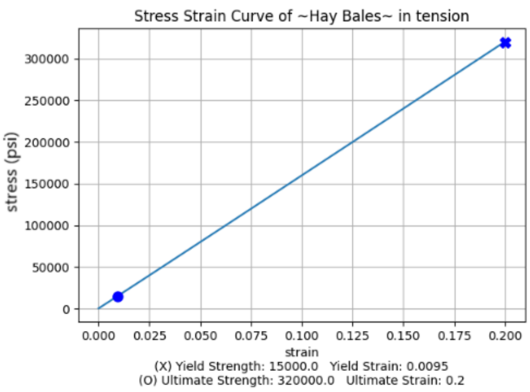
(Figure 2: stress-strain data of Western White Pine Wood)

Aluminium 6061

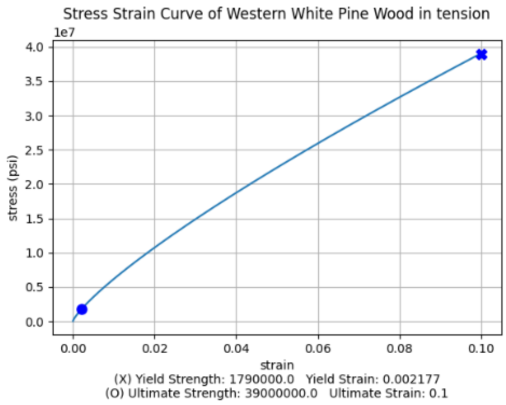
Strain	Stress	Strain Ultimate Strength	Stress Ultimate Strength	Stress Yield Strength	Strain Yield Strength
0	290000	0.1	290000000	240000000	0.005287671
0.000004	580000				
0.0000079	870000	**Young's Modulus**			
0.0000119	1160000	68000000000			
0.0000159	1450000				
0.0000199	1740000				
0.0000238	2030000				
0.0000278	2320000				
0.0000318	2610000				

(Figure 3: stress-strain data of Aluminium 6061)

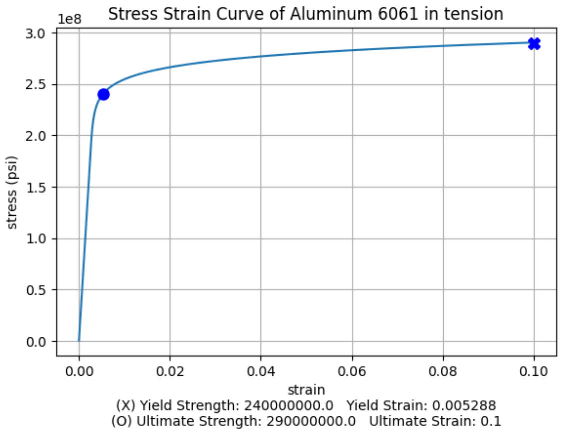
Graphs



(Figure 4: stress-strain curve of Hay Bale)



(Figure 5: stress-strain curve of Western White Pine Wood)



(Figure 6: stress-strain curve of Aluminium 6061)

Discussion – Suitability of 'The Three Little Pigs' Materials, Using Original Virtual Tools

The stress-strain data of the Hay Bale was observed to be tremendously smaller in magnitude compared to the other two materials, particularly Aluminium 6061 (Figure 1: Stress Yield Strength = 15000 pascal vs. Figure 3: Stress Yield Strength = 240000000 pascal). This matched well with contemporary comparisons of the materials, as pulling individual straw to its ultimate tensile strength is feasible by humans. Therefore, scaling to the size of typical Hay Bales, the results were as expected. Interestingly, the stress-strain curve of the Hay Bale displayed an unorthodox trend as it appeared essentially linearly (see Figure 4). This was due to the miniscule ultimate and yield strength, and young's modulus of the material – leaving minimal space for obvious necking. This pattern can be explained as the elastic region makes up a nearly insignificant portion of the curve. The Hay Bale can therefore be categorised as relatively brittle, coming close to snapping without much deviation in force applied beyond its plastic region. In turn, it is advisable Hay Bales are used for their advantageous properties, being its insulating and fire-retardant abilities, rather than for the sole foundation of a house.

The stress-strain data observed in Western White Pine wood saw improvement in roughly two orders of magnitude compared to the Hay Bale (Figure 1: Stress Ultimate Strength = 320000 pascal vs. Figure 2: Stress Ultimate Strength = 39000000 pascal). The background research found of Western White Pine wood in its application explained the comparative increase in strength, as it is reliably used for the construction of scaffolding of homes and other large-scale projects that require decent yield and tensile strengths for affordable prices. Like the Hay Bale, the stress-strain curve of Western White Pine wood appeared almost linear, with a slight curve close to and around the yield strength of the material. The pattern hinted to the wood's minimal elongating abilities, until eventually abruptly fracturing. These results were justified by Western White Pine wood's property as a softwood, having a relatively lower density and even grain, giving more leeway for elasticity before entering its plastic region.

As almost a control point of data, Aluminium 6061 was selected to compare the absurdity that was measured in the tensile and yield strengths of the Hay Bale and Western White Pine wood, and their reactions when under tensile stress. This point in research deviated furthest from the original inspiration, due to brick equivalents exhibiting nearly zero tensile strengths as they are such brittle materials. The values observed in Aluminium 6061 tensile and yield strengths are exponentially larger than the other two materials (Figure 3: Stress Ultimate Strength = 240000000 pascal & Stress Yield Strength = 240000000 pascal). However, compared to other ductile metals, these values are extremely regular. A statement that translated exactly in the pattern seen in the stress-strain curve of Aluminium 6061 as well. It required most of the force to be exerted around the yield strength, until finally necking to fracture over a relatively long distance but minimal force. Generally, the pattern described gave insight to the elasticity of Aluminium 6061, making up most of its stress-strain curve until the metal entered its plastic region (see Figure 6). The elastic attributes are ideal for sound construction of large-scale projects and designs that demand high durability in extreme situations.

Conclusion

In retrospect, usage of mostly non-metal and non-ductile materials for the sake of analysing the effectiveness of the materials used by the 'Three Little Pigs' exponentially increased the difficulty of research. Especially as research focused primarily on tensile and yield strengths. This reason is what prompted the creation of custom tools, to demonstrate the intuitive fact that such materials should not be used solely for the construction of a house. However, the custom tools gave way to further error, often outputting largely inaccurate stress-strain curves for materials with previously established and well researched tensile properties. These could have been circumvented given enough time to comprehend optimal and correct ways to simulate arduous behaviours. In repeating the experimentation in this report, more thorough research must be completed before simulating large quantities of data to avoid mistakes such as the mislabelling of graphs, as they are wrongly denoted as psi rather than pascals. Ultimately, all materials can and should be used in tandem for optimal results when constructing a house to best leverage each material's advantageous properties.

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Other Resources

(Screenshots of data outputted directly from python code)

```
NumPy version: 2.1.1
Pandas version: 2.2.2
        Strain     Stress  Unnamed: 2  Strain Yield Strength  Stress Yield Strength  Unnamed: 5  Strain Ultimate Strength  Stress Ultimate Strength
0          NaN        NaN         NaN                    0.2               320000.0         NaN                    0.0095                   15000.0
1     0.000000      320.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
2     0.000206      640.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
3     0.000411      960.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
4     0.000616     1280.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
...        ...        ...         ...                    ...                    ...         ...                       ...                       ...
997   0.199203   319040.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
998   0.199403   319360.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
999   0.199602   319680.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
1000  0.199801   320000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
1001  0.200000   320320.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN

[1002 rows x 8 columns]
```

```
NumPy version: 2.1.1
Pandas version: 2.2.2
        Strain       Stress  Unnamed: 2  Strain Yield Strength  Stress Yield Strength  Unnamed: 5  Strain Ultimate Strength  Stress Ultimate Strength
0          NaN          NaN         NaN                    0.1             39000000.0         NaN                  0.002216                 1790000.0
1     0.000000      39000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
2     0.000021      78000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
3     0.000049     117000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
4     0.000079     156000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
...        ...          ...         ...                    ...                    ...         ...                       ...                       ...
997   0.099503   38883000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
998   0.099627   38922000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
999   0.099752   38961000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
1000  0.099876   39000000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
1001  0.100000   39039000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN

[1002 rows x 8 columns]
```

```
NumPy version: 2.1.1
Pandas version: 2.2.2
        Strain        Stress  Unnamed: 2  Strain Yield Strength  Stress Yield Strength  Unnamed: 5  Strain Ultimate Strength  Stress Ultimate Strength
0          NaN           NaN         NaN                    0.1            290000000.0         NaN                  0.005288               240000000.0
1     0.000000      290000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
2     0.000004      580000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
3     0.000008      870000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
4     0.000012     1160000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
...        ...           ...         ...                    ...                    ...         ...                       ...                       ...
997   0.092425   289130000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
998   0.094263   289420000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
999   0.096139   289710000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
1000  0.098050   290000000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN
1001  0.100000   290290000.0         NaN                    NaN                    NaN         NaN                       NaN                       NaN

[1002 rows x 8 columns]
```