1343 **Code : 20SC01T**

Register Number | | | | | | | | | | |

I & II Semester Diploma Examination, June/July-2023

ENGINEERING MATHEMATICS

Time : 3 Hours] **[Max. Marks : 100**

Instructions : (i) Answer **one** full question from each section.

(ii) One full question carries **20** marks.

SECTION – I

1. (a) Solve for x, $\begin{vmatrix} 1 & 2 & 3 \\ 2 & x & 3 \\ 3 & 4 & 3 \end{vmatrix} = 0$ **4**

OR

If $A = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$, Find $A + A^T$.

(b) Using Cramer's rule, find the solution of the system of equations $2y - z = 0$, and $x + 3y = -4$, $3x + 4y = 4$ **6**

OR

Which of the matrix has no inverse ?

$A = \begin{bmatrix} 1 & 5 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 6 \\ -1 & -3 \end{bmatrix}$ $C = \begin{bmatrix} 3 & 2 \\ 12 & 8 \end{bmatrix}$

(c) Find the characteristic equation and characteristic roots value for the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ **5**

OR

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$, then verify that $(A + B)^T = A^T + B^T$

(d) Consider the matrix **5**

If $A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$, find A^{-1}.

OR

If $A = \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$ & $B = \begin{bmatrix} 9 & 2 \\ -3 & 5 \end{bmatrix}$, find AB.

SECTION – II

(Match the following)

P

2. (a) (A) Equation of a straight line passing through a given point (x, y) and having slope m is

(B) Equation of a straight line passing through two points (x_1, y_1) and (x_2, y_2) is

(C) The equation of a straight line whose x and y-intercepts are a, b respectively is

(D) If two lines are perpendicular then product of their slopes is equal to

Q

(1) $\frac{x}{a}+\frac{y}{b}=1$

(2) $y-y_1=m(x-x_1)$

(3) -1

(4) $\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$ 4

Answers :

P	Q
A	
B	
C	
D	

OR

(Match the following)

P

(A) If two lines with slopes m_1 and m_2 are parallel then 'θ' is

(B) Equation of a straight line whose slope is m and y intercept is C.

(C) Slope of line $ax + by + c = 0$

(D) Slope of a line joining two points $(x_1\ y_1)$ and $(x_2\ y_2)$.

Q

(1) $y = mx + c$

(2) 0 (zero)

(3) $\frac{y_2-y_1}{x_2-x_1}$

(4) $-\frac{a}{b}$

Answers :

P	Q
A	
B	
C	
D	

(b) Find the equation to the perpendicular to the line $6x - 5y - 2 = 0$ and passing through $(2, -3)$. 6

OR

Are the lines $4x + 6y + 7 = 0$ and $2x + 3y - 1 = 0$ parallel to each other ? Justify.

(c) Find the equation of straight line parallel to $5x + 6y - 10 = 0$ and passing through the point $(-3, 3)$. 5

OR

Are the lines $3x + 4y + 7 = 0$ and $28x - 21y + 50 = 0$ are perpendicular to each other ? Justify.

(d) Find the angle between the lines $x + 3y + 5 = 0$ and $4x + 2y - 7 = 0$ 5

OR

Find the equation of straight line which passes through the points $(-2, 3)$ and $(-5, 6)$.

SECTION – III

3. (a) Determine the value of $\cos(570°)$ and $\sin(330°)$. 4

OR

Convert 45 degree into radian and $\frac{11\pi}{5}$ radian into degree.

(b) If $A + B = \frac{\pi}{4}$ prove that $(1 + \tan A)(1 + \tan B) = 2$ 6

OR

Prove that $\sin 3A = 3\sin A - 4\sin^3 A$

(c) Given $\tan A = \frac{18}{17}$ and $\tan B = \frac{1}{35}$ show that $A - B = \frac{\pi}{4}$ 5

OR

Show that : $\frac{\cos(360° - A) \cdot \tan(360° + A)}{\cot(270° - A) \cdot \sin(90° + A)} = 1$

(d) Prove that $\cos 55° + \cos 65° + \cos 175° = 0$ 5

OR

Show that $\frac{\sin 40° + \sin 20°}{\cos 40° + \cos 20°} = \frac{1}{\sqrt{3}}$

SECTION – IV

4. (a) If $y = \sin x + \log x + e^x + \tan x$, then find $\frac{dy}{dx} = ?$ 4

OR

If $\frac{dy}{dx} = 4x^3 + 3x^2$, then find $\frac{d^2y}{dx^2}$ at $(1, 2)$

(b) Using chain rule of differentiation, find the derivative of the function $y = (3x + 8)^5$ **6**

OR

Using composite rule find the derivative of the function $y = \log(\sin(\log x))$

(c) The distance covered by a body in t seconds is given by $S = 4t - 5t^2 + 2t^3$, find the velocity and acceleration when t = 2 sec. **5**

OR

Distance travelled by a car is given by $S = 160t - 16t^2$ metre and time in seconds. When does the car stop ?

(d) Find the maximum and minimum values of the function $x^3 - x^2 - x = 0$. **5**

OR

Find the equation of the tangent to the curve $y = 2 - 3x + x^2$ at (1, 2)

SECTION – V

5. (a) Integrate : $\cos x + e^x + \frac{1}{x} + x^2$, w.r. to x. **4**

OR

The area under the curve $y = x^2$ between $x = 1$, and $x = 2$ is equal to ...

(b) Using the rule of integration by parts evaluate the integral $\int x \sin 2x . dx$ **6**

OR

Evaluate $\int \sin 2x \cos 3x \, dx$

(c) Find $\int_0^{\pi/2} \sin^2 x . dx$ **5**

OR

Evaluate $\int \sin^5 x \cos x \, dx$

(d) The area enclosed by the curve $y = x^2 + 1$, x-axis between $x = 1$, $x = 3$, calculate the area enclosed. **5**

OR

Find the volume generated by rotating the curve $y = \sqrt{x + 2}$ about x-axis between $x = 0$ and $x = 2$.

SCHEME OF VALUATION

I & II SEMESTER DIPLOMA EXAMINATION

Sub: Engg Mathematics June/ July 2023 Code: 20SCOIT

Q.NO	Matter	Marks
1 (a)	Expansion of determinant Simplifying and result **OR** Finding A^T Finding $A + A^T$ Result	**2** **2** **1** **2** **1**
1 (b)	Find Δ_x , Δ_y , Δ_z Finding x , y, z **OR** Find $\|A\| = -1$, A^{-1} exists Find $\|B\| = \cancel{-1}\,0$, B^{-1} exists Doesnot Find $\|C\| = \cancel{-1}\,0$, C^{-1} exists	 **3** **3** 2 2 2
1.c	$\|A - \lambda I\| = 0$ Finding characteristic equation Finding roots **OR** (A+B) $(A+B)^T$ A^T , B^T $A^T + B^T$	1 2 2 1 1 2 1
1.d	$\|A\|=7 \neq 0$ Formula $A^{-1} = \frac{adj\ A}{\|A\|}$ Find adj A Answer **OR** AB=$\begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 9 & 2 \\ -3 & 5 \end{bmatrix}$ Performing AB Answer	1 1 2 1 1 3 1

Q.NO	Matter	Marks
2(a)	Matching A-2 Award 1 to all for attending. B-4 C-1 D-3 **OR** Matching A-2 B-1 C-4 D-3	1 **1** 1 1 1 **1** 1 1
2 (b)	Finding $m = \frac{6}{5}$ Formula $(y - y_1) = m(x - x_1)$ Substitution and calculation Answer **OR** Finding $m_1 = \frac{-2}{3}$ $m_2 = \frac{-2}{3}$ Using condition Answer	1 1 3 1 2 2 1 1
2.c	**Finding** $m = \frac{-5}{6}$ **Formula** $(y - y_1) = m(x - x_1)$ Substitution and calculation Answer **OR** **Finding** $m_1 = \frac{-3}{4}$, $m_2 = \frac{4}{3}$ **Using condition** $m_1 \times m_2 = -1$ Answer	1 1 2 1 2 2 1
2.d	Finding $m_1 = \frac{-1}{3}$, $m_1 = -2$ Formula $\tan\theta = \left\|\frac{m_1 - m_2}{1 + m_1 m_2}\right\|$ Calculation Answer **OR** $\frac{(y - y_1)}{(x - x_1)} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ Substitution and calculation Answer	2 1 1 1 1 3 1

Q.NO	Matter	Marks
3.a	$\cos(570°) = \cos(6 \times 90° + 30°)$ $-\cos 30° = -\frac{\sqrt{3}}{2}$ $\sin(330°) = \sin(4 \times 90° - 30°)$ $-\sin 30° = -\frac{1}{2}$ **OR** $45° = \frac{\pi}{180} \times 45 = \frac{\pi}{4} = 0.7855\ rad$ $11\frac{\pi}{5} = \frac{180}{\pi} \times 11\frac{\pi}{5} = 36 \times 11 = 396°$	1 1 1 1 2 2
3. b	$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $\tan(A+B) = 1$ $\tan A + \tan B + \tan A \tan B = 1$ add 1 both the side, and calculations Answer **OR** $\sin 2A = 2\sin A \cos A$ $\cos 2A = 1 - 2sin^2 A$ $cos^2 A = 1 - sin^2 A$ $\sin(A+B) = \sin A \cos B + \cos A \sin B$ Calculation and answer	1 1 1 2 1 1 1 1 1 2
3. c	$\tan(A-B)$ Formula substitution and calculation Answer **OR** $\cos(360 - A) = cosA$ $\tan(360 + A) = tanA$ $Cos(270 - A) = tanA$ $Sin(90 + A) = CosA$ Results	1 3 1 1 1 1 1 1
3. d	Using Formula cosc+cosd Cos120=-sin30 Sin30=1/2 Calculation & answer **OR** Using formula SinC+SinD CosC+CosD Calculation Answer	1 1 1 2 1 1 2 1

Q.NO	Matter	Marks
4.a	Differentiation of each term **OR** $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ $\left(\frac{d^2y}{dx^2}\right)_{(1,2)} = 30$	1 mark =4m 2 2
4.b	$5(3x+8)^{5-1} \times \frac{d(3x+8)}{dx}$ $5(3x+8)^4 \times (3\frac{d(x)}{dx} + \frac{d(8)}{dx})$ Calculation & Results **OR** $\frac{d(\log[\sin(\log x)])}{dx} = \frac{1}{\sin(\log x)} \frac{d(\sin(\log x))}{dx}$ Remaining calculations Answer	2 2 2 1 4 1
4. c	$V = \frac{d(s)}{dt} = 4 - 10t + 6t^2$ V=8m/sec a=-10+12t a=14m/sec^2 **OR** $\frac{d(s)}{dt} == 160 - 32t$ When the car stop velocity becomes zero. V=0 calculation answer	1 1 2 1 1 1 2 1
4.d	$\frac{dy}{dx} = 3x^2 - 2x - 1$ $x = -\frac{1}{3}, \quad and \quad x = 1$ $\frac{d^2y}{dx^2} = 6x - 2$ Maximum value is $y = \left(\frac{5}{27}\right)$ Minimum value is $y = -1$ **OR** $\frac{dy}{dx} = -3 + 2x$ $m = \left(\frac{dy}{dx}\right)_{(1,2)} = -1$ $y - y_1 = m(x - x_1)$ Calculation and answer	1 1 1 1 1 1 1 1 2

Q.NO	Matter	Marks
5.a	For each integration **OR** $A = \int_a^b y\,dx$ Substitution and calculation Answer	1 marks each=4m 1 2 1
5.b	Integration by part formula Substitution Calculation Answer **OR** Integration by part formula Formula $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$ Substitution and calculation Answer	1 1 3 1 1 1 3 1
5. c	$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ And calculation $sin^2x + cos^2x = 1$ $I=\pi/4$ **OR** $t = \sin x$, $dt = \cos x\ dx$ Calculation Answer	1 2 1 1 2 2 1
5.d	$A = \int_a^b y\,dx$ Substitution and calculation Answer A=32/3 **OR** $V = \pi \int_a^b [y]^2\,dx$ Substitution Calculation V=6 π Cubic unit	1 3 1 1 1 2 1

OR
$A = \int_a^b y\,dx$ — 1
$= \int_0^{\pi/2} \sin^2 x\,dx = \int_0^{\pi/2} \frac{1-\cos 2x}{2}\,dx \rightarrow 1+1$
$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2}$ — 1 . Sub. & Ans $\rightarrow 1+1$

I AND II SEMESTER EXAMINATION JUNE/JULY 2023

ENGINEERING MATHEMATICS **CODE: 20SC01T**

1. (a) $1(3x-12)-2(6-9)+3(8-3x)=0$

$$3x-24+42-9x=0$$

$$18-6x=0 \Rightarrow \quad x=3$$

OR

A= $\begin{bmatrix}1 & 5\\2 & 3\end{bmatrix}$ A^T = $\begin{bmatrix}1 & 2\\5 & 3\end{bmatrix}$

A + A^T = $\begin{bmatrix}1 & 5\\2 & 3\end{bmatrix}$ + $\begin{bmatrix}1 & 2\\5 & 3\end{bmatrix}$ = + $\begin{bmatrix}1+1 & 5+2\\2+5 & 3+3\end{bmatrix}$

A + A^T = $\begin{bmatrix}2 & 7\\7 & 6\end{bmatrix}$

1. (b) 0x+2y-z =0 X+3y+0z =- 4 3x+4y +0z = 4

$\Delta =$ $\begin{vmatrix}0 & 2 & -1\\1 & 3 & 0\\3 & 4 & 0\end{vmatrix}$ = 0(0-0) -2(0-0) -1(4-9) =0-0-1(-5) = 5

Δ_x= $\begin{vmatrix}0 & 2 & -1\\-4 & 3 & 0\\4 & 4 & 0\end{vmatrix}$ = 0(0-0) -2(0-0) -1(-16-12) =0-0-1(-28) = 28

Δ_y= $\begin{vmatrix}0 & 0 & -1\\1 & -4 & 0\\3 & 4 & 0\end{vmatrix}$ = 0(0-0) -0(0-0) -1(4+12) =0-0-1(16) = -16

Δ_z= $\begin{vmatrix}0 & 2 & 0\\1 & 3 & -4\\3 & 4 & 4\end{vmatrix}$ = 0(12+16) -2(4+12) +0(4-9) =0-2(16)-0(-5) = - 32

$$x=\frac{\Delta x}{\Delta}=\frac{28}{5} \quad y=\frac{\Delta y}{\Delta}=\frac{-16}{5} \quad , \quad z=\frac{\Delta z}{\Delta}=\frac{-32}{5}$$

OR

$|A|=\begin{vmatrix}1 & 5\\0 & -1\end{vmatrix}=-1-0=-1$ $|B|=\begin{vmatrix}2 & 6\\-1 & -3\end{vmatrix}=-6+6=0$

$|C|=\begin{vmatrix}3 & 2\\12 & 8\end{vmatrix}=-24-24=0$ Matrix B and C does not have inverse.

1. (C) Given $\begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}$

Characteristic equation is $|A-\lambda I|=0$

$$\begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}1-\lambda & 2-0\\3-0 & 1-\lambda\end{bmatrix}=\begin{bmatrix}1-\lambda & 2\\3 & 1-\lambda\end{bmatrix}$$

$$\begin{vmatrix}1-\lambda & 2\\3 & 1-\lambda\end{vmatrix}=0 \Rightarrow (1-\lambda)(1-\lambda)-6=0$$

$$\Rightarrow \quad \lambda^2-2\lambda-5=0$$

$$\lambda=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-2)\pm\sqrt{(-2)^2-4\times1(-5)}}{2\times1}$$

$$\Rightarrow \quad \lambda=1+\sqrt{6} \text{ and } \lambda=1-\sqrt{6}$$

OR

A= $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ B= $\begin{bmatrix}2 & 0\\4 & 3\end{bmatrix}$

(A+ B) = $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}+\begin{bmatrix}2 & 0\\4 & 3\end{bmatrix}=\begin{bmatrix}1+2 & 2+0\\3+4 & 4+3\end{bmatrix}=\begin{bmatrix}3 & 2\\7 & 7\end{bmatrix}$

$(A+B)^T$ = $\begin{bmatrix}3 & 7\\2 & 7\end{bmatrix}$ ....(1)

$A^T=\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}$ $B^T=\begin{bmatrix}2 & 4\\0 & 3\end{bmatrix}$

$A^T+B^T=\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}+\begin{bmatrix}2 & 4\\0 & 3\end{bmatrix}=\begin{bmatrix}3 & 7\\2 & 7\end{bmatrix}$ ....(2)

From equation (1) & (2) $(A+B)^T=A^T+B^T$

1. (d) A= $\begin{bmatrix}2 & 1\\3 & 5\end{bmatrix}$ $\Rightarrow |A|=\begin{vmatrix}2 & 1\\3 & 5\end{vmatrix}=10-3=7\neq0$ $\therefore A^{-1}$ exists.

adj A = $\begin{bmatrix}5 & -1\\-3 & 2\end{bmatrix}$ $\Rightarrow A^{-1}=\frac{adj\,A}{|A|}=\frac{1}{7}\begin{bmatrix}5 & -1\\-3 & 2\end{bmatrix}$

OR

A= $\begin{bmatrix}2 & 1\\-3 & 5\end{bmatrix}$ B= $\begin{bmatrix}9 & 2\\-3 & 5\end{bmatrix}$

AB = $\begin{bmatrix}2 & 1\\-3 & 5\end{bmatrix}\begin{bmatrix}9 & 2\\-3 & 5\end{bmatrix}=\begin{bmatrix}18-3 & 4+5\\-27-15 & -6+25\end{bmatrix}=\begin{bmatrix}15 & 9\\-42 & 19\end{bmatrix}$

2. (a)

2a

(A) Equation of Straight line Passing through a given point (x_1, y_1) and having slope m is	(2) $(y - y_1) = m(x - x_1)$
Award one mark for this question for attending. because in question (x, y) is given	
(B) equation of straight line passing through two points (x_1, y_1) and (x_2, y_2) is	(4) $\frac{(y-y_1)}{(x-x_1)} = \frac{(y_2-y_1)}{(x_2-x_1)}$
(C) The equation of straight line whose x and y intercepts are a, b respectively is	(1) $\frac{x}{a} + \frac{y}{b} = 1$
(D) If two lines are perpendicular to each other then Product of their slope is equal to	(3) -1

OR

(A) if two lines with slopes m_1 and m_2 are parallel then θ is	(2) 0 (zero)
(B) equation of straight line whose slope is m and y intercept is C	(1) $y = mx + C$
(C) Slope of line is $ax + by + C = 0$	(4) $\frac{-a}{b}$
(D) slope of line joining two points (x_1, y_1) and (x_2, y_2)	(3) $\frac{(y_2-y_1)}{(x_2-x_1)}$

2. (b)

Given equation $6x - 5y - 2 = 0$

Slope of given line $m = \frac{-a}{b} = \frac{-6}{-5} = \frac{6}{5}$

Therefore slope of required line is $m = \frac{-5}{6}$

Equation of line passing through (2, -3) & having slope $m = \frac{-5}{6}$

$(y - y_1) = m(x - x_1)$, $(x_1, y_1) = (2, -3)$ $m = \frac{-5}{6}$

$[y - (-3)] = \frac{-5}{6}(x - 2)$ $\Rightarrow$ $5x + 6y + 8 = 0$

OR

Consider $4x + 6y + 7 = 0$ ....(1) $2x + 3y - 1 = 0$ ....(2)

Slope of line (1) $m_1 = \frac{-a}{b}$ $m_1 = \frac{-4}{6} = \frac{-2}{3}$

Slope of line (2) $m_2 = \frac{-a}{b}$, $m_2 = \frac{-2}{3}$

since $m_1 = m_2 = \frac{-2}{3}$ so the lines are parallel to each other.

2. (C) Given line $5x + 6y - 10 = 0$

Slope of given line $m = \frac{-a}{b}$ $\quad m = \frac{-5}{6}$

Since required line is parallel to $5x + 6y - 10 = 0$

So slope of required line is $m = \frac{-5}{6}$

$(x_1, y_1) = (-3, 3) \quad m = \frac{-5}{6}$

$(y - y_1) = m\,(x - x_1)$

$(y - 3) = \frac{-5}{6}(x + 3) \Rightarrow 6(y - 3) = -5\,(x + 3)$

$6y - 18 = -5x - 15 \Rightarrow 5x + 6y - 3 = 0$ required line.

OR

Consider $3x + 4y + 7 = 0$ .... (1) $\quad 28x - 21y + 50 = 0$ ....(2)

Slope of equation (1) $m_1 = \frac{-a}{b} = \frac{-3}{4}$ Slope of equation (2) $m_2 = \frac{-a}{b} = \frac{-28}{-21} = \frac{4}{3}$

Since product of slope of two perpendicular lines is equal to -1

$$m_1 \times m_2 = \frac{-3}{4} \times \frac{4}{3} = -1$$

$\therefore$ given lines are perpendicular to each other.

2. (d) let $x + 3y + 5 = 0$ ....(1) $\quad 4x + 2y - 7 = 0$ ....(2)

$m_1 = \frac{-a}{b} = \frac{-1}{3}$ $\quad m_2 = \frac{-a}{b} = \frac{-4}{2} = -2$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{-1}{3} + 2}{1 + \left(\frac{-1}{3}\right)(-2)}\right| = \left|\frac{\frac{-1+6}{3}}{1 + \frac{2}{3}}\right| = \left|\frac{\frac{5}{3}}{\frac{5}{3}}\right| = 1$$

$\tan\theta = 1 \Rightarrow \theta = \tan^{-1} 1 = 45°$

OR

Equation of straight line which passes through (x_1, y_1) & (x_2, y_2) is

$\frac{(y - y_1)}{(x - x_1)} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ and $(x_1, y_1) = (-2,\ 3)$ $\quad (x_2, y_2) = (-5, 6)$

$\frac{y-3}{x+2} = \frac{6-3}{-5+2} = -1$ $\quad \frac{y-3}{x+2} = -1$

$y - 3 = -1(x + 2) = -x - 2$

$y - 3 + x + 2 = 0 \Rightarrow x + y - 1 = 0$

3. (a)

$$\cos(570°) = \cos(6 \times 90° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$$

$$\sin(330°) = \sin(4 \times 90° - 30°) = -\sin 30° = -\frac{1}{2}$$

OR

$$45° = \frac{\pi}{180} \times 45 = \frac{\pi}{4} = 0.7855\ rad$$

$$11\frac{\pi}{5} = \frac{180}{\pi} \times 11\frac{\pi}{5} = 36 \times 11 = 396°$$

3. (b)

$$A + B = \frac{\pi}{4} \quad \Rightarrow \tan(A+B) = \tan\frac{\pi}{4} = 1$$

$$\tan(A+B) = 1$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1, \quad \tan A + \tan B = 1 - \tan A \tan B$$

$\tan A + \tan B + \tan A \tan B = 1$ add 1 both the side.

$$1 + \tan A + \tan B + \tan A \tan B = 1 + 1$$

$$1 + \tan A + \tan B\,(1 + \tan A) = 2$$

$$(1 + \tan A)(1 + \tan B) = 2$$

OR

$$\sin 3A = 3\sin A - 4sin^3 A$$

$$\sin 3A = \sin(2A + A) = \sin 2A \cos A + \cos 2A \sin A$$

$$= 2\sin A \cos A \cos A + (1 - 2sin^2 A)\sin A$$

$$= 2\sin A\, cos^2 A + \sin A - 2sin^3 A$$

$$= 2\sin A\,(1 - sin^2 A) + \sin A - 2sin^3 A$$

$$= 2\sin A - 2\,sin^3 A + \sin A - 2sin^3 A$$

$$\therefore \quad \sin 3A = 3\sin A - 4sin^3 A$$

3. (c) $\tan A = \frac{18}{17}$ $\qquad \tan B = \frac{1}{35}$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan(A-B)=\frac{\frac{18}{17}-\frac{1}{35}}{1+\frac{18}{17}\times\frac{1}{35}}=\frac{630-17}{595+18}=\frac{613}{613}=1$$

$$\tan(A-B)=1=\tan\frac{\pi}{4}$$

$$\Rightarrow\ (A-B)=\frac{\pi}{4}$$

OR

$$\frac{\cos(360-A)\tan(360+A)}{\cot(270-A)\sin(90+A)}=\frac{\cos(A)\tan(A)}{\tan(A)\cos(A)}=1$$

3. (d)

L.H.S $\cos 55° + \cos 65° + \cos 175° = \cos 55° + 2\cos(\frac{65+175}{2})\ \cos(\frac{65-175°}{2})$

$$= \cos 55° + 2\cos(120°)\ \cos(-55°)$$

$$= \cos 55° + 2\times\cos(90°+30°)\cos(55°)$$

$$= \cos 55° + 2\times(-\sin 30°)\cos(55°)$$

$$= \cos 55° - 2\times\frac{1}{2}\cos(55°)$$

$= \cos 55° - \cos 55° = 0$ R.H.S

OR

$$\frac{\text{Sin } 40°+\sin 20°}{\cos 40°+\cos 20°}=\frac{2\sin\left(\frac{40+20}{2}\right)\cos\left(\frac{40-20}{2}\right)}{2\cos\left(\frac{40+20}{2}\right)\cos\left(\frac{40-20}{2}\right)}$$

$$=\frac{\sin\left(\frac{40+20}{2}\right)}{\cos\left(\frac{40+20}{2}\right)}=\frac{\sin(30)}{\cos(30)}=\tan 30=\frac{1}{\sqrt{3}}$$

4. (a) $y=\sin x+\log x+e^x+\tan x$

$$\frac{dy}{dx}=\frac{d(\sin x)}{dx}+\frac{d\log x}{dx}+\frac{de^x}{dx}+\frac{d\tan x}{dx}$$

$$\frac{dy}{dx}=\cos x+\frac{1}{x}+e^x+sec^2x$$

OR

$y = 4x^3 + 3x^2$, $\frac{dy}{dx} = 4\frac{dx^3}{dx} + 3\frac{dx^2}{dx}$ Given, $\frac{dy}{dx} = 4x^3 + 3x^2$

$\frac{dy}{dx} = 4 \times 3x^2 + 3 \times 2x = 12x^2 + 6x$ ✓ $\therefore \frac{d^2y}{dx^2} = 4(3x^2) + 3(2x)$

$= 12x^2 + 6x$

$\frac{d^2y}{dx^2} = 12\frac{d(x^2)}{dx} + 6\frac{d(x)}{dx} = 12 \times 2x + 6 \times 1 = 24x + 6$ ✓

$\therefore \frac{d^2y}{dx^2}$ at $(1,2)$

$\left(\frac{d^2y}{dx^2}\right)_{(1,2)} = 24(1) + 6 = 30$ ✓

$\frac{d^2y}{dx^2} = 12(1)^2 + 6(1)$

$\frac{d^2y}{dx^2} = 12 + 6 = \underline{\underline{18}}$

4. (b)

$y = (3x+8)^5$

$\frac{dy}{dx} = 5(3x+8)^{5-1} \times \frac{d(3x+8)}{dx}$

$= 5(3x+8)^4 \times (3\frac{d(x)}{dx} + \frac{d(8)}{dx})$

$= 5(3x+8)^4 \times (3 \times 1 + 0)$

$= 15(3x+8)^4$ ✓

OR

$y = \log[\sin(\log x)]$

$\frac{dy}{dx} = \frac{d(\log[\sin(\log x)])}{dx} = \frac{1}{\sin(\log x)} \frac{d(\sin(\log x))}{dx}$

$\frac{dy}{dx} = \frac{1}{\sin(\log x)} (\cos(\log x)) \frac{d(\log x)}{dx}$

$\frac{dy}{dx} = \frac{\cos(\log x)}{\sin(\log x)} \left(\frac{1}{x}\right) = \underline{\frac{\cot(\log x)}{x}}$

4. (c) $S = 4t - 5t^2 + 2t^3$

$\frac{d(s)}{dt} = \frac{d(4t - 5t^2 + 2t^3)}{dt}$

$\frac{d(s)}{dt} = 4\frac{d(t)}{dt} - 5\frac{d(t^2)}{dt} + 2\frac{d(t^3)}{dt}$

$\frac{d(s)}{dt} = 4 \times 1 - 5 \times 2t + 2 \times 3t^2$ ✓

$V = \frac{d(s)}{dt} = 4 - 10t + 6t^2$ ✓

$V = \left(\frac{d(s)}{dt}\right)_{t=2} = 4 - 10 \times 2 + 6 \times 2^2 = 4 - 20 + 24 = 8$

$a = \frac{d(4-10t+6t^2)}{dt} = 0 - 10\frac{dt}{dt} + 6\frac{dt^2}{dt} = -10 \times 1 + 6 \times 2t = -10 + 12t$

$(a)_{t=2} = -10 + 12(2) = -10 + 24 = 14\, ms^{-2}$

OR

$S = 160t - 16t^2$

$\frac{d(s)}{dt} = V = \frac{d(160t-16t^2)}{dt} = 160\frac{d(t)}{dt} - 16\frac{d(t^2)}{dt}$

$\frac{d(s)}{dt} = 160 \times 1 - 16 \times 2t = 160 - 32t$ When the car stop

V= $0 = 160 - 32t$ velocity becomes zero. V=0

$t = \frac{160}{32} = 5\, sec$

4. (d) $y = x^3 - x^2 - x$, $\frac{dy}{dx} = 3x^2 - 2x - 1$

y is maximum or minimum, if $\frac{dy}{dx} = 0$

i.e $0 = 3x^2 - 2x - 1$ *or* $(3x+1)(x-1) = 0$

$\therefore x = -\frac{1}{3}$, *and* $x = 1$

$\frac{d^2y}{dx^2} = 6x - 2$, now if $x = -\frac{1}{3}$ $\frac{d^2y}{dx^2} = 6\left(\frac{-1}{3}\right) - 2$

$\frac{d^2y}{dx^2} = -4$

$\therefore$ y is maximum at $x = -\frac{1}{3}$

$y = x^3 - x^2 - x$

$y = \left(\frac{-1}{3}\right)^3 - \left(\frac{-1}{3}\right)^2 - \left(\frac{-1}{3}\right) = \left(\frac{-1}{27}\right) - \left(\frac{1}{9}\right) + \left(\frac{1}{3}\right) = \left(\frac{5}{27}\right)$

Maximum value is $y = \left(\frac{5}{27}\right)$

For $x = 1$ $\frac{d^2y}{dx^2} = 6(1) - 2 = 4 = +ve\ value$

$\therefore$ y is minimum at $x = 1$

$y = x^3 - x^2 - x \quad , y = 1^3 - 1^2 - 1 = -1$

Minimum value is $\quad y = -1$

OR

$y = 2 - 3x + x^2$ diff w.r.t x

$\frac{dy}{dx} = -3 + 2x \quad , \quad m = \left(\frac{dy}{dx}\right)_{(1,2)} = -3 + 2(1) = -1$

Equation of tangent $\quad y - y_1 = m(x - x_1)$

$y - 2 = -1(x - 1) \qquad y - 2 = -x + 1$

$y - 2 + x - 1 = 0 \qquad x + y - 3 = 0$

5. (a) $y = \cos x + e^x + \frac{1}{x} + x^2$

$\int y dx = \int \cos x \, dx + \int e^x dx + \int \frac{1}{x} dx + \int x^2 \, dx$

$= \sin x + e^x + \log x + \frac{x^3}{3} + C$

OR

Area A $= \int_a^b y \, dx = \int_1^2 x^2 \, dx = \left[\frac{x^3}{3}\right]_1^2 = \left[\frac{2^3}{3} - \frac{1^3}{3}\right] = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$

$\therefore A = \frac{7}{3}$ square units

5. (b) $\int uv \, dx = u \int v \, dx - \int \left[\frac{du}{dx} \times \int v \; dx\right] dx$

$\int x \sin 2x \, dx = x \int \sin 2x \, dx - \int \left[\frac{dx}{dx} \times \int \sin 2x \; dx\right] dx$

$= x\left(\frac{-\cos 2x}{2}\right) - \int \left(1 \times \left(\frac{-\cos 2x}{2}\right)\right) dx$

$$= x\left(\frac{-\cos 2x}{2}\right) + \frac{1}{2}\int \cos 2x \; dx$$

$$= -\frac{x\cos(2x)}{2} + \frac{\sin 2x}{4} + c$$

OR

$$\int uv\, dx = u\int v\, d \; - \int\left[\frac{du}{dx}\times\int v \; dx\right] dx$$

$$\int \sin 2x \cos 3x \, dx = \int \frac{1}{2}[\sin(2x+3x) + \sin(2x-3x)]dx$$

Here using formula $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$$\int \sin 2x \cos 3x \, dx = \frac{1}{2}\int \sin(5x)\, dx + \frac{1}{2}\int \sin(-x)\, dx$$

$$= \frac{1}{2}\left(-\frac{\cos 5x}{5}\right) + \frac{1}{2}\int(-\sin x)\, dx$$

$$= \frac{-\cos 5x}{10} - \frac{1}{2}(-\cos x) + C$$

$$= \frac{-\cos 5x}{10} + \frac{\cos x}{2} + C$$

5. (C) $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$

$I = \int_0^{\frac{\pi}{2}} \sin^2 x \, dx$ ....(1) $\quad \because \int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

$I = \int_0^{\frac{\pi}{2}} \sin^2\left(\frac{\pi}{2} - x\right) dx = \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$ ....(2)

Adding (1) & (2)

$$I + I = \int_0^{\frac{\pi}{2}} \sin^2 x \, dx + \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

$$2I = \int_0^{\frac{\pi}{2}}(\sin^2 x + \cos^2 x)\, dx = \int_0^{\frac{\pi}{2}} 1 \, dx = [x]_0^{\frac{\pi}{2}} = \left[\frac{\pi}{2} - 0\right]$$

$$\Rightarrow \quad 2I = \frac{\pi}{2} \quad \Rightarrow \quad I = \frac{\pi}{4}$$

OR

$\int \sin^5 x \cos x \, dx = \int t^5 \, dt$ $\qquad$ put $t = \sin x$

$= \frac{t^{5+1}}{5+1} + C$ $\qquad$ $\frac{dt}{dx} = \cos x$

$= \frac{t^6}{6} + C$ $\qquad$ $dt = \cos x \; dx$

$= \frac{(\sin x)^6}{6} + C$

5. (d) Area $= A = \int_a^b y\, dx = \int_1^3 (x^2+1)dx = \left[\frac{x^3}{3} + x\right]_1^3 = \frac{1}{3}[x^3]_1^3 + [x]_1^3$

$$= \frac{1}{3}[(3)^3 - (1)^3] + [3 - 1]$$

$$= \frac{1}{3}[27 - 1] + [2]$$

$$= \frac{26}{3} + 2 = \frac{32}{3}$$

OR

$$y = \sqrt{x+2}$$

$$V = \pi \int_a^b [y]^2\,dx = \pi \int_a^b [f(x)]^2\,dx = \pi \int_0^2 \left(\sqrt{x+2}\right)^2 dx$$

$$= \pi \int_0^2 (x+2)\,dx = \pi \int_0^2 (x)\,dx + 2\pi \int_0^2 1\,dx$$

$$= \pi \left[\frac{x^2}{2}\right]_0^2 + 2\pi [x]_0^2$$

$$= \frac{\pi}{2}[2^2 - 0^2] + 2\pi[2 - 0]$$

$$= \frac{\pi}{2}[4] + 2\pi[2] = 2\pi + 4\pi = 6\pi$$

$$V = 6\pi \ \ cubic\ unit$$

Note: Give full marks for any other alternate method.