Convolutional Nueral Network

September 25-26, 2018

Convolutional neural network

- Specially designed for data with grid-like structures (LeCun et al. 98)
  - 1D grid: sequential data
  - 2D grid: image
  - 3D grid: video, 3D image volume
- Beat all the existing computer vision technologies on object recognition on ImageNet challenge with a large margin in 2012

Convolutional neural network

We can compute the spatial size of the output volume as a function of the input volume size ($\boldsymbol{W}$), the receptive field size of the Conv Layer neurons ($\boldsymbol{F}$), the stride with which they are applied ($\boldsymbol{S}$), and the amount of zero padding used ($\boldsymbol{P}$) on the border. You can convince yourself that the correct formula for calculating how many neurons "fit" is given by $(\boldsymbol{W} - \boldsymbol{F} + \boldsymbol{2P})/\boldsymbol{S} + \boldsymbol{1}$. For example for a 7x7 input and a 3x3 filter with stride 1 and pad 0 we would get a 5x5 output. With stride 2 we would get a 3x3 output. Lets also see one more graphical example:

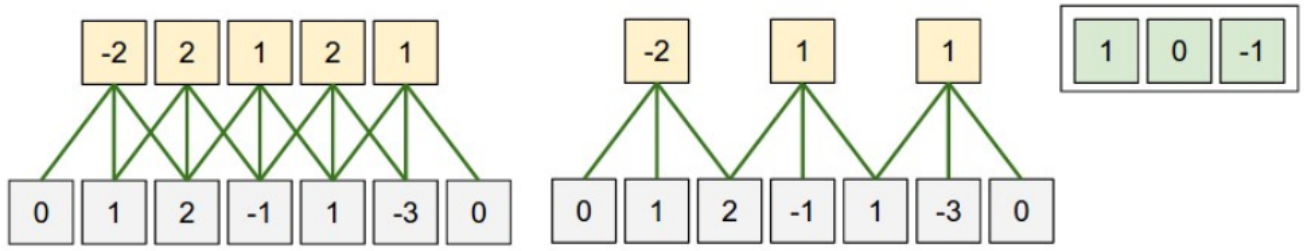
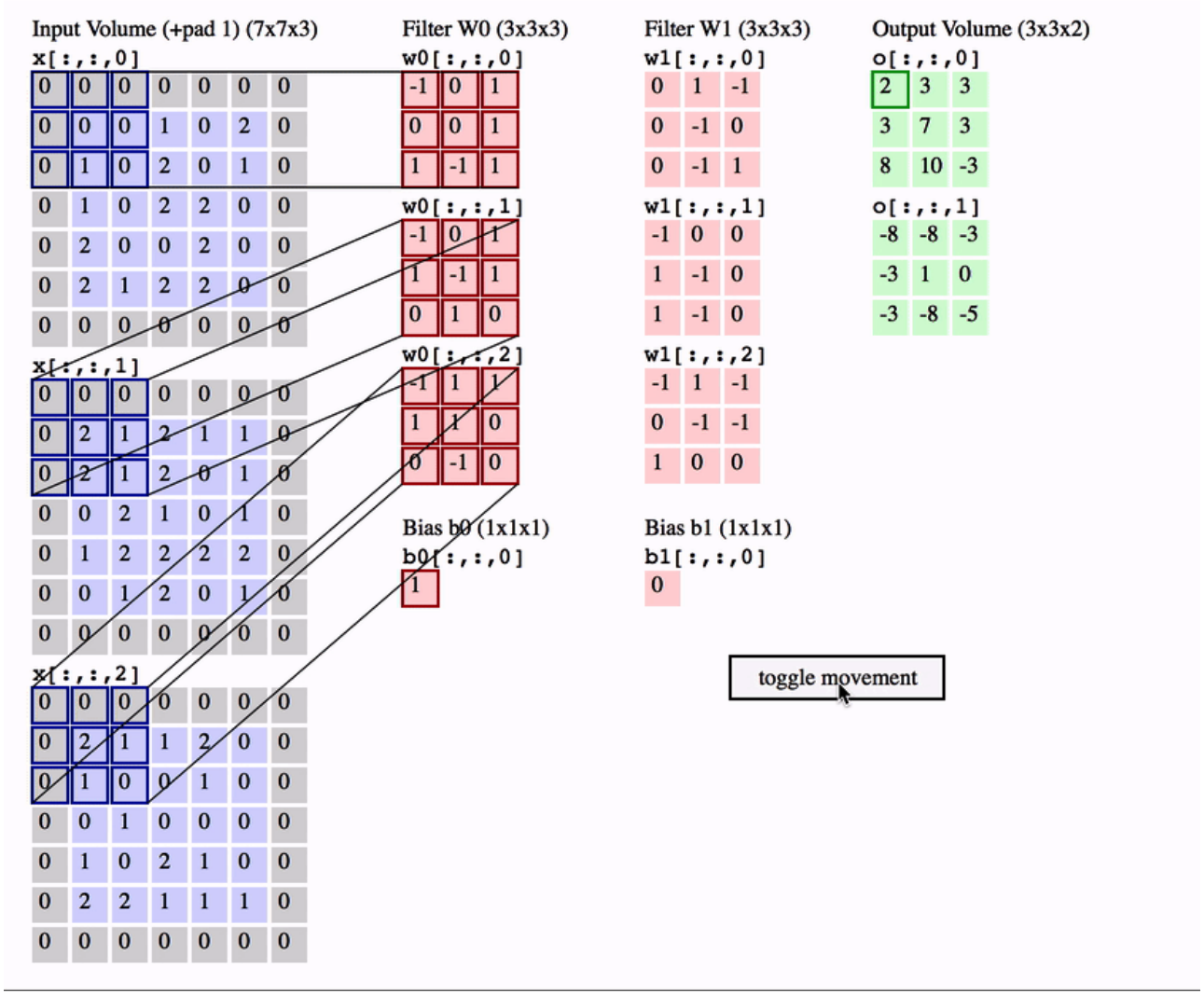


Illustration of spatial arrangement. In this example there is only one spatial dimension (x-axis), one neuron with a receptive field size of F = 3, the input size is W = 5, and there is zero padding of P = 1. **Left:** The neuron strided across the input in stride of S = 1, giving output of size (5 - 3 + 2)/1+1 = 5. **Right:** The neuron uses stride of S = 2, giving output of size (5 - 3 + 2)/2+1 = 3. Notice that stride S = 3 could not be used since it wouldn't fit neatly across the volume. In terms of the equation, this can be determined since (5 - 3 + 2) = 4 is not divisible by 3.
The neuron weights are in this example [1,0,-1] (shown on very right), and its bias is zero. These weights are shared across all yellow neurons (see parameter sharing below).

Conv1D

http://cs231n.github.io/convolutional-networks/#overview

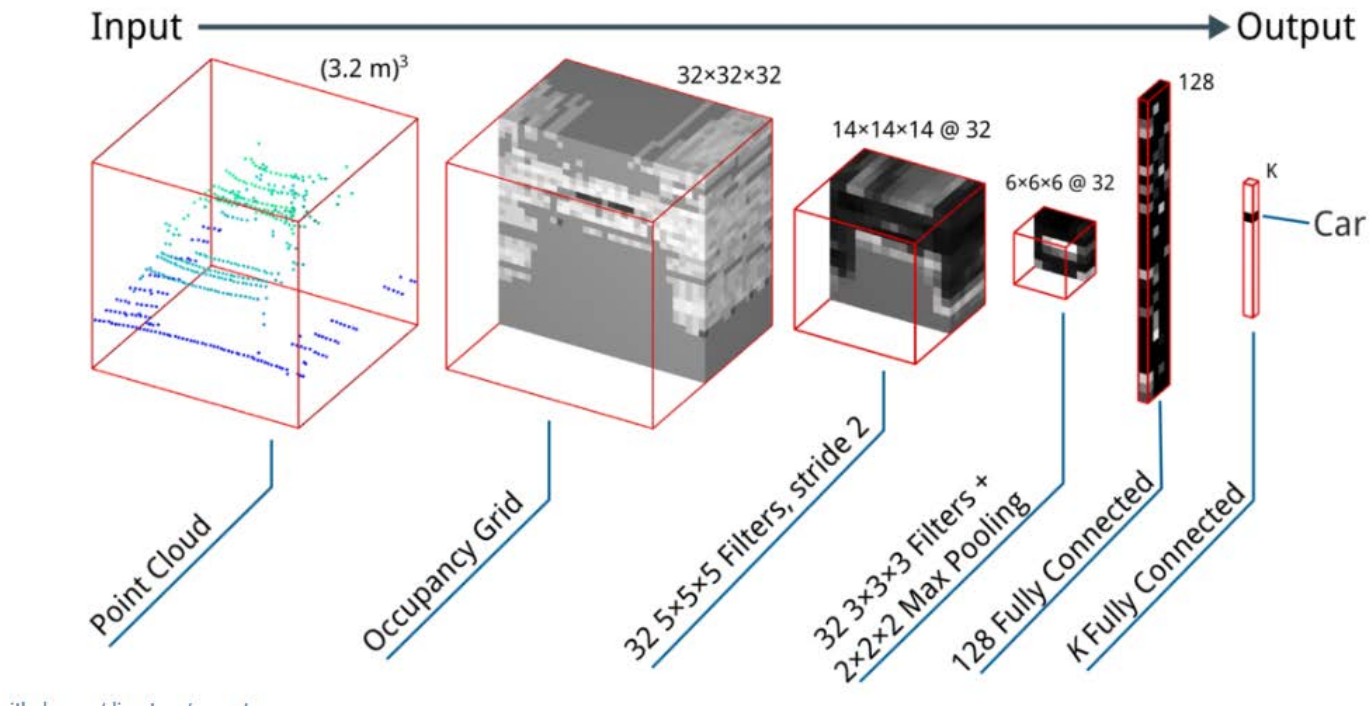
Convolutional neural network



Conv2d

http://cs231n.github.io/convolutional-networks/#overview

Convolutional neural network

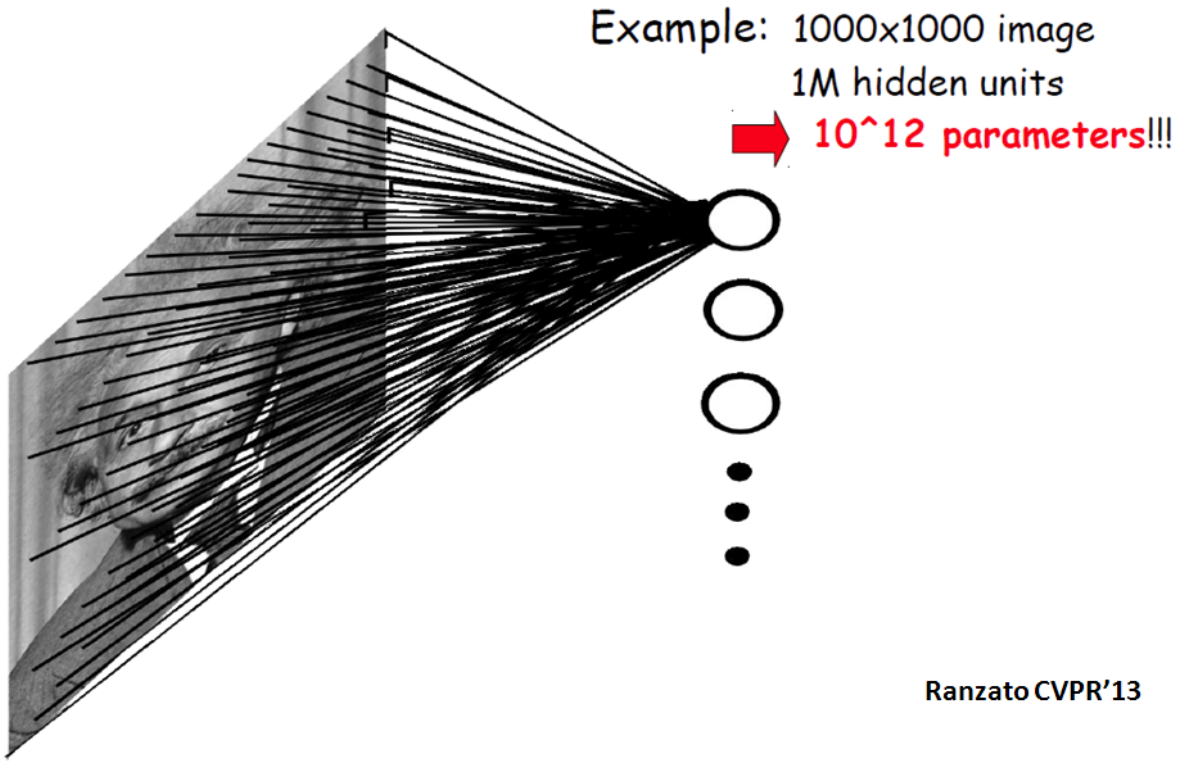


Conv3D

http://dimatura.net/research/voxnet/

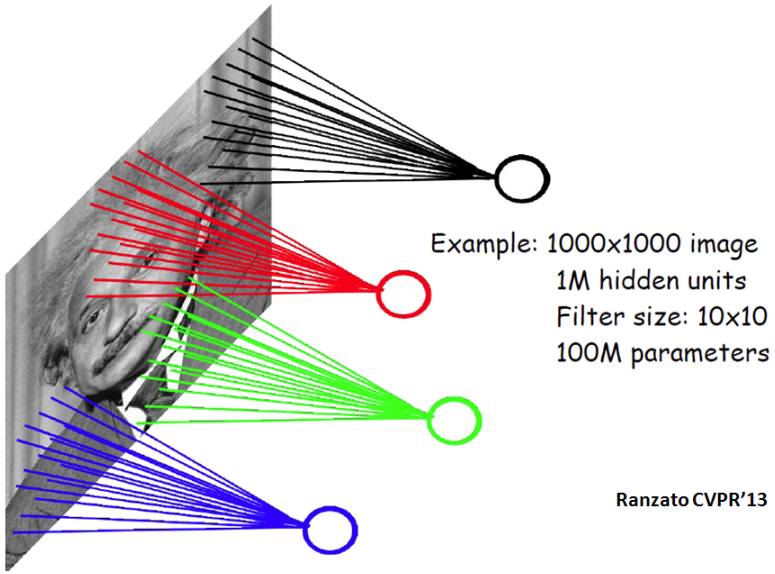
Problems of fully connected neural networks

- Every output unit interacts with every input unit
- The number of weights grows largely with the size of the input image
- Pixels in distance are less correlated



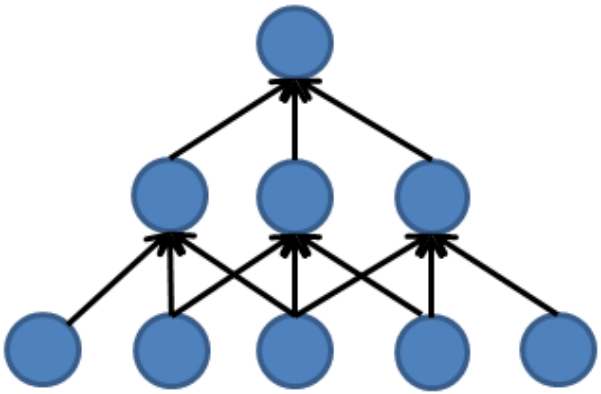
Locally connected neural networks

- Sparse connectivity: a hidden unit is only connected to a local patch (weights connected to the patch are called filter or kernel)
- It is inspired by biological systems, where a cell is sensitive to a small sub-region of the input space, called a receptive field. Many cells are tiled to cover the entire visual field.
- The design of such sparse connectivity is based on domain knowledge. (Can we apply CNN in frequency domain?)
  https://arxiv.org/pdf/1506.03767.pdf,
  https://arxiv.org/pdf/1612.00606.pdf
  https://www.zhihu.com/question/39689253



Locally connected neural networks

- The learned filter is a spatially local pattern
- A hidden node at a higher layer has a larger receptive field in the input
- Stacking many such layers leads to“filters”(not anymore linear) which become increasingly “global”



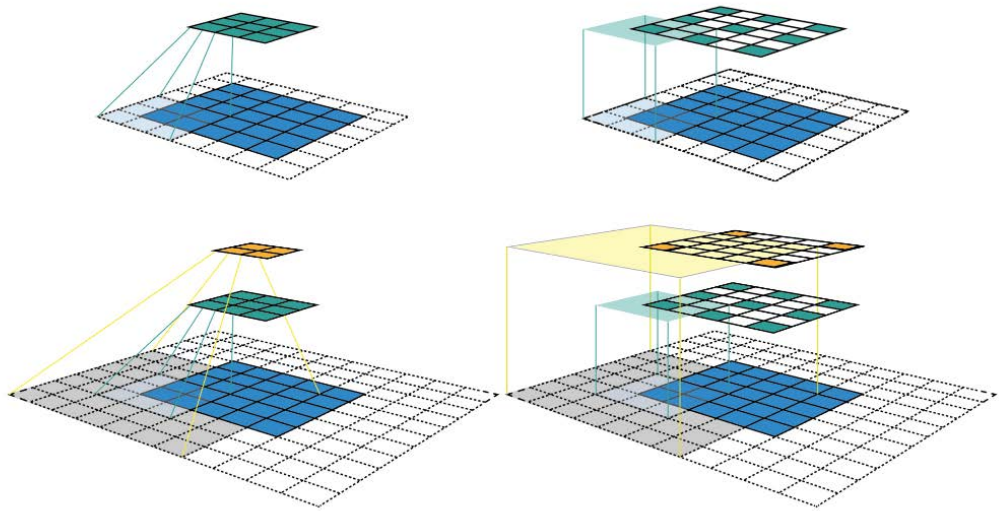
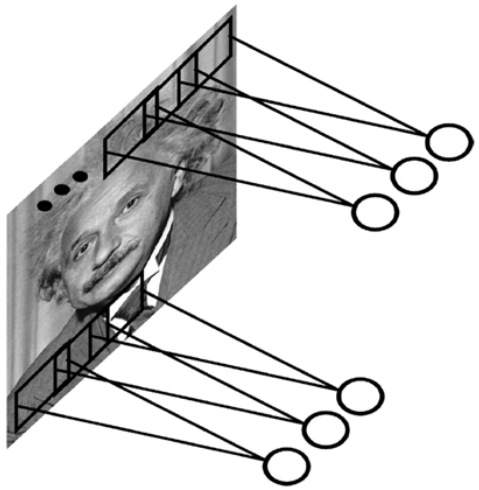


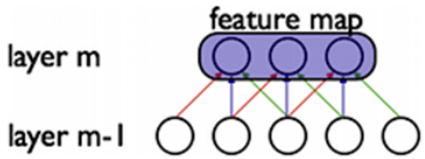
Figure 1: Two ways to visualize CNN feature maps. In all cases, we uses the convolution C with kernel size k = 3x3, padding size p = 1x1, stride s = 2x2. (Top row) Applying the convolution on a 5x5 input map to produce the 3x3 green feature map. (Bottom row) Applying the same convolution on top of the green feature map to produce the 2x2 orange feature map. (Left column) The common way to visualize a CNN feature map. Only looking at the feature map, we do not know where a feature is looking at (the center location of its receptive field) and how big is that region (its receptive field size). It will be impossible to keep track of the receptive field information in a deep CNN. (Right column) The fixed-sized CNN feature map visualization, where the size of each feature map is fixed, and the feature is located at the center of its receptive field.

https://medium.com/mlreview/a-guide-to-receptive-field-arithmetic-for-convolutional-neural-networks-e0f514068807

Shared weights

- Translation invariance: capture statistics in local patches and they are independent of locations
  - Similar edges may appear at different locations
- Hidden nodes at different locations share the same weights. It greatly reduces the number of parameters to learn $(100\text{M to } 100, \text{why?})$
- In some applications (especially images with regular structures), we may only locally share weights or not share weights at top layers



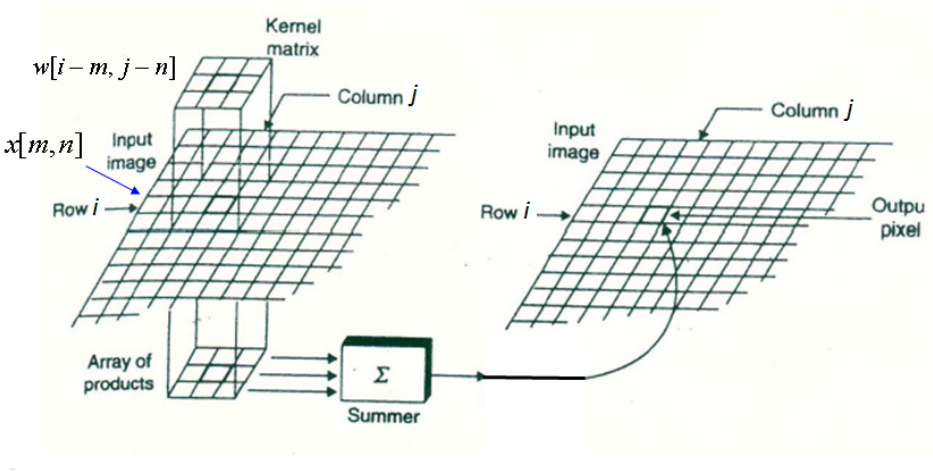


Weights with the same color have identical values

Convolution

- Computing the responses at hidden nodes is equivalent to convoluting the input image **x** with a learned filter **w**
- After convolution, a filter map *net* is generated at the hidden layer
- Parameter sharing causes the layer to have *equivariance* to translation. A function $f(x)$ is equivalent to a function g if $f(g(x)) = g(f(x))$
- Is convolution equivariant to changes in the scale or rotation? (page354)

$$net[i,j] = (x*w)[i,j] = \sum_m \sum_n x[m,n]w[i-m, j-n]$$

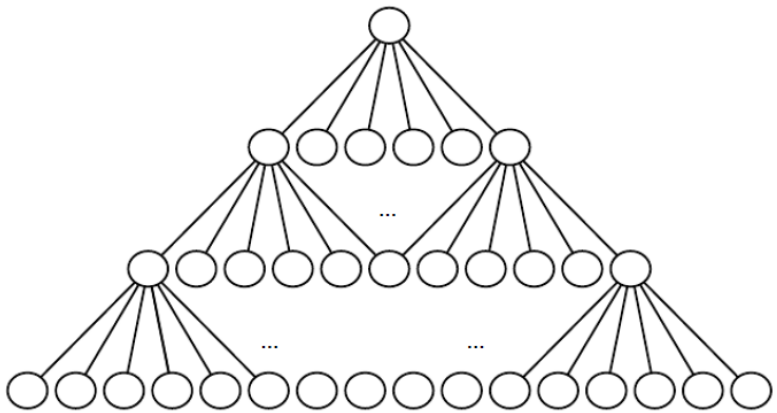


$$S(i,j) = (I*K)(i,j) = \sum_m \sum_n I(i+m, j+n)K(m,n). \qquad (9.6)$$

pape 348

Zero-padding in convolutional neural network

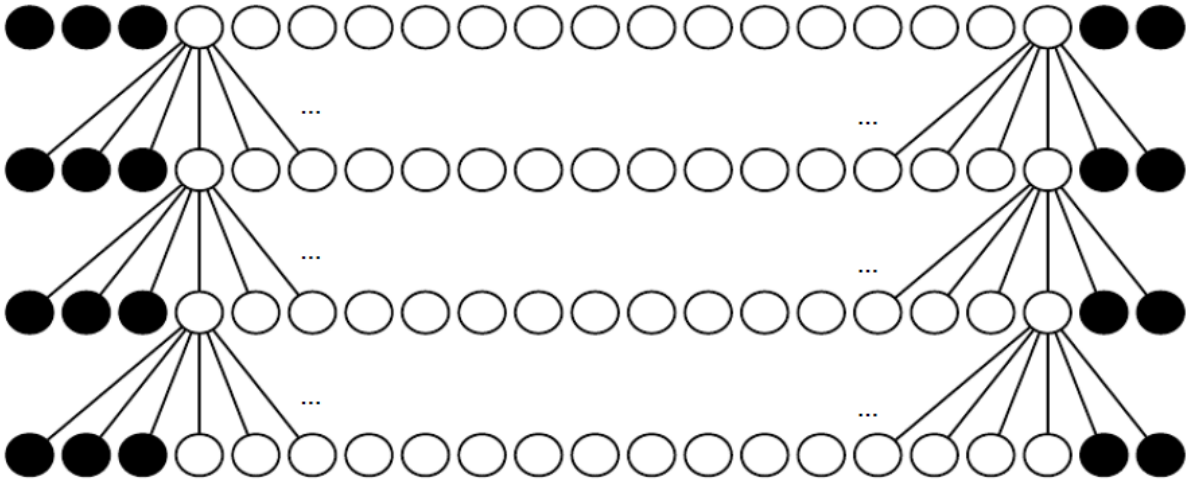
- The valid feature map is smaller than the input after convolution
- Implementation of neural networks needs to zero-pad the input **x** to make it wider
- Without zero-padding, the width of the representation shrinks by the filter width - 1 at each layer
- To avoid shrinking the spatial extent of the network rapidly, small filters have to be used



(Bengio et al. Deep Learning 2014)

Zero-padding in convolutional neural network

- By zero-padding in each layer, we prevent the representation from shrinking with depth. It allows us to make an arbitrarily deep convolutional network



(Bengio et al. Deep Learning 2014)

Downsampled convolutional layer

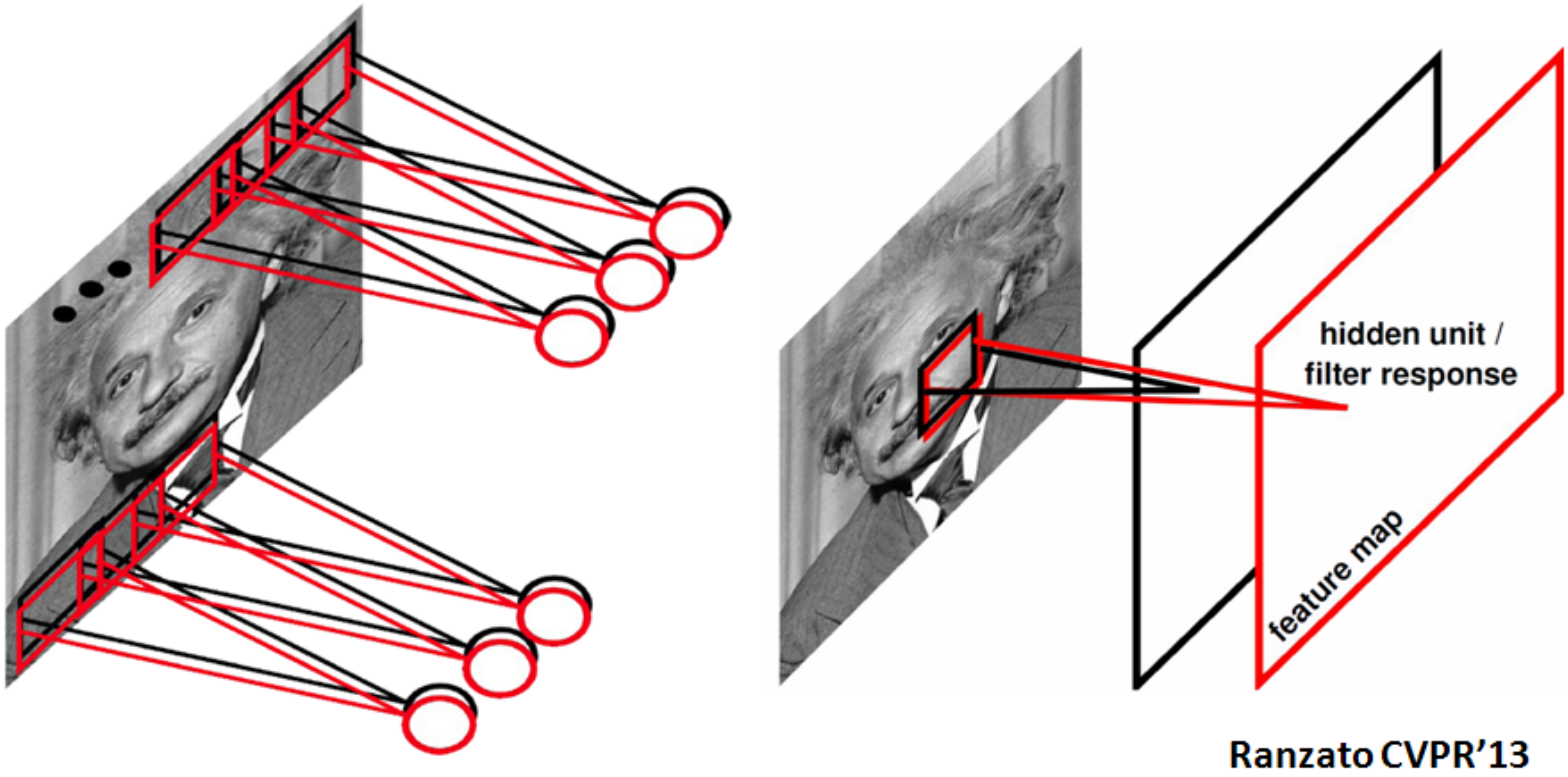
- To reduce computational cost, we may want to skip some positions of the filter and sample only every s pixels in each direction. A downsampled convolution function is defined as

$$net[i, j] = (\mathbf{x} * \mathbf{w})[i \times s, j \times s]$$

- s is referred as the *stride* of this downsampled convolution

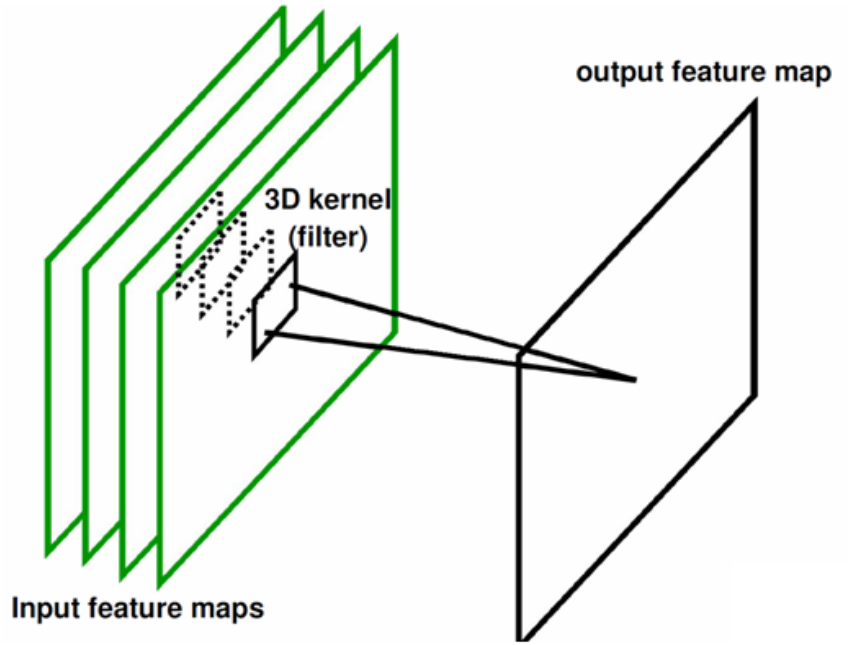
Multiple filters

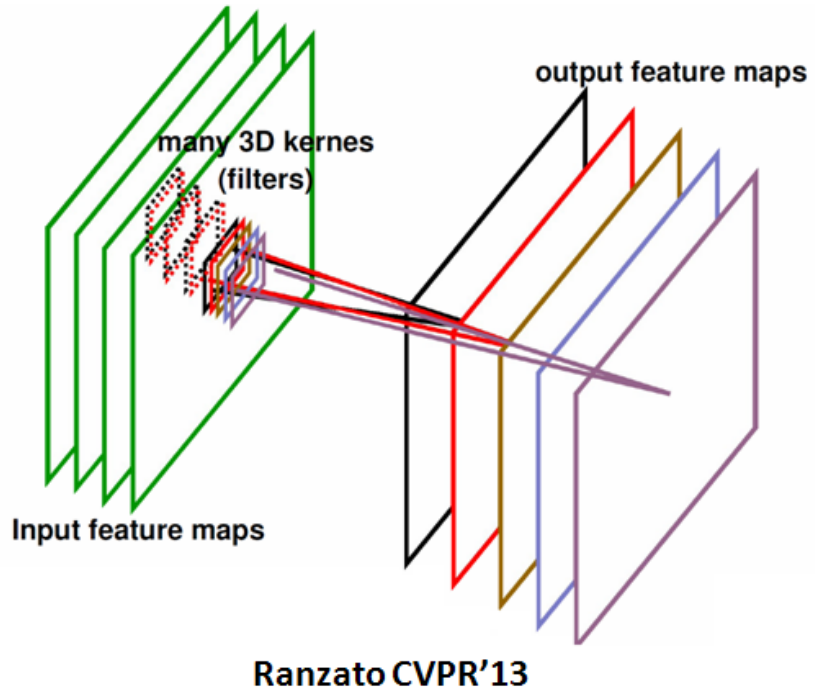
- Multiple filters generate multiple feature maps
- Detect the spatial distributions of multiple visual patterns



3D filtering when input has multiple feature maps

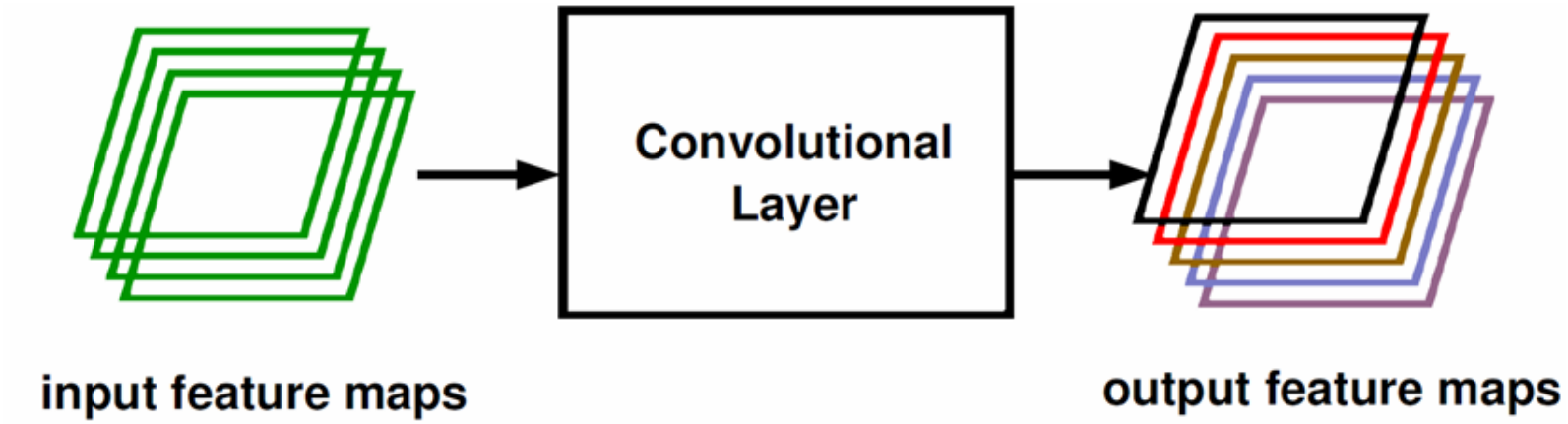
$$net = \sum_{k=1}^{K} \mathbf{x}^k * \mathbf{w}^k$$





Ranzato CVPR'13

Convolutional layer



Ranzato CVPR'13

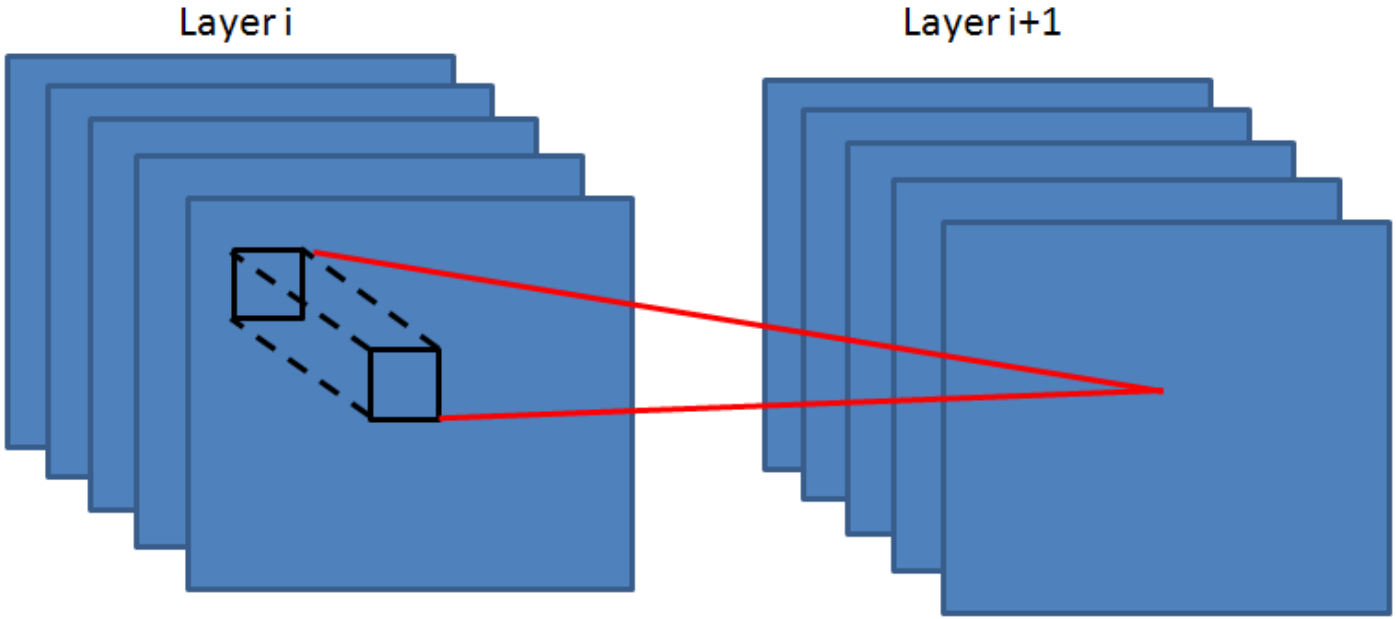
Nonlinear activation function

- tanh()
- Rectified linear unit

Local contrast normalization (LCN)

- Normalization can be done within a neighborhood along both spatial and feature dimensions

$$h_{i+1,x,y,k} = \frac{h_{i,x,y,k} - m_{i,N(x,y,k)}}{\sigma_{i,N(x,y,k)}}$$



Neural network normalization

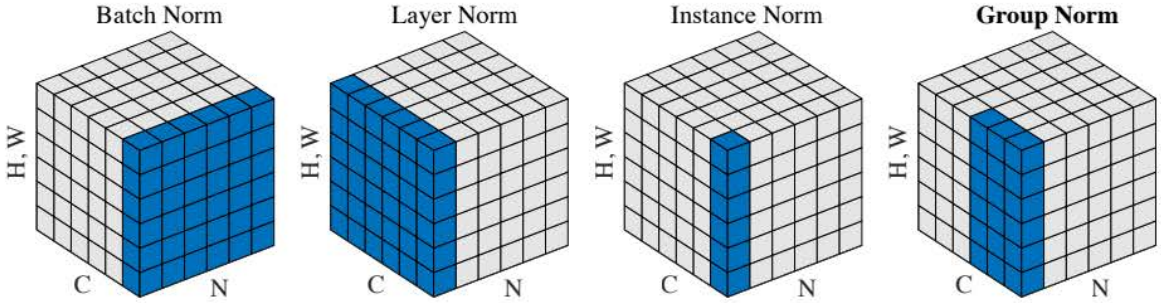


Figure 2. **Normalization methods**. Each subplot shows a feature map tensor, with N as the batch axis, C as the channel axis, and (H, W) as the spatial axes. The pixels in blue are normalized by the same mean and variance, computed by aggregating the values of these pixels.

The goal of batch norm is to *reduce internal covariate shift* by normalizing each mini-batch of data using the mini-batch mean and variance. For a mini-batch of inputs $\{x_1, \ldots, x_m\}$, we compute

$$\mu = \frac{1}{m}\sum_{i=1}^{m} x_i$$

$$\sigma^2 = \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu)^2$$

BN

and then replace each x_i with its normalized version

$$\hat{x}_i = \frac{x_i - \mu}{\sqrt{\sigma^2 + \epsilon}}$$

Instead of normalizing examples across mini-batches, layer normalization *normalizes features within each example*. For input x_i of dimension D, we compute

$$\mu = \frac{1}{D}\sum_{d=1}^{D} x_i^d$$

$$\sigma^2 = \frac{1}{D}\sum_{d=1}^{D}(x_i^d - \mu)^2$$

LN

and then replace each component x_i^d with its normalized version

$$\hat{x}_i^d = \frac{x_i^d - \mu}{\sqrt{\sigma^2 + \epsilon}}.$$

$\sqrt{\sigma^2 + \epsilon}$

Instance normalization

Instance norm (Ulyanov, Vedaldi, & Lempitsky, 2016) hit arXiv just 6 days after layer norm, and is pretty similar. Instead of normalizing all of the features of an example at once, instance norm normalizes features within each channel.

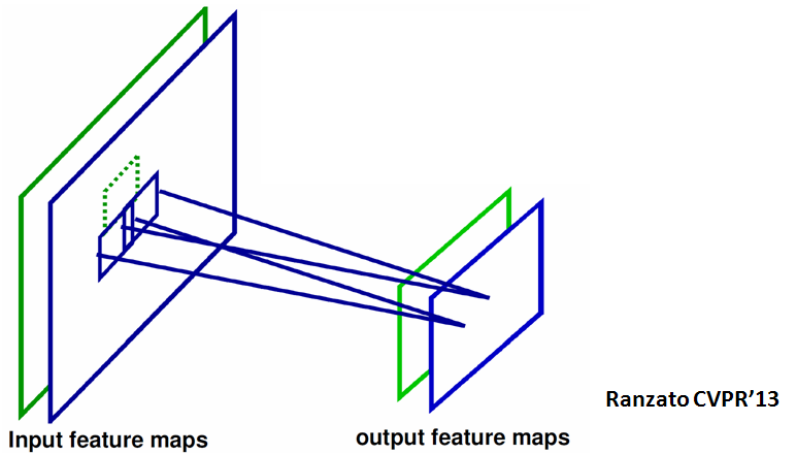
Group normalization

Group norm (Wu & He, 2018) is somewhere between layer and instance norm — instead of normalizing features within each channel, it normalizes features within pre-defined groups of channels.[4]

https://nealjean.com/ml/neural-network-normalization/

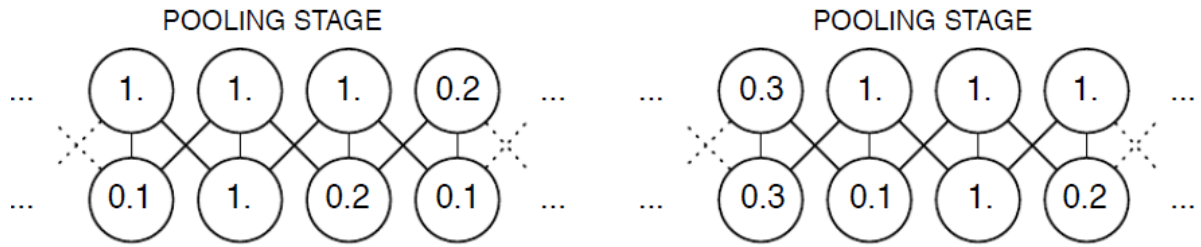
Pooling

- Max-pooling partitions the input image into a set of rectangles, and for each sub-region, outputs the maximum value
- Non-linear down-sampling
- The number of output maps is the same as the number of input maps, but the resolution is reduced
- Reduce the computational complexity for upper layers and provide a form of **translation invariance**
- Average pooling can also be used



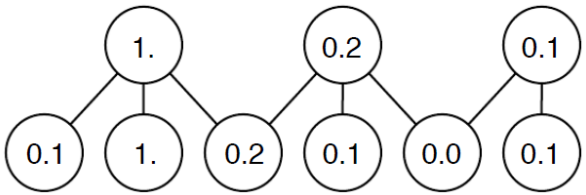
Pooling

- Pooling without downsampling (stride $s = 1$)
- Invariance vs. information loss (even if the resolution is not reduced)
- Pooling is useful if we care more about whether some feature is present than exactly there it is. It depends on applications.



(Bengio et al. Deep Learning 2014)

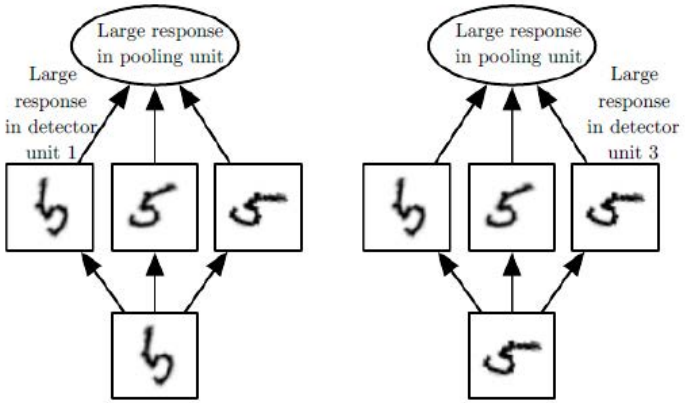
- Pooling with downsampling (commonly used)
- Improve computation efficiency

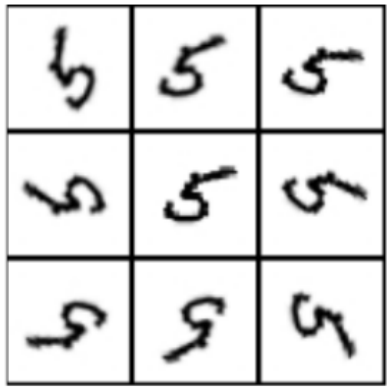


(Bengio et al. Deep Learning 2014)

Possible extension of CNN

- If we pool over the outputs of separately parameterized convolutions, the features can learn which transformations to become invariant to
- How to achieve scaling invariance?

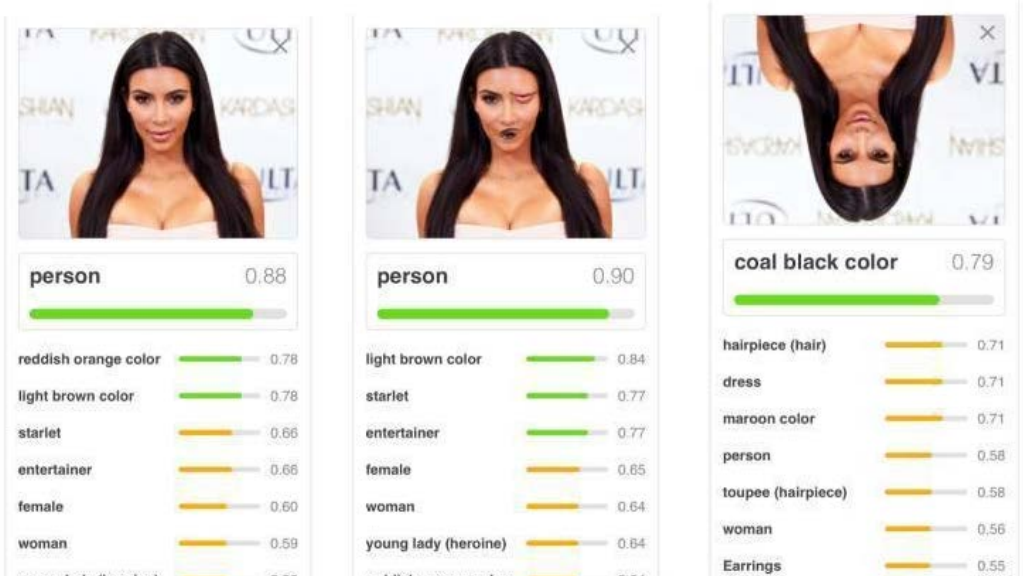


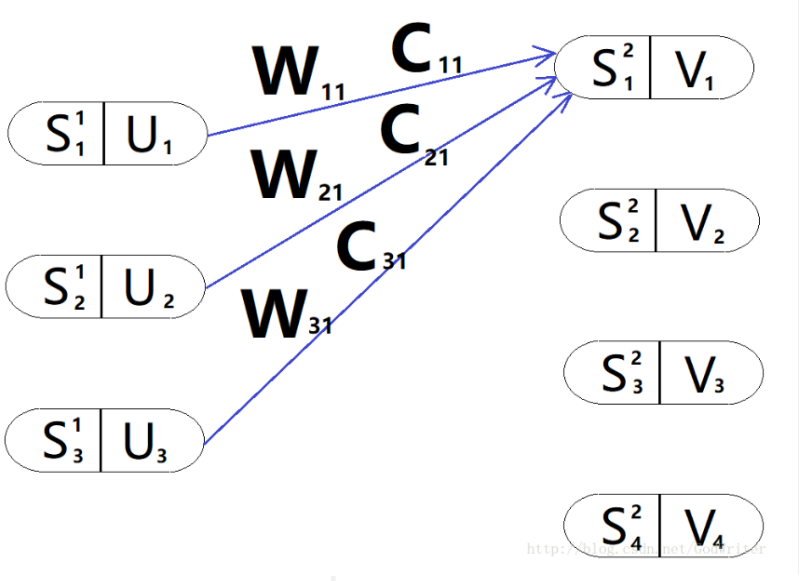


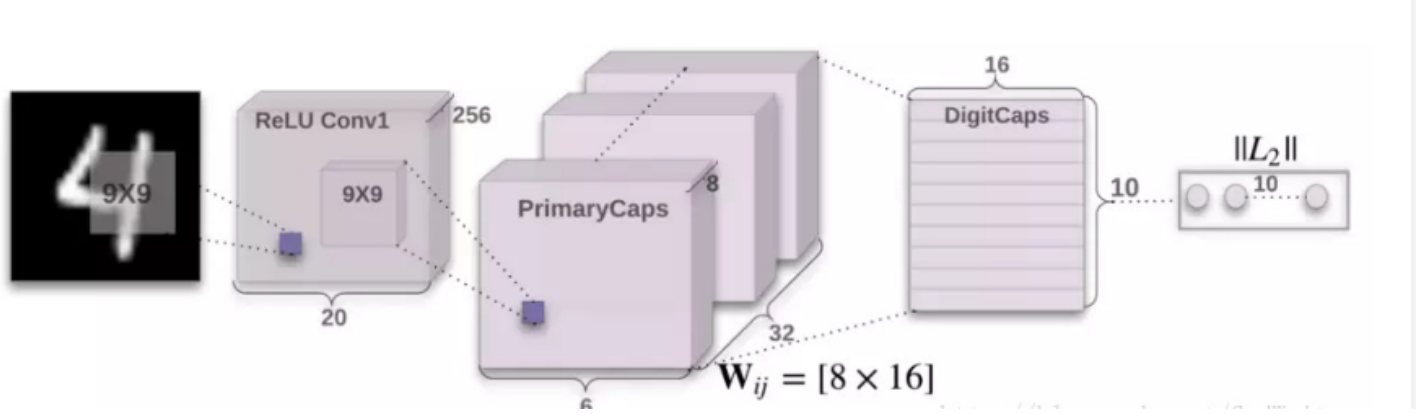
(Bengio et al. Deep Learning 2014)
Example of learned invariances: If each of these filters drive units that appear in the same max-pooling region, then the pooling unit will detect “5”s in any rotation. By learning to have each filter be a different rotation of the “5” template, this pooling unit has learned to be invariant to rotation. This is in contrast to translation invariance, which is usually achieved by hard-coding the net to pool over shifted versions of a single learned filter.

Possible extension of pooling

1. CapsNet (More accurate spatial relationship)
2. Tiled-Conv (Scale and rotation invariance, large scale and rotation???)







Possible extension of CNN

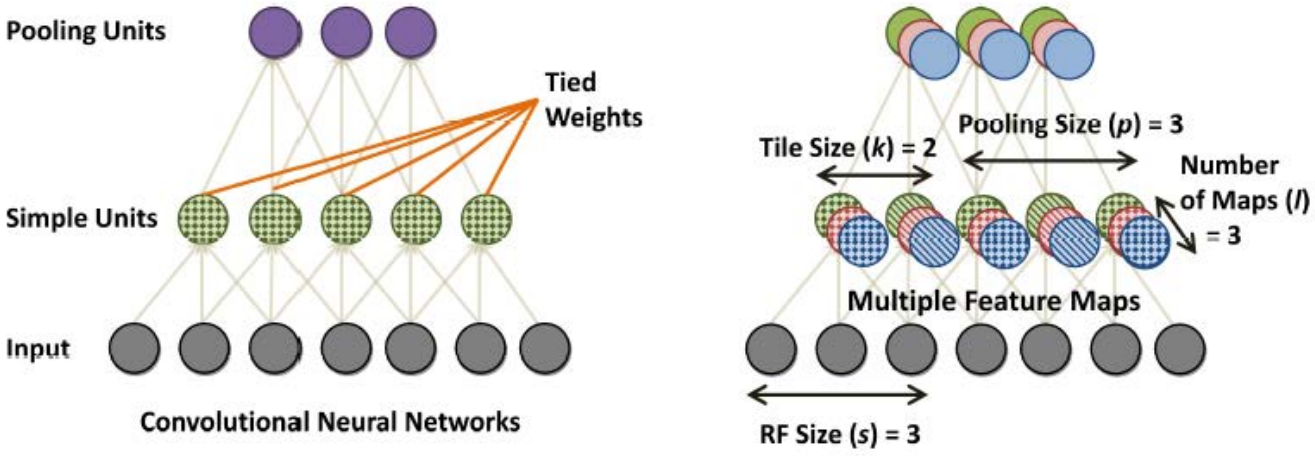


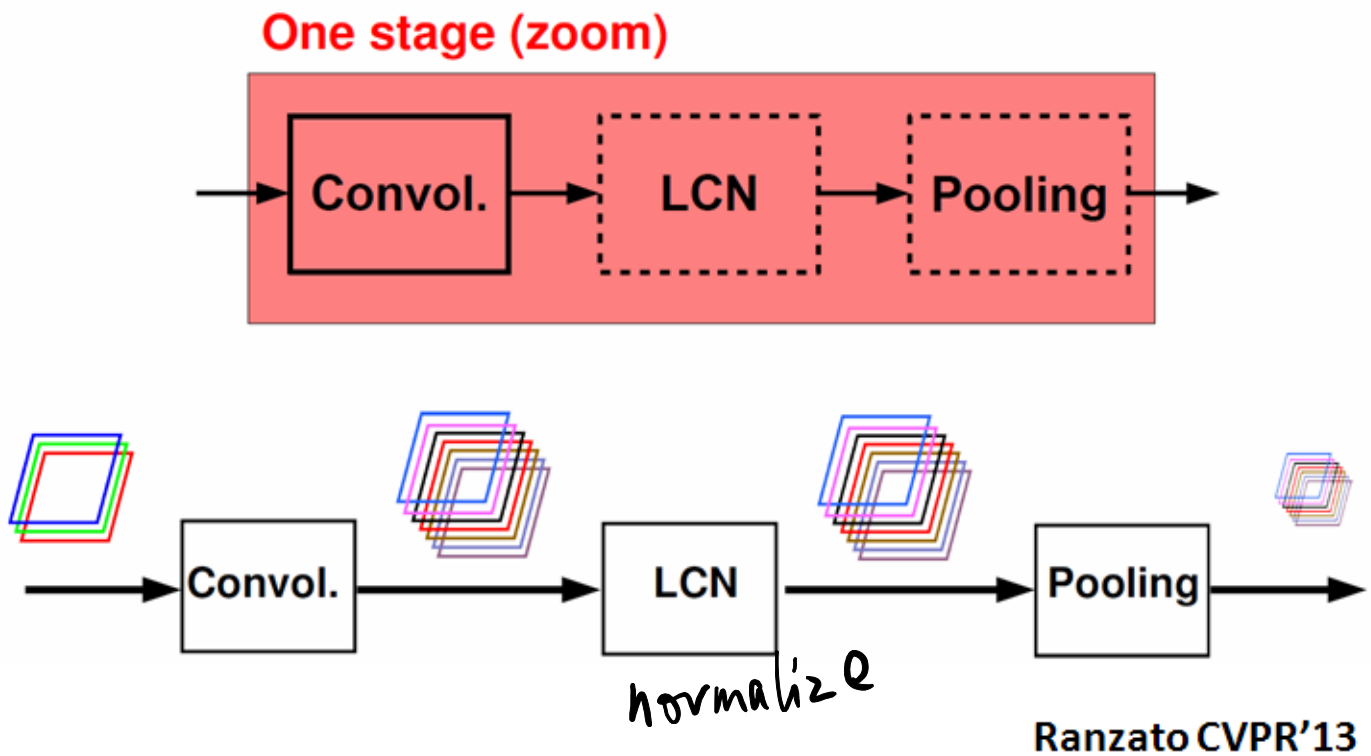
Figure 1: Left: Convolutional Neural Networks with local receptive fields and tied weights. Right: Partially untied local receptive field networks – Tiled CNNs. Units with the same color belong to the same map; within each map, units with the same fill texture have tied weights. (Network diagrams in the paper are shown in 1D for clarity.)

"By visualizing [28, 29] the range of optimal stimulus that activate each pooling unit in a Tiled CNN, we found units that were scale and rotationally invariant.9 We note that a standard CNN is unlikely to be invariant to these transformations."

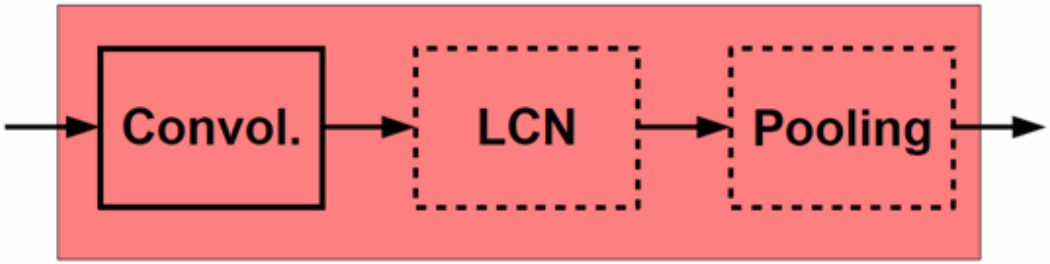
What about large scale and rotation invariant??? Open issue, final project???

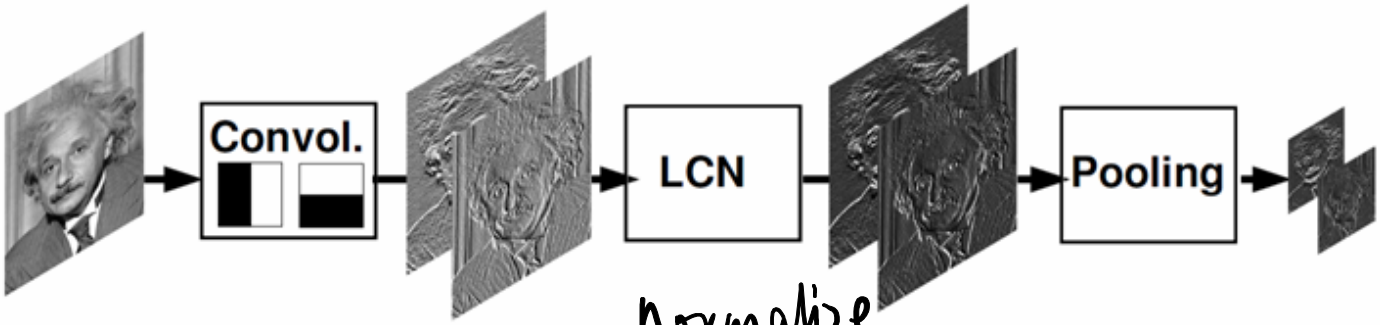
Typical architecture of CNN

- Convolutional layer increases the number of feature maps
- Pooling layer decreases spatial resolution
- LCN and pooling are optional at each stage



Typical architecture of CNN

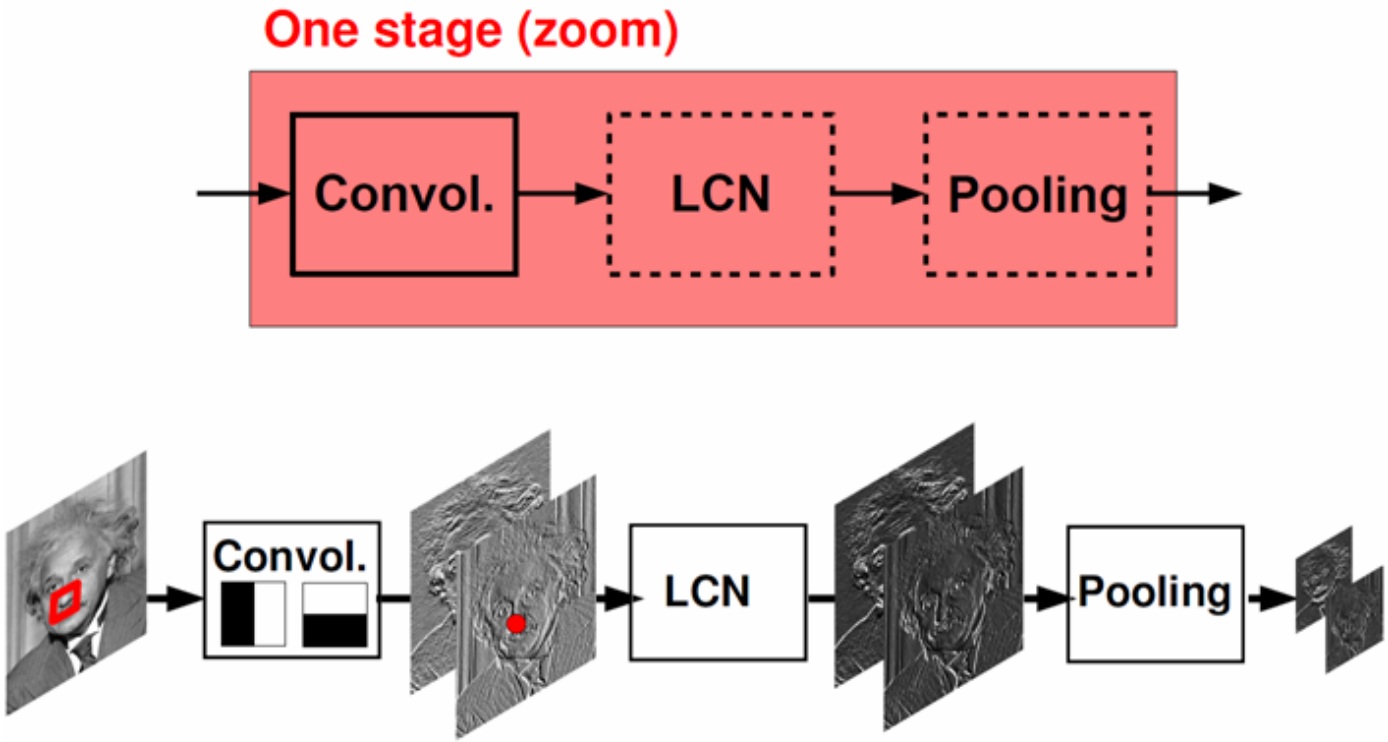




Example with only two filters.

Ranzato CVPR'13

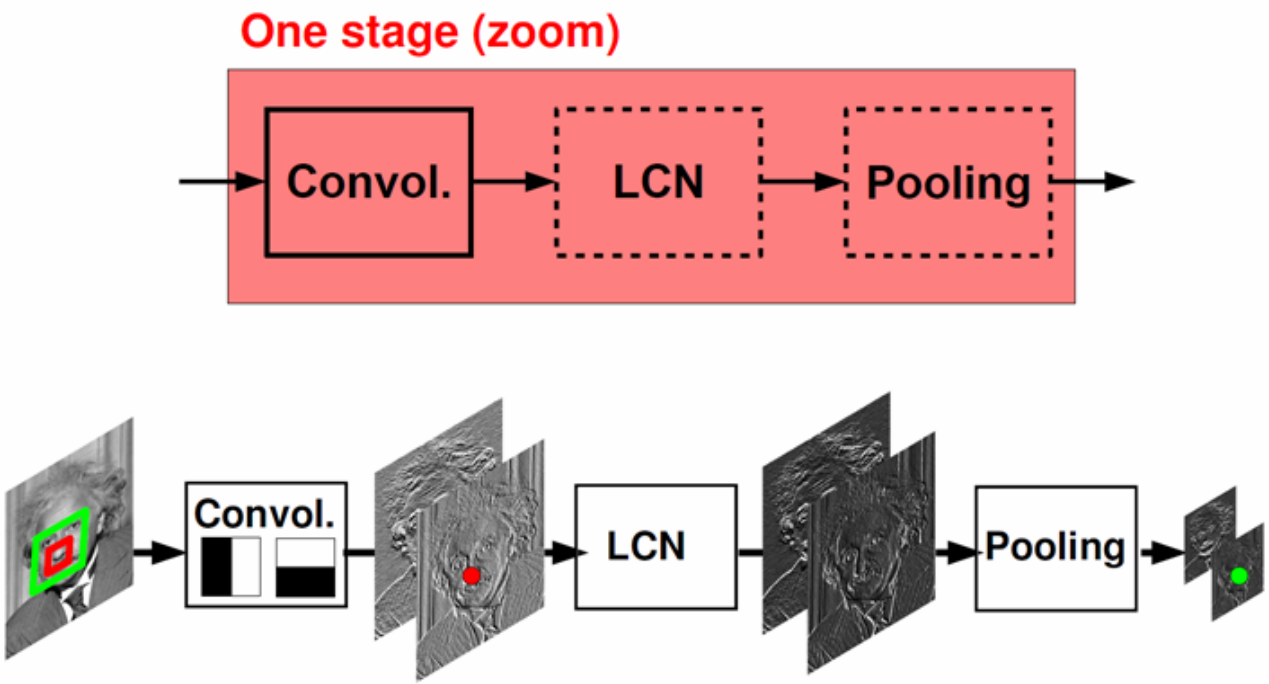
Typical architecture of CNN



A hidden unit in the first hidden layer is influenced by a small neighborhood (equal to size of filter).

Ranzato CVPR'13

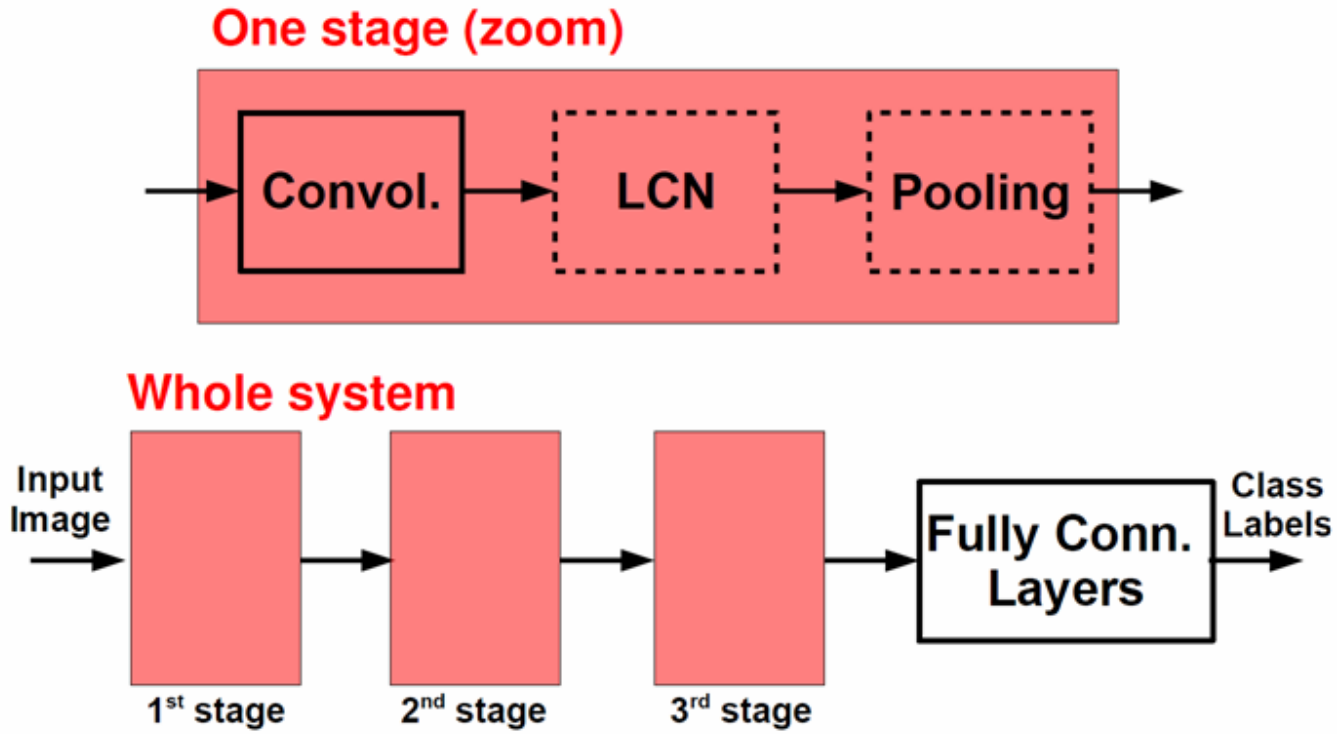
Typical architecture of CNN



A hidden unit after the pooling layer is influenced by a larger neighborhood (it depends on filter sizes and the sizes of pooling regions)

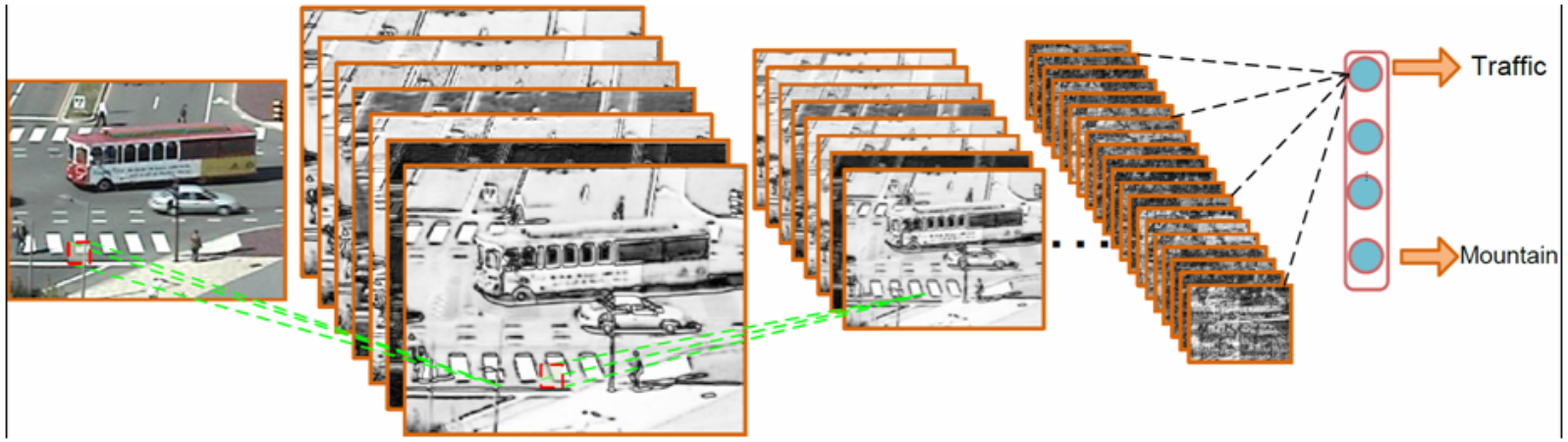
Ranzato CVPR'13

Typical architecture of CNN



After a few stages, residual spatial resolution is very small.
We have learned a descriptor for the whole image. **Ranzato CVPR'13**

Typical architecture of CNN



Convolution Pooling

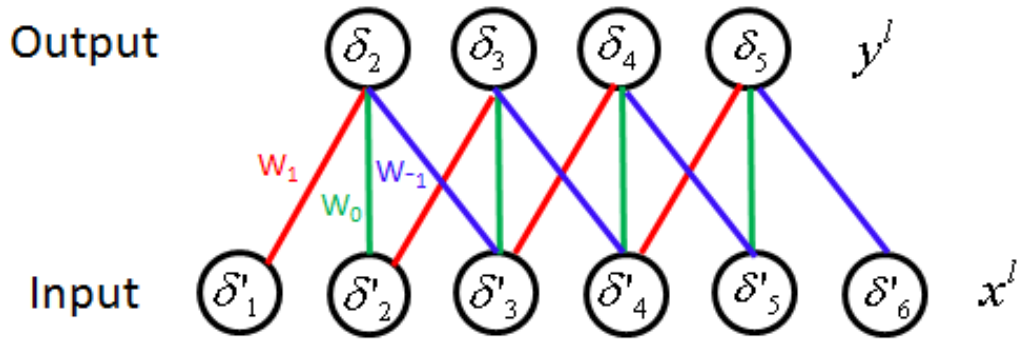
BP on CNN

- Calculate sensitivity (back propagate errors) $\delta = -\frac{\partial J}{\partial net}$ and update weights in the convolutional layer and pooling layer
- Calculating sensitivity in the convolutional layer is the same as multilayer neural network

Convolutional layer

$$net_i = \sum_{m=-d}^{d} w_m x_{i-m}$$

$$y_i = f(net_i)$$

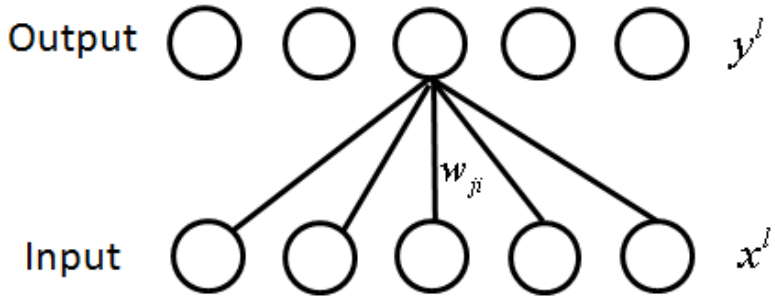


CNN has multiple convolutional layers. Each convolutional layer l has an input feature map (or image) $\mathbf{x}^l$ and also an output feature map $\mathbf{y}^l$. The sizes (n_x^l and n_y^l) of the input and output feature maps, and the filter size d^l are different for different convolutional layers. Each convultional layers has multiple filters, input feature maps and output feature maps. To simplify the notation, we skip the index (l) of the convolutional layer, and assume only one filter, one input feature map and one output feature map.

Calculate $\frac{\partial net}{\partial w}$ in the convulutional layer

- It is different from neural networks without weight sharing, where each weight W_{ij} is only related to one input node and one output node

Multilayer neural network without weight sharing



$$net_j = \sum_i w_{ji} x_i \qquad \frac{\partial net_j}{\partial w_{ji}} = x_i$$

- Taking 1D data as example, in CNN, assume the input layer $\mathbf{x} = [x_0, \ldots, x_{n_x-1}]$ is of size $n_x$and the filter $\mathbf{w} = [w_{-d}, \ldots, w_d]$ is of size $2 \times d + 1$. With weight sharing, each weight in the related with multiple input and output nodes

$$net_j = \sum_{m=-d}^{d} w_m x_{j-m}$$

Update filters in the convolutional layer

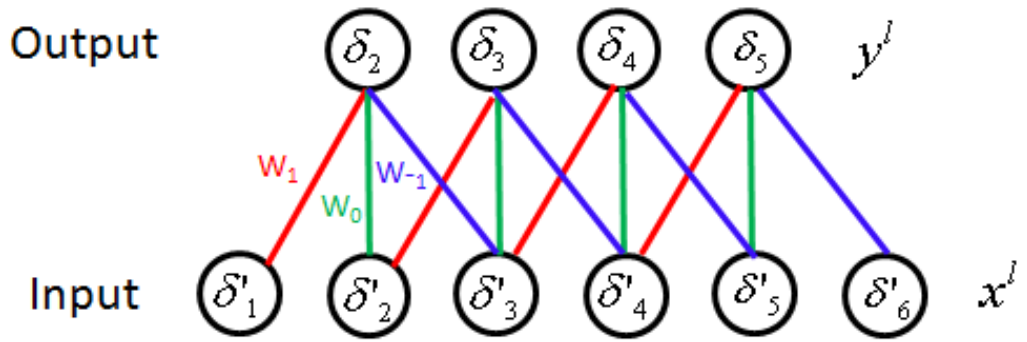
$$\frac{\partial J}{\partial w_m} = \sum_j \frac{\partial J}{\partial net_j} \frac{\partial net_j}{\partial w_m} = -\sum \delta_j x_{j-m}$$

- The gradient can be calculated from the correlation between the sensitivity map and the input feature map

Convolutional layer

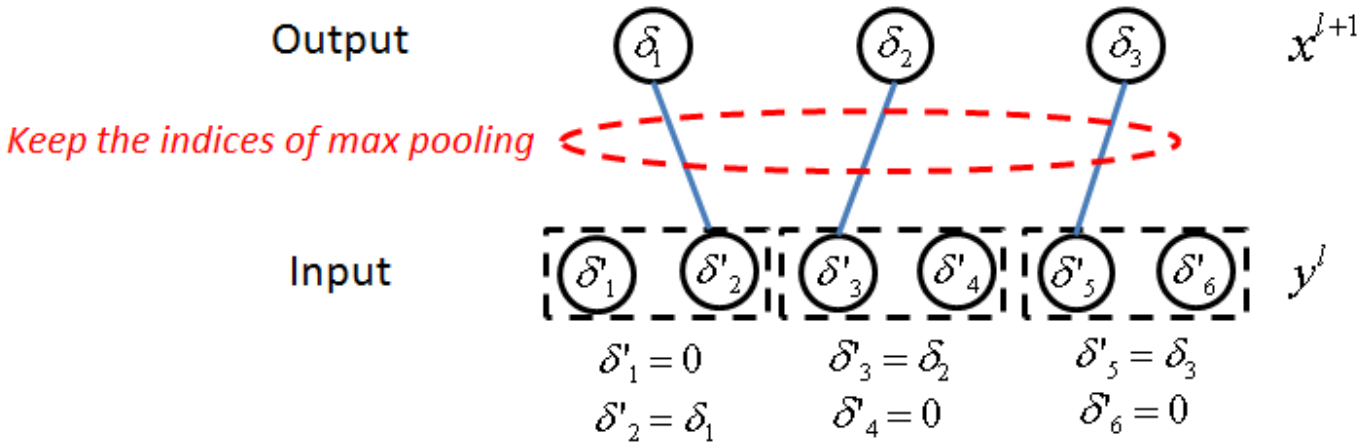
$$net_i = \sum_{m=-d}^{d} w_m x_{i-m}$$

$$y_i = f(net_i)$$



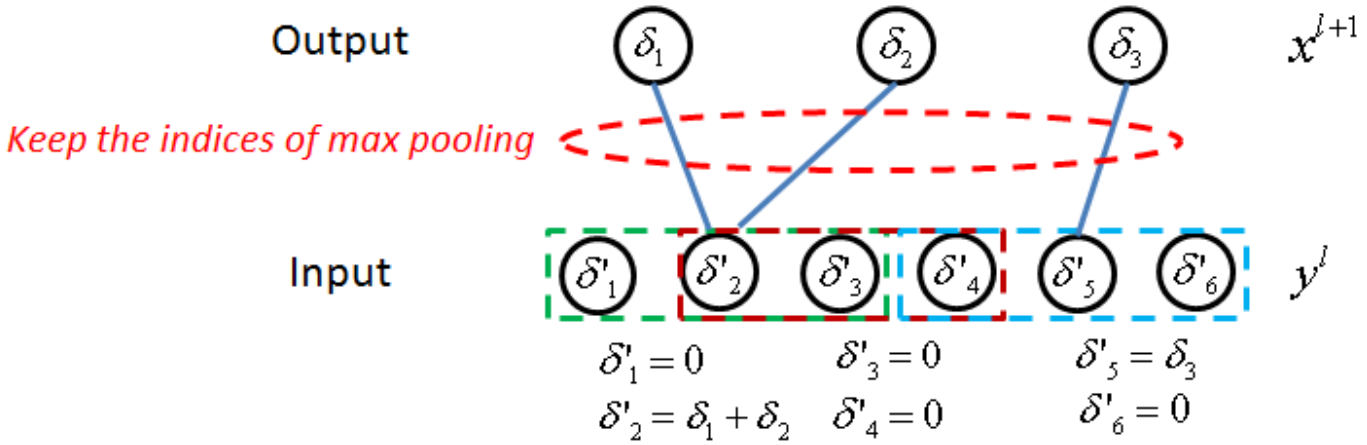
Calculate sensitivities in the pooling layer

- The input of a pooling layer l is the output feature map y^l of the previous convolutional layer. The output x^{l+1} of the pooling layer is the input of the next convolutional layer $l+1$
- For max pooling, the sensitivity is propagated according to the corresponding indices built during max operation. If max pooling regions are nonoverlapped,



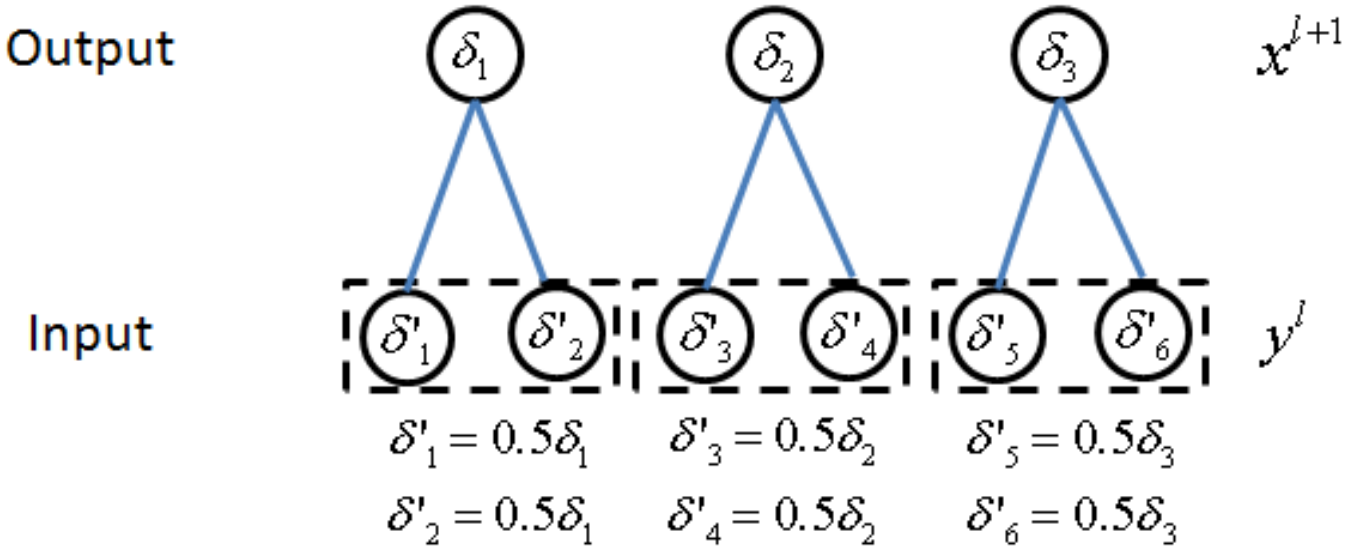
Calculate sensitivities in the pooling layer

- If pooling regions are overlapped and one node in the input layer corresponds to multiple nodes in the output layer, the sensitivities are added



Calculate sensitivities in the pooling layer

- Average pooling



- What if average pooling and pooling regions are overlapped?
- There is no weight to be updated in the pooling layer

CNN for object recognition on ImageNet challenge

- Krizhevsky, Sutskever, and Hinton, NIPS 2012
- Trained on one million images of 1000 categories collected from the web with two GPU. 2GB RAM on each GPU. 5GB of system memory
- Training lasts for one week
- Google and Baidu announced their new visual search engines with the same technology six months after that
- Google observed that the accuracy of their visual search engine was doubled

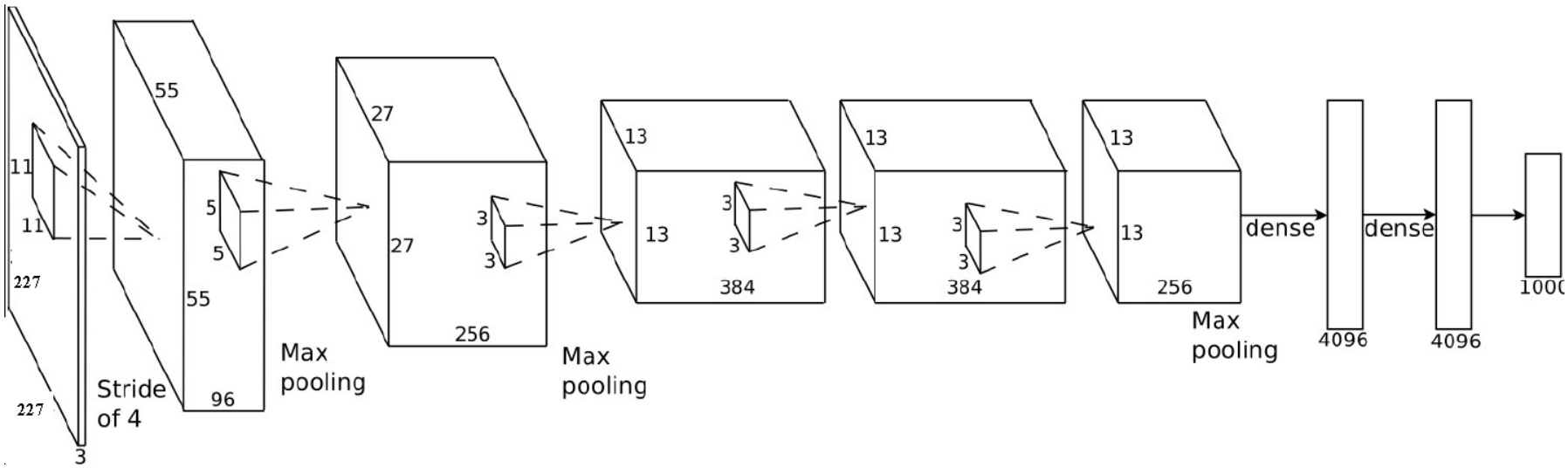
Rank	Name	Error rate	Description
1	**U. Toronto**	0.15315	Deep learning
2	U. Tokyo	0.26172	Hand-crafted features and learning models. Bottleneck.
3	U. Oxford	0.26979	
4	Xerox/INRIA	0.27058	

ImageNet


1000 object classes that we recognize
poster created by Fengjun Lv using VIPBase
images courtesy of ImageNet (http://www.image-net.org/challenges/LSVRC/2010/index)

Model architecture-AlexNet Krizhevsky 2012

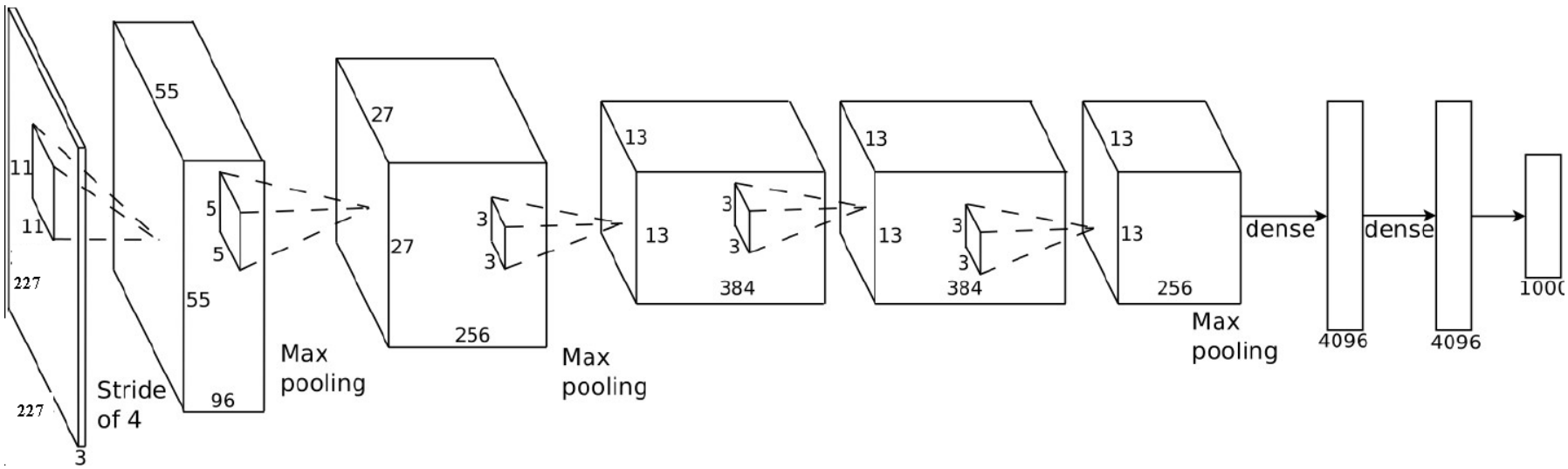
- 5 convolutional layers and 2 fully connected layers for learning features.
- Max-pooling layers follow first, second, and fifth convolutional layers
- The number of neurons in each layer is given by 253440, 186624, 64896, 64896, 43264, 4096, 4096, 1000
- 650000 neurons, 60000000 parameters, and 630000000 connections



(Krizhevsky NIPS 2014)

Model architecture-AlexNet Krizhevsky 2012

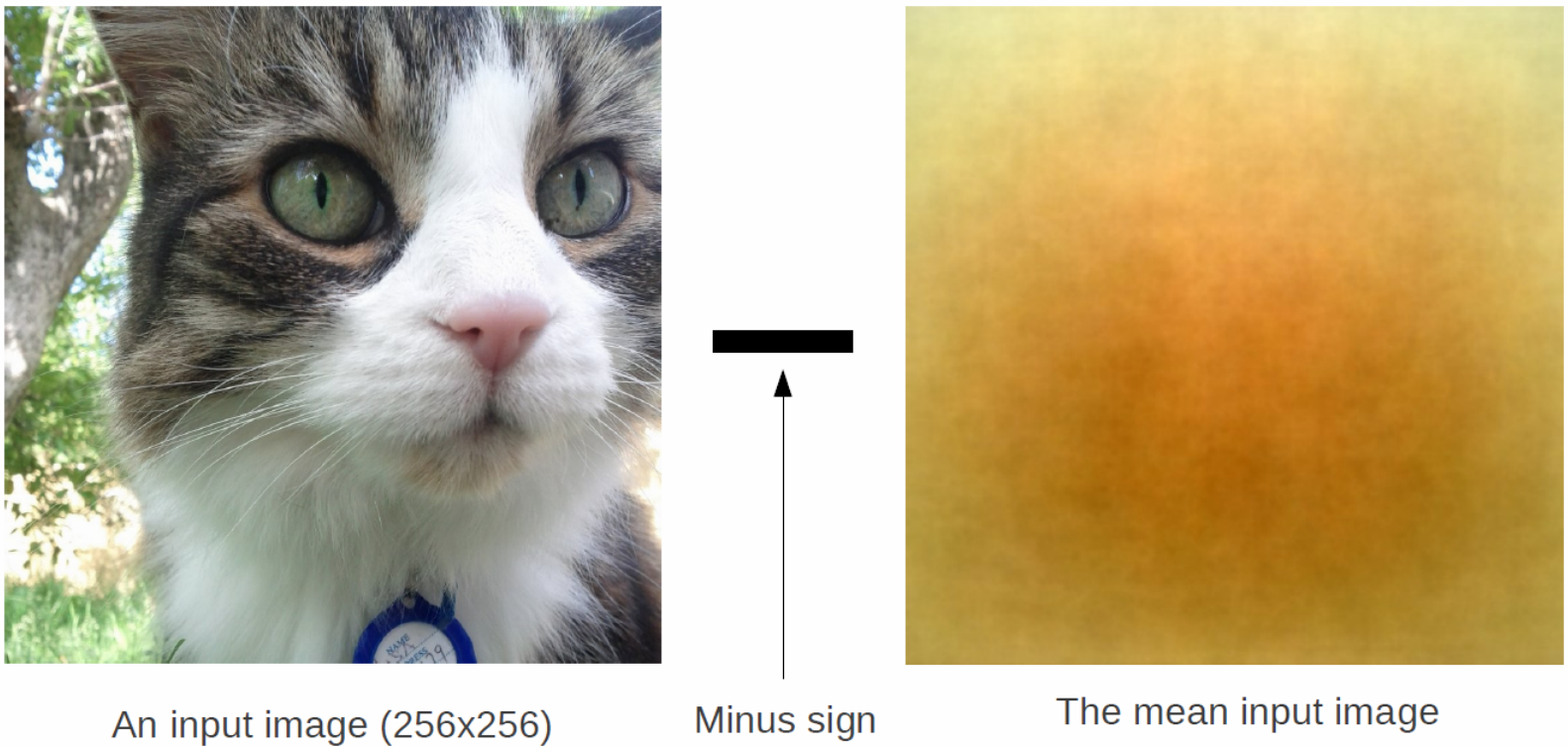
- The first time deep model is shown to be effective on large scale computer vision task.
- The first time a very large scale deep model is adopted.
- GPU is shown to be very effective on this large deep model.



(Krizhevsky NIPS 2014)

Technical details

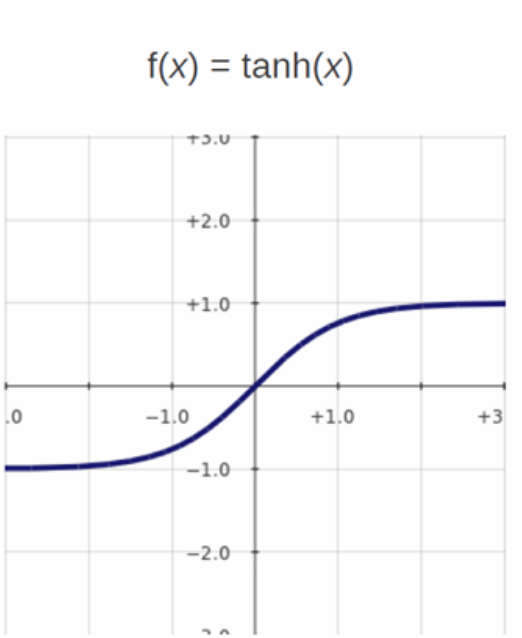
- Normalize the input by subtracting the mean image on the training set



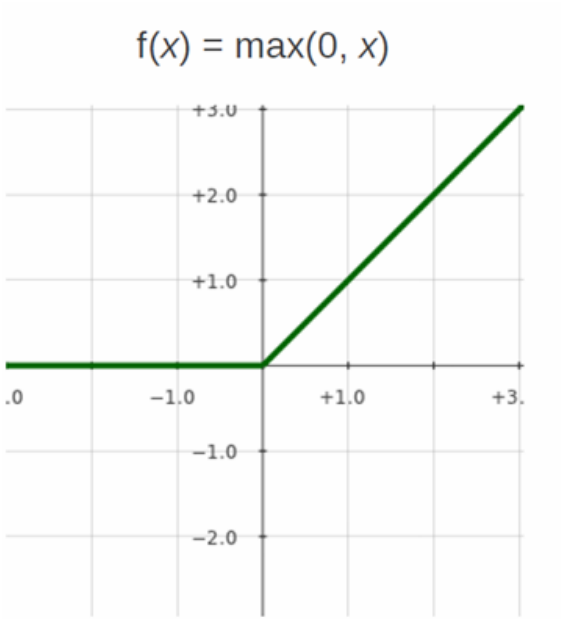
(Krizhevsky NIPS 2014)

Technical details

- Choice of activation function



Very bad (slow to train)



Very good (quick to train)

(Krizhevsky NIPS 2014)

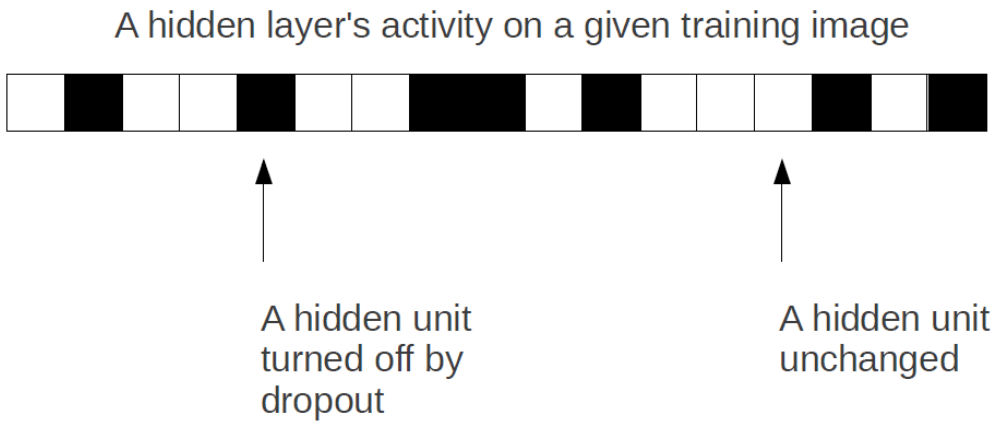
Technical details

- Data augmentation
  - The neural net has 60M real-valued parameters and 650,000 neurons
  - It overfits a lot. 224 $\times$ 224 image regions are randomly extracted from 256 images, and also their horizontal reflections

(Krizhevsky NIPS 2014)

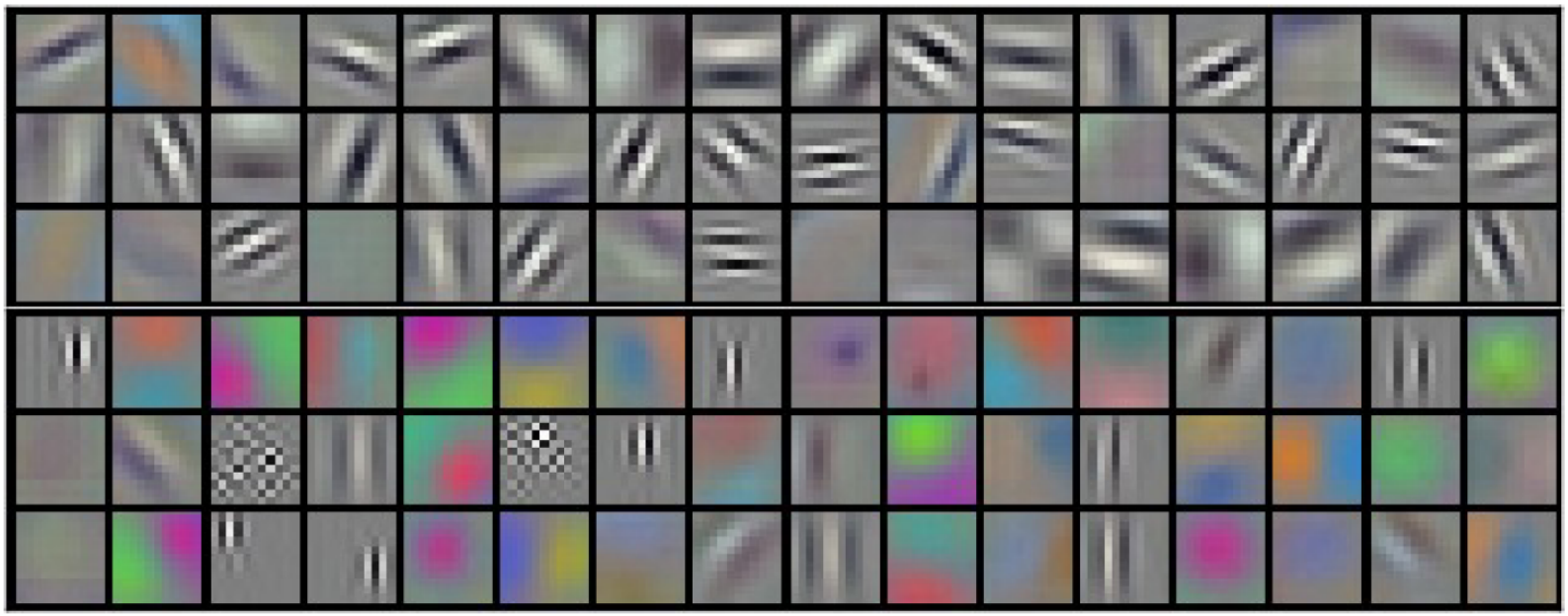
Technical details

- Dropout
  - Independently set each hidden unit activity to zero with 0.5 probability
  - Do this in the two globally-connected hidden layers at the net's output



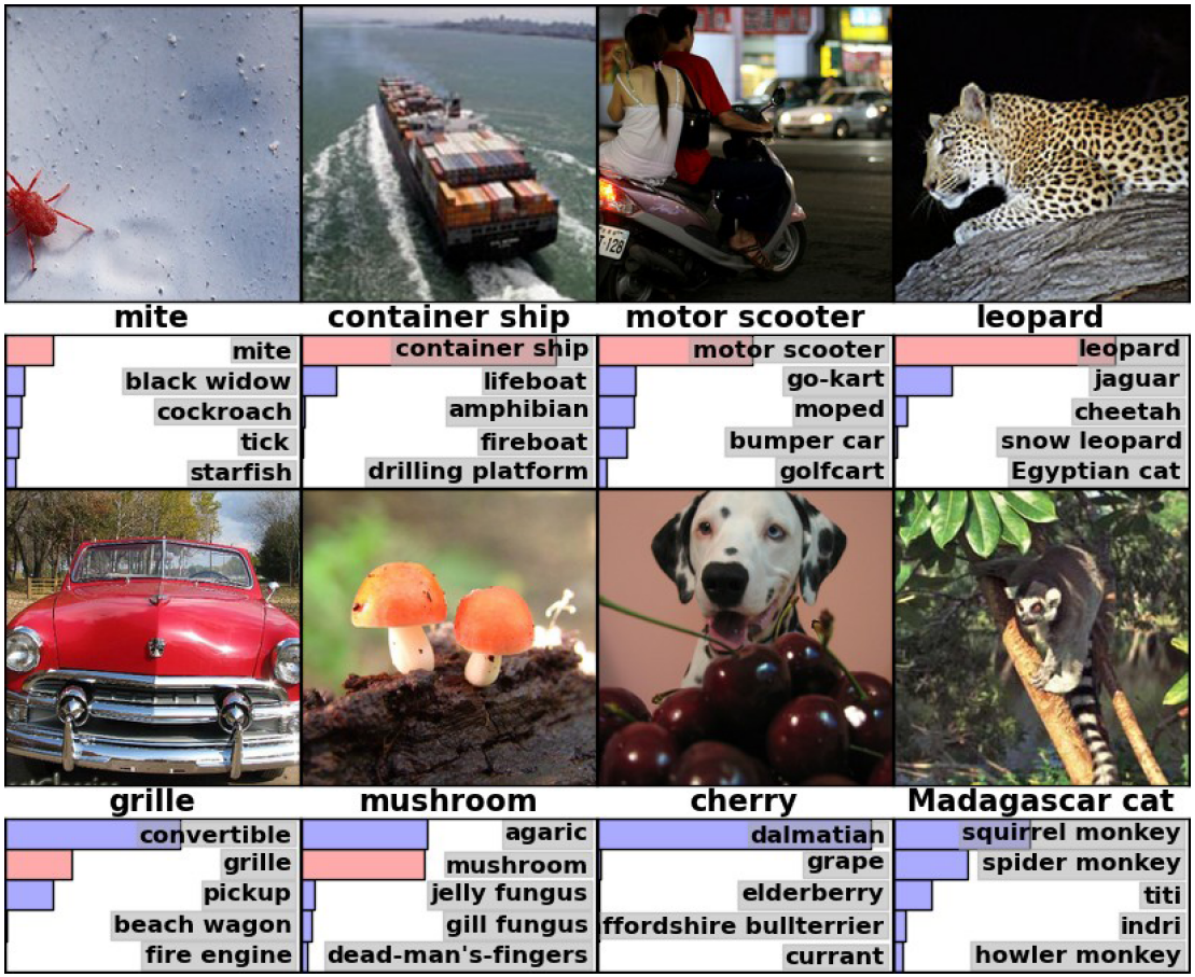
(Krizhevsky NIPS 2014)

96 learned low-level filters



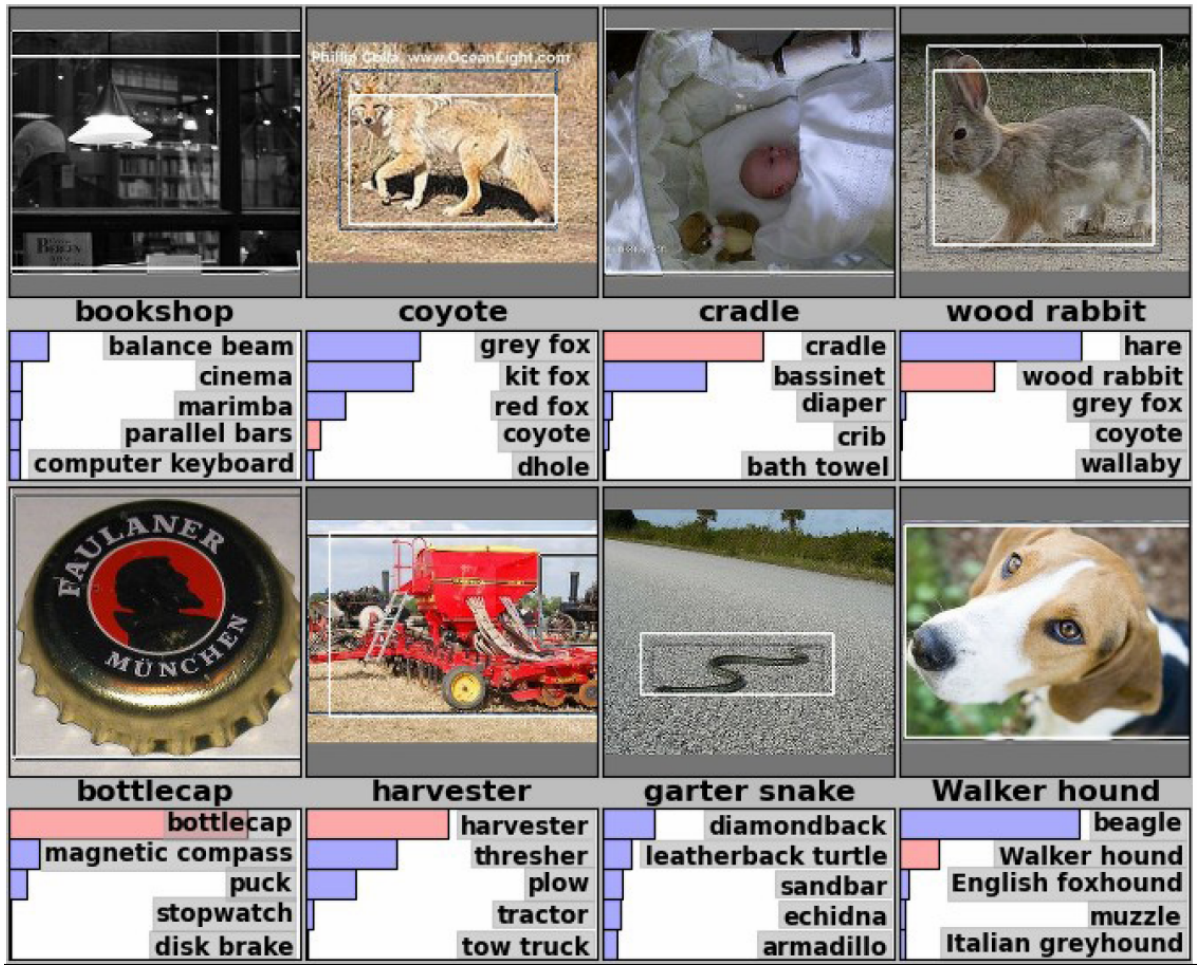
(Krizhevsky NIPS 2014)

Classification result



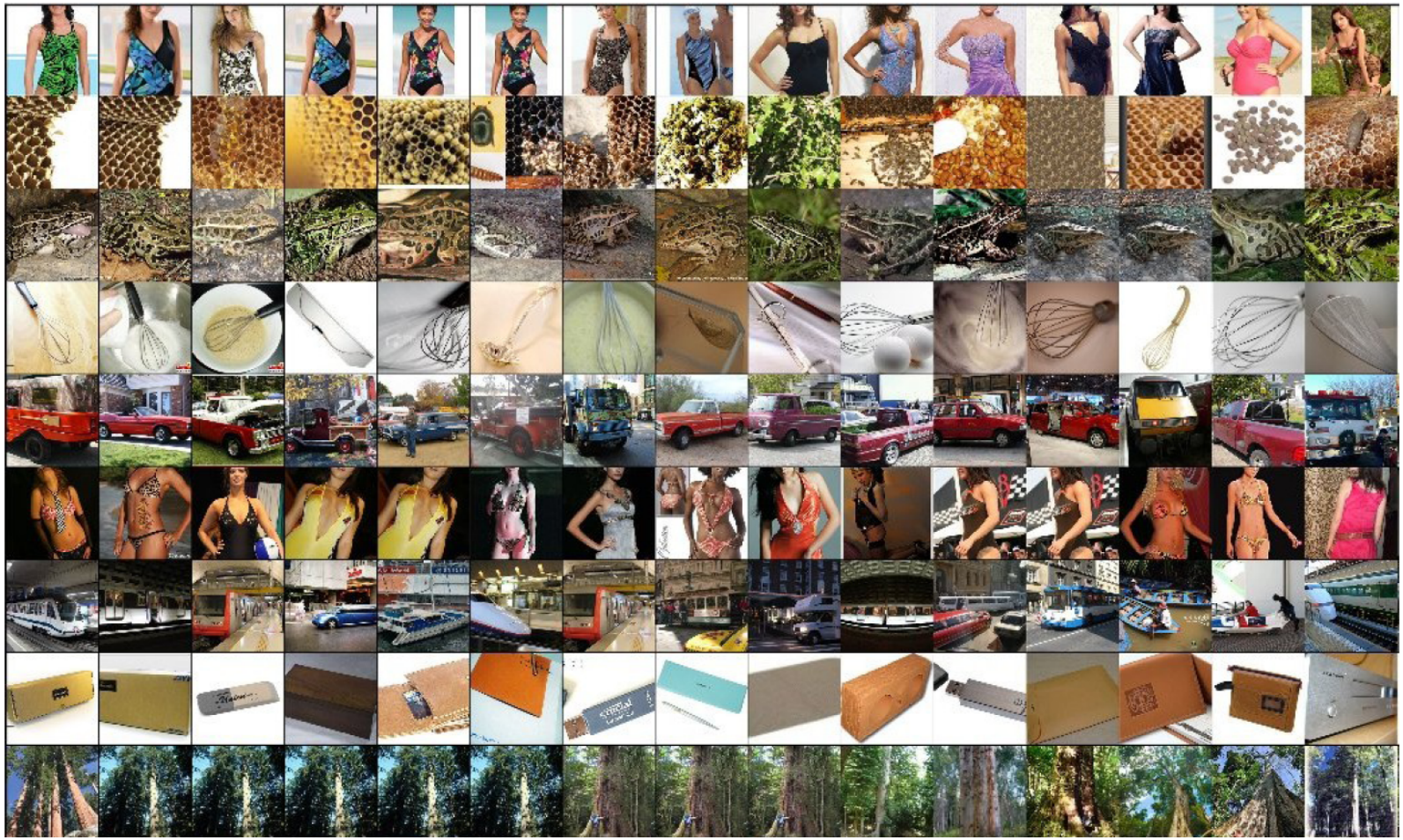
(Krizhevsky NIPS 2014)

Detection result



(Krizhevsky NIPS 2014)

Top hidden layer can be used as feature for retrieval



(Krizhevsky NIPS 2014)

Reading materials

- Y. Bengio, I. J. GoodFellow, and A. Courville, “Convolutional Networks,” Chapter 11 in “Deep Learning”, Book in preparation for MIT Press, 2014.
- Y. LeCun, L. Bottou, Y. Bengio, and P. Haffner,“Gradient-based learning applied to document recognition,” Proc. of the IEEE, 1998.
- J. Bouvrie, “Notes on Convolutional Neural Networks,” 2006.
- A. Krizhevsky, L. Sutskever, and G. E. Hinton, “ImageNet Classification with Deep Convolutional Neural Networks,” Proc. NIPS, 2012.
- M. Ranzato, “Neural Networks,” tutorial at CVPR 2013.