problem 01

(1) for example X_i, $net_{ik} = \omega_{ik}X_i + b_{ij}$, $z_{ik} = \frac{e^{net_{ik}}}{\sum_{k=1}^{c} e^{net_{ik}}}$.

the lost function: $L_i = -\sum_{k=1}^{c}[y_{ik}log(z_{ik})]$

the cost function: $C = \frac{1}{N}\sum_{i=1}^{N} L_i = -\frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{c}[y_{ik}log(z_{ik})]$

the gradient of hidden-to-output weights: $\frac{\partial C}{\partial net_{ik}} = \frac{\partial C}{\partial z_{ij}}\frac{\partial z_{ij}}{\partial net_{ik}}$

in this formula, $\frac{\partial C}{\partial z_{ij}} = -\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{c}[\frac{y_{ij}}{z_{ij}}]$

when $j = k$: $\frac{\partial z_{ik}}{\partial net_{ik}} = \frac{\partial}{\partial net_{ik}}\frac{e^{net_{ik}}}{\sum_{k=1}^{c} e^{net_{ik}}} = \frac{e^{net_{ik}}\sum_{k=1}^{c} e^{net_{ik}} - e^{2net_{ik}}}{(\sum_{k=1}^{c} e^{net_{ik}})^2} = z_{ik} - z_{ik}^2$

when $j \neq k$: $\frac{\partial z_{ij}}{\partial net_{ik}} = \frac{\partial}{\partial net_{ik}}\frac{e^{net_{ij}}}{\sum_{k=1}^{c} e^{net_{ik}}} = -\frac{e^{net_{ij}}e^{net_{ik}}}{(\sum_{k=1}^{c} e^{net_{ik}})^2} = -z_{ij}z_{ik}$

take them into the original fomula, $\frac{\partial C}{\partial net_{ik}} = -\frac{1}{N}\sum_{i=1}^{N}(\sum_{j\neq k}^{c}\frac{y_{ij}}{z_{ij}}(-z_{ij}z_{ik}) + \frac{y_{ik}}{z_{ik}}(z_{ik} - z_{ik}^2)) = -\frac{1}{N}\sum_{i=1}^{N}(\sum_{j=1}^{c}(-y_{ij}z_{ik}) + y_{ik})$

(2)don't really understand

problem 02

(1)

$$h_{11} = f_{11}(x_1) = \begin{cases} -1, & x_1 \leq 0.5 \\ 1, & x_1 > 0.5 \end{cases}$$

$$h_{12} = f_{12}(x_2) = \begin{cases} -1, & x_2 \leq 0.5 \\ 1, & x_2 > 0.5 \end{cases}$$

$$g = h_{11} * h_{12}$$

(2)

$$h_{21} = f_{21}(x_1) = \begin{cases} 1, & 1 < x_1 \leq 1.5 \\ -1, & x_1 > 1.5 \end{cases}$$

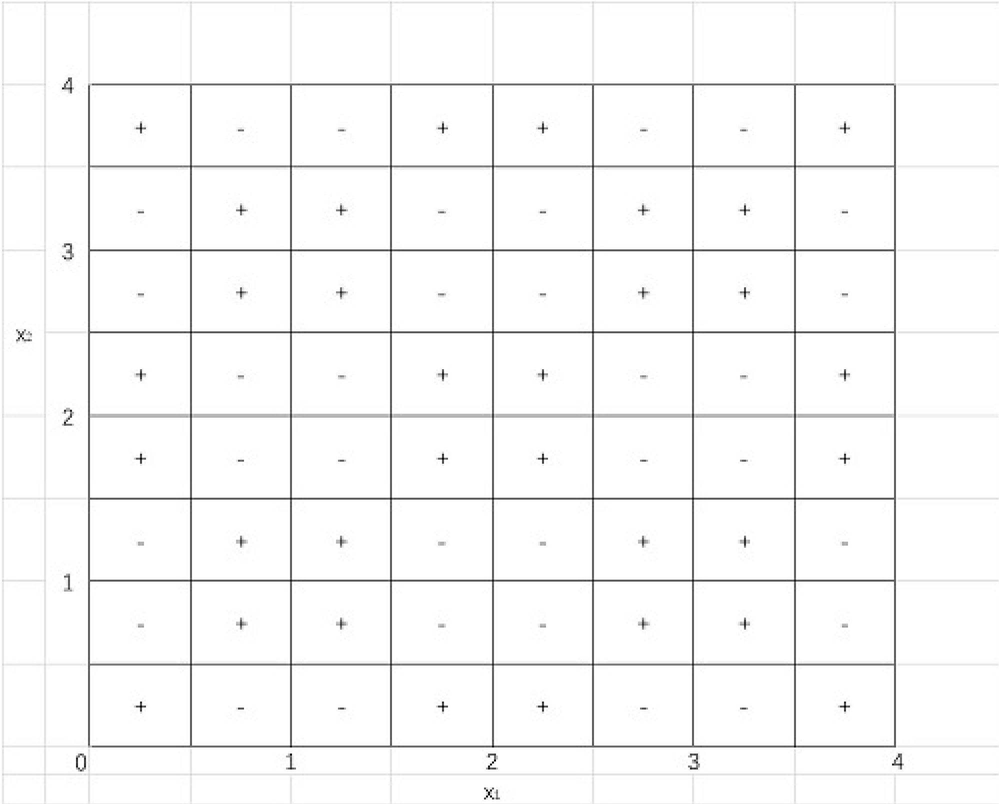
$$= -f_{11}(x_1 - 1)$$

$$h_{22} = f_{22}(x_2) = \begin{cases} 1, & 1 < x_2 \leq 1.5 \\ -1, & x_2 > 1.5 \end{cases}$$

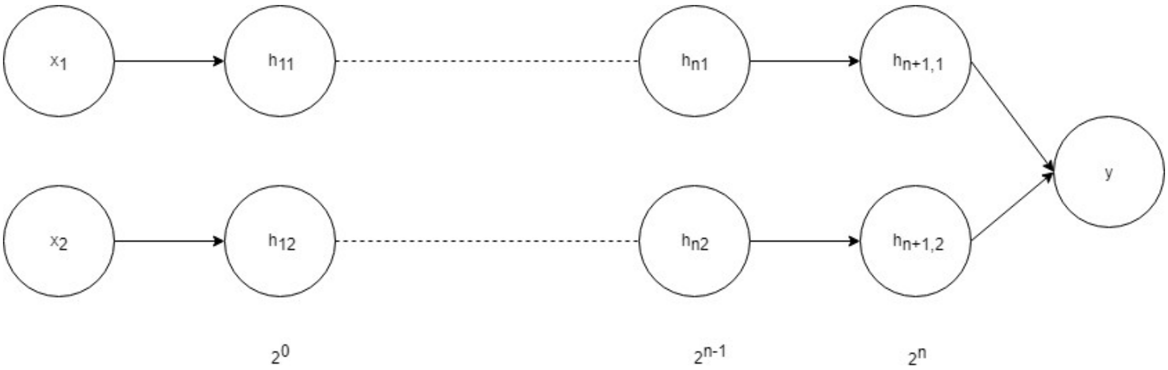
$$= -f_{12}(x_2 - 1)$$

$$g = h_{21} * h_{22}$$

(3)data are mirror symmetry to the 2^n axis.



(4)

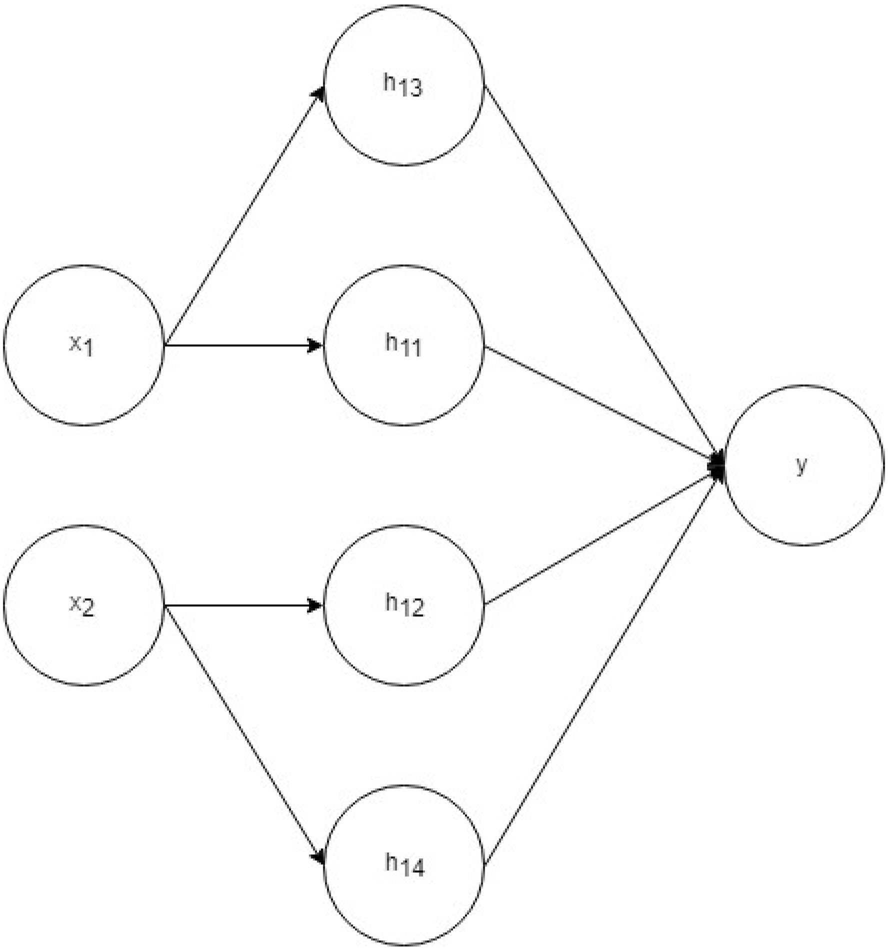


when $n \geq 1$:

$$h_{n1} = f_{n1}(x_1) = -f_{n-1}(x_1 - 2^{n-1})$$

$$h_{n2} = f_{n2}(x_2) = -f_{n-1}(x_2 - 2^{n-1})$$

(5)



$$h_{11} = (-1)^{x_1|0.5}$$

$$h_{13} = (-1)^{x_1|1}$$

$$h_{12} = (-1)^{x_2|0.5+1}$$

$$h_{14} = (-1)^{x_2|1+1}$$

$$y = h_{11} * h_{12} * h_{13} * h_{14}$$