Week 2: - Resonance

Resonant circuit: A resonant circuit consists of R, L, and C elements and whose frequency response characteristic changes with changes in frequency.

Resonance: Resonance is the tendency of a system to oscillate at a greater amplitude at some frequencies than at others.

The frequency at which the response amplitude is maximum is called resonant frequency.

Electrical resonance:

It is the condition when the voltage across a circuit becomes in phase with the current supplied to the circuit.

OR

It is the condition when the magnitude of capacitive reactance becomes equal to that of inductive reactance.

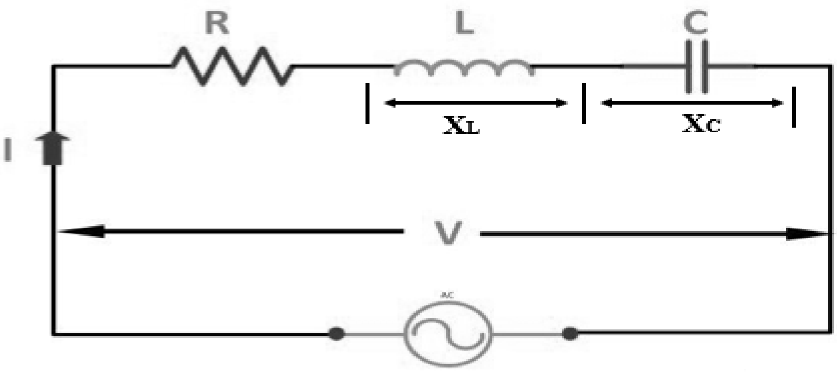
At resonance, the circuit behaves like a resistive circuit. As a result of resonance, maximum current flows through the RLC circuit.

Types of resonance circuit:

(i) Series resonance circuit. (ii) Parallel resonance circuit.

Series resonance circuit:

Series resonance circuit consists of a resistor of resistance *R*, an inductor of inductance *L* and a capacitor of capacitance *C* connected across an alternating voltage source as shown below.



- The capacitive and inductive reactance of the circuit is given by

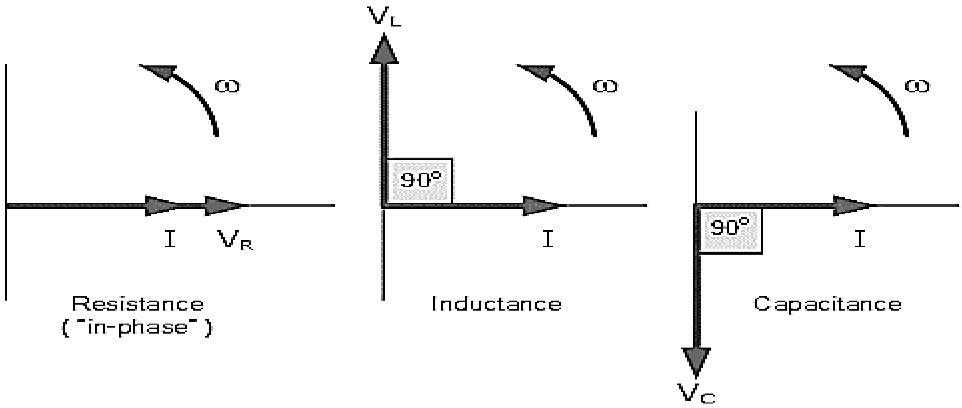
  Capacitive reactance $X_C = \frac{1}{2\pi fC}$

  Inductive reactance $X_L = 2\pi f L$

  Where, X_L increases with increase in frequency and X_C decreases with decrease in frequency.
- As the instantaneous current flows through the circuit element, X_L and X_C changes as they are a function of the supply frequency. Therefore sinusoidal response of a series RLC circuit will therefore vary with frequency, f.

- When $X_L = X_C$ then the RLC Series circuit comes to the resonance condition.
- The frequency at which resonance arises is called ***Resonant Frequency.***

Phasor diagram of capacitor and inductor:



The instantaneous voltage across a pure resistor, V_R is "in-phase" with current

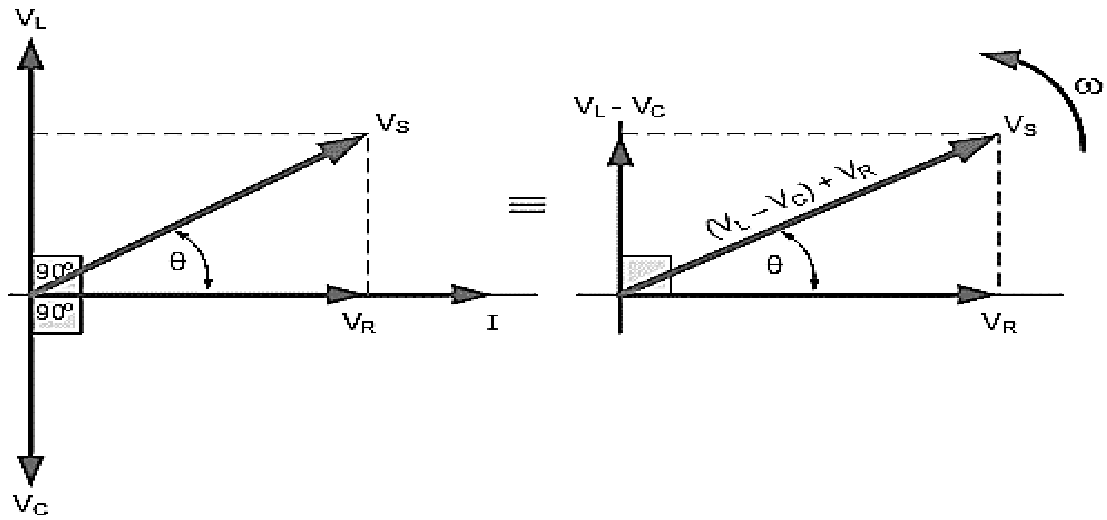
The instantaneous voltage across a pure inductor, V_L "leads" the current by 90^o

The instantaneous voltage across a pure capacitor, V_C "lags" the current by 90^o

Therefore, V_L and V_C are 180^o "out-of-phase" and in opposition to each other.

Condition for resonance or Phasor diagram of series RLC circuit:

The Phasor diagram of series RLC circuit is as below



From Pythagoras theorem, we have

$$V_S^{\,2} = V_R^{\,2} + (V_L - V_C)^2$$

$$V_S = \sqrt{[V_R^{\,2} + (V_L - V_C)^2]} = I \cdot \sqrt{[R^2 + (X_L - X_C)^2]}$$

$V_S = I \cdot Z$, where $Z = \sqrt{[R^2 + (X_L - X_C)^2]}$

For maximum voltage $\boxed{\mathbf{Z=R}}$, therefore $\boxed{\mathbf{X_L = X_C}}$ is the condition for resonance

Expression for resonance frequency:

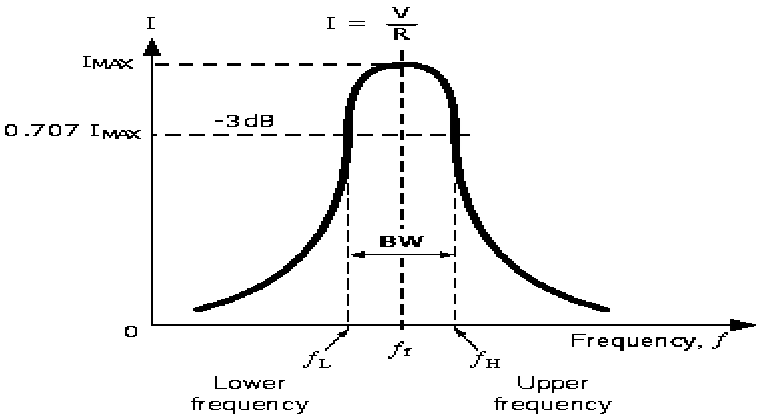
We know that, at resonance $X_L = X_C$

$$2\pi f_r L = 1/ 2\pi f_r C$$

$$f_r^2 = 1/ 4\pi^2 LC$$

$$f_r = 1/ 2\pi\sqrt{LC}$$

Resonance plot for series resonance circuit:



- The frequency response curve of a series resonance circuit is as shown above.
- The magnitude of the current is a function of frequency
- Graph shows us that the response starts at near to zero, reaches maximum value at the resonance frequency when $I_{MAX} = I_R$ and then drops again to nearly zero as f becomes infinite.
- Series resonance circuit is also known as an ***Acceptor Circuit*** because at resonance, the impedance of the circuit is at its minimum so it accepts the current whose frequency is equal to its resonant frequency.

Characteristics of series resonance circuit:

- Resonance occurs when $X_L = X_C$
- Resonance frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$
- Impedance is minimum and purely resistive at resonance.
- Current is maximum at resonance.
- Frequencies other than $\boldsymbol{f_r}$ are rejected. Only $\boldsymbol{f_r}$ is accepted. Hence it is called the acceptor circuit.