WEEK1: - NETWORK THEOREMS

Introduction:

Network: - A network is a collection of interconnected components (resistors, capacitors, inductors and voltage or current sources).

Network analysis is the process of finding the voltages across, and the currents through, all network components.

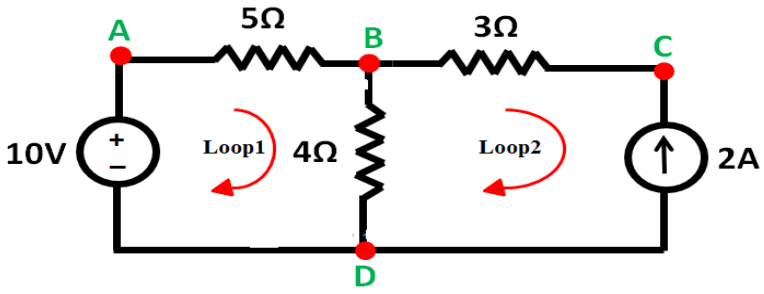
Network terminology:-

Component: A device with two or more terminals through which current flow.

Node: Node is a terminal to which Bone or more circuit elements are connected.

Branch: Branch is a component joining two nodes.

Loop: Loop is any closed path in the circuit formed by branches.



Nodes: A, B, C and D.

Branches: AB, BC, CD and AD.

Loop: Loop1 (A, B, D and A)

Loop2 (B, C, D and B)

Ohm's law:-

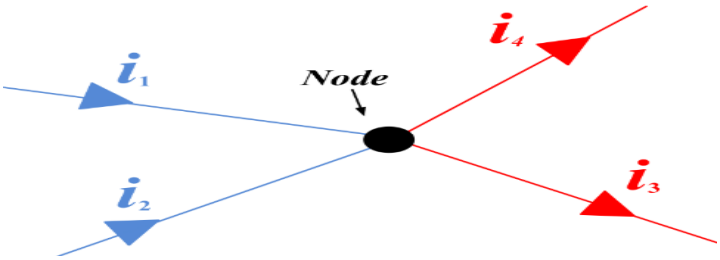
Ohm's law states that the current through a conductor between two points is directly proportional to the voltage across the two points.

$$V \propto I$$

Therefore, **V = RI** where R is a constant called resistance.

Kirchhoff's Current Law:-

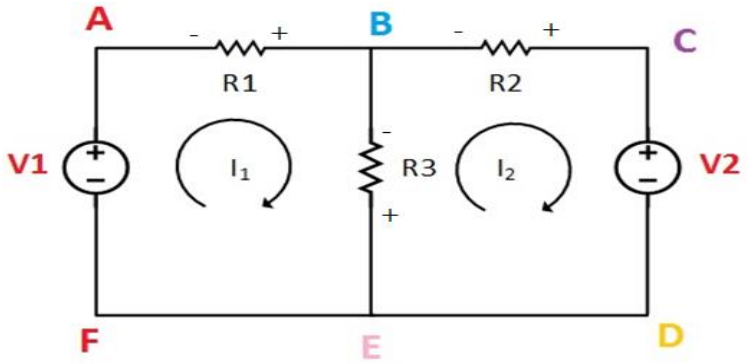
The sum of all currents entering a node is equal to the sum of all currents leaving the node.



i1+ i2 = i3 + i4

i.e., i1+ i2 - i3 - i4 =0

KVL (Kirchhoff's Voltage Law):- KVL states that the sum of voltages in an enclosed circuitry is always equal to 0.



For Loop ABEFA,

$V_1 + I_1R_1 + (I_1 - I_2) R_3 = 0$

For Loop BCDEB,

$I_2R_2 - V_2 - (I_1 - I_2) R_3 = 0$

1.1 Superposition theorem:-

Superposition theorem states that the voltage or current in an element of a linear bilateral circuit equals the algebraic sum of the responses produced by each source separately.

OR

The voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

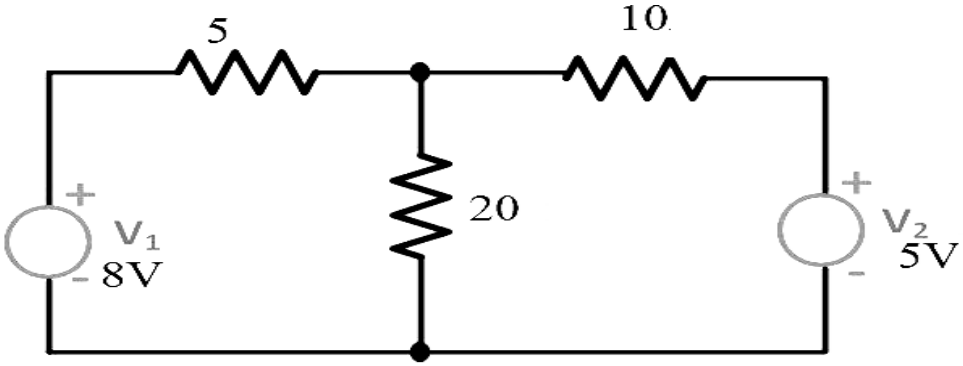
Steps to analyze circuit using Superposition theorem:

Step 1: Turn on only one independent source and turn off (short circuit for voltage source and open circuit for current source) all other independent sources. Find the output current or voltage for that source using KVL or KCL or other basic method.

Step 2: Do step 1 for all the independent source separately.

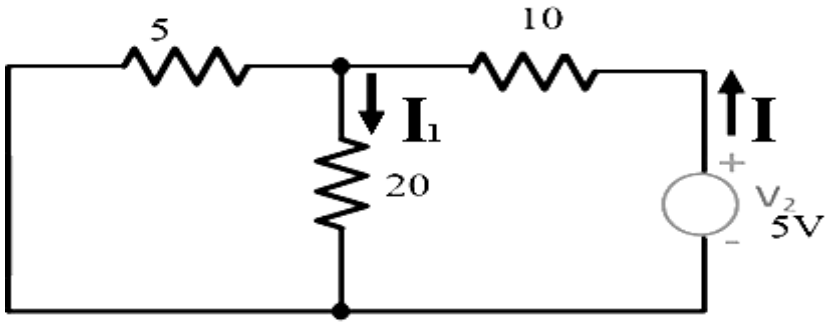
Step 3: Add all the contributions of independent sources algebraically and find total contribution.

Problem 1) Find the current flowing through 20Ω using superposition theorem.



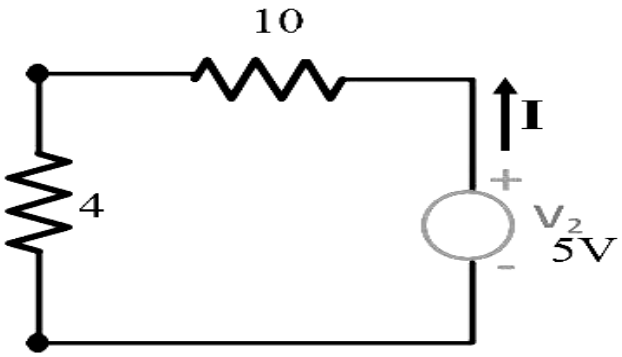
Solution:

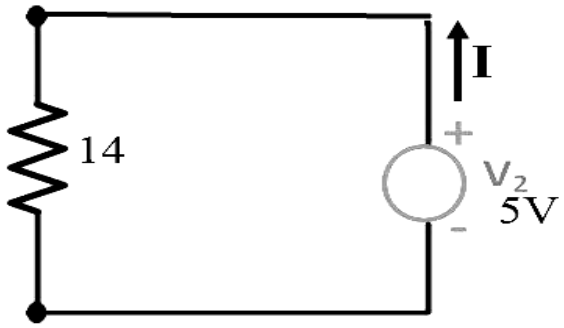
Step 1: Remove (short) the 8V power supply, the new circuit becomes as shown below and measure the current through 20Ω resistor.



Here 20 and 5 are in parallel, therefore resultant resistance is = 5||20 = (5 X 20) / (5+20) = 4Ω

This 4Ω is in series with 10Ω, therefore series resistance is 4+10 =14Ω



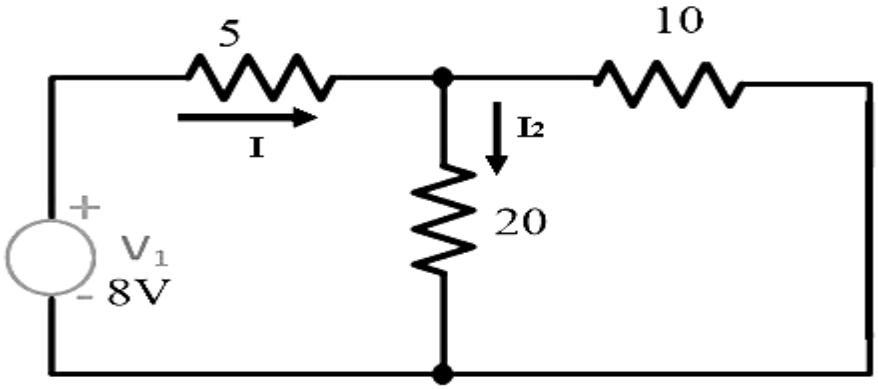


The current I=V / R = 5/14 = 0.357A

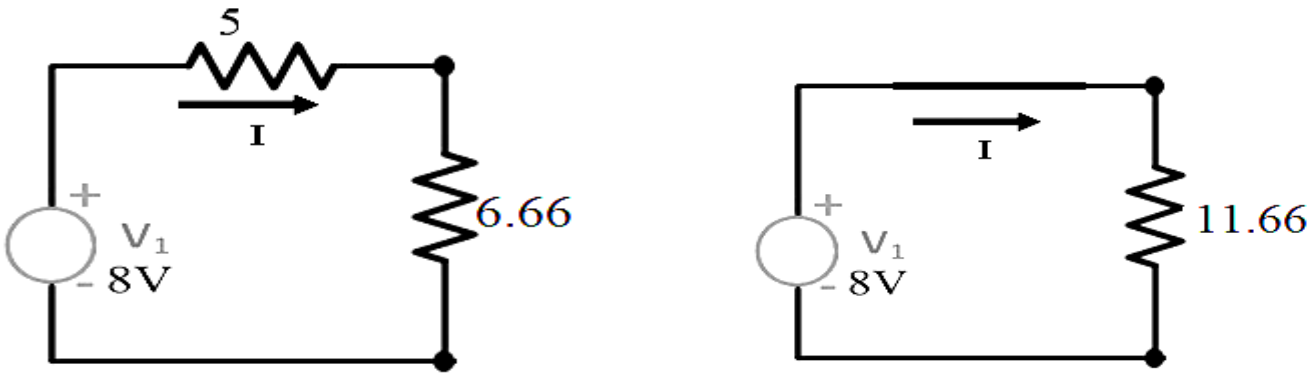
Therefore current through 20k due to 5V supply is given by,

I_1 = 0.357 X 5 / (5+20) = 0.0714A

Step 2: Remove (short) the 5V power supply, the new circuit becomes as shown below and measure the current through 20Ω resistor.



Here 20Ω and 5Ω are in parallel, therefore resultant resistance is =10Ω||20Ω = (10 X 20) / (10+20) = 6.66Ω



The current I=V / R = 8/11.66 = 0.685A

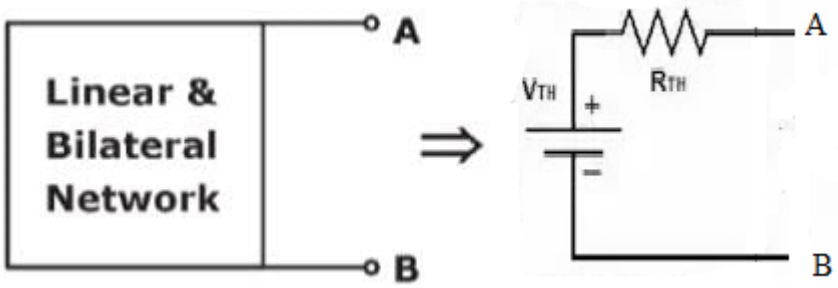
Therefore current through 20k due to 8V supply is given by,

I_1 = 0.685 X 10 / (10+20) = 0.2283A

Step 3: Therefore from superposition theorem current flowing through 20Ω is given by, $I_T = I_1 + I_2$

I_T = 0.0714A + 0.2283A = 0.2997A

Thevenin's Theorem:- Thevenin's theorem states that it is possible to simplify any linear complex circuit, to an equivalent circuit with a single voltage source V_{TH} and a series resistance R_{TH}.



Steps to analyze circuit using Thevenin's theorem:

Step 1: To find Thevenin's voltage V_{TH}

- Open the load resistor
- Calculate the voltage across load terminal AB. This is the Thevenin's voltage V_{TH}.

Step 2: To find open circuit resistance R_{TH}

- Open all current and short all voltage sources.
- Calculate the resistance across load terminal AB. This is the series resistance R_{TH}.

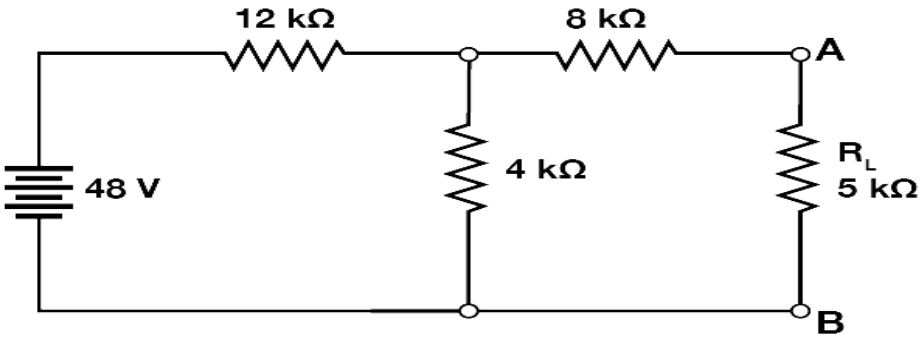
Step 3: Draw Thevenin's circuit

- Draw the circuit by connecting R_{TH} in series with V_{TH} and reconnect the load resistor across terminal AB. This is the Thevenin's circuit for the given complex circuit.

Step 4: Find current (I_L) through load resistor using Ohm's law

i.e., $I_L = V_{TH} / (R_{TH} + R_L)$

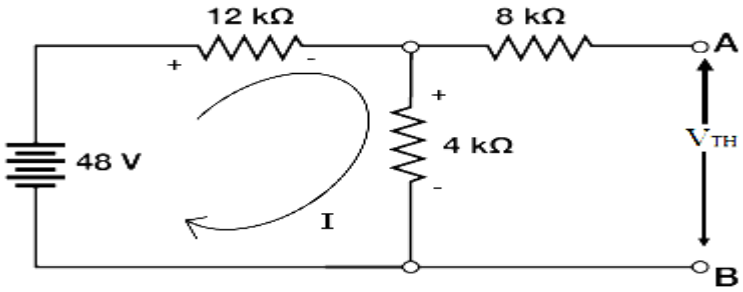
PROBLEM 1: Find V_{TH}, R_{TH} and the load current I_L flowing through and load voltage across the load resistor in the circuit below using Thevenin's Theorem.



SOLUTION:

Step 1: Find Thevenin's voltage V_{TH}

Remove the load resistor R_L

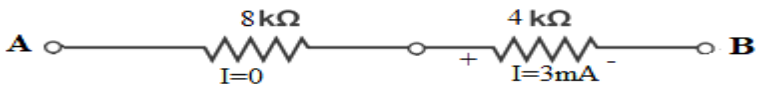


Applying KVL to the above loop we have,

$48-(12k\Omega*I) – (4k\Omega*I) = 0$

$I = 3mA$

The Thevenin's voltage across AB is given by



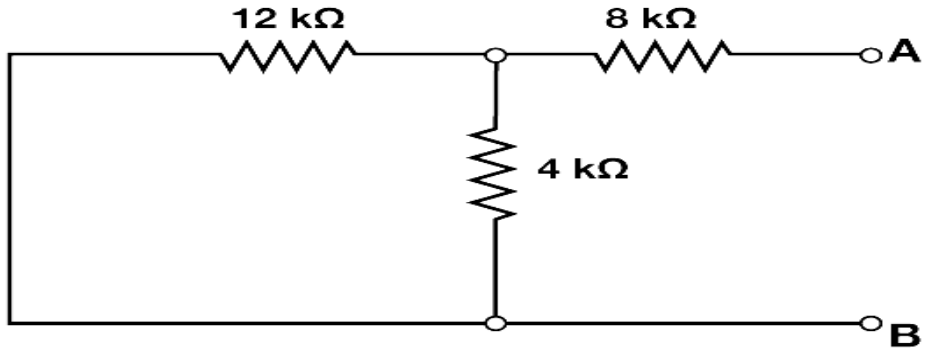
V_{TH} = Voltage across 8kΩ + Voltage across 4kΩ

V_{TH} = (0 X 8kΩ) + (3mA X 4kΩ) = (0 X 8kΩ) + (3mA X 4kΩ)

V_{TH} = 12V

Step 2: Find open circuit resistance R_{TH}

Remove load resistor R_L and short all voltage sources, calculate open circuit resistance across AB

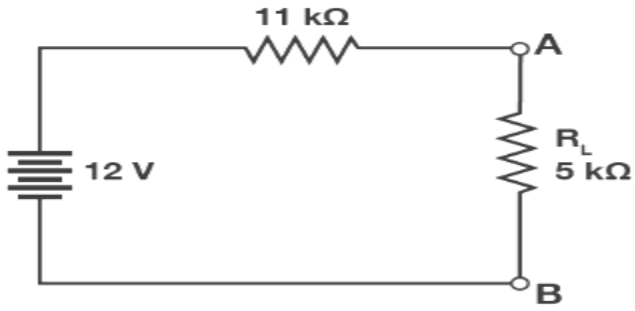


R_{TH} = 12kΩ || 4kΩ + 8kΩ = [(12*4) / (12+4)] + 8kΩ

R_{TH} = 3kΩ + 8kΩ

R_{TH} = 11kΩ

Step 3: Now, connect the R_{TH} in series with Voltage Source V_{TH} and the load resistor as shown in the figure.



Step 4: Calculate the load voltage and load current using Ohm's law as follows:

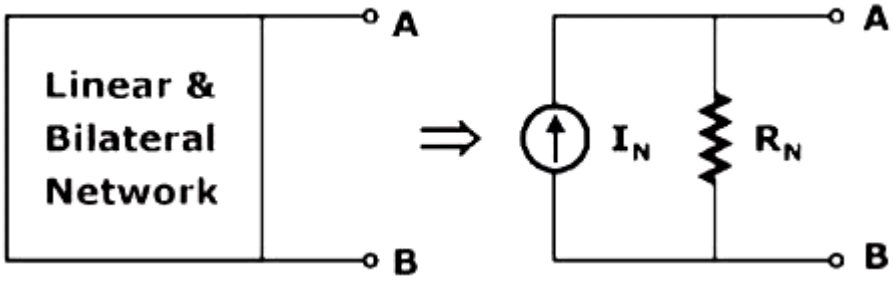
$I_L = V_{TH} / (R_{TH} + R_L)$

I_L = 12 V / (11 kΩ + 5 kΩ) = 12 V/16 kΩ = **0.75 mA**

The load voltage is determined as follows:

V_L = 0.75 mA x 5 kΩ = 3.75 V

Norton's Theorem:- Norton's theorem states that it is possible to simplify any linear complex circuit, to an equivalent circuit with a single current source I_N and a parallel resistance R_N.



Steps to analyze circuit using Norton's theorem:

Step 1: To find Norton's current I_N

- Short the load resistor
- Calculate the short circuit current I_N.

Step 2: To find open circuit resistance R_N

- Open all current and short all voltage sources.
- Calculate the resistance across load terminal AB. This is the series resistance R_{TH}.

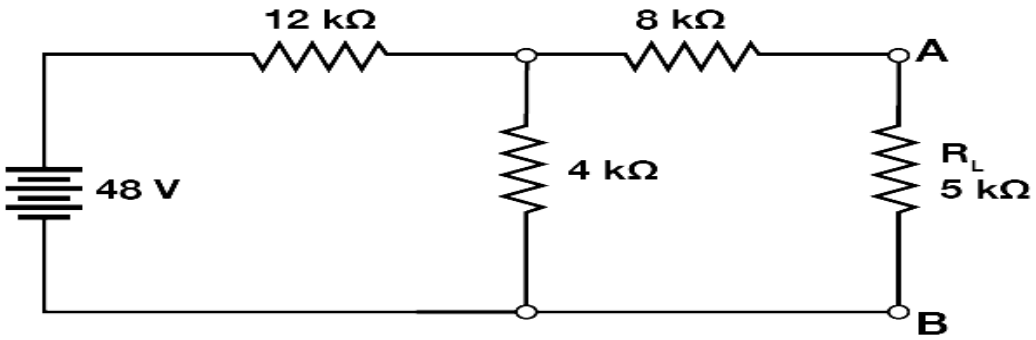
Step 3: Draw Norton's circuit

- Draw the circuit by connecting R_N in parallel with I_N and reconnect the load resistor across terminal AB. This is the Norton's circuit for the given complex circuit.

Step 4: Find current (I_L) through load resistor using Ohm's law

$I_L = I_N / [R_N / (R_N + R_L)$

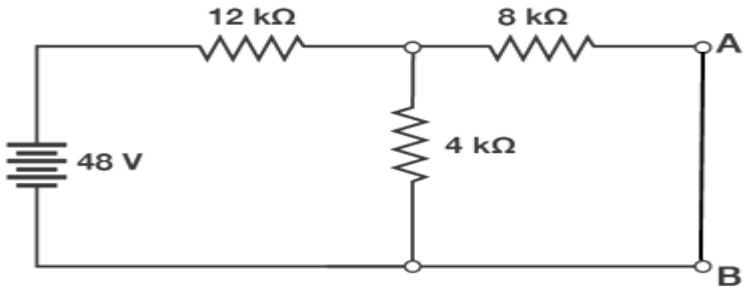
PROBLEM 1: Find I_N, R_N and the load current I_L flowing through and load voltage across the load resistor in the circuit below using Norton's Theorem.



SOLUTION:

Step 1: Find Norton's voltage I_N

Short the load resistor R_L



The resistance of above circuit is given by, Re = (8kΩ || 4kΩ) + 12kΩ

$$Re = \frac{8k \times 4k}{8k+4k} + 12k = 2.66k + 12k$$

$$Re = 14.66\ k\Omega$$

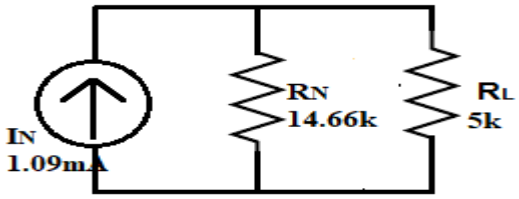
Total current I in the above circuit is given by Ohm's law,

$$I = \frac{V}{Re} = \frac{48}{14.66\ k} = 3.274mA$$

Norton's current $I_N = 3.274mA \times \frac{4k}{4k+8k} = 3.274 \times 10^{-3} \times 0.333 = 1.09\ mA$

I_N =1.09 mA

Step 3: Now, connect the R_N in parallel with current Source I_N and the load resistor as shown in the figure.



Step 4: Calculate the load voltage and load current using Ohm's law as follows:

$$I_L = 1.09mA \times \frac{14.66k}{14.66k+5k}$$

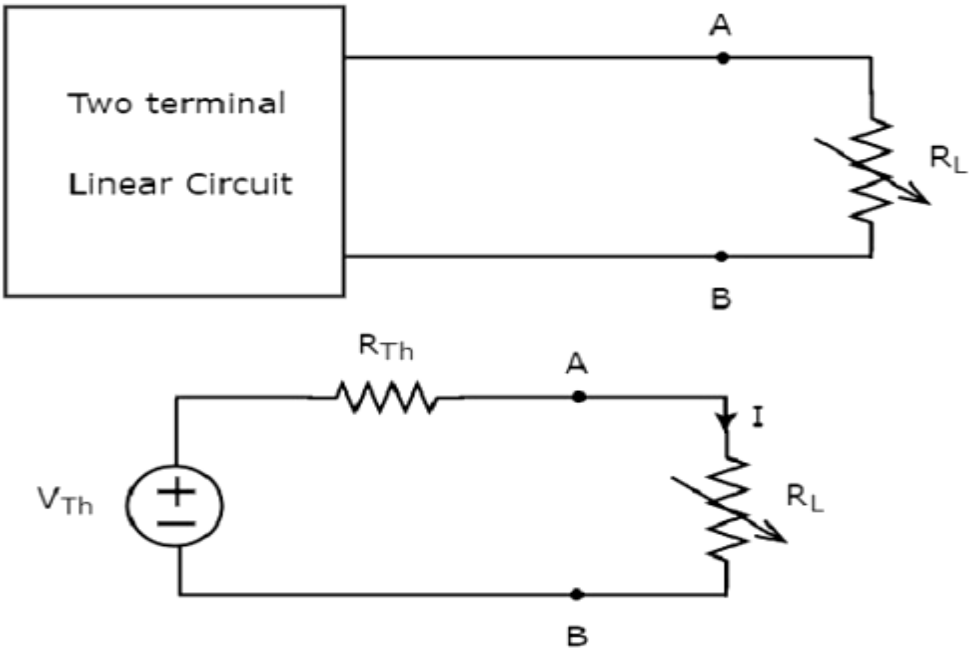
I_L = 0.812 mA

The load voltage is determined as follows:

$V_L = 0.812$ mA x 5 kΩ = 4.06 V

Maximum Power Transfer Theorem:-

The **maximum power transfer theorem** states that, the load receives maximum power when the load resistance is equal to the internal resistance of the source network as seen from the load terminal.

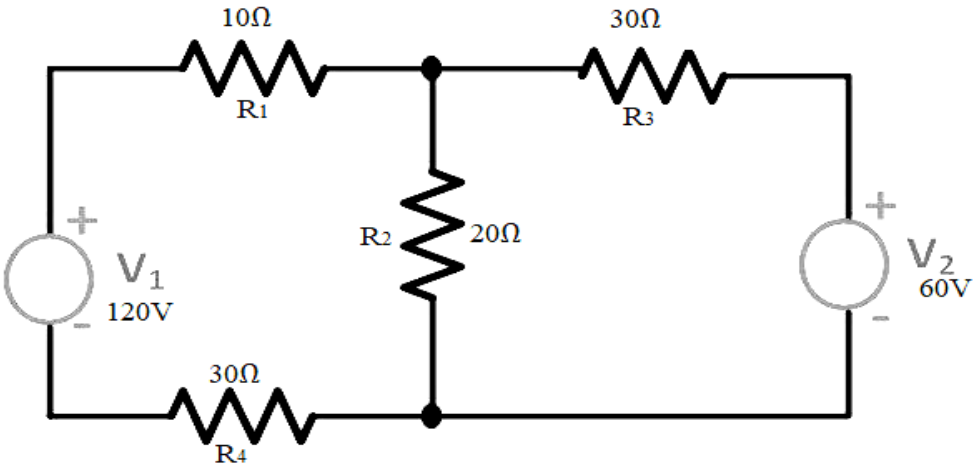


The maximum power transferred to the load when $R_L = R_{Th}$ is given by

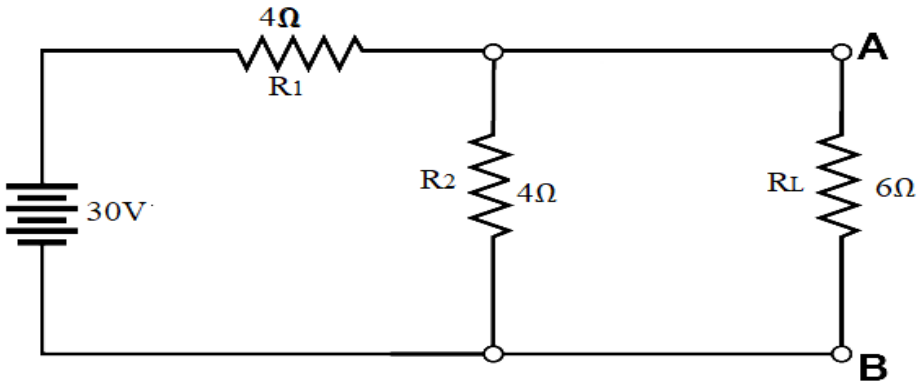
$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

ACTIVITY FOR TUTORIALS

1) Find the current flowing through R_2 in the below circuit using superposition theorem.



2) Find the current through resistance $R_L = 6\Omega$ using Thevenin's theorem.



3) Find the current through resistance 12Ω in the circuit using Norton's theorem.